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Probing a QCD String Axion with Precision Cosmological Measurements

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ABSTRACT

String and M-theory compactifications generically have compact moduli which can potentially act as the QCD axion. However, as demonstrated here, such a compact modulus can not play the role of a QCD axion and solve the strong CP problem if gravitational waves interpreted as arising from inflation with Hubble constant $H_{\rm inf} \gtrsim 10^{13}$ GeV are observed by the PLANCK polarimetry experiment. In this case axion fluctuations generated during inflation would leave a measurable isocurvature and/or non-Gaussian imprint in the spectrum of primordial temperature fluctuations. This conclusion is independent of any assumptions about the initial axion misalignment angle, how much of the dark matter is relic axions, or possible entropy release by a late decaying particle such as the saxion; it relies only on the mild assumption that the Peccei-Quinn symmetry remains unbroken in the early universe.

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Contents

1 Introduction

M-theory and perhaps its perturbative string theory limits represent the best hope for a fundamental theory which integrates matter fields, gauge interactions, and gravity. This theory makes many interesting predictions for physics at the string or eleven-dimensional Planck scales. These scales could in principle be anywhere between the electroweak and four-dimensional Planck scales. However, the only compelling indirect hint for these scales comes from the unification of gauge couplings due to four-dimensional renormalization group running below the unification scale, $M_U \sim 2 \times 10^{16}$ GeV [1]. In this case the fundamental scale is so far above any energy scale available in the laboratory that direct probes of string/M-theory seem practically impossible. However, energies in the early universe might have been comparable to these scales. For example, the energy density during inflation could have been as large as the unification scale. It is therefore worth investigating in detail whether any features of string/M-theory could give effects in the very early universe which might be detected in the present and coming generation of precision cosmological measurements.

Previous discussions have been rather pessimistic as to the possibility of discerning effects from such large energy scales with cosmological measurements [2]. However, in this paper we show that there is at least one feature of many string and M-theory vacua which can be probed by precision cosmological measurements – a string/M-theory QCD axion. This conclusion turns out to be nearly independent of any cosmological assumptions such as the initial axion misalignment angle, how much of the dark matter is relic axions, or dilution of relic axions by a late entropy release.

A generic feature of supersymmetric string and M-theory compactifications is the existence of complex moduli fields. The compact components of these moduli ultimately arise in one description or another from p-form gauge fields on non-trivial p-cycles in an internal manifold. The vacuum angles for low energy gauge groups often transform inhomogeneously under shifts of these compact moduli. In the case of QCD such a compact modulus then gives a realization of the Peccei-Quinn symmetry in Nambu-Goldstone mode [3]. Supersymmetry breaking generically lifts moduli. While not guaranteed, it is possible that the compact components of moduli are protected from obtaining a potential from supersymmetry breaking to a fairly high order by approximate or exact discrete symmetries [4]. In the case of QCD the leading contributions to the potential for such a compact modulus can then arise from infrared QCD effects. Such a compact modulus then acts as an invisible axion and provides an elegant solution to the strong CP problem.

In the classes of vacua mentioned above in which gauge coupling unification arises from four-dimensional renormalization group running, the Peccei-Quinn scale for a string/M-theory QCD axion is generally of order the unification scale, $f_a \sim M_U$.

For example, as presented in appendix A, the Peccei-Quinn scale for the canonically normalized model-independent string axion is

$$f_a = \sqrt{2}\,\frac{\alpha_U}{4\pi} M_P \sim 10^{16}\ \text{GeV} \tag{1}$$

where $\alpha_U = g_U^2/4\pi$ is the unified value of the fine structure constant and $M_p \simeq 2.4 \times 10^{18}$ GeV is the four-dimensional reduced Planck mass. We therefore focus on Peccei-Quinn scales $f_a \sim 10^{16}$ GeV, but consider the general case of large f_a. The interactions of such axions are exceedingly weak, first because of the derivative coupling, and second because of the large Peccei-Quinn scale. Even if a relic condensate of such an axion comprised all the dark matter it seems practically impossible to detect directly in laboratory experiments.

Precision cosmological measurements do, however, provide the possibility to probe a string/M-theory QCD axion. Quantum fluctuations are imprinted on such an axion during inflation. These fluctuations eventually contribute to temperature fluctuations in the Cosmic Microwave Background Radiation (CMBR) with the distinctive characteristics of isocurvature [5] and non-Gaussian [6] components. The magnitude of these fluctuations are controlled by the energy scale of inflation. At present there is no direct experimental indication for this scale. However, upcoming experiments sensitive to the polarization of the CMBR, such as the PLANCK Surveyor [7] or CMBPol could conceivably detect background primordial gravitational waves. The presence of a gravitational wave signal in CMBR fluctuations would represent a true triumph for CMBR experiments, and if interpreted as arising from inflation would imply a rather high inflationary energy scale. Most importantly for the present discussion, it would establish the magnitude of primordial axion fluctuations imprinted during inflation. And for a given Peccei-Quinn scale, f_a, this would provide a lower limit on the isocurvature and non-Gaussian components in the CMBR fluctuations. So if gravity waves are detected in future CMBR fluctuations experiments, then the existence of a string/M-theory QCD axion can be tested by a search for isocurvature and/or non-Gaussian components also in CMBR fluctuations.

The notion that information might be gained about axions from the CMBR is not new [6,8–13]. However, these works either assume that the axion makes up all of the dark matter (or even the entire critical density) [8,9,11–13], do not allow for the possibility of dilution of the axion condensate by a late entropy release [6,8–13], and do not consider separately isocurvature and non-Gaussian contributions to CMBR temperature fluctuations [6,8–13].

The goal here is to assess how well precision CMBR measurements can probe a string/M-theory QCD axion *independent* of any cosmological assumptions. In order to do this the correlation between the magnitude of inflationary gravity wave contributions to CMBR fluctuations, and axion induced isocurvature and non-Gaussian components of CMBR fluctuations must be derived for the widest possible range of cosmological assumptions: First, the initial axion misalignment angle must be taken to be arbitrary within allowed constraints. Second, no assumption must be made about how much of the dark matter is made of relic axions. And third, any allowed, but otherwise arbitrary, late entropy release which could dilute any relic axions must be allowed. Obtaining a definitive statement about how well precision CMBR measurements can probe a string/M-theory QCD axion then amounts to employing the cosmological assumptions which *minimize* the axion induced isocurvature and non-Gaussian contributions to the CMBR fluctuations for a given magnitude of inflationary gravity wave contributions to CMBR fluctuations and Peccei-Quinn scale. This corresponds to assuming a vanishingly small average misalignment angle, with the relic axion condensate arising only from inflationary induced fluctuations of the axion field [10], along with the maximum allowed dilution of this condensate by a late entropy release. In this case, any fluctuation induced relic axions turn out to make up at most a very small fraction of the dark matter. If this most pessimistic of cosmological assumptions can be probed experimentally, then all possible assumptions can be probed: the observational effects of an axion are necessarily larger for non-vanishing average initial misalignment angle and non-maximal dilution by an entropy release. The motivation for employing the widest possible set of cosmological assumptions is that in the era of precision cosmological measurements, ideally it should not be necessary to make *ad hoc* untestable assumptions about early universe cosmology in order to obtain definitive tests.

Even with the above assumptions which minimize the observational effects of relic axions we come to a strong conclusion:

> *A string/*M*-theory compact modulus can not play the role of a* QCD *axion and solve the strong* CP *problem if gravitational waves interpreted as arising from inflation with a Hubble constant* $H_{\rm inf} \gtrsim 10^{13}$ GeV *are observed by the* PLANCK *polarimetry experiment.*

This conclusion requires only the current bounds on isocurvature and non-Gaussian contributions to CMBR fluctuations and relies on the expectation that the maximum possible Peccei-Quinn scale for a string/M-theory axion is the four-dimensional Planck scale $f_a \lesssim M_p$ [14]. In terms of an arbitrary Peccei-Quinn scale these bounds would require at least $f_a \gtrsim 3\times 10^{20}$ GeV if PLANCK observed gravity waves corresponding to an inflation scale of $H_{\rm inf} \simeq 10^{13}$ GeV, and at least $f_a \gtrsim 5\times 10^{25}$ GeV if gravity waves were observed just below the current WMAP bound of $H_{\rm inf} \lesssim 3\times 10^{14}$ GeV [15]. These bounds do require the mild assumption that the Peccei-Quinn symmetry remains unbroken in the early universe. They would be further strengthened by (likely) improvements in the bound on the isocurvature components of CMBR fluctuations.

In the next section the production of relic axions is reviewed in all the relevant cosmological scenarios including the effects of inflationary induced axion fluctuations and the possibility of dilution by an entropy release due a late decaying particle. The specific case of the late decay of the saxion superpartner of the axion is considered, but shown to generally lead to additional cosmological problems rather than a successful dilution of the axion condensate. The standard results are also extended to the case of large Peccei-Quinn scales. In section 3 the isocurvature and non-Gaussian contributions to CMBR temperature fluctuations arising from inflationary axion fluctuations are presented. The current bounds and possibilities for future improvements on the CMBR probes of the Hubble constant during inflation and isocurvature and non-Gaussian components are also discussed. In section 4 the rather stringent bounds on the Peccei-Quinn scale which would result from the current isocurvature and non-Gaussian CMBR temperature fluctuation bounds are presented in the case that primordial gravity waves interpreted as arising from inflation are in fact observed in future experiments. The implications for a string/M-theory QCD axion are also considered. In section 5 the scenarios for realizing the Peccei-Quinn mechanism to which the stringent bounds would apply are spelled out. Possible loop holes for these bounds are also addressed. In section 6 the strong conclusion that a string/M-theory QCD axion is not consistent with the observation of primordial gravity waves by the PLANCK polarimetry experiment is reviewed. And a corollary is presented regarding the implications of observation of isocurvature and/or non-Gaussian components of CMBR temperature fluctuations would have if interpreted as arising from a QCD axion.

The relation between the unified fine structure constant and properly normalized Peccei-Quinn scale for the model independent string axion is derived in appendix A. The parameterization of the finite temperature axion mass resulting from the dilute instanton calculation of the $\theta_{\rm QCD}$-dependence of the finite temperature free energy is given in appendix B. The relation between the dimensionless skewness defined in section 3.3 and a parameterization of non-Gaussian components of temperature fluctuations commonly used in the literature is derived in appendix C.

2 Relic Axions

Relic axions can be produced in the early universe through a variety of mechanisms: thermal production, axion string radiation, or condensate formation from misalignment of the initial QCD vacuum energy [16, 17, 18, 19]. For the large Peccei-Quinn scales relevant here, thermal production is unimportant. In addition, since any string/M-theory modulus which contains a QCD axion is present before and during inflation any axion strings are inflated away. This leaves coherent production from misalignment as the dominant mechanism for producing relic string/M-theory QCD axions.

The formation of a QCD axion condensate is reviewed in section 2.1, with special attention paid to the case of large Peccei-Quinn scale relevant to a string/M-theory axion. It is shown that there are sizeable uncertainties in the calculation of the axion relic density due to strong QCD effects for Peccei-Quinn scales in the region of interest, but that this does not affect the final conclusions. An updated version of the classic cosmological bound on the Peccei-Quinn scale for an average misalignment angle is presented taking into account recent determinations of the dark matter density. This bound is significantly exceeded for $f_a/N \sim 10^{16}$ GeV, the value that might be expected for a string/M-theory QCD axion. So the possibility of accommodating such an axion by reducing relic production with a small misalignment angle is considered in section 2.2. The complimentary or additional possibility of reducing relic axion density by a late entropy release is discussed in section 2.3. It is shown that the maximum possible dilution by a late decaying particle is alone not sufficient to accommodate $f_a/N \sim 10^{16}$ GeV with average misalignment angle. Such an axion requires a small misalignment angle. The inflationary quantum fluctuations imprinted on an axion are reviewed in section 2.4. These fluctuations are shown to lead to a minimum relic density of axions which depends on the Hubble constant during inflation and the Peccei-Quinn scale.

2.1 Formation of the Axion Condensate

A relic axion condensate is formed in the early universe from misalignment of the initial QCD vacuum angle [16] when the axion begins to oscillate at $3H \simeq m_a(T_{\rm osc})$, where H is the Hubble parameter. Given the temperature dependence of the axion mass $m_a(T)$, this condition along with the Friedman equation, $3H^2M_p^2 = \rho$, and thermal energy density in a radiation dominated era $\rho = (\pi^2/30)g_*T_{\rm osc}^4$ defines the temperature, $T_{\rm osc}$, at which the axion begins to oscillate

$$T_{\rm osc} \simeq (10/g_*\pi^2)^{1/4}(m_a(T_{\rm osc})M_p)^{1/2} \tag{2}$$

For oscillation temperatures just above $\Lambda_{\rm QCD}$, $g_* \simeq 61.75$. The parameterization of the high temperature dependence of the axion mass for $T \gtrsim \Lambda_{\rm QCD}$ is reviewed in Appendix B, and may be written [20, 17]

$$\xi(T) \equiv \frac{m_a(T)}{m_a} \simeq C\left(\frac{\Lambda_{\rm QCD}}{200\ {\rm MeV}}\right)^{1/2}\left(\frac{\Lambda_{\rm QCD}}{T}\right)^4 \qquad {\rm for}\ \ T \gtrsim \Lambda_{\rm QCD} \tag{3}$$

where $C \simeq 0.018$ is a parameter defined in Appendix B. With this, the oscillation temperature is then

$$T_{\rm osc} \sim 150 \text{ MeV} \left(\frac{C}{0.018}\right)^{1/6} \left(\frac{\Lambda_{\rm QCD}}{200 \text{ MeV}}\right)^{3/4} \left(\frac{10^{16} \text{ GeV}}{f_a/N}\right)^{1/6} \quad \text{for } T_{\rm osc} \gtrsim \Lambda_{\rm QCD} \quad (4)$$

This expression for the oscillation temperature is only appropriate for $T_{\rm osc} \gtrsim \Lambda_{\rm QCD}$ corresponding to $f_a/N \lesssim 2 \times 10^{15}$ GeV. The case of $T_{\rm osc} \lesssim \Lambda_{\rm QCD}$ for larger Peccei-Quinn scales is considered separately below.

The local axion number density in the condensate when it is formed is

$$n_a \simeq f_c \frac{1}{2} \xi(T_{\rm osc}) m_a (f_a/N)^2 (\theta_i^2 + \sigma_\theta^2) f(\theta_i^2) \quad (5)$$

where $m_a = m_a(0)$ is the zero temperature mass, f_c is a correction for temperature dependence of the axion mass, $f_c \simeq 1.44$ [17] for the $\xi(T)$ parameterization (3), and $f(x)$ is a correction for anharmonic effects of the axion potential; for $x \ll 1$, $f(x) \to 1$. And $\theta_i \equiv \langle\theta\rangle$ is the initial QCD vacuum angle zero-mode averaged over the presently observable universe, and $\sigma_\theta^2 \equiv \langle(\theta - \langle\theta\rangle)^2\rangle$ is the mean square fluctuations of the initial QCD vacuum angle, produced for example during an early epoch of inflation. Since the number density redshifts like the entropy density, the local axion density today is given by

$$\rho_a \simeq (n_a/s) m_a s_0 \gamma \quad (6)$$

where $s = (2\pi^2/45) g_{*s} T_{\rm osc}^3$ is the thermal entropy density when the axion begins to oscillate, $s_0 \simeq (2\pi^2/45) g_{*s0} T_0^3$ is the thermal entropy density today with $g_{*s0} \simeq 3.91$ and $T_0 \simeq 2.73$ K. The factor γ is a possible dilution factor due to entropy release after the axion begins to oscillate, $\gamma = S_{\rm osc}/S_0$. Using all this with the oscillation temperature (4) leads to a relic axion density, $\Omega_a = \rho_a/(3H_0^2 M_p^2)$, of

$$\Omega_a h^2 \sim 2 \times 10^4 \left(\frac{0.018}{C}\right)^{1/6} \left(\frac{200 \text{ MeV}}{\Lambda_{\rm QCD}}\right)^{3/4} \left(\frac{f_a/N}{10^{16} \text{ GeV}}\right)^{7/6} \left(\theta_i^2 + \sigma_\theta^2\right) f(\theta_i^2)\gamma \quad (7)$$

where $h = H_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1})$. This expression for the relic axion density is only valid for at least $f_a/N \lesssim 2 \times 10^{15}$GeV.

Since the expression (7) for the relic axion density uses the high temperature expression (4) for the oscillation temperature, it does not apply for $T_{\rm osc} \lesssim \Lambda_{\rm QCD}$. For oscillation temperatures in this range (corresponding to large Peccei-Quinn scales), the axion begins to oscillate in effectively the zero temperature potential and another expression for the relic density is obtained. For oscillation temperatures just below $\Lambda_{\rm QCD}$, $g_* \simeq 10.75$. With this, and for a general temperature dependence for the axion mass, $\xi(T)$, the oscillation temperature (2) is

$$T_{\rm osc} \sim 950 \text{ MeV } \left(\frac{10^{16} \text{ GeV}}{f_a}\right)^{1/2} \xi(T_{\rm osc})^{1/2} \quad \text{for } T_{\rm osc} \lesssim \Lambda_{\rm QCD} \tag{8}$$

This expression for the oscillation temperature is only appropriate for $T_{\rm osc} \lesssim \Lambda_{\rm QCD}$, corresponding to at least $f_a/N \gtrsim 2 \times 10^{17}\, \xi(T_{\rm osc})$ GeV. For $T_{\rm osc} \lesssim \Lambda_{\rm QCD}$, the relic axion density from (5) and (6) is

$$\Omega_a h^2 \simeq 5 \times 10^3 \left(\frac{f_a/N}{10^{16} \text{ GeV}}\right)^{3/2} \left(\theta_i^2 + \sigma_\theta^2\right) f(\theta_i^2)\gamma\, f_c \xi(T_{\rm osc})^{-1} \tag{9}$$

While there is some residual temperature dependence of the axion mass for $T_{\rm osc} \lesssim \Lambda_{\rm QCD}$, this is unimportant and $f_c\xi(T_{\rm osc})^{-1} \simeq 1$ is a good approximation well into this regime. The relic density (9) is only valid for at least $f_a/N \gtrsim 2 \times 10^{17}$ GeV. Note that is has a different parametric dependence on f_a/N than (7) appropriate for small Peccei-Quinn scales.

Note that the relic axion densities (7) and (9) calculated for oscillations temperatures (4) and (8) above and below $\Lambda_{\rm QCD}$ do not have overlapping regions of validity. This indicates that in the transition region, 2×10^{15} GeV $\lesssim f_a/N \lesssim 2 \times 10^{17}$ GeV, neither expression is completely accurate. This is in fact to be expected for $T_{\rm osc} \sim \Lambda_{\rm QCD}$. The high temperature expression (4) is derived using a dilute instanton gas approximation [20] as described in Appendix B. This likely receives non-trivial corrections from multi-instantons for $T_{\rm osc}$ somewhat above $\Lambda_{\rm QCD}$. Likewise the low temperature expression (8) likely receives non-trivial corrections from spontaneous chiral symmetry breaking effects for $T_{\rm osc}$ just slightly below $\Lambda_{\rm QCD}$. It is perhaps unfortunate that for a Peccei-Quinn scale relevant to a string/M-theory axion, strong QCD effects are important during formation of the axion condensate. However, for f_a/N well outside the transition region the true expression for the axion relic density asymptotes to the expressions given above, and should smoothly interpolate between these limiting forms. And the bounds described below turn out to apply to all Peccei-Quinn scales up to a very large value which is well outside the transition region. So the final conclusions are not sensitive to unknown strong QCD effects. The relic densities (7) and (9) turn out to be equal for $f_a/N \simeq 6 \times 10^{17}$ GeV. This is in fact outside the transition region given above, due mainly to the abrupt change in the number of degrees of freedom at a temperature $T \sim \Lambda_{\rm QCD}$. In what follows we simply use (7) for Peccei-Quinn scales lower than this value and (9) for higher scales. Also note that the expressions for the relic densities (7) and (9) only apply if the universe is radiation dominated when the axion condensate forms. The case of matter domination is presented in section 2.3.

Most of the relic axions have momenta much less than the zero temperature axion mass, $p_a \ll m_a$, and so act as cold dark matter (CDM). The present CDM energy density has been accurately determined in a ΛCDM cosmology by recent measurements to be $\Omega_{\rm CDM}h^2 = 0.113 \pm 0.010$ [21]. At most, relic axions could saturate this density and make up all the CDM. Alternately, the CDM might consist of more than one species, with relic axions contributing only part of the total observed CDM density. So the relic axion density is bounded from above by

$$\Omega_a h^2 \lesssim 0.12 \tag{10}$$

The average mean square initial misalignment angle over all inflationary domains is $\langle\theta^2\rangle = \pi^2/3$, for which the anharmonic correction is $f(\pi^2/3) \simeq 1.2$ [17]. So for the average misalignment angle the relic axion density (7) without any dilution, $\gamma = 1$, along with the bound (10), require that the Peccei-Quinn scale is bounded from above by

$$f_a/N \lesssim 3 \times 10^{11} \text{ GeV} \quad \text{for} \ \langle\theta^2\rangle = \pi^2/3 \ \text{and} \ \gamma = 1 \tag{11}$$

This is somewhat smaller than the canonical $f_a/N \lesssim 10^{12}$ GeV bound arising from $\Omega_a h^2 \lesssim 1$ quoted in the older literature [16] mainly due to the recent improvement in measurement of the CDM density quoted above.

2.2 Small Initial Misalignment Angle

A QCD string/M-theory axion with $f_a/N \sim 10^{16}$ GeV is clearly inconsistent with the bound (11) for the average misalignment angle and without any dilution of the axion condensate after its formation. One possibility which could accommodate such an axion is that the average misalignment angle within the observable universe is small. For the relic axion density (7) with $\gamma = 1$, the bound (10) then requires

$$(\theta_i^2 + \sigma_\theta^2)^{1/2} \lesssim 2 \times 10^{-3} \left(\frac{10^{16}\text{ GeV}}{f_a/N}\right)^{7/12} \tag{12}$$

In fact, with random initial conditions prior to an early epoch of inflation, there are at present distant regions of the universe which sample all values of initial misalignment angles. So it is possible in principle that $\theta_i^2 + \sigma_\theta^2$ happens to be sufficiently small in our observable universe to allow $f_a/N \sim 10^{16}$ GeV. Regions with misalignment angles which are larger(smaller) than ours then have a larger(smaller) Hubble constant at a given temperature.

In this scenario the question naturally arises though as to why our initial misalignment angle is so small. Linde has argued that with f_a as large as considered here there may be a dynamical selection effect which makes the observation of small misalignment angle likely [9] (for a recent discussion see [22]). At fixed large f_a, the density of gravitationally collapsed objects such as galaxies grows as a rapid power of θ_i^2 [9]. Since very dense galaxies may be inhospitable to observers such as ourselves, it may then be more likely that a given observer measures a small misalignment angle. However, quantifying the probability distribution of observers as a function of galactic density remains an open problem. In addition, such arguments could not explain an initial misalignment angle significantly smaller than the bound (12).

Independent of the question of possible selection effects for the observed value of the initial misalignment angle, it is possible in principle that the misalignment angle in the observable universe is so small for whatever reason that relic axions comprise a vanishingly small fraction of the CDM. In this case it might appear that there are no observational effects. However, quantum fluctuations of the axion during inflation lead to a non-vanishing mean square fluctuation of the QCD vacuum angle, $\sigma_\theta \neq 0$, as discussed in section 2.4. This gives a lower limit on the relic axion density (7) or (9), and therefore on the observable effects. The magnitude of these effects depends crucially on the Hubble constant during inflation, as discussed below. The goal here is to establish how well a QCD string/M-theory axion can be probed by precision cosmological measurements independent of any assumptions about the initial misalignment angle. To do this, the Hubble constant during inflation must therefore be measured by independent means in order to establish a minimum relic axion density, as discussed in section 3.1.

2.3 Dilution of the Axion Condensate

Another possibility which might help accommodate a QCD string/M-theory axion is that there is a late entropy release with a reheat temperature $T_{\rm RH} < T_{\rm osc}$ which dilutes the axion condensate after its formation [23]. For the relic axion density (7) with average misalignment angle $\langle \theta^2 \rangle = \pi^2/3$, the relic density bound (10) requires a dilution factor of

$$\gamma \lesssim 2 \times 10^{-6} \left(\frac{10^{16} \text{ GeV}}{f_a/N} \right)^{7/6} \tag{13}$$

A late entropy release must satisfy a number of non-trivial constraints in order to successfully dilute the axion condensate. First, the entropy release must occur after the condensate is formed at $T_{\rm osc} \sim \Lambda_{\rm QCD}$. An entropy release at a higher temperature does not modify the relic axion density. Second, the entropy release must not adversely modify the successful predictions of relic light element abundances from big bang nucleosynthesis (BBN). The ^{4}He abundance is determined in standard radiation dominated BBN mainly by the primordial baryon to entropy ratio, n_b/s, and the neutron to proton ration, n/p, when the weak interactions drop out of equilibrium at $T \sim 1$ MeV. Standard BBN then generally requires that any entropy release terminate in a radiation domination era with a reheat temperature of at least $T_{\rm RH} \gtrsim 1$ MeV and possibly higher. Finally, the entropy release must not over-dilute the primordial relic baryon density.

The case of an entropy release by a late decaying particle is considered below. The special case of a late decaying saxion superpartner of the axion is also presented. The very unlikely possibilities of a first order phase transition or very late inflation which could dilute the axion condensate are commented on in section 5.

2.3.1 Late Decaying Particle

A very natural way to obtain a late entropy release is by the late decay of a massive non-relativistic particle. In order to have a non-trivial dilution of the axion condensate the late decaying particle must dominate the energy density for at least $T_{\rm osc} \gtrsim \Lambda_{\rm QCD}$. This can occur in a number of ways. For a scalar particle, a relic condensate formed from an initial misalignment of the scalar field from the minimum of the potential can dominate at a very early epoch. For either a scalar or fermion particle, relic particles produced during reheating after inflation, or simply from thermal freeze out can dominate at an early epoch. For the discussion here, detailed models for the origin of the late decaying relic particles are not necessary. The dilution of the axion condensate can be parameterized solely in terms of the particle reheat temperature as described below.

For a late decaying particle the BBN bounds on the reheat temperature are slightly more stringent than the case of a general entropy release discussed above. An exponentially small number of late decaying particles survive to temperatures below the reheat temperature. If hadronic decay channels are available (which is inevitable in the thermalization process) then these very late decays can enrich n/p even for $T_{\rm RH} \gtrsim 1$ MeV. And this can be a significant effect since n_b/s is so small. For a late decaying particle with $m \sim 1$ TeV, a reheat temperature of $T_{\rm RH} \gtrsim 6$ MeV is required in order to ensure that the ^{4}He abundance is not too large [24]. So in order to dilute the axion condensate while keeping the successful predictions of BBN a late decaying particle must first, dominate the energy density for at least $T \gtrsim \Lambda_{\rm QCD}$ and second, have a reheat temperature in the range 6 MeV $\lesssim T_{\rm RH} \lesssim \Lambda_{\rm QCD}$.

The relic axion density which results with the late decay of a massive particle ϕ can be calculated from the axion fractional energy density when the axion condensate is formed. If a massive non-relativistic particle dominates the energy density, the universe is matter dominated at this epoch with $\rho_\phi = n_\phi m_\phi \simeq 3H^2 M_p^2$. The number density in the axion condensate when it forms at $3H \simeq m_a(T_{\rm osc})$ is given by (5). Since the axion and massive particle number densities both redshift like matter, the ratio n_a/n_ϕ is constant after the axion condensate is formed and before the massive particle decays. Using all this, the ratio of axion density $\rho_a = m_a n_a$ to massive particle density during this epoch is

$$\frac{\rho_a}{\rho_\phi} \simeq f_c \frac{3}{2} \xi(T_{\rm osc})^{-1} \frac{(f_a/N)^2}{M_p^2} \left(\theta_i^2 + \sigma_\theta^2\right) f(\theta_i^2) \tag{14}$$

When the massive particle decays its energy is converted to thermal radiation. In the sudden decay approximation the resulting thermal energy and entropy densities are $\rho_\phi \simeq (\pi^2/30) g_* T_{\rm RH}^4$ and $s \simeq (2\pi^2/45) g_* T_{\rm RH}^3$. The relic axion number density to entropy density ratio at this epoch is then

$$\frac{n_a}{s} = \frac{\rho_a/m_a}{\rho_\phi} \frac{\rho_\phi}{s} \tag{15}$$

with $\rho_\phi/s \simeq (3/4) T_{\rm RH}$. Since the number density redshifts like the entropy density after the late decay, the local axion density today is given by (6) using the ratio (15). Combining all this, the relic axion density with the late decay of a massive particle is [19]

$$\Omega_a h^2 \simeq 35 \left(\frac{T_{\rm RH}}{6~{\rm MeV}}\right) \left(\frac{f_a/N}{10^{16}~{\rm GeV}}\right)^2 \left(\theta_i^2 + \sigma_\theta^2\right) f(\theta_i^2) \quad f_c \xi(T_{\rm osc})^{-1} \tag{16}$$

Note that in this scenario appropriate to a matter dominated era, the relic axion density has a parametric dependence on f_a/N which is different than both (7) and (9) appropriate to a radiation dominated era.

Starting from a very early radiation dominated era, the temperature for a given Hubble parameter during the subsequent late decaying particle matter dominated era is smaller than that for the same Hubble parameter of a radiation dominated era. So as long as the late decaying particle dominates the energy density at a temperature at least somewhat above $\Lambda_{\rm QCD}$, the temperature that the axion begins to oscillate, $T_{\rm osc}$, is at least somewhat below $\Lambda_{\rm QCD}$. The axion condensate then forms in the zero temperature potential, and $f_c \xi(T_{\rm osc})^{-1} \simeq 1$ is a good approximation. This implies that if the late decaying particle comes to dominate the energy density for a temperature well above $\Lambda_{\rm QCD}$ (which is likely in most scenarios) then the relic abundance (16) does not suffer unknown uncertainties from strong QCD effects in the region of interest, unlike the relic densities (7) and (9) appropriate to a radiation dominated era.

For the average mean square misalignment angle of $\langle \theta^2 \rangle = \pi^2/3$, the relic density (16) with maximum dilution, $T_{\rm RH} \simeq 6$ MeV, along with bound (10), requires that the Peccei-Quinn scale is bounded from above by

$$f_a/N \ \lesssim\ 3 \times 10^{14}~{\rm GeV} \qquad {\rm for}\ \ \langle \theta^2 \rangle = \pi^2/3 \ \ {\rm and}\ \ {\rm Maximum~Dilution} \tag{17}$$

A string/M-theory QCD axion with $f_a/N \sim 10^{16}$ GeV is inconsistent even with this maximum dilution bound. So dilution from late decay of a massive particle turns out alone to be insufficient to accommodate such an axion with generic misalignment angle. A small misalignment angle is therefore required in any cosmological scenario with a Peccei-Quinn scale which far exceeds the bound (17).

2.3.2 Saxion Decay

In any supersymmetric model the pseudoscalar axion has a scalar saxion superpartner. For the model independent string axion the saxion is just the dilaton. In the supersymmetric limit, Peccei-Quinn symmetry ensures that the entire axion supermultiplet remains massless to all orders in perturbation theory. However, with supersymmetry and $U(1)_R$ breaking, only the axion mass is protected by Peccei-Quinn symmetry, and the saxion potential is determined by supersymmetry breaking. The saxion mass is then naturally much larger than the axion mass. A coherent condensate of the saxion can form at a very early epoch from saxion misalignment and dominate the energy well before the axion condensate forms. The saxion condensate eventually decays, with the decay products rethermalizing to a standard radiation dominated era. So a supersymmetric axion model naturally contains all the elements which could in principle dilute the axion condensate [25, 26]. However, as shown below, the reheat temperature is generally too low for $f_a/N \sim 10^{16}$ GeV. This complicates rather than improves the cosmological scenario.

The detailed cosmological scenario which is obtained with a supersymmetric axion model depends on the saxion reheat temperature. This is determined by the saxion decay width, Γ, and mass, and is therefore sensitive to details of supersymmetry breaking. In the sudden decay approximation the saxion condensate decays when $\Gamma \simeq H$ with its energy converted to thermal radiation, $(\pi^2/30) g_* T_{RH}^4 \simeq 3\Gamma^2 M_p^2$. The decay width depends on the saxion couplings which are model dependent. However, if the anomalous axion–gluon coupling (74) given in Appendix A arises directly in the high energy theory (rather than from integrating out low energy standard model fields) as is the case for large classes of string/M-theory axions including the model independent string axion, the saxion–gluon coupling is determined by supersymmetry

$$\frac{s}{f_a/N} \frac{g_s^2}{32\pi^2} F^a_{\mu\nu} F^{a\mu\nu} \tag{18}$$

where the gluon fields are canonically normalized. This coupling gives a decay rate for $s \to gg$ of

$$\Gamma(s \to gg) = \frac{\alpha_s^2}{64\pi^3} \frac{m_s^3}{(f_a/N)^2} \tag{19}$$

In the sudden decay approximation the decay rate (19) gives a reheat temperature of

$$T_{\rm RH} \simeq 1 \times 10^{-2} \text{ MeV } \left(\frac{m_s}{\rm TeV}\right)^{(n+1)/2} \left(\frac{10^{16} \text{ GeV}}{f_a/N}\right) \tag{20}$$

with $n = 2$ corresponding to the two derivatives in the coupling (19). As another example, the DFS [27] class of axion models couple only to the visible sector through the Higgs sector. The couplings to standard model fields then come from mixings with Higgs bosons. In this case the leading coupling of the saxion are to heavy quarks

$$c \, \frac{s}{f_a/N} \, m_Q \overline{Q} Q \tag{21}$$

For the DFS axion $c = \frac{1}{6}$ for each quark and $N = 6$. Couplings to gluons include an additional loop factor and are therefore suppressed. The coupling (21) gives a decay rate for $s \to \overline{Q}Q$ of

$$\Gamma(s \to \overline{Q}Q) = \frac{3c^2}{16\pi} \frac{m_s m_Q^2}{(f_a/N)^2} \left(1 - \frac{4m_Q^2}{m_s^2}\right)^{3/2} \tag{22}$$

For a DFS saxion heavier than the top quark, the dominant decay is to tops, $s \to t\bar{t}$, with a reheat temperature from (22) which numerically turns out to be essentially identical to (20) with $n = 0$ corresponding to the absence of derivatives in the coupling (21).

For a saxion mass of $m_s \sim 1$ TeV, the reheat temperature (20) is consistent with the standard BBN requirement of $T_{\rm RH} \gtrsim 6$ MeV [24] only for at least $f_a/N \lesssim 2 \times 10^{13}$ GeV. So for $f_a/N \sim 10^{16}$ GeV appropriate to a string/M-theory QCD axion, the rather late decay of a saxion condensate formed at an early epoch leads to an additional saxion problem which must be solved, rather than contributing successfully to dilution of the axion condensate. A very large saxion mass could however increase the reheat temperature sufficiently to be consistent with BBN. For example, in the case of an saxion with high energy coupling (18) and Peccei-Quinn scale $f_a/N \sim 10^{16}$ GeV, a saxion mass of $m_s \gtrsim 70$ TeV would just barely give $T_{\rm RH} \gtrsim 6$ MeV. Although as discussed in section 2.3.1, dilution by late decay alone is not sufficient to accommodate such an axion without small initial misalignment angle. And obtaining such a heavy saxion seems difficult in most models of supersymmetry breaking with superpartners just above the electroweak scale. In many models of supersymmetry breaking the saxion mass is in fact much lighter than assumed above, $m_s \ll$ TeV, leading to an even more severe saxion problem. In addition, the axino fermionic partner of the axion has, for a given mass, a similar decay rate and reheat temperature to those given above [28]. So if sufficiently produced, relic axinos can also lead to a problem. However, in some scenarios for supersymmetry breaking the axino can be the lighest supersymmetric particle, and might be a CDM candidate [28].

Any complete cosmological scenario which includes a supersymmetric QCD axion must address the potential problems associated with the saxion and axino partners, presumably by limiting the initial cosmological production of these states. Since the goal here is to assess how well precision cosmological measurements can probe a string/M-theory QCD axion independent of specific cosmological assumptions, detailed models which address the saxion and axino problems will not be presented. Instead, we show below that in any model in which these problems are in fact solved, precision CMBR measurements can in principle definitively probe such an axion.

2.4 Inflationary Axion Fluctuations

The relic axion density is proportional to the quadrature sum of the zero-mode and root mean square fluctuations of the initial QCD vacuum angle, $\theta_i^2 + \sigma_\theta^2$. This combination amounts to an arbitrary initial condition for the mean square axion field when the axion condensate is formed. But in an inflationary cosmology the mean square axion fluctuations are related to the Hubble constant during inflation. In this section, inflationary production of axion fluctuations are reviewed. The minimum relic axion densities associated with these fluctuations [10] for the scenarios described in the previous sections are presented. These minimum relic axion densities are crucial in determining a lower limit on the observational effects described in the subsequent sections.

All massless fields undergo de Sitter fluctuations during an inflationary epoch. At zero temperature the leading Peccei-Quinn breaking effects which lift the axion potential must be due to QCD instantons in order to obtain a successful solution of the strong CP problem. These effects are exponentially suppressed during inflation; and as long as the Peccei-Quinn symmetry is not explicitly broken in the early universe, the axion is effectively massless during inflation. The de Sitter power spectral density of fluctuations of a massless field such as the axion

$$\mathcal{P}_a(k) \equiv \frac{k^3}{2\pi^2}\langle|\delta a_{\mathbf{k}}|^2\rangle \tag{23}$$

is related to the Hubble constant during inflation by

$$\mathcal{P}_a = \left(\frac{H_{\rm inf}}{2\pi}\right)^2 \tag{24}$$

where the $\mathbf{k}$-space density is $V d^3k/(2\pi)^3 = V dk k^2/2\pi^2$ and V is the quantization volume. The power spectral density (23) is nothing other than the mean square fluctuations of the field, $\mathcal{P}_a = \sigma_a^2$. So the root mean square fluctuations of the axion field and initial misalignment angle are related to the Hubble constant by

$$\sigma_a = T_{\rm GH} = \frac{H_{\rm inf}}{2\pi} \quad \Rightarrow \quad \sigma_\theta = \frac{H_{\rm inf}}{2\pi(f_a/N)} \tag{25}$$

where $T_{\rm GH}$ is the Gibbons-Hawking temperature [29]. Numerically, the variance of the initial misalignment angle is related to the Hubble constant during inflation and Peccei-Quinn scale by

$$\sigma_\theta \simeq 1.6 \times 10^{-4} \left(\frac{H_{\rm inf}}{10^{13}~{\rm GeV}} \right) \left(\frac{10^{16}~{\rm GeV}}{f_a/N} \right) \tag{26}$$

This inflationary produced variance provides a lower bound on the combination $\theta_i^2 + \sigma_\theta^2$ that appears in the expressions for the axion relic density.

In order to establish a lower limit for the observational effects of relic axion fluctuations in the CMBR discussed in the next section, it is useful to determine the minimum relic axion density as a function of Hubble scale during inflation for each late time cosmological scenario described in the previous subsections. Consider first the case in which the axion condensate forms during a radiation dominated era without any dilution by a late decaying particle. For a Peccei-Quinn scale of $f_a/N \lesssim 6 \times 10^{17}$ GeV, the relic density (7) with $C = 0.018$ and $\Lambda_{\rm QCD} = 200$ MeV, and for an average initial misalignment angle which is small compared with the variance induced by inflationary fluctuations (26), namely $\theta_i^2 \lesssim \sigma_\theta^2$, is bounded by

$$\Omega_a h^2 \gtrsim 5 \times 10^{-4} \left(\frac{H_{\rm inf}}{10^{13}~{\rm GeV}} \right)^2 \left(\frac{10^{16}~{\rm GeV}}{f_a/N} \right)^{5/6} \qquad {\rm for} \quad \theta_i^2 \lesssim \sigma_\theta^2 \tag{27}$$

With the same late time cosmological scenario, but with at least $f_a/N \gtrsim 6 \times 10^{17}$ GeV, the relic density (9) with $f_c \xi(T_{\rm osc})^{-1} \simeq 1$ and with small average initial misalignment angle, $\theta_i^2 \lesssim \sigma_\theta^2$, gives a bound on the relic axion density of

$$\Omega_a h^2 \gtrsim 1 \times 10^{-4} \left(\frac{H_{\rm inf}}{10^{13}~{\rm GeV}} \right)^2 \left(\frac{10^{16}~{\rm GeV}}{f_a/N} \right)^{1/2} \qquad {\rm for} \quad \theta_i^2 \lesssim \sigma_\theta^2 \tag{28}$$

The lower bounds (27) and (28) for small and large Peccei-Quinn scales respectively depend on *inverse* powers of f_a/N because the variance in initial misalignment angle (25) is inversely proportional to the Peccei-Quinn scale. And both scale like the square of the Hubble constant during inflation since the relic density scales like the square variance of axion field, and the variance scales like the Hubble constant.

Next consider the case of a late entropy release in which the axion condensate is formed during a matter dominated era with a later transition to standard radiation domination by reheating from a massive particle. In this case the relic density (16) for an average initial misalignment angle which is small compared with the variance induced by inflationary fluctuations (26), namely $\theta_i^2 \lesssim \sigma_\theta^2$, is bounded by

$$\Omega_a h^2 \gtrsim 9 \times 10^{-7} \left(\frac{T_{\rm RH}}{6~{\rm MeV}} \right)^2 \left(\frac{H_{\rm inf}}{10^{13}~{\rm GeV}} \right)^2 \qquad {\rm for} \quad \theta_i^2 \lesssim \sigma_\theta^2 \tag{29}$$

Note that this bound on the residual relic axion density induced by inflationary fluctuations is *independent* of the Peccei-Quinn scale because in this case the relic density (16) is proportional to $(f_a/N)^2$ while the variance in initial misalignment angle (25) squared is proportional to $(f_a/N)^{-2}$.

The bound (29) is only applicable if the axion condensate is formed during the matter dominated era before the dominating particle decays, which requires $3H(T_{\rm RH}) \lesssim m_a(T_{\rm RH})$. For a small enough axion mass the axion condensate is formed in the radiation dominated era subsequent to the late particle decay. This will occur for $3H(T_{\rm RH}) \gtrsim m_a(T_{\rm RH})$ corresponding to a Peccei-Quinn scale

$$f_a/N \;\gtrsim\; 2\times 10^{20}\left(\frac{6\,{\rm MeV}}{T_{\rm RH}}\right)^2\;{\rm GeV} \tag{30}$$

In this extreme case the relic density (9) for formation of the axion condensate in the zero temperature potential in a radiation dominated era is applicable. And the bound (28) on the relic axion density from inflationary induced axion fluctuations is obtained. Such a large Peccei-Quinn scale would seem unnatural and does not occur in any known string/M-theory compactifications [14]. However, it is useful to consider this regime in order to assess the strength of the bounds described in section 4.

3 Axion Signatures in the CMBR

A string/M-theory QCD axion with $f_a/N \sim 10^{16}$ GeV is likely to be practically impossible to detect in the laboratory. However, inflationary produced fluctuations of such an axion do give rise to two distinctive signatures in CMBR temperature fluctuations which are in principle measurable. The first is an isocurvature component of temperature fluctuations [5] which are discussed in section 3.2. The second is a non-Gaussian component of temperature fluctuations [6] discussed in section 3.3.

Both of these signatures depend on the relic axion density. The minimum residual relic densities arising from inflationary axion fluctuations presented in section 2.4 are therefore a crucial ingredient in establishing lower bounds for the observable isocurvature and non-Gaussian effects in the CMBR temperature fluctuations. These lower bounds, however, depend on the Hubble constant during inflation. So it is necessary to have some independent measurement which can in principle establish the Hubble constant during inflation. This can in fact be accomplished by a measurement of primordial inflationary gravity wave contributions to CMBR temperature fluctuations as described section 3.1 [30].

3.1 Gravity Waves

All massless fields undergo quantum de Sitter fluctuations during inflation, as discussed in section 2.4. This includes the metric as well as an axion. So a measurement of the magnitude of metric fluctuations generated during inflation would establish the magnitude of the canonically normalized axion fluctuations generated during inflation. And for a given Peccei-Quinn scale this would then establish a lower limit for the magnitude of observable effects of a string/M-theory QCD axion in CMBR temperature fluctuations described in the following subsections.

The power spectral density of inflationary dimensionless metric fluctuations may be written

$$\mathcal{P}_h(k) \equiv \frac{k^3}{2\pi^2} 2\langle |h_{+\mathbf{k}}|^2 + |h_{\times\mathbf{k}}|^2 \rangle \tag{31}$$

where h_+ and $h_\times$ are the two polarization modes of the metric perturbations. This power is related to the Hubble constant during inflation by [15],

$$\mathcal{P}_h = 2\left(\frac{2}{M_p}\frac{H_{\rm inf}}{2\pi}\right)^2 \tag{32}$$

This is identical to the power spectral density of axion fluctuations (24) up to a normalization related to canonical normalization of metric fluctuations and overall normalization chosen in (31).

Metric perturbations generated during inflation appear as gravity waves re-entering the horizon after inflation. These gravity waves induce tensor B-modes in CMBR temperature fluctuations. In order to characterize the magnitude of the power in gravity waves it is useful to define the ratio of tensor to scalar metric perturbations. Here we adopt the normalization used by the WMAP collaboration [15, 31] in which the tensor to scalar ratio is defined by the ratio of spectral power densities at some reference scale k_0,

$$r = \frac{\mathcal{P}_h(k_0)}{\mathcal{P}_{\mathcal{R}}(k_0)} \tag{33}$$

where

$$\mathcal{P}_{\mathcal{R}}(k) \equiv \frac{k^3}{2\pi^2}\langle |\mathcal{R}_{\mathbf{k}}|^2 \rangle \tag{34}$$

is the power spectral density in scalar metric perturbations and $\mathcal{R}$ is the gauge invariant dimensionless curvature perturbation [32]. It is worth noting that the definition (33) has the advantage that the relation between r and the Hubble constant during inflation does not depend on cosmological parameters such as the present dark matter or dark energy fractions (in contrast to some other definitions of the tensor to scalar ratio in the literature). The scalar curvature perturbation spectral power density measured from CMBR temperature fluctuations by WMAP under the assumption that they arise predominantly from scalar metric fluctuations is [15]

$$\mathcal{P}_{\mathcal{R}}(k_0) \simeq 2.1 \times 10^{-9} \qquad \text{at} \;\; k_0 = 0.002 \text{ Mpc}^{-1} \tag{35}$$

With this, the variance of inflationary induced fluctuations in the initial QCD misalignment angle (26) may be related to the scalar to tensor ratio (33) using (32) by

$$\sigma_\theta \simeq 1.7 \times 10^{-3} \; \sqrt{r} \; \left(\frac{10^{16} \text{ GeV}}{f_a/N} \right) \tag{36}$$

For a given Peccei-Quinn scale this gives a relation between an observable quantity and the variance of the initial misalignment angle.

The current WMAP bound on the scalar to tensor ratio from the non-observation of B-mode tensor components in CMBR temperature fluctuations is $r \lesssim 1.3$ [15]. With the definition (33) and (32) this corresponds to an upper limit on the Hubble constant during inflation of $H_{\rm inf} \lesssim 2.8 \times 10^{14}$ GeV. Future experiments will be capable of probing significantly smaller values of the scalar to tensor ratio r and concomitantly smaller values of the Hubble constant during inflation. The PLANCK polarimetry experiment could ultimately be sensitive at the one sigma level to at best roughly $r \gtrsim 5 \times 10^{-4}$ corresponding to $H_{\rm inf} \gtrsim 5 \times 10^{12}$ GeV [33, 34]. This represents a very optimistic sensitivity for a positive observation. So throughout we will consider $H_{\rm inf} \gtrsim 10^{13}$ GeV as a reasonable benchmark for a positive observation of primordial gravity waves by the PLANCK polarimetry experiment. If achieved this would represent a significant improvement over the current WMAP bound. If PLANCK does indeed observe tensor B-modes at any level between the current WMAP bound and its ultimate sensitivity, this would imply a lower bound on the axion relic density for a given Peccei-Quinn scale in the various cosmological scenarios discussed in section 2.4. The ultimate sensitivity to primordial gravity waves achievable in future CMBR experiments is limited not only by issues of instrumentation, but also from the fact that tensor B-mode fluctuations can be generated at second order in primordial scalar fluctuations from gravitational lensing effects. This provides an irreducible background for *any* future probe of primordial gravity waves in CMBR experiments. In terms of the scalar to tensor ratio this has been estimated to be roughly $r \gtrsim 7 \times 10^{-5}$ corresponding to $H_{\rm inf} \gtrsim 2 \times 10^{12}$ GeV [33, 35].

3.2 Isocurvature Perturbations

Quantum fluctuations generated during inflationary can lead to observable signatures in CMBR temperature fluctuations in a number of ways. Fluctuations of the inflaton field give rise to fluctuations in the scalar curvature and lead to adiabatic fluctuations in the CMBR. Fluctuations of the metric lead to tensor B-model fluctuations in the CMBR, as discussed above. Inflationary induced fluctuations of an axion field turn out to lead isocurvature fluctuations in the CMBR [5] as reviewed below. This distinctive signature provides a probe for the existence of an axion field during inflation.

Isocurvature perturbations correspond to fluctuations in the local equation of state of some species, $\delta(n_i/s) \neq 0$, with no fluctuation in the total energy density, $\delta\rho = 0$. These are in some sense orthogonal to adiabatic perturbations which correspond to fluctuations in the total energy density, $\delta\rho \neq 0$, with no fluctuation in the local equation of state, $\delta(n_i/s) = 0$. During inflation the potential for a QCD axion is essentially flat as long as the Peccei-Quinn symmetry remains unbroken during this epoch, as discussed above. Fluctuations in the axion field therefore do not affect the total density and are isocurvature.

In order to discuss both adiabatic and isocurvature fluctuations it is useful to define the fractional fluctuation in the number density of the i-th species divided by the entropy density

$$S_i \equiv \frac{\delta(n_i/s)}{n_i/s} = \frac{\delta n_i}{n_i} - 3\frac{\delta T}{T} \tag{37}$$

where the second equality follows from $s \propto T^3$. Throughout, all fields other than the axion are assumed to undergo adiabatic fluctuations, $S_i = 0$ for $i \neq a$. Relaxing this assumption would only allow for a smaller axion contribution for a fixed total isocurvature contribution to CMBR temperature fluctuations, and so could only result in stronger bounds than those given in section 4. As described above, isocurvature fluctuations of the axion on length scales larger than the horizon do not modify the total density on these scales, $\delta\rho_{\text{iso}} \simeq 0$. After the axion condensate forms, the total energy density is a sum over non-relativistic species including the axion plus radiation

$$\rho = \sum_i m_i n_i + m_a n_a + \rho_r \tag{38}$$

where $\rho_r \propto T^4$ is the radiation energy density, and throughout $\sum_i$ is over all non-relativistic species but the axion. The number density and temperature fluctuations for an isocurvature fluctuation on scales large compared with the horizon in this epoch are then related by

$$\delta\rho_{\text{iso}} = \sum_i m_i \delta n_i + m_a \delta n_a + 4\rho_r \frac{\delta T}{T} \simeq 0 \tag{39}$$

Since all species other than the axion are assumed to fluctuate adiabatically, $S_i = 0$, the definition (37) relates the number density fluctuation in each species to the initial isocurvature temperature fluctuation by $\delta n_i = 3n_i(\delta T/T)$. With this, the isocurvature constraint (39) then relates the isocurvature temperature fluctuation to the axion number density fluctuations after the axion condensate is formed

$$\frac{\delta T}{T} \simeq -\frac{\rho_a}{3\sum_i \rho_i + 4\rho_r}\frac{\delta n_a}{n_a} \tag{40}$$

where $\delta\rho_a = m_a \delta n_a$. The relation (37) may then be used to write the axion induced temperature fluctuation on super-horizon scales in terms of the fractional axion fluctuation S_a

$$\frac{\delta T}{T} \simeq -\frac{\rho_a}{3(\sum_i \rho_i + \rho_a) + 4\rho_r}\, S_a \tag{41}$$

This relation is more useful than (40) for arbitrary times after the axion condensate forms since S_a rather than $\delta n_a/n_a$ is approximately constant for fluctuations on super-horizon size scales [36].

The fractional axion fluctuation S_a can be related directly to the inflationary induced fluctuations in the axion field. When the axion condensate is formed at a temperature of $T_{\rm osc} \sim \Lambda_{\rm QCD}$, in any of the cosmological scenarios discussed in section 2, it comprises a small fraction of the total energy density, $\rho_a \ll \sum_i \rho_i + \rho_r$. From (40) it follows that just after the axion condensate is formed, the axion induced fractional temperature fluctuations are much smaller than the fractional axion number density fluctuations, $(\delta T/T)_{\rm init} \ll (\delta n_a/n_a)_{\rm init}$. From this it follows that the fractional fluctuation S_a defined in (37) just after the axion condensate is formed may then be related to fluctuations in the initial misalignment angle

$$S_a \simeq \frac{\delta n_a}{n_a}_{\rm init} \simeq \frac{\delta(\theta^2)}{\langle\theta^2\rangle} = \frac{(\langle\theta\rangle + \delta\theta)^2 - \langle\theta^2\rangle}{\langle\theta^2\rangle} = \frac{2\theta_i\delta\theta + (\delta\theta)^2 - \sigma_\theta^2}{\theta_i^2 + \sigma_\theta^2} \tag{42}$$

where $\delta\theta$ and σ_θ are understood to be due to the initial inflationary induced fluctuations in the misalignment angle, θ_i is the average misalignment angle over the currently observable universe, and $n_a \propto \theta^2$ ignoring anharmonic effects (this is a very good assumption for $f_a/N \sim 10^{16}$ GeV since $\theta \ll \pi$ is required in this case). And since S_a is approximately constant for super-horizon size fluctuations [36], the relation (42) between S_a and the initial misalignment angle fluctuations remains good for super-horizon modes for arbitrary times after the axion condensate forms. For later reference, with the relation (42), the mean square of the fractional axion fluctuation on super-horizon scales is related to the mean square fluctuations of, and average value of, the initial misalignment angle by

$$\langle S_a^2\rangle = 2\sigma_\theta^2 \frac{2\theta_i^2 + \sigma_\theta^2}{(\theta_i^2 + \sigma_\theta^2)^2} \tag{43}$$

where $\delta\theta = \theta - \langle\theta\rangle$ is assumed to be Gaussian distributed, $\theta_i = \langle\theta\rangle$, and $\sigma_\theta^2 = \langle(\theta - \langle\theta\rangle)^2\rangle$.

As discussed below, the most relevant axion induced temperature fluctuations are those on the largest angular scales. These temperature fluctuations enter the horizon well into the matter dominated era for which $\sum_i \rho_i + \rho_a \gg \rho_r$. So from (41) the axion induced initial temperature fluctuations on large angular scales at horizon crossing are related to the axion isocurvature fluctuation S_a by

$$\frac{\delta T}{T}_{\text{horizon}} \simeq -\frac{1}{3}\frac{\Omega_a}{\Omega_m} S_a \tag{44}$$

where $\Omega_a = \rho_a/\rho_c$ is the fractional axion density and $\Omega_m = (\sum_i \rho_i + \rho_a)/\rho_c$ is the fractional density of all non-relativistic matter including baryons, axions, and any other species which contribute to cold dark matter, $\Omega_m = \Omega_{\text{CDM}} + \Omega_b$. From WMAP the total matter density in a ΛCDM cosmology is measured to be [21]

$$\Omega_m h^2 = 0.135^{+0.008}_{-0.009} \tag{45}$$

An experimental observation of temperature fluctuations corresponds to the initial fluctuation at horizon crossing (44) plus an additional Sachs-Wolfe contribution [37] from the integrated redshift between the surface of last scattering and detector. The latter contribution turns out to be five times smaller [38] than that from the initial temperature fluctuation (44),

$$\frac{\delta T}{T}_{\text{SW}} \simeq -\frac{1}{15}\frac{\Omega_a}{\Omega_m} S_a \tag{46}$$

The total observable isocurvature temperature fluctuation is then related to the axion isocurvature fluctuation by

$$\frac{\delta T}{T}_{\text{iso}} \simeq -\frac{6}{15}\frac{\Omega_a}{\Omega_m} S_a \tag{47}$$

The expression (47) relating axion induced temperature fluctuations with the fractional axion fluctuation applies strictly only on the largest angular scales. The spectrum of temperature fluctuations is modified substantially on smaller scales corresponding to multipoles well beyond the Sachs-Wolfe plateau. This is illustrated in Fig. 1 which gives the CMBR temperature fluctuation power spectrum for pure adiabatic or pure isocurvature initial fluctuations in a ΛCDM cosmology generated by CMBFAST version 4.3 [39]. The power in isocurvature fluctuations falls only by a factor of two for roughly $\ell \sim 50$ but continues to fall rapidly for larger ℓ. This rapid decrease in isocurvature contributions to the power in CMBR temperature fluctuations compared with adiabatic contributions arises for two reasons. First, the growth of sub-horizon size isocurvature and adiabatic temperature fluctuations are completely out of phase with one another. So while the magnitude of adiabatic fluctuations are growing towards the first peak, the magnitude of isocurvature fluctuations are

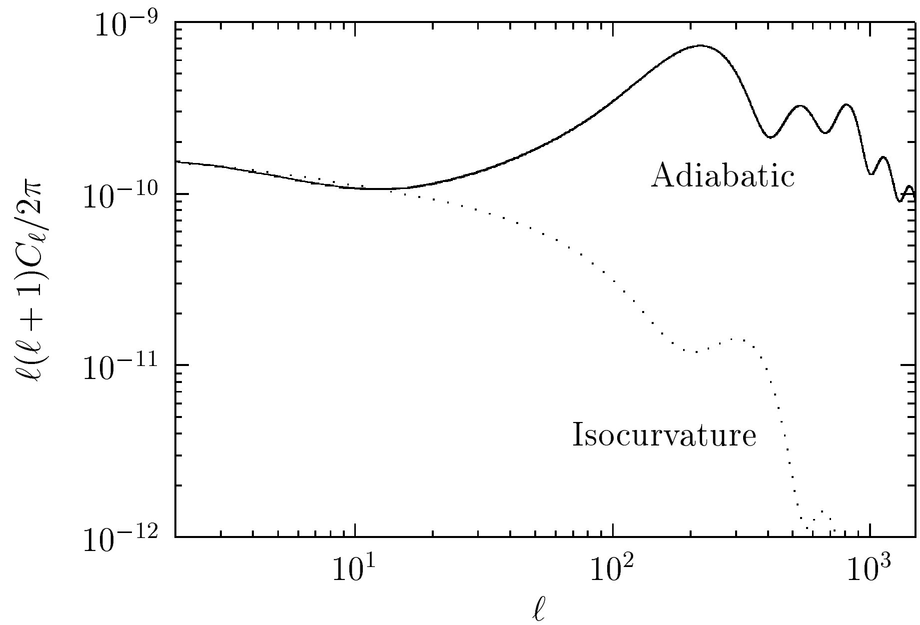

Figure 1: Temperature fluctuation power spectrum with pure adiabatic (solid line) or pure isocurvature (dashed line) initial fluctuations, $\Delta T(\theta,\phi)/T = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta,\phi)$, $C_\ell = \langle |a_{\ell m}|^2 \rangle$. ΛCDM cosmology with $\Omega_\Lambda = 0.73$, $\Omega_{\rm CDM} = 0.22$, $\Omega_b = 0.05$, $H = 72$ km s^{-1} Mpc^{-1}, and spectral index $n = 1$.

falling towards a first minimum. Secondly, fluctuations well beyond the Sachs-Wolfe plateau enter the horizon during radiation domination. Unlike adiabatic fluctuations, the temperature component of isocurvature fluctuations during this epoch are suppressed by the radiation density, as can be seen in the denominator of (41). This follows since well back into the radiation dominated era the isocurvature condition (39) can be satisfied for fixed magnitude of axion density fluctuation with vanishing small temperature fluctuation. This fall off of isocurvature temperature fluctuations with multipoles beyond the Sachs-Wolfe plateau is important in considering how well such fluctuations can be probed either by a direct measure of the power spectrum or search for non-Gaussian components discussed in the next subsection.

In order to characterize the observable effects of isocurvature fluctuations it is useful to define the ratio of average power in the isocurvature component to the average total power in CMBR temperature fluctuations

$$\alpha \equiv \frac{\left\langle (\delta T/T)^2_{\rm iso} \right\rangle}{\left\langle (\delta T/T)^2_{\rm tot} \right\rangle} \tag{48}$$

The total root mean square CMBR temperature fluctuation was measured by COBE to be [40]

$$\left\langle (\delta T/T)^2_{\rm tot} \right\rangle^{1/2} \simeq 1.1 \times 10^{-5} \tag{49}$$

Because of the rapid fall off of isocurvature power, the ratio (48) receives contributions mainly from low multipoles on the Sachs-Wolfe plateau. And since, as discussed above, axion induced isocurvature temperature fluctuations are fairly well approximated by (47) for these low multipoles, the isocurvature power ratio (48) may be approximately related to the mean square fluctuations and average value of the initial misalignment angle using (43) by

$$\alpha \simeq \left(\frac{6}{15}\right)^2 \frac{(\Omega_a/\Omega_m)^2 \langle S_a^2 \rangle}{\left\langle (\delta T/T)^2_{\rm tot} \right\rangle} \simeq \left(\frac{6}{15}\right)^2 \frac{(\Omega_a/\Omega_m)^2}{\left\langle (\delta T/T)^2_{\rm tot} \right\rangle} \, 2\sigma_\theta^2 \frac{2\theta_i^2 + \sigma_\theta^2}{\left(\theta_i^2 + \sigma_\theta^2\right)^2} \tag{50}$$

A full calculation of the relation between α and the variance and average value of the initial misalignment angle utilizing the full isocurvature power spectrum illustrated in Fig. 1 would not differ significantly from the approximation (50).

It is instructive to consider the numerical relation (50) between the axion fluctuation induced isocurvature power fraction and the variance and average initial misalignment angle in each of the cosmological scenarios for formation of the axion condensate discussed in section 2. For the case of smaller values of the Peccei-Quinn scale in which the axion condensate forms during a radiation dominated era in the high temperature axion potential and without any dilution after its formation, the relic axion density is given by (7) in section 2.1. This relic density with $C = 0.018$ and $\Lambda_{\rm QCD} = 200$ MeV and ignoring anharmonic effects, along with the WMAP total matter density measurement (45) and COBE total root mean square CMBR temperature fluctuation measurement (49) gives a fractional isocurvature power (50) of

$$\alpha \sim 6 \times 10^{19} \left(\frac{f_a/N}{10^{16}\ {\rm GeV}}\right)^{7/3} \sigma_\theta^2 \left(2\theta_i^2 + \sigma_\theta^2\right) \tag{51}$$

For the case of larger Peccei-Quinn scales in which axion condensate forms during a radiation dominated era but in effectively the zero temperature axion potential and again without any dilution after its formation, the relic axion density is given by (9) in section 2.1. This relic density with $f_c \xi(T_{\rm osc})^{-1} \simeq 1$ and ignoring anharmonic effects, along with (45) and (49) gives a fractional isocurvature power (50) of

$$\alpha \simeq 3.6 \times 10^{18} \left(\frac{f_a/N}{10^{16}\ {\rm GeV}}\right)^{3} \sigma_\theta^2 \left(2\theta_i^2 + \sigma_\theta^2\right) \tag{52}$$

Finally, for the case in which the axion condensate forms during a matter dominated era with subsequent late decay of the dominating particle with a reheat temperature $T_{\rm RH} \lesssim \Lambda_{\rm QCD}$, the relic density is given by (16) in section 2.3.1. This relic density with $f_c \xi(T_{\rm osc})^{-1} \simeq 1$ and ignoring anharmonic effects, along with (45) and (49) gives a fractional isocurvature power (50) of

$$\alpha \simeq 1.8 \times 10^{14} \left(\frac{T_{\rm RH}}{6 \text{ MeV}} \right) \left(\frac{f_a/N}{10^{16} \text{ GeV}} \right)^4 \sigma_\theta^2 \left(2\theta_i^2 + \sigma_\theta^2 \right) \tag{53}$$

Note that the fractional isocurvature power is strictly bounded by $\alpha \leq 1$. So for a string/M-theory axion with $f_a/N \sim 10^{16}$ GeV the large pre-factors in (51)–(53) imply that even just the magnitude of the observed temperature fluctuations (49) places a stringent limit on the combination $\sigma_\theta^2(2\theta_i^2 + \sigma_\theta^2)$ of variance and average value of the initial misalignment angle in any cosmological scenario for the formation of the axion condensate in which the axion exists during inflation.

The isocurvature power ratio (48) may be measured or bounded by fitting the observed CMBR temperature fluctuation power spectrum with a linear combination of adiabatic and isocurvature components with some assumptions about the underlying cosmological model. Using WMAP data and an analysis in which the tilt of both the adiabatic and isocurvature components are allowed to vary over fairly large ranges, the constraint $\alpha \lesssim 0.4$ has been obtained [41]. This analysis is very conservative and weakens the bound due to some degeneracies in parameters. The magnitude of inflationary induced axion isocurvature fluctuations depends on the Hubble constant during inflation. And in most theories of inflation the Hubble constant during inflation is by definition a very slowly changing quantity. This results in a nearly scale invariant spectrum of axion induced isocurvature temperature fluctuations, $n_{\rm iso} \simeq 1$. If this prior were imposed, the resulting bound on α using WMAP data would likely be at the many percent level. Since most of the power in isocurvature temperature fluctuations is restricted to low multipoles, future measurements will not improve knowledge of the power spectrum there since the current measurements are already cosmic variance limited even beyond the first peak. However, future measurements at higher multipoles along with additional information from polarization, such as from PLANCK, will resolve degeneracies in the power spectrum such as the adiabatic tilt. And this may well allow the fractional isocurvature power α to be bounded at the percent level. It is worth noting that gravity waves also contribute to CMBR temperature mainly at low multipoles on the Sachs-Wolfe plateau. If gravity waves are indeed detected in a future CMBR experiment, then even less of the budget of observed power at low multipoles can be due to a isocurvature component which would further strengthen the bound on α.

3.3 Non-Gaussianity

Primordial inflationary fluctuations in a QCD axion field give rise to CMBR temperature fluctuations. These fluctuations have the distinctive feature of being isocurvature

[5] with an angular power spectrum which differs from that of adiabatic fluctuations, as described in the previous subsection. Axion induced CMBR temperature fluctuations turn out also to have a non-Gaussian component [6] as reviewed below. This additional distinctive feature provides another probe for the existence of a QCD axion field during inflation.

Quantum fluctuations of a free massless field generated during inflation are Gaussian distributed. Slow roll inflation generally requires an inflaton with exceedingly small self coupling and so its fluctuations act essentially like a free field. Inflaton fluctuations in most models are therefore Gaussian to a very high degree. The adiabatic temperature fluctuations which ultimately result from inflaton induced fluctuations of the scalar curvature are then also Gaussian distributed. As discussed above, as long as the Peccei-Quinn symmetry remains unbroken, a QCD axion is also essentially a free massless field during inflation and so the initial misalignment angle undergoes Gaussian distributed fluctuations. However, the axion induced CMBR temperature fluctuations are proportional to fluctuations in the axion number density when the axion condensate is formed, as discussed in section 3.2. But the number density is proportional to the square of the misalignment angle ignoring anharmonic effects, $n_a \propto \theta^2$, while it is the fluctuations in the misalignment angle, $\delta\theta$, which are Gaussian distributed. This non-linearity introduces a non-Gaussian component in the axion induced temperature fluctuations.

On large angular scales the axion induced temperature fluctuations (47) may be related to fluctuations in the initial misalignment angle through the relation (42) as

$$\frac{\delta T}{T}_{\text{iso}} \simeq -\frac{6}{15}\frac{\Omega_a}{\Omega_m}\frac{2\theta_i\delta\theta + (\delta\theta)^2 - \sigma_\theta^2}{\theta_i^2 + \sigma_\theta^2} \tag{54}$$

The first term in the numerator proportional to $\delta\theta$ is Gaussian distributed with zero mean, while the remaining terms together, $(\delta\theta)^2 - \sigma_\theta^2$, have a χ^2 distribution with zero mean. So primordial Gaussian distributed fluctuations in the initial misalignment angle lead in general to a combination of Gaussian and χ^2 distributed isocurvature temperature fluctuations. In the limit of large average misalignment angle compared with the variance, $\theta_i \gg \sigma_\theta$, the axion induced temperature fluctuations (54) are mostly Gaussian, whereas in the opposite limit, $\theta_i \ll \sigma_\theta$, they are mostly χ^2. The relative importance of the Gaussian and χ^2 fluctuations is determined by the ratio θ_i/σ_θ. The temperature fluctuation (54) in these two limits is

$$\frac{\delta T}{T}_{\text{iso}} \simeq -\frac{6}{15}\frac{\Omega_a}{\Omega_m}\begin{cases} 2\delta\theta/\theta_i & \theta_i \gg \sigma_\theta \\ (\delta\theta)^2/\sigma_\theta^2 - 1 & \theta_i \ll \sigma_\theta \end{cases} \tag{55}$$

For $\theta_i \gg \sigma_\theta$ the magnitude of the axion induced temperature fluctuations are suppressed by the relic axion density and additionally suppressed on average by σ_θ/θ_i.

However, for $\theta_i \ll \sigma_\theta$ they are only suppressed by the relic axion density since the fractional axion fluctuations are order one in this case. The latter limit turns out to be most important in obtaining the most conservative bounds discussed in section 4.

Gaussian fluctuations with zero mean are completely defined in terms of the two point function. All odd point functions vanish, and all even point functions are products of the two point function. So in order to test for non-Gaussianity it is necessary to measure three or higher point functions. In the present context the simplest possibility to characterize a non-Gaussian component in CMBR temperature fluctuations is to measure the average three point function of the temperature fluctuation. This is conveniently normalized as a dimensionless skewness

$$S_3 \equiv \frac{\left\langle (\delta T/T)^3 \right\rangle}{\left\langle (\delta T/T)^2 \right\rangle^{3/2}} \tag{56}$$

For a pure Gaussian distribution the skewness vanishes while for a pure χ^2 distribution with zero mean $S_3 = -\sqrt{8}$. Of course the full three point function bi-spectrum contains additional information which would be invaluable in discerning the origin of non-Gaussianity in CMBR temperature fluctuations if ever observed. However, in the absence of such positive measurements the dimensionless skewness (56) provides a good dimensionless normalized measure to characterize the magnitude of non-Gaussian components.

In order to evaluate the dimensionless skewness (56) we consider the special case in which CMBR temperature fluctuations are an uncorrelated sum of adiabatic plus isocurvature contributions

$$\frac{\delta T}{T} = \frac{\delta T}{T}_{\rm ad} + \frac{\delta T}{T}_{\rm iso} \tag{57}$$

This is the case if inflaton fluctuations are responsible for the dominant adiabatic temperature fluctuations observed in the CMBR, and the only other massless field during inflation is a QCD axion. Inclusion of gravity wave tensor B-mode contributions from inflationary metric fluctuations would not modify the results below. And inclusion of other sources of non-Gaussianity would only allow for a smaller axion contribution for a fixed total skewness of CMBR temperature fluctuations, and so could only result in stronger bounds than those given in section 4. Since the inflationary axion fluctuation induced isocurvature temperature fluctuations have both Gaussian and χ^2 components, the total temperature fluctuation (57) may be written in terms of a sum of distributions

$$\frac{\delta T}{T} = \frac{\delta T_I}{T} + \frac{\delta T_a}{T} = \phi_I + \phi_a + \psi_a \tag{58}$$

where ϕ_I parameterizes the Gaussian distributed inflaton induced temperature fluctuations, ϕ_a the Gaussian distributed component of the axion induced temperature fluctuations, and ψ_a the χ^2 distributed non-Gaussian component of the axion induced temperature fluctuations. Each distribution by definition has zero mean, $\langle\phi_I\rangle = \langle\phi_a\rangle = \langle\psi_a\rangle = 0$. Note that ϕ_a and ψ_a are not independent since they both arise from axion fluctuations whereas ϕ_I is independent of both ϕ_a and ψ_a since it arises from uncorrelated fluctuations of an independent field. So cross correlations between ϕ_I and ϕ_a or ψ_a vanish. With this, the two and three point temperature fluctuation correlation functions from (58) are then

$$\left\langle \frac{\delta T}{T}\frac{\delta T}{T}\right\rangle = \left\langle \frac{\delta T_I}{T}\frac{\delta T_I}{T}\right\rangle + \left\langle \frac{\delta T_a}{T}\frac{\delta T_a}{T}\right\rangle = \langle\phi_I\phi_I\rangle + \langle\phi_a\phi_a\rangle + \langle\psi_a\psi_a\rangle \tag{59}$$

$$\left\langle \frac{\delta T}{T}\frac{\delta T}{T}\frac{\delta T}{T}\right\rangle = \left\langle \frac{\delta T_a}{T}\frac{\delta T_a}{T}\frac{\delta T_a}{T}\right\rangle = 3\,\langle\phi_a\phi_a\psi_a\rangle + 3\,\langle\phi_a\psi_a\psi_a\rangle + \langle\psi_a\psi_a\psi_a\rangle \tag{60}$$

The three point function of course only depends on the axion fluctuation induced isocurvature components since by assumption the adiabatic inflaton fluctuations are purely Gaussian distributed.

Now the dimensionless skewness defined in (56) is an average over all angular scales. Axion isocurvature contributions through the three-point function (60) however come mainly from low multipoles on the Sachs-Wolfe plateau, just as for the isocurvature power discussed in section 3.2. On these large angular scales the relation between the axion induced isocurvature temperature fluctuation and fluctuations of the initial misalignment angle is fairly well approximated by (54). The relation between the axion induced Gaussian and χ^2 distributed temperature fluctuation distributions ϕ_a and ψ_a in (58) and the fluctuations in the initial misalignment angle may then be approximated by

$$\phi_a \simeq -\frac{6}{15}\frac{\Omega_a}{\Omega_m}\frac{2\theta_i\delta\theta}{\theta_i^2+\sigma_\theta^2} \qquad \text{Gaussian} \tag{61}$$

$$\psi_a \simeq -\frac{6}{15}\frac{\Omega_a}{\Omega_m}\frac{(\delta\theta)^2-\sigma_\theta^2}{\theta_i^2+\sigma_\theta^2} \qquad \chi^2 \tag{62}$$

With this, the three point function (60) and dimensionless skewness (56) for the axion induced isocurvature component of temperature fluctuations may be approximately related to the mean square fluctuations and average value of the initial misalignment angle by

$$S_{3,\text{iso}} \simeq -8\left(\frac{6}{15}\right)^3 \frac{(\Omega_a/\Omega_m)^3}{\left\langle(\delta T/T)^2_{\text{tot}}\right\rangle^{3/2}}\;\sigma_\theta^4\,\frac{3\theta_i^2+\sigma_\theta^2}{(\theta_i^2+\sigma_\theta^2)^3} \tag{63}$$

A full calculation of the relation between $S_{3,\text{iso}}$ and the variance and average value of the initial misalignment angle utilizing the full isocurvature power spectrum would not differ significantly from the approximation (63). Note that this skewness is necessarily negative as a result of the χ^2 nature of the non-Gaussian component of axion fluctuations. This would be the first test that a positive measurement of non-Gaussianity in CMBR temperature fluctuations is in fact due to a QCD axion.

It is instructive to consider the numerical relation between the axion fluctuation induced isocurvature skewness (63) and the variance and average value of the initial misalignment angle in each of the cosmological scenarios for formation of the axion condensate discussed in section 2. For the case of smaller values of the Peccei-Quinn scale in which the axion condensate forms during a radiation dominated era in the high temperature axion potential and without any dilution after its formation, the relic axion density is given by (7) in section 2.1. This relic density with $C = 0.018$ and $\Lambda_{\rm QCD} = 200$ MeV and ignoring anharmonic effects, along with the WMAP total matter density measurement (45) and COBE total root mean square CMBR temperature fluctuation measurement (49) gives an isocurvature skewness (63) of

$$S_{3,\rm iso} \sim -1 \times 10^{30} \left(\frac{f_a/N}{10^{16}\ \rm GeV} \right)^{7/2} \ \sigma_\theta^4 \left(3\theta_i^2 + \sigma_\theta^2 \right) \tag{64}$$

For the case of larger Peccei-Quinn scales in which axion condensate forms during a radiation dominated era but in effectively the zero temperature axion potential and again without any dilution after its formation, the relic axion density is given by (9) in section 2.1. This relic density with $f_c \xi(T_{\rm osc})^{-1} \simeq 1$ and ignoring anharmonic effects, along with (45) and (49) gives an isocurvature skewness (63) of

$$S_{3,\rm iso} \simeq -1.9 \times 10^{28} \left(\frac{f_a/N}{10^{16}\ \rm GeV} \right)^{9/2} \ \sigma_\theta^4 \left(3\theta_i^2 + \sigma_\theta^2 \right) \tag{65}$$

Finally, for the case in which the axion condensate forms during a matter dominated era with subsequent late decay of the dominating particle with a reheat temperature $T_{\rm RH} \lesssim \Lambda_{\rm QCD}$, the relic density is given by (16) in section 2.3.1. This relic density with $f_c \xi(T_{\rm osc})^{-1} \simeq 1$ and ignoring anharmonic effects, along with (45) and (49) gives an isocurvature skewness (63) of

$$S_{3,\rm iso} \simeq -6.7 \times 10^{21} \left(\frac{T_{\rm RH}}{6\ \rm MeV} \right)^3 \left(\frac{f_a/N}{10^{16}\ \rm GeV} \right)^6 \ \sigma_\theta^4 \left(3\theta_i^2 + \sigma_\theta^2 \right) \tag{66}$$

Note that the axion fluctuation induced isocurvature skewness is negative definite and strictly bounded by $S_{3,\rm iso} \geq -\sqrt{8}$, and as discussed below is already bounded to be significantly smaller in magnitude. So for a string/M-theory axion with $f_a/N \sim 10^{16}$ GeV, similar to the case of isocurvature power fraction, the large prefactors in (64)–(66) imply that even just the magnitude of the observed temperature fluctuations (49) places a stringent limit on the combination $\sigma_\theta^4(3\theta_i^2 + \sigma_\theta^2)$ of variance and average value of the initial misalignment angle in any cosmological scenario for the formation of the axion condensate in which the axion exists during inflation.

The dimensionless skewness (56) may be measured or bounded from the observed CMBR temperature fluctuation three point function. As reviewed in appendix C, it has become common in the literature to parameterize possible non-Gaussianity in terms of a non-linear product of Gaussian-distributed temperature fluctuations proportional to a dimensionless parameter f_{NL}. As derived in (91) of appendix C, the dimensionless skewness may be related to this parameter at leading order by

$$S_3 \simeq 18 f_{NL} \left\langle (\delta T/T)^2_{\rm tot} \right\rangle^{1/2} \tag{67}$$

The WMAP bound on f_{NL} for multipoles $\ell < 65$ is roughly $f_{NL} = -150^{+200}_{-150}$ [42]. Since the dimensionless skewness for axion induced non-Gaussianity is strictly negative, $S_{3,\rm iso} \leq 0$, we conservatively take $f_{NL} > -300$ for this case. With the total root mean square CMBR temperature fluctuation (49) measured by COBE [40] this bound then corresponds to $S_{3,\rm iso} \gtrsim -6 \times 10^{-2}$.

Since the magnitude of isocurvature temperature fluctuations falls rapidly beyond the Sachs-Wolfe plateau, as discussed in section 3.2, the inclusion of multipoles much beyond $\ell \gtrsim 65$ would not significantly improve the search for, or bound on, a non-Gaussian component in isocurvature temperature fluctuations. And since current measurements at these low multipoles are already cosmic variance limited, future measurements will also not significantly improve the current bound on a non-Gaussian component in isocurvature temperature fluctuations (although the bound on a non-Gaussian component in adiabatic temperature fluctuations, which do not fall as rapidly at large multipoles, would be improved). In addition, future reductions in degeneracies of parameters describing the dominant adiabatic power spectrum from measurements at higher multipoles will also not help improve the bound on a non-Gaussian component in isocurvature temperature fluctuations. So unlike the fractional isocurvature power, α, discussed in section 3.2, the bound on the isocurvature skewness, $S_{3,\rm iso}$ is unlikely to improve significantly.

4 Constraints on the Axion from the CMBR

It seems unlikely that a string/M-theory QCD axion with $f_a/N \sim 10^{16}$ GeV will ever be probed directly in laboratory experiments. Such an axion can, however, give rise to in principle measurable isocurvature [5] and non-Gaussian [6] signatures in CMBR temperature fluctuations, as reviewed in section 3. In this section we show that a detection of primordial gravity waves interpreted as arising from an early epoch of inflation places a lower limit on these observable effects. The current bounds on isocurvature and non-Gaussian components of CMBR temperature fluctuations are then shown to be sufficient to rule out a string/M-theory QCD axion if gravity waves interpreted as arising from inflation with a Hubble constant $H_{\rm ing} \gtrsim 10^{13}$ GeV are observed by future experiments.

Consider first the axion fluctuation induced signature of an isocurvature component in CMBR temperature fluctuations. The goal here is to assess how well CMBR experiments can probe a string/M-theory QCD axion independent of any cosmological assumptions. This requires considering in turn all the relevant cosmological scenarios for formation of the axion condensate presented in section 2. In addition it requires considering the smallest possible axion induced isocurvature component in CMBR temperature fluctuations consistent with other future CMBR measurements; this implies the weakest possible bound. The numerical relations between the fractional isocurvature power α and variance and average value of the initial misalignment angle for the various cosmological scenarios for formation of the axion condensate considered in section 2 are given in (51)–(53) of section 3.2. In each case the fractional isocurvature power is proportional to the combination $\sigma_\theta^2(2\theta_i^2+\sigma_\theta^2)$. This is clearly minimized in the limit in which the average misalignment angle is much smaller than the variance of inflationary fluctuations in the initial misalignment angle, $\theta_i^2 \ll \sigma_\theta^2$. In this limit of small average misalignment angle, the relic axion density is due mostly to inflationary fluctuations of the axion field as discussed in section 2.4. And the fractional axion fluctuations are order one in this limit, $\langle S_a^2\rangle \simeq 2$ from (43).

Axion induced isocurvature components in CMBR temperature fluctuations of course require primordial fluctuations in the axion field. As long as the Peccei-Quinn symmetry remains unbroken during inflation, a string/M-theory QCD axion undergoes quantum fluctuations during this epoch with magnitude (25) determined by the inflationary Hubble constant. As discussed in section 3.1 all massless fields undergo quantum fluctuations during inflation including the metric which ultimately gives rise to tensor B-modes in CMBR temperature fluctuations. Observation of these modes interpreted as arising from inflation would establish the Hubble constant during inflation and the variance of fluctuations in the axion initial misalignment angle for a given Peccei-Quinn scale, as given in (36) in terms of the tensor to scalar ratio. This would in turn establish a lower limit for the isocurvature power fraction. So obtaining a definitive lower limit on an axion induced isocurvature component in CMBR temperature fluctuations independent of cosmological assumptions requires the observation of primordial gravity waves interpreted as arising from inflation.

The lower limit on α in terms of the Hubble constant during inflation for a given cosmological scenario for the formation of the axion condensate may be obtained from (51)–(53) in the limit $\theta_i^2 \ll \sigma_\theta^2$ discussed above, along with the relation (36) between the variance of the axion initial misalignment angle and inflationary Hubble

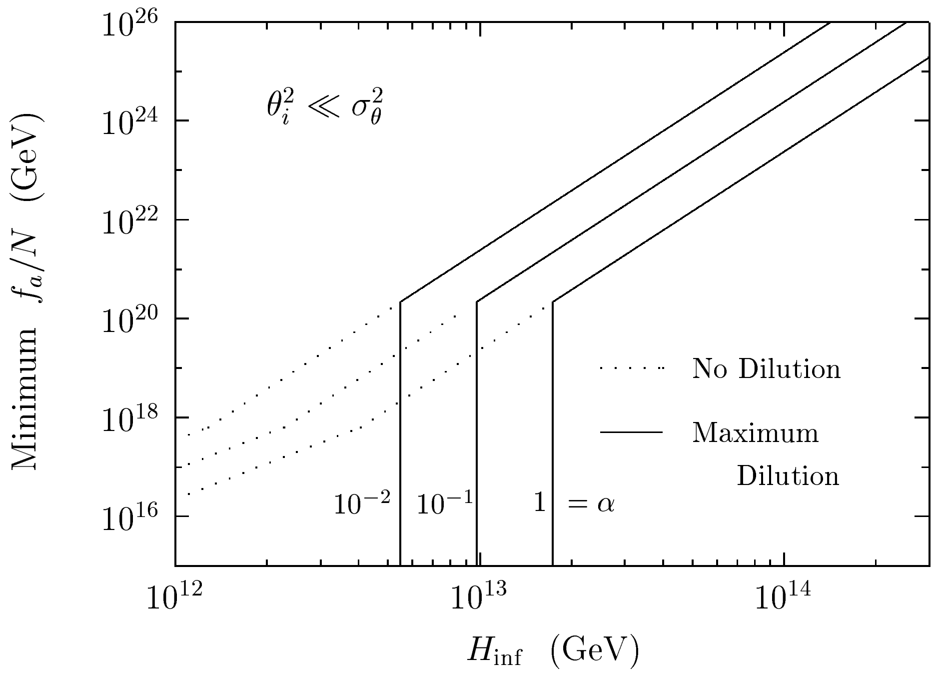

Figure 2: Minimum allowed axion Peccei-Quinn scale f_a/N as a function of $H_{\rm inf}$ for isocurvature temperature fluctuation power fractions $\alpha = \langle(\delta T/T)^2_{\rm iso}\rangle/\langle(\delta T/T)^2\rangle = 1, 10^{-1}, 10^{-2}$ in the limit that the average initial misalignment angle is small compared to inflation induced fluctuations, $\theta_i^2 \ll \sigma_\theta^2 = (H_{\rm inf}/(2\pi f_a/N))^2$. For $\theta_i^2 \gtrsim \sigma_\theta^2$ the minimum allowed f_a/N grows like θ_i^2. The dotted lines correspond to no dilution of the axion condensate after formation while the solid lines correspond to maximum possible dilution by a late decaying particle with $T_{\rm RH} \simeq 6$ MeV.

constant. This may also be obtained directly from (50) in the limit $\langle S_a^2\rangle \simeq 2$ discussed above, along with the minimum fluctuation dominated relic axion densities (27)–(29) in terms of the inflationary Hubble constant. If gravity wave contributions to CMBR temperature fluctuations are in fact observed and interpreted as arising from inflation, then the lower limit on α can be inverted to give a minimum allowed Peccei-Quinn scale in terms of the implied Hubble constant during inflation and the present bound on the isocurvature power fraction.

The simplest cosmological scenario to consider is obtained if the axion condensate is formed during the radiation dominated era without any dilution after its formation. In this case for small values of Peccei-Quinn scale the isocurvature power fraction (51) with $\theta_i^2 \ll \sigma_\theta^2$ and the relation (36) yields a lower limit on the Peccei-Quinn scale as outlined above in terms of α and the Hubble constant during inflation of

$$f_a/N \gtrsim \frac{6 \times 10^{18}}{\alpha^{3/5}} \left(\frac{H_{\rm inf}}{10^{13}~{\rm GeV}}\right)^{12/5} ~{\rm GeV} \qquad \theta_i^2 \lesssim \sigma_\theta^2 \tag{68}$$

In the same case for large values of Peccei-Quinn scale, the isocurvature power fraction (52) with $\theta_i^2 \ll \sigma_\theta^2$ and the relation (36) yields a lower limit on the Peccei-Quinn scale of

$$f_a/N \gtrsim \frac{2.4 \times 10^{19}}{\alpha} \left(\frac{H_{\rm inf}}{10^{13}~{\rm GeV}}\right)^{4} ~{\rm GeV} \qquad \theta_i^2 \lesssim \sigma_\theta^2 \tag{69}$$

These bounds on the Peccei-Quinn scale are plotted in Fig. 2 as the diagonal dotted and solid lines for $\alpha = 1, 10^{-1}, 10^{-2}$. The bounds for large and small Peccei-Quinn scale coincide for $f_a/N \sim 6 \times 10^{17}$ GeV. As discussed in section 2.1 for Peccei-Quinn scales in this transition region the relic axion density calculation suffers unknown strong QCD uncertainties but should asymptotically approach the limiting expressions for scales well outside the transition region.

In the remaining cosmological scenario to consider, the axion condensate is formed during a matter dominated era with subsequent decay of the dominating particle. In this case with $\theta_i^2 \ll \sigma_\theta^2$, the maximum possible dilution consistent with BBN discussed in section 2.3.1, $T_{\rm RH} \simeq 6$ MeV, and the relation (36) between the variance of the initial misalignment angle and Hubble constant during inflation, the isocurvature power fraction (53) is bounded by

$$\alpha \lesssim 1.1 \times 10^{-1} \left(\frac{T_{\rm RH}}{6~{\rm MeV}}\right) \left(\frac{H_{\rm inf}}{10^{13}~{\rm GeV}}\right)^{4} \qquad \theta_i^2 \lesssim \sigma_\theta^2 \tag{70}$$

Note that this bound is *independent* of the Peccei-Quinn scale since the explicit $(f_a/N)^4$ dependence in (53) cancels the $(f_a/N)^{-4}$ dependence in σ_θ^4. This can also be seen directly in (50) with $\langle S_a \rangle \simeq 2$ as discussed above in this limit, along with the minimum inflationary fluctuation induced relic axion density (29) which is independent of Peccei-Quinn scale as discussed in section 2.4. This implies that if the bound (70) is not satisfied, then for an axion which exists during inflation, all Peccei-Quinn scales are excluded for which the axion condensate forms during the matter dominated era before subsequent decay of the dominating particle. As discussed in section 2.4, this occurs with maximum dilution, $T_{\rm RH} \sim 6$ MeV, for Peccei-Quinn scales $f_a/N \lesssim 2 \times 10^{20}$ GeV. For larger Peccei-Quinn scales the axion condensate forms during the subsequent radiation dominated era after reheating from decay of the dominating particle. So for $f_a/N \gtrsim 2 \times 10^{20}$ GeV the bound (69) is recovered in any scenario for formation of the axion condensate. The bounds obtained from (70) for $\alpha = 1, 10^{-1}, 10^{-2}$ with maximum dilution, $T_{\rm RH} \sim 6$ MeV, are plotted as vertical solid lines in Fig. 2. Since the axion condensate forms in essentially the zero temperature potential in this case, these bounds do not suffer large QCD uncertainties.

Now consider the axion fluctuation induced signature of a non-Gaussian component in CMBR temperature fluctuations. The numerical relations between the axion induced isocurvature skewness, $S_{3,\text{iso}}$, and variance and average value of the initial misalignment angle for the various cosmological scenarios for formation of the axion condensate considered in section 2 are given in (64)–(66) of section 3.3. In each case the skewness is proportional to the combination $\sigma_\theta^4(3\theta_i^2 + \sigma_\theta^2)$. Just as for the isocurvature power fraction, this combination is minimized for $\theta_i^2 \ll \sigma_\theta^2$ which yields in turn the weakest possible bound. In this limit of small initial misalignment angle, the relic axion density is due mostly to inflationary fluctuations of the axion field as discussed in section 2.4. And the axion induced isocurvature temperature fluctuations are approximately χ^2 distributed as discussed in section 3.3. The lower limit on $S_{3,\text{iso}}$ may be obtained in the $\theta_i^2 \ll \sigma_\theta^2$ limit from (64)–(66) along with the relation (36) between the variance of the axion initial misalignment angle and inflationary Hubble scale. This may also be obtained directly from (63) in this limit along with the minimum fluctuation dominated relic axion densities (27)–(29) in terms of the inflationary Hubble constant. Just as for the isocurvature power fraction, if gravity wave contributions to CMBR temperature fluctuations are in fact observed and interpreted as arising from inflation, then the lower limit on $S_{3,\text{iso}}$ can be inverted to give a minimum allowed Peccei-Quinn scale in terms of the implied Hubble constant during inflation and the present bound on the isocurvature skewness.

The simplest cosmological scenario for formation of the axion condensate is that it formed during the radiation dominated era without any dilution after its formation. In this case for small values of Peccei-Quinn scale the isocurvature skewness (64) yields a lower limit of the Peccei-Quinn scale in terms of $S_{3,\text{iso}}$ and the Hubble constant during inflation, as described above, of

$$f_a/N \gtrsim \frac{8 \times 10^{18}}{(-S_{3,\text{iso}})^{2/5}} \left(\frac{H_{\text{inf}}}{10^{13}\ \text{GeV}}\right)^{12/5} \text{GeV} \qquad \theta_i^2 \lesssim \sigma_\theta^2 \tag{71}$$

In the same case for large values of Peccei-Quinn scale, the isocurvature skewness (65) with $\theta_i^2 \ll \sigma_\theta^2$ and the relation (36) yields a lower limit on the Peccei-Quinn scale of

$$f_a/N \gtrsim \frac{4.8 \times 10^{19}}{(-S_{3,\text{iso}})^{2/3}} \left(\frac{H_{\text{inf}}}{10^{13}\ \text{GeV}}\right)^{4} \text{GeV} \qquad \theta_i^2 \lesssim \sigma_\theta^2 \tag{72}$$

These bounds on the Peccei-Quinn scale are plotted in Fig. 3 as the diagonal dotted and solid lines for $-S_{3,\text{iso}} = 10^{-1}, 10^{-2}$. The bounds for large and small Peccei-Quinn scale coincide for $f_a/N \sim 6 \times 10^{17}$ GeV, and suffer unknown QCD uncertainties in the transition region.

In the remaining cosmological scenario to consider, the axion condensate is formed during a matter dominated era with subsequent decay of the dominating particle. In this case with $\theta_i^2 \ll \sigma_\theta^2$, the maximum possible dilution consistent with BBN discussed

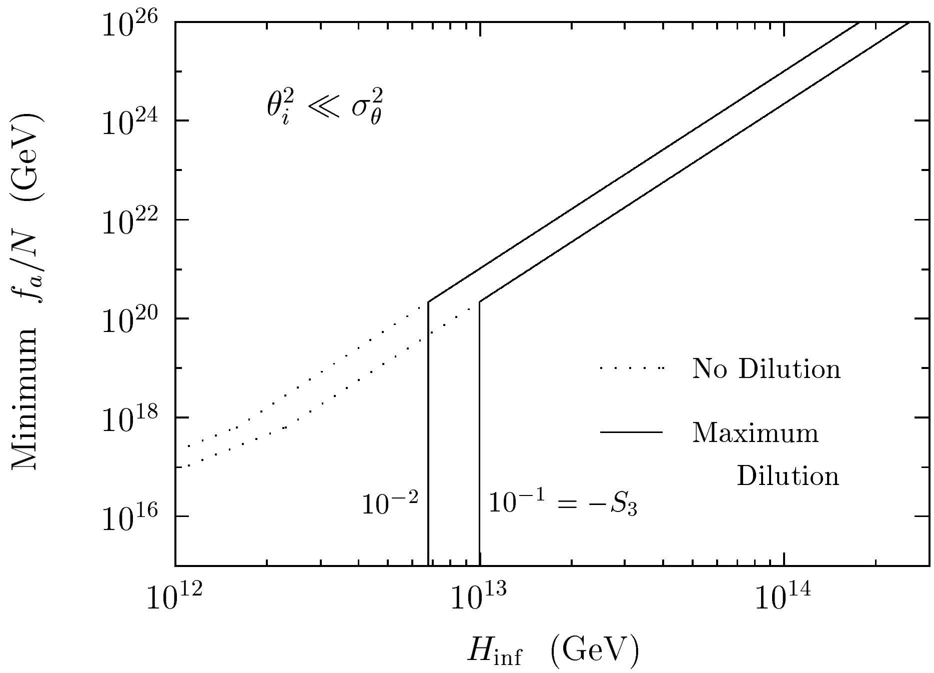

Figure 3: Minimum allowed axion Peccei-Quinn scale f_a/N as a function of $H_{\rm inf}$ for isocurvature temperature fluctuation skewness $S_3 = \langle(\delta T/T)^3_{\rm iso}\rangle / \langle(\delta T/T)^2\rangle^{3/2} = -10^{-1}, -10^{-2}$ in the limit that the average initial misalignment angle is small compared with inflation induced fluctuations, $\theta_i^2 \ll \sigma_\theta^2 = (H_{\rm inf}/(2\pi f_a/N))^2$. For $\theta_i^2 \gtrsim \sigma_\theta^2$ the minimum allowed f_a/N grows like $\theta_i^{4/3}$. The dotted lines correspond to no dilution of the axion condensate after formation while the solid lines correspond to maximum possible dilution by a late decaying particle with $T_{\rm RH} \simeq 6$ MeV.

in section 2.3.1, $T_{\rm RH} \simeq 6$ MeV, and the relation (36) between the variance of the initial misalignment angle and Hubble constant during inflation, the isocurvature skewness (66) is bounded by

$$-S_{3,{\rm iso}} \lesssim 1.1 \times 10^{-1} \left(\frac{T_{\rm RH}}{6 \text{ MeV}}\right)^3 \left(\frac{H_{\rm inf}}{10^{13} \text{ GeV}}\right)^6 \qquad \theta_i^2 \lesssim \sigma_\theta^2 \tag{73}$$

Just as for the isocurvature power fraction in this scenario for formation of the axion condensate, this bound is *independent* of the Peccei-Quinn scale since here the explicit $(f_a/N)^6$ dependence in (66) cancels the $(f_a/N)^{-6}$ dependence in σ_θ^6. This implies that if the bound (73) is not satisfied, then for an axion which exists during inflation, all Peccei-Quinn scales are excluded for which the axion condensate forms during the matter dominated era before subsequent decay of the dominating particle. As discussed in section 2.4, this occurs with maximum dilution, $T_{\rm RH} \sim 6$ MeV, for Peccei-Quinn scales $f_a/N \lesssim 2 \times 10^{20}$ GeV. For larger Peccei-Quinn scales the axion condensate forms during the subsequent radiation dominated era after reheating from decay of the dominating particle. So for $f_a/N \gtrsim 2 \times 10^{20}$ GeV the bound (72) is recovered in any scenario for formation of the axion condensate. The bounds obtained from (73) for $-S_{3,\rm iso} = 10^{-1}, 10^{-2}$ with maximum dilution, $T_{\rm RH} \sim 6$ MeV, are plotted as vertical solid lines in Fig. 3. Since the axion condensate forms in essentially the zero temperature potential in this case, these bounds do not suffer large QCD uncertainties.

The bounds arising from isocurvature and non-Gaussian contributions to CMBR temperature fluctuations presented in Figs. 2 and 3 on the Peccei-Quinn scale for an axion which exists during inflation are quite severe if primordial gravity waves are observed in future experiments. As discussed in section 3.1, the search for tensor B-mode components in CMBR temperature fluctuations with the PLANCK polarimetry experiment may reach an ultimate sensitivity to the Hubble constant during inflation of $H_{\rm inf} \gtrsim 5 \times 10^{12}$ GeV [33, 34], with a likely benchmark for a positive observation of $H_{\rm inf} \gtrsim 10^{13}$ GeV. Even with the rather conservative current bound on the isocurvature power fraction discussed in section 3.2 of $\alpha \gtrsim 0.4$ [41], if the PLANCK experiment did in fact observe tensor B-modes interpreted as arising from inflation with $H_{\rm inf} \gtrsim 10^{13}$ GeV, then (69) would require that the Peccei-Quinn scale be bounded by at least $f_a/N \gtrsim 6 \times 10^{19}$ GeV. And in the same case the current bound on the isocurvature skewness discussed in section 3.3 of $-S_3 \lesssim 6 \times 10^{-2}$ [42] with (72) would require that the Peccei-Quinn scale be bounded by at least $f_a/N \gtrsim 3 \times 10^{20}$ GeV. If CMBR temperature fluctuation tensor B-modes were discovered with magnitude not too far below the current WMAP implied limit on the Hubble constant during inflation discussed in section 3.1 of $H_{\rm inf} \gtrsim 2.8 \times 10^{14}$ GeV [15] the bounds would be even more severe. For example, if it turned out that $H_{\rm inf} = 2 \times 10^{14}$ GeV the current isocurvature power fraction bound would require $f_a/N \gtrsim 10^{25}$ GeV, while the current isocurvature skewness bound would require $f_a/N \gtrsim 5 \times 10^{25}$ GeV.

If primordial gravity waves are in fact observed, the bound on the Peccei-Quinn scale from the present bound on a non-Gaussian component in CMBR temperature fluctuations would be somewhat more stringent than that from the present bound on the isocurvature power fraction. However, as discussed at the end of sections 3.2 and 3.3, the bound on isocurvature non-Gaussianity is not expected to improve in the future, while the bound on isocurvature power fraction should improve considerably either through a re-analysis of current WMAP data and/or future CMBR temperature fluctuation data at larger multipoles. So ultimately the bound from the isocurvature power fraction would likely be somewhat better. In comparing the isocurvature power fraction with isocurvature skewness bounds note that the parametric dependence on the Hubble constant during inflation of (68) and (71) for low Peccei-Quinn scales are identical, as are those of (69) and (72) for high Peccei-Quinn scales. In the case that the axion condensate is formed during a matter dominated era, the parametric dependence of the bounds (70) and (73) on the Hubble constant during inflation and reheat temperature after the axion condensate forms are different however.

For lower values of the Hubble constant during inflation the implied bounds on the Peccei-Quinn scale given in Figs. 2 and 3 are reduced. But as long as $H_{\rm inf} \gtrsim 10^{13}$ GeV, even with the current bounds on the CMBR isocurvature power fraction and isocurvature skewness, the bounds imply that in this case the only allowed scenario for formation of the axion condensate would be during the radiation dominated era at a temperature lower than minimum allowed reheat temperature of $T_{\rm RH} \sim 6$ MeV implied by consistency with BBN. In this regime the axion condensate forms in essentially the zero temperature potential and the relic density calculation does not suffer any strong QCD uncertainties. So the bounds presented here that would apply with a positive measurement of primordial gravity waves by PLANCK are largely free from such uncertainties.

The bounds discussed above on the Peccei-Quinn scale which would result if primordial gravity waves are observed in a future experiment are quite severe. However, it should be noted again that these bounds are the weakest possible given all scenarios for formation of the axion condensate, and average initial misalignment angle. These weakest possible bounds are obtained in only an extraordinarily tiny sliver of the full parameter space of initial misalignment angles. Outside these tiny slivers even stronger bounds would apply. For example, even for the minimum Hubble constant during inflation of roughly $H_{\rm inf} \sim 10^{13}$ GeV to which the PLANCK polarimetry experiment will be sensitive for a positive measurement, the bound on the Peccei-Quinn scale quoted above of $f_a/N \gtrsim 3 \times 10^{20}$ GeV which would result from the current bound on the isocurvature skewness, corresponds to an initial misalignment angle of at most $|\theta_i| \lesssim 5 \times 10^{-9}$. For fixed $H_{\rm inf}$ and isocurvature skewness, the bound on the Peccei-Quinn scale grows like $\theta_i^{4/3}$ for $\theta_i^2 \gtrsim \sigma_\theta^2$, and for fixed isocurvature power fraction like θ_i^2. This also implies that even if the bounds were just barely saturated, relic axions would comprise a very small fraction of the CDM. This can be seen directly, for example, in the relation (50) between the isocurvature power fraction, fractional axion fluctuations, relic axion density, and magnitude of the total mean square temperature fluctuations. Since as discussed above, $\langle S_a^2 \rangle \simeq 2$ results in the weakest possible bound, the current bound of $\alpha \lesssim 0.4$ in this limit, along with the COBE measurement (49) of the magnitude of temperature fluctuations, implies that if the bounds are just saturated then $\Omega_a \lesssim 1 \times 10^{-5}\ \Omega_m$ at least.

Finally, let us consider what implications the stringent bounds on the Peccei-Quinn scale given above, if realized by a future measurement of primordial gravity waves, would have on a string/M-theory QCD axion. First, since a string/M-theory QCD axion exists during inflation these bounds would apply. In all cases for an observation of gravity waves which implied a Hubble constant during inflation of

$H_{\rm inf} \gtrsim 10^{13}$ GeV, definitive bounds on the Peccei-Quinn scale would result as discussed above. For this Hubble constant during inflation, the current bound on isocurvature power fraction would require at least $f_a/N \gtrsim 6 \times 10^{19}$ GeV and the current bound on the isocurvature skewness would require at least $f_a/N \gtrsim 3 \times 10^{20}$ GeV. These bounds would exceed the four-dimensional Planck scale of $M_p \simeq 2.4 \times 10^{18}$ GeV. Now obtaining a Peccei-Quinn scale which is in excess of the four-dimensional Planck scale seems problematic in a fundamental theory of gravity. The maximum scale for compact moduli in string/M-theory is strongly believed to be the four-dimensional Planck scale [14]; no counter example is known. With this restriction we are led to the strong conclusion that if primordial gravity waves interpreted as arising from inflation are observed with an implied Hubble constant $H_{\rm inf} \gtrsim 10^{13}$ GeV, which includes the range of expected sensitivity of the PLANCK CMBR polarimetry experiment, then a string/M-theory compact modulus can not play the role of the QCD axion and solve the strong CP problem.

5 Remaining Axion Windows

The bounds presented in section 4 rather convincingly require a QCD axion which exists during inflation to have a Peccei-Quinn scale of at least $f_a/N \gtrsim 3 \times 10^{20}$ GeV if gravity waves interpreted as arising from inflation with Hubble constant $H_{\rm inf} \gtrsim 10^{13}$ GeV are observed by the PLANCK polarimetry experiment, and at least $f_a/N \gtrsim 5 \times 10^{25}$ GeV if gravity waves are observed just below the current WMAP bound. As discussed above, these bounds are the weakest possible considering all relevant cosmological scenarios for formation of the axion condensate and all possible average initial misalignment angles, and are therefore quite conservative. However, it is worth discussing under what conditions these bounds do not apply and possible loopholes for evading the bounds.

In some scenarios for realizing the Peccei-Quinn mechanism, the QCD axion arises from spontaneous breaking of an anomalous global $U(1)$ symmetry. In this case the Peccei-Quinn phase transition to the broken phase could in principle take place after inflation. The axion field would then not exist during inflation, so the CMBR temperature fluctuations bounds discussed here, which depend on the existence of the axion during inflation, would not apply. However, in such a scenario global axion cosmic strings are formed during the Peccei-Quinn phase transition by the Kibble mechanism. These cosmic strings reach a scaling distribution through axion radiation and chopping off string loops. The cosmic string radiated axions ultimately contribute to the relic axion density. The total number density of string radiated relic axions can conservatively be estimated to be roughly an order of magnitude more than those arising from coherent production in the zero mode [44]. And in this scenario since the QCD vacuum angle winds around the axion strings $\theta \in (0, 2\pi N]$, the average initial misalignment angle squared is $\langle \theta^2 \rangle = \pi^2/3$. So without a dilution from a late decaying particle, the bound (11) discussed in section 2.1, including cosmic string radiated axions, would be strengthened to roughly $f_a/N \lesssim$ few $\times 10^{10}$ GeV. With a late decaying particle with the lowest possible reheat temperature consistent with BBN constraints, $T_{\rm RH} \sim 6$ MeV, the bound (17) discussed in section 2.3 would likewise be strengthened to at best $f_a/N \lesssim 10^{14}$ GeV. The lower bound on the Peccei-Quinn scale in any scenario for realizing the Peccei-Quinn mechanism with a QCD axion comes from the cooling rate of supernova SN1987A which requires roughly $f_a/N \gtrsim 10^{10}$ GeV [45]. These upper cosmological bounds and lower astrophysical bound define the open window for a QCD axion in a cosmological scenario with a Peccei-Quinn phase transition after inflation.

In scenarios in which the QCD axion does indeed exist during inflation, the most important consideration for applicability of the bounds discussed in section 4 is the observation of gravity waves either from tenor B-mode contributions to CMBR temperature fluctuations or other means. If these are not observed then the allowed axion window can be quite wide with in principle no cosmological upper bound. In this case the Hubble constant during inflation might in fact be unobservably small. Searches for isocurvature or non-Gaussian components of CMBR temperature fluctuations then do not provide definitive probes for a QCD axion which exists during inflation. However, without dilution of the axion condensate after formation, exceeding the bound (11) of $f_a/N \lesssim 3\times10^{11}$ GeV does requires a small misalignment angle. And as discussed above this is not possible in a scenario with a Peccei-Quinn phase transition after inflation, and so would require that the axion exist during inflation and happen to have a small initial misalignment angle. Even with a late decaying particle with the lowest possible reheat temperature consistent with BBN constraints, $T_{\rm RH} \sim 6$ MeV, exceeding the bound (17) of $f_a/N \lesssim 3 \times 10^{14}$ GeV also requires a small misalignment angle, and so could only be realized if the axion existed during inflation.

It is also worth considering whether there are any possible loop holes to the bounds presented in section 4 in the case that gravity waves interpreted as arising from inflation are in fact observed in future experiments. As emphasized, application of the CMBR temperature fluctuation isocurvature and/or non-Gaussian bounds requires the existence of a QCD axion during inflation which undergoes quantum fluctuations, which in turn requires that the Peccei-Quinn symmetry remain unbroken during inflation. It is possible in principle, however, that the Peccei-Quinn symmetry could be broken during inflation, but restored after inflation. In this case the axion would possess a non-trivial potential during inflation which could in principle suppress quantum fluctuations if $m_{a,{\rm inf}} \gtrsim H_{\rm inf}$, where $m_{a,{\rm inf}}$ is the mass scale of the induced axion potential during inflation. Such a scenario requires a significant breaking of Peccei-

Quinn symmetry during inflation. This can be obtained in field theory models for the origin of a QCD axion designed specifically for this purpose [46]. But for large Peccei-Quinn scales the minimum of the axion potential during inflation would have to be quite close to the zero temperature minimum in order for the relic axion density to be consistent with the observed CDM density.

For a string/M-theory QCD axion it is not clear how to obtain such a large violation of Peccei-Quinn symmetry in the early universe, consistent with the requisite exceedingly small violation in the current epoch for a successful solution of the strong CP problem. One possibility might be a spontaneous breaking of electroweak $SU(2)_L \times U(1)_Y$ gauge symmetry during inflation by a large expectation value along the $H_u H_d$ D-flat direction. If the expectation value were sufficiently large that all the resulting standard model quark masses where larger than the Hubble constant, $m_q \gtrsim H_{\rm inf}$, then an appropriately small dilaton expectation value during inflation could in principle lead to gluino condensation for $SU(3)_C$ during inflation with $\Lambda_{\rm QCD,inf} \gg H_{\rm inf}$. If in addition $U(1)_R$ symmetry were maximally broken during inflation then a gluino mass of at most $m_3 \lesssim H_{\rm inf}$ could be generated [43]. If all these conditions were met, then mixing of the string/M-theory QCD axion with the phase of the gluino condensate would lead to a potential for the axion during inflation with mass scale $m_{a,\rm inf}^2 \sim m_3 \Lambda_{\rm QCD,inf}^3/(f_a/N)^2$. For $\Lambda_{\rm QCD,inf}$ sufficiently large, $m_{a,\rm inf} \gtrsim H_{\rm inf}$ might be possible which would suppress quantum fluctuations. This seems the only reasonable scenario for suppressing quantum fluctuations of a string/M-theory QCD axion during inflation. While rather contrived, it may be kept in mind as a possible loop hole to the bounds discussed here in the case that primordial gravity waves are in fact observed, and considered in more detail at that time. However, it does suffer the problem that during inflation the sum of the phases of the Higgs condensate $H_u H_d$ and the phase of the $U(1)_R$ breaking gluino mass m_3 would have to be quite close to the sum of these phases at zero temperature. Otherwise the minimum of the axion potential during inflation would not be close to the zero temperature minimum, and would lead to a relic axion density in excess of the observed CDM density.

Another possible loophole is the form of a late entropy release which could dilute the axion condensate after formation. The general case of dilution by a late decaying particle was considered in section 2.3.1, and the largest such dilution allowed by the successful predictions of BBN was included in the bounds presented in section 4. An arbitrary entropy release at a temperature below that at which the axion condensate forms, $T_{\rm osc} \sim \Lambda_{\rm QCD}$, but with reheat temperature well in excess of $T_{\rm RH} \gtrsim 1$ MeV could leave BBN unmodified while diluting the axion condensate by an arbitrary amount. This might in principle occur from a strongly first order phase transition in this temperature range. However, such a strong phase transition would require new physics at this energy scale with significant tree-level coupling to standard model fields in order to achieve successful reheating. This is certainly ruled out by the non-observation of such physics in laboratory experiments. An extreme version of such an entropy release would be a very late inflation with Hubble constant $H \gtrsim m_a$, but with a reheat temperature $T_{\rm RH} \gtrsim 1$ MeV [47]. A reheat temperature this large with such a low Hubble constant would again require significant tree-level coupling of the inflaton sector at the minimum of its potential to standard model fields at this energy scale, and is also certainly ruled out by laboratory experiments. Even if not already ruled out, such scenarios would have to cope with the possible problem of over dilution or production of the baryon asymmetry. So an arbitrary entropy release after formation of the axion condensate does not seem to be a credible loop hole.

6 Conclusion

String/M-theory vacua generically have compact moduli which can potentially act as the QCD axion and provide an elegant solution of the strong CP problem. In large classes of vacua, such as with unification of the gauge couplings by four-dimensional renormalization group running, the Peccei-Quinn scale for a string/M-theory QCD axion can be large, $f_a/N \sim 10^{16}$ GeV. Obtaining an acceptable cosmological scenario to accommodate such an axions can be problematic. In order for relic axion production to be consistent with measurement of the present CDM density, a small initial misalignment angle to minimize the relic axion density, and possibly an late entropy release to dilute it further, are required. In addition, the late decays of relic saxions and axinos present problems in many scenarios for supersymmetry breaking, which must be addressed. But it is certainly possible to construct cosmological scenarios with a string/M-theory QCD axion in which all the requisite conditions are met and problems avoided.

The goal here was to assess how well precision cosmological measurements could probe a string/M-theory QCD axion in any cosmological scenario which can accommodate such an axion. This required considering all possible scenarios which could in principle minimize the observational effects of relic axions, consistent with possible future precision cosmological measurements. Establishing a minimal magnitude for observational effects requires an independent measurement which would imply a lower limit on the density of relic axions. Such a measurement would be provided by the observation of primordial gravity waves interpreted as arising from inflation. A measurement of the power in primordial gravity waves would establish the magnitude of inflationary induced quantum fluctuations of the axion field. For a given Peccei-Quinn scale this would in turn establish a minimum density in the axion condensate when formed, arising from at least the inflationary axion fluctuations. Minimizing the relic axion density resulting from this formation requires in turn considering a scenario with an entropy release by a late decaying particle which dilutes the axion condensate by the maximally allowed amount consistent with the successful predictions of BBN.

A measurement of the power in primordial gravity waves would also establish the minimal contribution of relic axion fluctuations to both isocurvature and non-Gaussian components in CMBR temperature fluctuations. Bounds already exist from WMAP on isocurvature and non-Gaussian contributions to CMBR temperature fluctuations. Given even these current bounds, we have shown that an observation of primordial gravity waves interpreted as arising from an early epoch of inflation with Hubble constant $H_{\rm inf} \gtrsim 10^{13}$ GeV would imply for a QCD axion which exists during inflation, a bound on the Peccei-Quinn scale which is in substantial excess of the four-dimensional Planck scale. However, the Peccei-Quinn scale for a string/M-theory compact modulus is strongly believed to be bounded from above by the four-dimensional Planck scale [14]. The above range of power in primordial gravity waves is expected to be within the reach of sensitivity of the PLANCK polarimetry experiment to tensor B-mode contributions to CMBR temperature fluctuations. In the future, space based gravity wave interferometers may also reach this level of sensitivity in direct searches for background primordial gravity waves. All this leads to the following strong conclusion:

> *A string/*M*-theory compact modulus can not play the role of a* QCD *axion and solve the strong* CP *problem if gravitational waves interpreted as arising from inflation with a Hubble constant* $H_{\rm inf} \gtrsim 10^{13}$ GeV *are observed in future experiments.*

The sensitivity to isocurvature components in CMBR temperature fluctuations are expected to improve considerably beyond the current bound. This implies the possibility for an interesting corollary to the above conclusion:

> *If isocurvature and/or non-Gaussian contributions to* CMBR *temperature fluctuations are observed and interpreted as coming from a string/M-theory* QCD *axion, then primordial gravity waves arising from an early epoch of inflation with* $H_{\rm inf} \gtrsim 10^{13}$ GeV *will not be observed.*

These conclusions require only the assumption that the Peccei-Quinn symmetry remains unbroken in the early universe. With this mild assumption the bounds which imply these conclusions are quite stringent even though they are the weakest possible considering all relevant scenarios for the formation of the axion condensate and initial misalignment angle.

The stringent bounds on the Peccei-Quinn scale which would result from a positive measurement of primordial gravity waves would apply generally to any QCD axion model in which the axion exists during inflation. And the bounds would imply that any such axion with a Peccei-Quinn scale which exceeded the implied bounds would comprise at most a very small fraction of the CDM.

The coming generation of precision cosmological observations will provide a wealth of data on cosmological evolution. This data can have an direct impact on our understanding of very early universe cosmology. As demonstrated here, these observations can also have a direct impact on our understanding of fundamental physics at the highest possible energy scales.

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A The Model Independent String Axion

All perturbative string vacua posses a model-independent axion supermultiplet which couples universally to all gauge kinetic terms. The relation between the Peccei-Quinn scale, four-dimensional Planck scale, and unified gauge coupling for this axion may be determined from the form of the Lagrangian. The gauge kinetic terms after compactification to four dimensions and field redefinitions to supersymmetric Einstein frame are of the form [48]

$$\int d^2\theta \ \frac{1}{4} S \ W^{a\alpha} W^a_\alpha + h.c. \supset \frac{1}{4g^2} F^a_{\mu\nu} F^{a\mu\nu} + \frac{i \ \theta}{32\pi^2} F^a_{\mu\nu} \widetilde{F}^{a\mu\nu} \tag{74}$$

where W^a_α is the superfield strength, $F^a_{\mu\nu}$ the field strength, $\widetilde{F}^a_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} F^{a\rho\sigma}$ the dual field strength, and

$$S = \frac{1}{g^2} + \frac{i \ \theta}{8\pi^2} \tag{75}$$

the bosonic component of the axion supermultiplet. The real part of the axion multiplet is related to the unified value of the gauge coupling by

$$\text{Re } S = \frac{\alpha_U^{-1}}{4\pi}$$

where $\alpha_U = g_U^2/4\pi$. And with the normalization (74) the θ-angle, $\theta = 8\pi^2 \text{Im } S$ is periodic mod 2π.

Starting from any of the ten-dimensional string theories, the axion multiplet Lagrangian kinetic terms in supersymmetric Einstein frame after compactification to four dimensions are [48]

$$-\int d^4\theta \ M_p^2 \ln(S^\dagger + S) \supset M_p^2 \frac{\partial_\mu(\text{Im}S)\partial^\mu(\text{Im}S)}{(2\text{Re}S)^2} = \frac{M_p^2}{(8\pi^2)^2}\frac{2}{(\alpha_U^{-1}/2\pi)^2}\frac{1}{2}\partial_\mu\theta\partial^\mu\theta \quad (76)$$

where M_p is the four-dimensional reduced Planck mass, $M_p = m_p/\sqrt{8\pi}$. In any model the vacuum angle is related to the canonically normalized axion field and Peccei-Quinn scale by

$$\theta = \frac{a}{f_a/N}$$

where N is the axion anomaly coefficient. The axion Lagrangian kinetic term is then

$$\frac{1}{2}\partial_\mu a \partial^\mu a = \frac{1}{2}\frac{f_a^2}{N^2}\partial_\mu\theta\partial^\mu\theta \quad (77)$$

For the model independent string axion $N = 1$ for gauge groups realized at the first Kac-Moody level. Comparing (76) and (77) the Peccei-Quinn scale is then related to the four-dimensional Planck mass and unified gauge coupling by

$$f_a = \sqrt{2}\ \frac{\alpha_U}{4\pi} M_P \sim 10^{16}\ \text{GeV} \quad (78)$$

Previous discussions [4, 49, 26] of the model independent axion Peccei-Quinn scale differ from this definition by a factor proportional to g_U^{-2} because of improperly normalized θ-angle.

B Zero and Finite Temperature Axion Mass

At zero temperature the physical axion mass results mainly from the small pion component induced by mixing with the QCD pseudo-Goldstone multiplet. This may be calculated by either current algebra [50] or chiral Lagrangian techniques with the result

$$m_a \simeq \frac{2\sqrt{z}}{1+z}\frac{f_\pi m_\pi}{f_a/N} \simeq 1.2\times 10^{-9}\ \text{eV}\ \frac{10^{16}\ \text{GeV}}{f_a/N} \quad (79)$$

where $z = m_u/m_d \simeq 0.56$ is the ratio of up to down current quark masses, and $f_\pi \simeq 93$ MeV is the pion decay constant. The leading corrections to (79) are $\mathcal{O}(m_\pi^2/m_K^2)$ from Kaon mixing.

At finite temperatures above the chiral symmetry breaking scale, $T \gtrsim \Lambda_{\rm QCD}$, the axion mass may be determined from the $\theta_{\rm QCD}$-dependence of the thermal free energy. At sufficiently high temperature, $T \gg \Lambda_{\rm QCD}$, this may be reliably related in the dilute instanton gas approximation to the single instanton amplitude. In this approximation the $\theta_{\rm QCD}$-dependent free energy density resulting from summing over both instantons and anti-instantons is related to the integral over instanton density $n(\rho)$ at size ρ by [20]*

$$F(\theta, T) \simeq -2\cos(\theta) \int d\rho \; n(\rho, T) \tag{80}$$

For $SU(N_c)$ gauge group with N_f light quark flavors the finite temperature instanton density in this approximation is [20]

$$n(\rho, T) = \frac{C_{N_c}}{\rho^5} \left(\frac{4\pi^2}{g^2(\rho)} \right)^{2N_c} (\chi\rho)^{N_f} (\det \, m) \; e^{-8\pi^2/g^2(\rho)} e^{-f(\rho T)} \tag{81}$$

where $C_{N_c} = (0.260\ 156)\chi^{-(N_c-2)}/[(N_c-1)!(N_c-2)!]$, $\chi = 1.338\ 76$, m is the light quark mass matrix, and

$$f(x) = \frac{\pi^2}{3}(2N_c + N_f)x^2 + 12A(x) \left[1 + \frac{1}{6}(N_c - N_f) \right] \tag{82}$$

where $A(x)$ is very well fit by the expression

$$A(x) \simeq -\frac{1}{12} \ln \left(1 + \frac{\pi^2}{3} x^2 \right) + \alpha \left[1 + \gamma(\pi x)^{-3/2} \right]^{-8} \tag{83}$$

where $\alpha = 0.012\ 897\ 64$ and $\gamma = 0.158\ 58$. The next to leading order two-loop renormalization group improvement of the scale dependent gauge coupling, $g^2(\rho)$, may be related to $\Lambda_{\rm QCD}$, defined as the pole of the inverse gauge coupling defined at this order, by [20]

$$\frac{8\pi^2}{g^2(\rho)} \simeq b \ln(1/\rho\Lambda_{\rm QCD}) + \frac{b_2}{b} \ln\ln(1/\rho\Lambda_{\rm QCD}) \tag{84}$$

where $b = \frac{1}{3}(11N_c - 2N_f)$ is the coefficient of the one-loop gauge β-function, and $b_2 = \frac{1}{3}[17N_c^2 - N_f(13N_c^2 - 3)/2N_c]$. At next to leading order in the instanton density (81) the two-loop improvement for the scale dependence of the gauge coupling (84) must be included in the instanton amplitude $e^{-8\pi^2/g^2(\rho)}$, but the scale dependence of the $4\pi^2/g^2(\rho)$ prefactor need only be evaluated at the one-loop level.

*Expression (5.6) of Ref. [20] for the θ-dependence of the free energy should contain a factor of 2 for summing over both instantons and anti-instantons.

With the normalizations given above, the finite temperature axion mass squared,

$$F(a,T) \simeq -m_a^2(T)(f_a/N)^2 \cos[a/(f_a/N)] = \frac{1}{2}m_a^2(T)a^2 + \cdots \tag{85}$$

in the dilute instanton gas approximation at next to leading order is then

$$m_a^2(T) \simeq 2C_{N_c}\left(\frac{b}{2}\right)^{2N_c} \chi^{N_f}\frac{\Lambda_{\rm QCD}^4}{(f_a/N)^2}\frac{\det m}{\Lambda_{\rm QCD}^{N_f}}\left(\frac{\Lambda_{\rm QCD}}{T}\right)^{N_f+b-4}$$
$$\int dx\; x^{N_f+b-5}\left[\ln\left(\frac{T}{x\Lambda_{\rm QCD}}\right)\right]^{2N_c-b_2/b} e^{-f(x)} \tag{86}$$

where $x=\rho T$. In the high temperature limit the integral is dominated by instantons of size roughly of order $\rho \sim \pi/T$. The integral in (86) has been evaluated numerically for the physically relevant case of $N_c=3$ and various N_f with the result that the temperature dependent axion mass may be fit by the functional form [17]

$$\xi(T) \equiv \frac{m_a(T)}{m_a} \simeq C\left(\frac{\Lambda_{\rm QCD}}{200\ {\rm MeV}}\right)^{\alpha}\left(\frac{\Lambda_{\rm QCD}}{T}\right)^{\beta}\left[1-\ln\left(\frac{\Lambda_{\rm QCD}}{T}\right)\right]^{d} \tag{87}$$

For the normalizations given above and correctly including both the instanton and anti-instanton contributions with $N_f=3$ light quark flavors relevant to the oscillations temperatures which result for $f_a/N \sim 10^{16}$ GeV, the numerical parameters of the fit (87) are $C \simeq 0.018$, $\alpha=\frac{1}{2}$, $\beta \simeq 4.0$, and $d \simeq 1.2$ [17]. For oscillation temperatures not too much larger than $\Lambda_{\rm QCD}$ the $\ln(\Lambda_{\rm QCD}/T)$ term coming from next to leading order effects may be neglected. This approximation is used in deriving the oscillation temperature given in section 2.

C Parameterizing Non-Gaussian Fluctuations

Scalar temperature fluctuations in the CMBR may generally be related to a gauge invariant potential function by $\delta T/T = \Phi/3$. One manner in which non-Gaussian temperature fluctuations could arise is that the potential Φ is a sum of terms which are linear and quadratic in an underlying Gaussian distributed potential function. As discussed in section 3.3 temperature fluctuations induced by axion fluctuations are in fact of this form. It has become common in the literature to parameterize this possibility by a sum of linear and non-linear contributions to the total potential

$$\Phi = \Phi_L + f_{NL}\left(\Phi_L^2 - \left\langle \Phi_L^2 \right\rangle\right) \tag{88}$$

where Φ_L is the underlying Gaussian distributed potential function, and f_{NL} is a constant parameterizing the strength of the non-linear fluctuations quadratic in Φ_L.

This parameterization of non-Gaussian fluctuations may be related to the dimensionless skewness (56) defined in section 3.3. To see this, first consider the mean square variance of the temperature fluctuation with the parameterization (88)

$$\left\langle (\delta T/T)^2 \right\rangle = \frac{\sigma_{\Phi_L}^2}{3^2} \left(1 + 2 f_{NL}^2 \sigma_{\Phi_L}^2\right) \simeq \frac{\sigma_{\Phi_L}^2}{9} \tag{89}$$

where $\sigma_{\Phi_L}^2 = \langle \Phi_L^2 \rangle$ and the second approximate equal sign results if the non-Gaussian non-linear contributions to the potential are small compared with the linear Gaussian contributions, $f_{NL}^2 \sigma_{\Phi_L}^2 \ll 1$. Next consider the mean cubic variance of the temperature fluctuations with the parameterization (88). To leading order in f_{NL} it is a product of two linear Gaussian contributions and one non-linear quadratic Gaussian contribution to the potential [c.f. the leading term on the right hand side of (60)]

$$\left\langle (\delta T/T)^3 \right\rangle \simeq \frac{3}{3^3} \left\langle \Phi_L \Phi_L f_{NL} \left(\Phi_L^2 - \left\langle \Phi_L^2 \right\rangle\right) \right\rangle = \frac{6}{27} f_{NL} \sigma_{\Phi_L}^4 \tag{90}$$

where $\langle \Phi_L^4 \rangle = 3\sigma_{\Phi_L}^4$. The dimensionless skewness (56) is then related to f_{NL} at leading order by

$$S_3 = \frac{\left\langle (\delta T/T)^3 \right\rangle}{\left\langle (\delta T/T)^2 \right\rangle^{3/2}} \simeq 6 f_{NL} \sigma_{\Phi_L} \simeq 18 f_{NL} \left\langle (\delta T/T)_{\rm tot}^2 \right\rangle^{1/2} \tag{91}$$

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