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WHY DO TOTAL CROSS SECTIONS GROW WITH ENERGY? *

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ABSTRACT

By using exact inclusive sum rules we infer that the growth with energy of the total pp cross section is connected with the mechanism which is responsible for the appearance of a sharp peak near the kinematical boundary in the process $p + p \rightarrow p$ + anything. We discuss this mechanism in the context of Regge models and suggest tests of this idea which involve K^+p scattering experiments.

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The apparent rise of the total cross section for pp scattering at ISR energies[1,2] has caused renewed interest in the question of whether asymptotic cross sections are finite. Clearly two possibilities are open: either the present trend will persist indefinitely and hence $\sigma_{pp}(\infty)$ is unbounded, or else the rise in σ_{pp} seen at the ISR is transient and will eventually disappear. In this latter case, the value of $\sigma_{pp}(\infty)$ remains an open question.

It is, of course, possible to construct models[3] of various degrees of believability, which can accommodate either of the above possibilities and which can give reasonable fits to the existing data. A more difficult task, however, is to discover a compelling enough mechanism which provides a natural and consistent explanation of why and at what energy the observed phenomena occurs. Our purpose in this note is to discuss such a mechanism which is suggested by data on inclusive reactions.

Let us denote the inclusive cross section for the process $a + b \rightarrow c + X$ by

$$E_c \frac{d\sigma_{ab}^{c}}{d^3 p_c} = f_{ab}^{c} \tag{1}$$

Then we can write the sum rule which expresses the conservation of energy[4], in the CM system, as

$$\sum_c \int \frac{d^3 p_c}{E_c} E_c f_{ab}^{c} = \sqrt{s}\ \sigma_{ab}(s) \quad . \tag{2}$$

In the above the dominant contributions arise from the fragmentation regions of a and b since the weighting factor of E_c effectively suppresses the pionization region.

We remark that if the f_{ab}^{c} show limiting behaviour and if they are not too singular near the kinematical limits, which of course are energy dependent, then the total cross section, $\sigma_{ab}(s)$, is energy independent. The converse statement does not follow: energy independence of $\sigma_{ab}(s)$ does not imply that the f_{ab}^{c} scale. However, if there is some energy dependence in $\sigma_{ab}(s)$ then, necessarily, some f_{ab}^{c} must not be limiting and/or some f_{ab}^{c} are singular near the edge of phase space.

As is well known by now[5], recent data on inclusive pp scattering obtained at the ISR indicate that in the fragmentation region f_{pp}^{c} scale within an uncertainty of perhaps 20%.[6] For our purposes, however, what is more important is that the small energy variations that occur in the data between conventional accelerator energies and ISR energies do not appear to have a definite sign for all f_{pp}^{c} over all phase space. This suggests that the energy dependence seen in $\sigma_{pp}(s)$ is not connected with possible energy variations of the f_{pp}^{c} (i.e. non-scaling effects), but rather that this behaviour is due to the singular contribution of some f_{pp}^{c} near the kinematical boundary. This point of view is reinforced by two observations:

1) At ISR energies a very sharp peak near the kinematical limit is seen in f_{pp}^{p},[7] in contrast with the situation at accelerator energies.[8]

2) If one estimates[9] the resultant increase in the integral under the peak in the sum rule, one finds a contribution of approximately 2 mb. (As we shall see later, this gives rise to an increase of approximately 4 mb in the total cross section).

These arguments do not "prove" that the increase in $\sigma_{pp}(s)$ is due to the same physical effects which cause the development of the peak for f^{p}_{pp} near the edge of phase space. However, we feel that this is a very plausible connection since both effects are of a comparable order of magnitude and occur roughly in the same energy range. With respect to this last point we should remark that although the rise in $\sigma_{pp}(s)$ starts at a somewhat later energy than the appearance of the peak of f^{p}_{pp} this behaviour is most likely due to the temporary decrease of several channels, in particular σ^{el}_{pp}, seen for energies up to the NAL range. In other words, in our picture the rise in the peak of f^{p}_{pp} causes a growth with energy of σ^{inel}_{pp}. This growth is compensated for a limited energy range by a decline in σ^{el}_{pp} leading to a temporarily constant total cross section. We should note, finally, that we could have also arrived at a connection between the rise of the peak in f^{p}_{pp} and the growth of $\sigma_{pp}(s)$ by using the proton multiplicity sum rule

$$< n_p > \sigma_{pp}(s) = \int \frac{d^3p_p}{E_p} f^{p}_{pp} \qquad (3)$$

and the experimental observation that $< n_p >$ is approximately constant.[10]

The relation that we have inferred between the rise of $\sigma_{pp}(s)$ and the development of the peak in f^{p}_{pp} near the kinematical boundary was based solely on plausible experimental observations and exact sum rules. If this observation is correct, and we believe this to be the case, then only theoretical models which contain a mechanism that produces a peak in f^{p}_{pp} are viable candidates to explain the growth of $\sigma_{pp}(s)$ at ISR energies. From this viewpoint both geometrical models, at least for this energy range, and any Regge models that do not produce a peak in f^{p}_{pp} near the kinematical limit are excluded from serious consideration. Chew[11] has emphasized recently that modified two component models[12] of particle production which include the possibility of diffractively exciting high mass states, and hence models which in the Regge framework can produce a peak in f^{p}_{pp} near the kinematical boundary, can lead to growth in the inelastic cross section in a limited energy range. (In fact, Chew originally suggested that this was the mechanism which was responsible for the apparent constancy of σ_{tot} even though σ_{el} appeared to decrease.) We believe that our observation, via the inclusive sum rules, provides additional "experimental" backing for the validity of this picture of high energy production. We shall elaborate on this below.

Processes in which one excites diffractively a high mass state can, in Regge models, give rise to a logarithmic growth in energy in the cross section for a limited energy range, provided certain conditions are satisfied:[11,13]

1) The Pomeranchuk singularity is dominantly a pole of intercept at, or very close to, one.

2) If one approximates the fall off in the momentum transfer t of the cross section for diffractively exciting a large mass state by e^{bt}, then $1 \gg \frac{2\alpha'}{b} \ln \frac{s}{s_0}$ where α' is the slope of the Pomeranchuk trajectory and $s_0 \simeq 1\ (\mathrm{GeV})^2$.

That this is indeed the case can be readily understood since 1) allows one to write the differential cross section for such a process as

$$\frac{d\sigma}{dt dM^2} \propto \frac{1}{s}\ e^{bt} \left(\frac{s}{M^2}\right)^{2\alpha(0)-1}\ e^{2\alpha' t \ln(s/M^2)} \tag{4}$$

and 2) guarantees that the dominant behaviour comes from the "singular term" $(s/M^2)^{2\alpha(0)-1}$.[14] We should note that Eq. (4) is relevant only when both M^2 and s/M^2 are large. This means that these processes become important only at high energy. A crude estimate of this energy gives $s \gtrsim 100\ \mathrm{GeV}^2$ but the exact number may depend on the reaction.

The presence of dominating short-range correlation effects at the ISR[15], and indications of phenomenological fits based on two component models[16] at NAL energies seem to substantiate the fact that the Pomeranchuk singularity is mostly a pole in this energy range. Furthermore, the fact that total cross sections do not fall indicates $\alpha(0) \simeq 1$. Finally, the inequality in 2) also appears to be satisfied up to the ISR range.[13] Since conditions 1) and 2) seem to be experimentally true it follows that in the above models one can simply attribute the growth of $\sigma_{pp}(s)$ as being due to the occurrence of diffractive excitation of high mass states, provided that the magnitude of these processes are large enough. The crucial point is that our use of the energy sum rule, or alternatively a direct examination of the magnitude of the cross section in the peak of f^p_{pp},[9] assures one that this is indeed the case.

We should comment here on a point alluded to before. The full contribution to the total cross section from the diffractive excitation of the proton is given by the integral of f^p_{pp} over the forward and backward peaks[17]

$$\sigma_{SDE} = \int_{\substack{\text{forward}\\ \text{peak}}} \frac{d^3p_p}{E_p} f^p_{pp} + \int_{\substack{\text{backward}\\ \text{peak}}} \frac{d^3p_p}{E_p} f^p_{pp} = 2 \int_{\substack{\text{forward}\\ \text{peak}}} \frac{d^3p_p}{E_p} f^p_{pp} . \tag{5}$$

In the energy sum rule the contribution of the peaks in f^p_{pp} to the total cross section is

$$\int_{\substack{\text{forward}\\ \text{peak}}} \frac{d^3p_p}{E_p} \left(\frac{E_p}{\sqrt{s}}\right) f^p_{pp} + \int_{\substack{\text{backward}\\ \text{peak}}} \frac{d^3p_p}{E_p} \left(\frac{E_p}{\sqrt{s}}\right) f^p_{pp} = 2 \int_{\substack{\text{forward}\\ \text{peak}}} \frac{d^3p_p}{E_p} \left(\frac{E_p}{\sqrt{s}}\right) f^p_{pp} \tag{6}$$

Since, however, in the peak $E_p \simeq \sqrt{s}/2$ we have

$$2 \int_{\substack{\text{forward}\\ \text{peak}}} \frac{d^3p_p}{E_p} \frac{E_p}{\sqrt{s}} f^p_{pp} \cong \int_{\substack{\text{forward}\\ \text{peak}}} \frac{d^3p_p}{E_p} f^p_{pp} \tag{7}$$

The mismatch between Eq. (7) and Eq. (5) can be easily understood by realizing that in any given process where the proton is produced near the kinematical edge by momentum conservation in the CM the produced proton only takes half the available energy. Thus one must add to the contribution to σ_{tot} shown in (7) an equal contribution arising from the energy carried by the diffractively excited state. The actual increase in σ_{pp} associated with the development of the peak in f^p_{pp} is then given by Eq. (5) and hence $\Delta\sigma_{pp} \simeq 4$ mb.[18]

We want to emphasize that the conditions spelled out above under which Regge models can give rise to a (temporarily?) growing cross section do not involve the much subtler questions of whether the Pomeranchuk intercept is precisely at 1 or at $1-\epsilon$, or whether the triple Pomeranchuk vertex $g_{ppp}(t)$ vanishes or not at $t=0$. These questions have to do with the consistency of exchanging pole Pomeranchuk singularities at infinite energy.[19] At finite energy, e.g. the ISR, what the answers to these important questions are does not appear to be crucial for our arguments. At energies which are higher than the ISR, these consistency questions will play a role and will finally determine whether diffractive excitation processes will continue to give rise to a growing contribution to the total cross section, or whether they will give a constant contribution or disappear altogether.[20] The answers to these questions are not known.

The mechanism that we have discussed here as the cause of the rise in the total pp cross section can be studied in K^+p experiments. The total cross section for K^+p scattering seems to grow with energy already at Serpukhov energies.[21] From our point of view this growth is correlated with the possibility of diffractively exciting either the proton or the kaon into high mass states. Thus an interesting test of our ideas would be to observe sizeable growing diffractive peaks in the processes $p + K^+ \rightarrow p + X$ and $K^+ + p \rightarrow K^+ + X$ in the energy range 20-70 GeV/c. One can estimate the increase in σ_{K^+p} due to the development of these peaks by using the magnitude of the triple Pomeron coupling obtained in pp scattering[9] and factorization. This increase is about 0.4 mb in the above energy range and 0.6 mb from 70 to 300 GeV/c. The remarkable constancy of σ_{K^+p} at low energies may be the reason why one observes the increase earlier in this reaction. One can also calculate for other processes the increase in total cross sections due to the mechanism discussed above. However, for non-exotic processes this increase may not be apparent until quite high energies because of the effect of secondary Regge trajectories.

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References and Notes

1. U. Amaldi et al., CERN Preprint.
2. S. E. Amendolia et al., CERN Preprint.
3. Various possible models are discussed by G. L. Kane, invited talk at the "Symposium on the Pomeron," Argonne National Laboratory, February 1973, and by F. Zachariasen, Review talk at the "NAL Conference on Diffractive Phenomena," National Accelerator Laboratory, March 1973.
4. T. T. Chou and C. N. Yang, Phys. Rev. Letters 25, 1072 (1970).
5. See, for example, the Rapporteur talk of M. Jacob, "Proceedings of the XVI International Conference on High Energy Physics," Batavia, Illinois, 1972.
6. To be sure, some f_{pp}^{c} like $f_{pp}^{\bar{p}}$ show a larger energy dependence, but their contribution is only minimal in the sum rule.
7. M. G. Albrow et al., CERN Preprint.
8. J. V. Allaby et al., Contribution to the "Fourth International Conference on High Energy Collisions," Oxford (England), April 1972.
9. See A. Capella, SLAC-PUB-1198.
10. M. Antinucci et al., CERN Preprint.
11. G. F. Chew, Berkeley Preprint LBL-1556.
12. K. Wilson, Cornell Report CLNS-131 (unpublished) 1970. See also, for example, W. R. Frazer, R. D. Peccei, S. S. Pinsky and C. I. Tan, Phys. Rev., to be published.
13. W. R. Frazer and D. R. Snider, NAL Preprint, NAL-PUB-73/15.
14. We should note that diffractive excitation models of the type discussed by R. Hwa and C. Lam, Phys. Rev. Letters 27, 1098 (1971), and M. Jacob and R. Slansky, Phys. Letters 37B, 408 (1971), also give rise to a peak

near the kinematical boundary. However, this peak does not scale and, furthermore, its integral does not increase with energy. The mechanism discussed in the text attributes the presence of this peak to what is conventionally called a PPP term. Phenomenological arguments supporting this latter interpretation can be found in ref. 9 and in A. Capella, W. Hogaasen and V. Rittenberg, SLAC-PUB-1176.

15. See, for example, G. Bellettini in "Proceedings of the XVI International Conference on High Energy Physics," Batavia, Illinois (1972), and H. Dibon et al., CERN Preprint.

16. See, for example, the second reference of ref. 12, K. Fialkowski and H. Miettinen, Phys. Letters 43B, 61 (1973), and H. Harari and B. Rabinovici, Phys. Letters 43B, 49 (1973).

17. There may be a small amount of double counting in Eq. (8). See ref. 11 for a discussion of this point.

18. The contribution from the diffractively excited state presumably gives rise to a non-scaling part of the sum rule. However, this contribution is a direct consequence of the existence of the peak. Furthermore, it is spread among all the secondary particles and also over the entire phase space and therefore cannot be easily identified given the accuracy of the data.

19. For a discussion of these and related matters see, for example, the Rapporteur Reports of V. N. Gribov and F. E. Low in the "Proceedings of the XIV International Conference on High Energy Physics," Batavia, Illinois, 1972.

20. That these consistency questions indeed enter at really high energy can be appreciated by remarking that they will become relevant at energies in which $1 << \frac{2\alpha'}{b} \ell n\, s/s_0$.

21. See, for example, G. Giacomelli, Rapporteur Report in the "Proceedings of the XVI International Conference on High Energy Physics," Batavia, Ill., 1972.