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PARTIAL WAVE ANALYSIS IN 2 → 3 BODY REACTIONS AND COMMENTS ON INELASTIC FINAL STATES IN $\pi\pi$ AND $K\pi$ SCATTERING*

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ABSTRACT

Multiparticle states derived from $\pi\pi$ and $K\pi$ scattering are discussed and the techniques for analysis of 3 body final states are described. The role that inelastic reactions play in determination of resonance parameters is investigated and the conclusion reached that they only provide information on the specific inelastic channel couplings. Thus the identification of resonances coupling to $\pi\pi$ or $K\pi$ systems is most easily made by observation of the elastic and charge exchange reactions.

INTRODUCTION

In the future analysis of $\pi\pi$ and $K\pi$ scattering will be performed at higher energies. This has two immediate consequences in that we expect (1) higher J^P states and (2) increased inelasticity. Thus we might expect that the inelastic channels will become an increasingly important tool in identifying new resonance states.

In this talk I want to (1) demonstrate that viable techniques exist for the analysis of 2 → 3 body reactions; (2) consider the available data in $K\pi \rightarrow K\pi\pi$ scattering and the application of such techniques; (3) discuss the contribution such inelastic information might make to $\pi\pi$ and $K\pi$ scattering; and finally (4) come to some conclusion about the best way of proceeding in the study of $\pi\pi$ and $K\pi$ scattering.

To illustrate many of these points I will draw on experience one has gained from studies[1] of

$$\pi N \rightarrow \pi\pi N \qquad (1)$$

INELASTIC REACTIONS IN $\pi\pi$ AND $K\pi$

In Table I, I have made a list (not exhaustive) of the inelastic reactions one might expect in $\pi\pi$ and $K\pi$ scattering, taking into account the restrictions of G-parity in the former case. In the case of $K\pi$ scattering, $K\pi\pi$ could well be an important state at low c.m. energies. Thus I will use this as an indication of what might be gained by observation and analysis of an inelastic reaction. The analysis methods for stable 2 body inelastic

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reactions $K\bar{K}$, $K\eta$ (I also include $\pi\omega$, $K\omega$) are well known and hence I will pay little attention to these.

I might note at this point that the 0^+ wave in $K\pi$ scattering can only lead to inelastic final states with ≥ 4 particles, a clear difference from the same J^P state in $\pi\pi$ scattering.

Table I Inelastic final states in $\pi\pi$ and $K\pi$ scattering

	$\pi\pi$	$K\pi$
2	$\pi\pi$ $K\bar{K}$	$K\pi$ $K\eta$
3	$K\bar{K}\pi$	$K\pi\pi$ ($K^*\pi$, $K\rho$ ...) $KK\bar{K}$ ($M > 1500$)
4 etc.	$\pi\pi\pi\pi(\pi\omega)$	$K\pi\pi\pi(K\omega)$

THE ANALYSIS OF 3 BODY FINAL STATES

Formalism and method. The formalism for this has been developed[2, 3] to deal with

$$\pi N \rightarrow \pi\pi N \tag{2}$$

and later applied to

$$A_1, A_2 \ldots \rightarrow 3\pi \ . \tag{3}$$

The main ingredients are

(1) There exists strong resonance production in the final state, e.g., K^*, ρ. This is then introduced as an integral part of the model.

(2) The amplitude for the observation of a state JPM is written as a coherent sum over all of the possible intermediate quasi-two body states

$$A(K\pi\pi) = \sum_{JPM} f^{JPM}(\omega, t)\ G^{JPM}(\omega_1, \omega_2, \alpha, \beta, \gamma) \tag{4}$$

where $f^{JPM}(\omega, t)$ = amplitude for formation of the state JPM with mass ω and 4 momentum transfer t; $G^{JPM}(\omega_1, \omega_2, \alpha, \beta, \gamma)$ = amplitude for the decay of this state into $K\pi\pi$ ($\omega_1, \omega_2, \alpha, \beta, \gamma$ - the 5 variables necessary to describe the state). The decay amplitude is then

$$G^{JPM} = \sum_{Lj} X_{Lj}^{JPM}\ B_j(\omega_j) \tag{5}$$

where X_{Lj}^{JPM} contains all the angular momentum decomposition and $B_j(\omega_j)$ contains the final state enhancement factor for the pair of particles labelled j, e.g., Watson final state factor[5] or Breit-Wigner.

This is represented in Fig. 1.

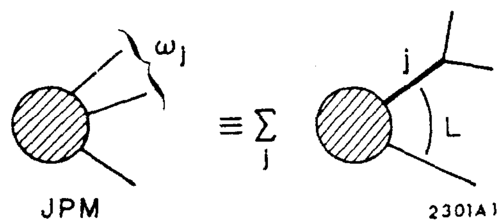

Fig. 1. Decay of state JPM to 3 bodies via intermediate isobar.

(3) We can then write the differential cross section as

$$d^5\sigma(\omega_1^2, \omega_2^2, \alpha, \beta, \gamma) \propto \sum_{\substack{JPM \\ J_1P_1M_1}} f^{JPM} G^{JPM} \left[f^{J_1P_1M_1} G^{J_1P_1M_1} \right]^* \tag{6}$$

$$\propto \sum \rho^{JP\,J_1P_1}_{M\,M_1} G^{JPM} G^{J_1P_1M_1\,*} \tag{7}$$

The object is then to determine

$$f^{JPM}(\omega, t) \quad \text{or} \quad \rho^{JP\,J'P'}_{M\,M'}(\omega, t) \tag{8}$$

and in order to make maximum use of the data this is usually done by the maximum likelihood method.

Results. The success of this method is best demonstrated in the results in $\pi\pi N$ partial wave analyses. There one finds that (1) Good fits to the data are obtained. However large numbers of events are necessary to obtain stable solutions, e.g., in $\pi\pi N$ we use $\sim$5000-10,000 at each energy; and (2) The expected resonance structures are observed. This is seen in Fig. 2 where I show the F_{15} partial wave amplitudes — the F_{15} being clearly seen in the $\pi\Delta$, $N\rho$ and $N\sigma$ channels.

I might also add that the analyses of $A \to 3\pi$ data have also clearly identified the resonant structure in the $J^PM = 2^+, 1$ waves (the A_2).

Conclusion. The conclusion is that one now has comparatively well understood methods of analyzing such reactions and one can consider applying them to the 3 body channels in $\pi\pi$ and $K\pi$ scattering.

THE DATA AND ITS UTILIZATION FROM $K\pi$ INELASTIC REACTIONS

The data. In Fig. 3, I show the data that exists on the reaction

$$K^+p \to K^o\pi^+\pi^-\Delta^{++} \tag{9}$$

with incident 10 GeV/c K^+ mesons.[6] This is a large bubble chamber experiment and produces such meagre statistics.

The utilization of the data. In the spirit of $\pi\pi$ and $K\pi$ partial wave analyses one would optimistically like to follow one of two paths. (1) Amplitude extrapolation from the physical region to the pion pole — this means that one must determine the amplitudes at different t values. Now in inelastic final states one in general can have both natural and unnatural spin parity series (cf, $\pi\pi$ or $K\pi$ elastic scattering where one only has natural spin parity) so that the amplitude extraction is more difficult. From the $\pi\pi N$ experience this means $\gtrsim$ 1000 events for each m, t bin. (2) Amplitudes from

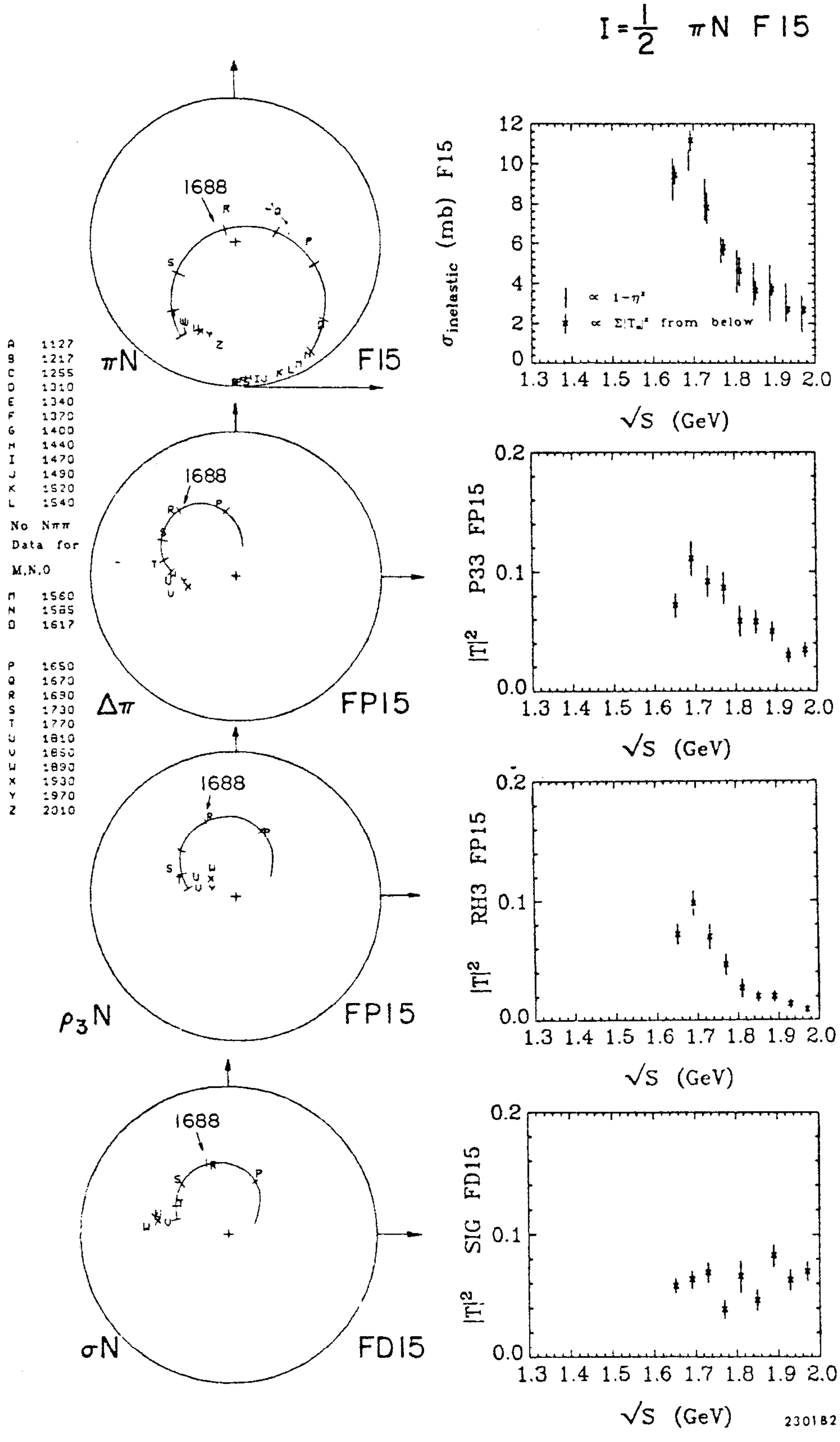

Fig. 2. The F_{15} partial wave.

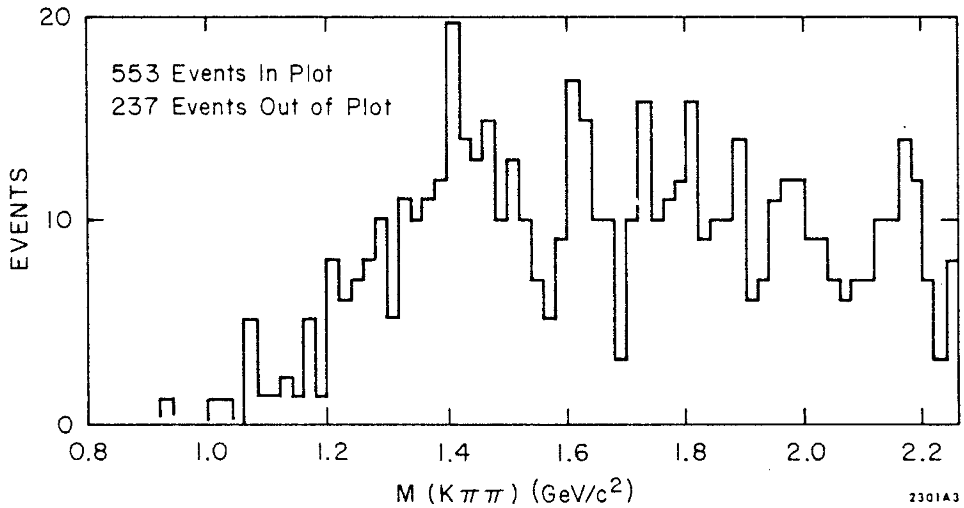

Fig. 3. $K^0\pi^+\pi^-$ invariant mass spectrum from $K^+p \to K^0\pi^+\pi^-\Delta^{++}$ at 10 GeV/c (other π^+p not in Δ^{++}).

fits in the physical region with a specific model — again one will require a lot of data but furthermore one also needs good models for other production processes besides π exchange. After all there may be Q's, etc. lurking in the background.

Thus (1) and (2) both imply the necessity of more data than will be available for a long time together with a complete phenomenological theory for (2). However, I do believe that (2) offers more hope for the near future.

At present (1) and (2) are very optimistic but one can follow a more modest path by attempting to extrapolate the inelastic cross section to the pion pole. Of course one will not then know the spin parity composition but the reduction in the amount of data required makes this feasible. In fact this has been attempted for reaction (9) using data at 5.0 and 8.25 GeV/c and the results are shown in Fig. 4. The presence of the K*(1400) is clearly seen on essentially a zero background. The superimposed curve is a Breit-Wigner derived from the current K*(1400) parameters.

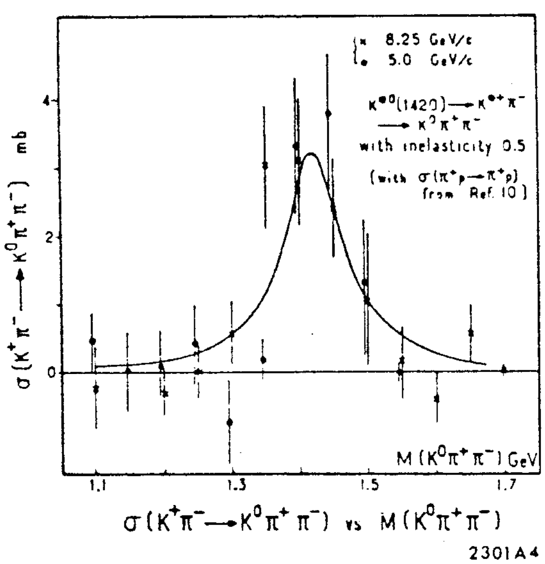

Fig. 4. Extrapolated inelastic $K\pi$ cross section versus $M(K^0\pi^+\pi^-)$ at 5.0 and 8.25 GeV/c using elastic (π^+p) cross section from a π-nucleon phase-shift analysis.

Conclusion. It seems reasonable to attempt to obtain the extrapolated inelastic cross section although anything more ambitious will require at least an order of magnitude increase in the quantity of data.

THE INFORMATION CONTAINED IN INELASTIC PARTIAL WAVE AMPLITUDES

It is now important to understand the impact on analyses of the absence of detailed information on the inelastic partial wave amplitudes. Again as a guide I would like to take some of our work on the πN system in discussing this question.

Resonance observation. It is first worth noting that all of the resonances listed in the PDG tables have been observed first in stable 2 body partial wave analyses, i.e., $\pi N \rightarrow \pi N$, $\bar{K}N \rightarrow \bar{K}N$, $\Lambda\pi$, $\Sigma\pi$. Indeed our analysis has essentially only added one new state, the $D_{13}(1700)$ (already hinted at by EPSA) and removed another, the $P_{33}(\sim 1700)$.

Resonance parameters (pole position in the T-matrix). One might expect that the quantitative definition would improve by adding the inelastic channel information. We have been making coupled channel K-matrix analyses of the different πN partial waves and so we are in a position to investigate this question by performing the analysis with or without the inelastic data. Of course if the partial wave is inelastic sensible fits can only be obtained if an inelastic channel is introduced to preserve unitarity. In Fig. 5

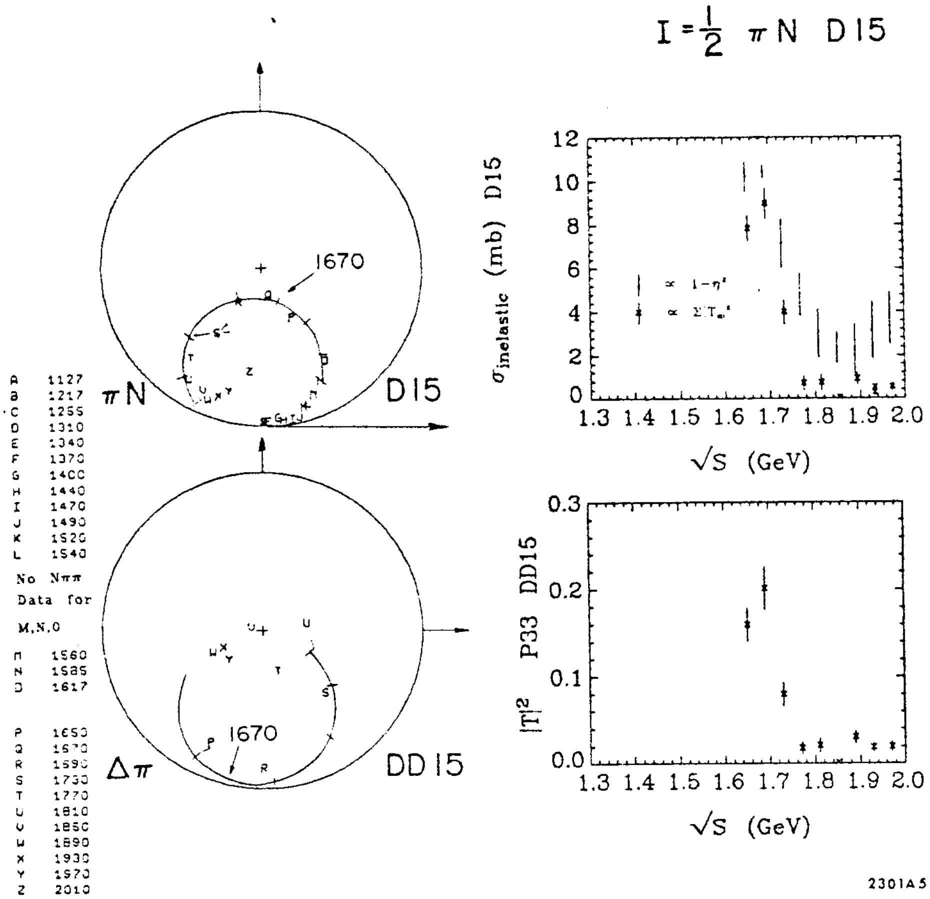

Fig. 5. D_{15} partial wave.

and Fig. 6 we see the Argand diagrams for the two partial waves D_{15} and F_{35}, both exhibiting large inelasticity but in two different channels, $\pi\Delta$ and $N\rho$ respectively. In analyzing just the elastic data we have considered the "junk" channel in both cases to be $\pi\Delta$, clearly a bad assumption for the F_{35}.

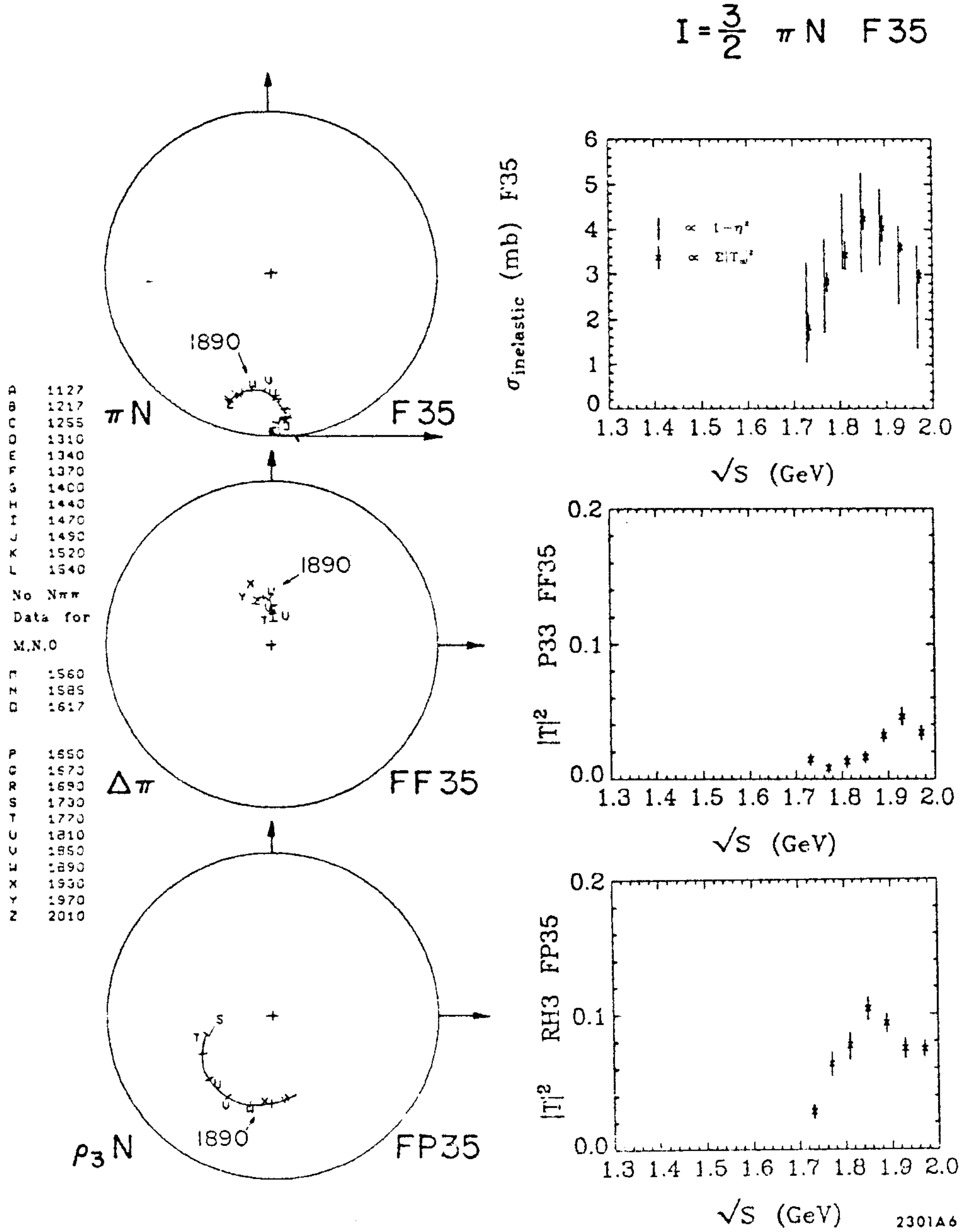

Fig. 6. F_{35} partial wave.

The results of these analyses is contained in Table II.

The conclusions from this work are, I think, obvious (1) Resonance pole positions are well determined by the elastic data alone; and (2) Little change is caused by the introduction of the inelastic data.

Table II Partial wave pole positions

Partial wave	Elastic data only	Elastic and inelastic
D_{15}	$1660 - i\,\frac{140}{2}$ (Junk = $\pi\Delta$)	$1666 - i\,\frac{159}{2}$ (Major channel = $\pi\Delta$)
F_{35}	$1810 - i\,\frac{275}{2}$ (Junk = $\pi\Delta$)	$1824 - i\,\frac{282}{2}$ (Major channel = $N\rho$)

Resonance couplings. Clearly one will never know the inelastic channel to which a resonance couples unless one measures inelastic channels. It is very important to know the couplings to specific decays for the evaluation of resonance classifications or symmetry schemes in general. However, one should realize that $\pi\pi N$ experience has taught us that this is really the only contribution.

Thus one is forced to conclude from these empirical observations that (1) The qualitative existence and quantitative evaluation of resonances which couple to stable 2 body channels are most easily determined by studying those channels; and (2) Other more complicated inelastic channels provide information only on the couplings to specific channels, although this in itself is very valuable data.

CONCLUSIONS

The following points are the conclusions we can draw: (1) The quantities of data required for partial wave analyses in multiparticle states derived from $\pi\pi$ and $K\pi$ scattering are large, making such projects unfeasible. We might be able to attempt such analyses in the future, first by assuming that certain resonances are present with a specific production mechanism and then secondly determining their coupling constants. (2) The extrapolation of the total inelastic cross section to the pion pole, which requires less data, may well provide a useful constraint in future partial wave analyses of the elastic scatterings. In this case it would be important to study

$$K^+ p \rightarrow (MM)\,\Delta^{++}$$

or

$$\pi^+ p \rightarrow (MM)\,\Delta^{++}$$

in order to obtain the total inelastic cross section. (3) Stable 2 body data and in particular the elastic channels — identify resonances and give good estimates of the pole parameters. This means that the value of the inelastic data lies in its ability to give the couplings to very specific final states.

Finally one should not infer that it is of little interest to study these higher multiplicity final states. Clearly one can never see G = -1, S = 0 boson resonances in 2π final states. However, if one wishes to identify resonances which do couple to the elastic channel in $\pi\pi$ or $K\pi$ scattering then studying those elastic channels will produce the most dramatic reward.

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