SLAC-PUB-1235
(T/E)
April 1973

PIONIC TRANSITIONS AS TESTS OF THE CONNECTION BETWEEN CURRENT AND CONSTITUENT QUARKS*

Frederick J. Gilman

California Institute of Technology
Pasadena, California 91109

and

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

Moshe Kugler†

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

Sydney Meshkov

California Institute of Technology
Pasadena, California 91109

and

National Bureau of Standards
Washington, D.C. 20234

ABSTRACT

A proposed connection between current and constituent quarks is discussed and tested through comparison with the magnitudes and signs of amplitudes for pionic transitions between hadrons.

(Submitted for publication.)

---

*Work supported in part by the U. S. Atomic Energy Commission.

†On leave of absence from the Weizmann Institute, Rehovot, Israel.

Quarks have been used in two distinct ways[1] in particle physics, each of which may be associated with a different $SU(6)_W$ algebra. One $SU(6)_W$ algebra, that of strong interactions,[2] uses a constituent quark basis to describe the behavior of hadrons. The other $SU(6)_W$ algebra, that of currents,[3] consists of integrals over current densities which are assumed to commute like bilinear products of current quark fields. A possible mathematical connection between these two different $SU(6)_W$ algebras has been formulated recently by H. J. Melosh,[4] and leads to a number of consequences for current matrix elements between hadron states.[5]

With the additional assumption of the PCAC hypothesis, these current matrix elements are related to the most commonly observed transitions between hadron states, i.e., the emission of pions. Several authors[6,7] have already employed PCAC to make an experimental comparison of relations among current matrix elements taken between hadron states with a given helicity λ. In this paper we make the additional assumption that hadron states with different values of the constituent quark spin (and λ) can be related by the $SU(6)_W$ of strong interactions. This assumption considerably reduces the number of independent matrix elements. With the assumptions of PCAC and of $SU(6)_W$ relations among constituent quark spin states, we shall test Melosh's proposed connection[4,5] between the two $SU(6)_W$ algebras using both the magnitudes and signs of the amplitudes for pionic transitions between hadrons.

The specific matrix elements we consider are of the form $\langle \text{hadron}'|Q_5^\alpha|\text{hadron}\rangle$. Here Q_5^α is one of the sixteen vector and axial vector charges, Q^α and Q_5^α, which make up the familiar chiral $SU(3)\times SU(3)$ algebra, a subalgebra of the $SU(6)_W$ of currents. We label an irreducible representation (I.R.) of chiral $SU(3)\times SU(3)$ as $(A,B)_{S_z}$, where A and B are the representations of $Q^\alpha + Q_5^\alpha$ and $Q^\alpha - Q_5^\alpha$, respectively, and S_z is the eigenvalue of Q_5^o, the singlet axial-vector charge. S_z corresponds to the intrinsic quark spin projection in a quark model, but may be defined in a general way as above. The operator Q_5^α then transforms simply under the SU(3)×SU(3) of currents as $(8,1)_0 - (1,8)_0$.

We assume that the observed hadron states are (at least to good approximation) identifiable with those in the constituent quark model ($q\bar{q}$ for mesons and qqq for baryons), and therefore belong to simple I.R.'s of the $SU(6)_W$ of strong interactions. The spectrum of observed meson and baryon states provides good evidence for this. Hadron states thus transform as simple I.R.'s under the SU(3)×SU(3) of strong interactions.

Following Melosh,[4] we assume that a unitary transformation, V, connects the two different algebras. Then

$$\begin{aligned} |\text{hadron}\rangle &= |\text{I.R., constituents}\rangle \\ &= V\,|\text{I.R., currents}\rangle \quad . \end{aligned} \tag{1}$$

Therefore we may rewrite the matrix element of interest as

$$\begin{aligned} &\langle \text{hadron}'|Q_5^\alpha|\text{hadron}\rangle \\ &\qquad = \langle \text{I.R.}', \text{ currents } | V^{-1} Q_5^\alpha V |\text{I.R., currents}\rangle \quad . \end{aligned} \tag{2}$$

In the free quark model one finds[4] that $V^{-1}Q_5^\alpha V$ is quite simple. It transforms as a sum of the $(8,1)_0 - (1,8)_0$ and $(3,\bar{3})_1 - (\bar{3},3)_{-1}$ representations of the SU(3)×SU(3) of currents and as a $\underline{35}$ of the $SU(6)_W$ of currents. It is this simple property of $V^{-1}Q_5^\alpha V$ which we abstract from the free quark model and proceed to test using pionic transitions between hadrons.

For this purpose we assume that the matrix elements of Q_5 are related to those of the pion field by the PCAC hypothesis. The decay width for hadron' $\rightarrow$ hadron + π is then given in narrow resonance approximation by

$$\Gamma = \frac{c}{2J'+1} \frac{p_\pi (M'^2 - M^2)^2}{M'^2} \sum_\lambda |\langle \text{hadron}' \, \lambda \, | Q_5 | \, \text{hadron} \, \lambda \rangle|^2 , \qquad (3)$$

where c is a constant related to the pion decay rate and the isotopic spin of the hadrons, p_π is the pion momentum, and the sum extends over the possible common helicities, λ, of the hadrons. We have no arbitrary choice of phase space factors and the width is fixed directly by the matrix elements of Q_5 (up to the validity[8] of PCAC).

The constituent quark states with different values of the quark spin are related by the $SU(6)_W$ of strong interactions. Therefore, the matrix elements of all hadron states in a given SU(6) multiplet are related, and the quantities of interest in Eq. (2) depend on at most two independent reduced matrix elements. These correspond to the $(8,1)_0 - (1,8)_0$ and $(3,\bar{3})_1 - (\bar{3},3)_{-1}$ pieces of $V^{-1}Q_5^\alpha V$, each belonging to a $\underline{35}$ of the $SU(6)_W$ of currents. For each matrix element of Q_5^α we write the initial and final hadron states with $J_z = \lambda$ in terms of states with definite S_z. This involves coupling internal quark L and S to form total J for each hadron. The matrix element of the $(8,1)_0 - (1,8)_0$ or $(3,\bar{3})_1 - (\bar{3},3)_{-1}$ term can then be written as a reduced matrix element times the product of quark angular momentum, $SU(6)_W$, SU(3), and W-spin Clebsch-Gordan coefficients.

In this paper we do not assume that the $(8,1)_0 - (1,8)_0$ piece of $V^{-1}Q_5^\alpha V$ is proportional to Q_5^α as in the work of Gilman and Kugler.[6] However, unlike Refs. 6 and 7, we make a stronger assumption by employing $SU(6)_W$ to relate states with different values of the quark spin.

We first consider the decays of the $\underline{35}$ L=1 mesons into the $\underline{35}$ L=0 mesons. The two independent reduced matrix elements are determined by normalizing to $\Gamma(A_2 \rightarrow \pi\rho) = 77$ MeV and requiring that $\Gamma_{\lambda=0}(B \rightarrow \pi\omega) = 0$, in agreement with experiments which show a dominantly transverse decay.[9] This latter condition makes the reduced matrix element of the $(8,1)_0-(1,8)_0$ term vanish.[6] Thus all decay rates are proportional. The resulting predictions for the various decays of L=1 mesons are shown in Table I. In constructing the table we have assumed that the η is pure octet and have employed Zweig's rule[10] to relate the $SU(6)_W$ $\underline{1}$ and $\underline{35}$ parts of the ω, f, and σ states with $\lambda=0$. As can be seen, the agreement with experiment is good where comparison is possible.

We have explored the pionic decays of other meson multiplets, e.g., $L=0 \rightarrow L=0$, $L=2 \rightarrow L=0$, $L=1 \rightarrow L=1$, and $L=2 \rightarrow L=1$.[11] One generally finds for transitions between hadrons with different values of internal (quark) angular momentum, L' and L, that the relative orbital angular momentum, ℓ, between the pion and final hadron obeys the rule[12]

$$||L-L'| - 1| \leq \ell \leq |L+L' + 1| \quad . \tag{4}$$

In addition, if L'=L then the $(8,1)_0-(1,8)_0$ term is purely p-wave. No such simplification occurs in general for the $(3,\bar{3})_1-(\bar{3},3)_{-1}$ term. In the particular case of $L=1 \rightarrow L=1$ meson decays, the $(3,\bar{3})_1-(\bar{3},3)_{-1}$ term is also pure p-wave, whereas both p and f-wave amplitudes might be expected. In the case $L=0 \rightarrow L=0$ only the $(8,1)_0-(1,8)_0$ term can contribute and the predicted relative couplings for $\rho \rightarrow \pi\pi$, $K^* \rightarrow \pi K$, and $\omega \rightarrow \pi\rho$ are in good agreement with experiment.

Encouraged by the meson results, we turn to baryons. For $\underline{56}$ $L=0 \rightarrow$ $\underline{56}$ L=0 transitions only the $(8,1)_0-(1,8)_0$ term contributes and the amplitudes are in satisfactory agreement with experiment. For $\underline{70}$ $L=1 \rightarrow \underline{56}$ L=0 decays, linear combinations of the two reduced matrix elements correspond to s- and d-wave amplitudes for decay into πN or $\pi\Delta$. The analysis[13] of the reaction

$\pi N \to \pi\pi N$ allows us to compare both the relative signs and magnitudes of $N^* \to \pi N$ and $N^* \to \pi\Delta$ amplitudes. The quark spin S=1/2 and 3/2 states having the same total quantum numbers within the $\underline{70}$ may be mixed.[14] However, the sums over such mixed states of squares of the Q_5 matrix elements are independent of mixing, and we compare these with experiment. The predictions for widths are given in Table II, where we have used combined widths of the two D_{13} states and two S_{11} states decaying into πN to fix the d- and s-wave amplitudes, respectively. The predicted relative signs of amplitudes in $\pi N \to N^* \to \pi\Delta$ are compared with experiment in Table III.

A similar analysis of $\underline{56}$ L=2 $\to$ $\underline{56}$ L=0 decays relates the two independent reduced matrix elements to p- and f-wave πN and $\pi\Delta$ decay amplitudes. In Table II we present the predicted widths, fixing the f- and p-wave amplitudes by the $F_{15}(1688) \to \pi N$ and $P_{31}(1860) \to \pi N$ decay rates, respectively. The predictions for relative signs are again in Table III.

A study of Table II shows that while there are many successes, there are also predicted widths which are in disagreement with experiment by factors of 2 to 3. For example, $\Gamma(D_{15} \to \pi\Delta)/\Gamma(D_{15} \to \pi N)$ is smaller than predicted (by a factor 2.5), and the experimental situation is rather solid. This is one of the worst discrepancies — in most other cases the agreement is better. Some of the discrepancies may be due to the use of the narrow resonance approximation to which $\pi\Delta$ decays are notably sensitive. We also neglect mixing between different SU(6) multiplets. From this standpoint we may regard Table II as a reasonable first approximation.

Table III poses stringent tests of our assumptions. It contains two kinds of relations: 1) those that involve the same partial wave in both the incoming (πN) and outgoing ($\pi\Delta$) states have definite relative signs independent of what values the reduced matrix elements of the $(8,1)-(1,8)$ and $(3,\bar{3})_1-(\bar{3},3)_{-1}$ terms have; 2) those that involve different initial and final partial waves depend on these values and may indicate which term is dominant. The present data analysis[13] disagrees with relations of both the first and second types for decays of the 70 L=1 baryons. If this is the only solution for the $\pi N \rightarrow \pi\Delta$ phase shifts, the theory faces serious difficulty.

In our approach, the algebraic properties of the matrix elements of Q_5 are identical to those for pion coupling constants obtained in certain quark models[15] and in ℓ-broken $SU(6)_W$ calculations.[16] However, our results, e.g., Tables I and II, differ from previous calculations[15,16] in that PCAC imposes an unambiguous connection between the matrix elements of Q_5 and the widths, which does not contain arbitrary ℓ dependent centrifugal barrier factors. Our predictions for the signs of amplitudes coincide with those of the cited models.[17] Thus difficulties stemming from Table III are common to all these approaches.

By considering matrix elements of the vector current, we have extended our considerations to photon transitions. Again, our results turn out to be algebraically identical to explicit quark model calculations.[18] For example, the radiative decays from 70 L=1 $\rightarrow$ 56 L=0 depend on two independent matrix elements, those of $(8,1)_0+(1,8)_0$ and $(3,\bar{3})_1+(\bar{3},3)_{-1}$ terms. These correspond respectively, to the convection current and magnetic moment terms in quark models. The relative signs and magnitudes of the transition amplitudes predicted in this case are in agreement with experiment.[19]

A priori, we do not relate π with ρ transitions between hadron states, as in some quark models.[15,16] However, with the assumption of vector meson dominance, we can relate the ρ transitions to those of the photon discussed above. Demanding consistency between the two ways of treating $A_2 \rightarrow \pi\rho$, for example, then leads to interesting connections between the reduced matrix elements involved in π transitions and those in ρ transitions.[11]

In summary, we have used the simple form of the transformed axial-vector charge proposed by Melosh, together with the assumptions of PCAC and the $SU(6)_W$ relations between constituent quark states with different values of S_z, to analyze all pionic transition amplitudes between hadrons. The resulting theory is (1) simple, in that there are only two terms in the transformed Q_5, (2) systematic, since one can treat all the baryons and mesons which are identifiable as qqq or $q\bar{q}$ states on the same footing; and last, but not least, (3) definite, with the transformed Q_5 having a clear origin and structure with a known relation between matrix elements of Q_5 and decay rates, and with different hadronic matrix elements of Q_5 related by Clebsch-Gordan coefficients. The results for decay widths, particularly those of mesons, are encouraging. However, the relative signs of the amplitudes in $\pi N \to \pi\Delta$ are a crucial test, and the theory is in conflict with the results of the present experimental analysis. If this disagreement persists, we have to face the possibilities that: (1) there is large mixing of SU(6) multiplets,[20] invalidating our identification of the observed hadrons with simple quark model states; (2) the use of $SU(6)_W$ to relate different quark spin states is wrong, and only a weaker symmetry holds, or (3) the algebraic properties of $V^{-1}Q_5^{\alpha}V$ abstracted from the free quark model do not hold in nature.

Acknowledgements

Two of us (FJG and SM) thank the members of the high energy theory group at Caltech for their hospitality. We are grateful to M. Gell-Mann for his advice and encouragement, and to D. Horn and H. J. Melosh for discussions. We appreciate the patience and help of R. Cashmore in understanding the experimental situation.

REFERENCES

1. M. Gell-Mann, Schladming Lectures 1972 (unpublished).
2. H. J. Lipkin and S. Meshkov, Phys. Rev. Letters 14, 670 (1965), and Phys. Rev. 143, 1269 (1966);

   K. J. Barnes, P. Carruthers, and F. von Hippel, Phys. Rev. Letters 14, 82 (1965).
3. R. Dashen and M. Gell-Mann, Phys. Letters 17, 142 (1965).
4. H. J. Melosh IV, Caltech thesis (1973) (unpublished).
5. See also E. Eichten, J. Willemsen, and F. Feinberg, SLAC preprint (1973) (unpublished) and S. P. deAlwis, CERN preprint TH. 1601 (1973) (unpublished).
6. F. J. Gilman and M. Kugler, Phys. Rev. Letters 30, 518 (1973).
7. A. J. G. Hey and J. Weyers, CERN preprint TH. 1614 (1973) (unpublished).
8. Intrinsic to the use of PCAC is a $\sim$10% error for matrix elements. Massive or massless final pions have momenta which typically differ by less than 10% for any given decay which we treat in this paper.
9. See the discussion and references in F. J. Gilman and M. Kugler, Ref. 6. A detailed discussion will be presented elsewhere.
10. G. Zweig, CERN preprints TH. 401 and TH. 412 (1964)(unpublished).
11. F. J. Gilman, M. Kugler and S. Meshkov, to be published.
12. D. Horn and S. Meshkov, unpublished.
13. D. Herndon et al., LBL preprint LBL-1065 (1972) (unpublished) and R. Cashmore, private communication. See also U. Mehtani et al., Phys. Rev. Letters 29, 1634 (1973).
14. For a recent discussion, see D. Faiman and D. Plane, Nucl. Phys. B50, 379 (1972).

15. For example, see D. Faiman and A. W. Hendry, Phys. Rev. 173, 1720 (1968) and R. P. Feynman, M. Kislinger and F. Ravndal, Phys. Rev. D3, 2706 (1971).

16. J. C. Carter and M.E.M. Head, Phys. Rev. 176, 1808 (1968); L. Micu, Nucl. Phys. B10, 521 (1969); R. Carlitz and M. Kislinger, Phys. Rev. D2, 336 (1970); D. Horn and Y. Ne'eman, Phys. Rev. D1, 2710 (1970); E. W. Colglazier and J. L. Rosner, Nucl. Phys. B27, 349 (1971); W. P. Petersen and J. L. Rosner, Phys. Rev. D6, 820 (1972).

17. See D. Faiman and D. Plane, Ref. 14, and D. Faiman and J. Rosner, CERN preprint TH. 1636 (1973) (unpublished). We disagree with the experimental signs quoted by the latter authors, and their resulting conclusions, presumably because of isospin phase conventions of Ref. 13 unknown to them.

18. L. A. Copley, G. Karl, and E. Obryk, Phys. Letters 29B, 117 (1969) and Nucl. Phys. B13, 303 (1969); R. P. Feynman et al., Ref. 15.

19. R. G. Moorhouse and H. Oberlack, Phys. Letters 43B, 44 (1973).

20. See in this connection D. Faiman, J. L. Rosner, and J. Weyers, CERN preprint TH. 1622 (1973) (unpublished).

21. P. Söding et al., Phys. Letters 39B, 1 (1972).