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CONSERVED CURRENTS, THEIR COMMUTATORS AND THE SYMMETRY STRUCTURE OF RENORMALIZABLE THEORIES OF ELECTROMAGNETIC, WEAK AND STRONG INTERACTIONS*

Marvin Weinstein

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

Abstract

This paper discusses the role played by conserved currents in fixing the structure of currently popular renormalizable theories of strong electromagnetic and weak interactions. The major objective of this work is to show that these theories correspond to another kind of symmetry—which we call a Higgs-type symmetry—and to clarify the relation of this scheme to the already familiar normal and Goldstone symmetries. In order to do this, we introduce a language which makes no reference to any specific Lagrangian formalism and so avoids questions of whether or not hadrons are composite and whether or not the Goldstone bosons (massless particles of these theories) necessarily have massive partners. For pedagogical reasons, we discuss the original Weinberg model of leptons and a model coupling leptons and hadrons from the current algebra point of view.

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1. INTRODUCTION

In an earlier paper[1] we showed how focusing attention on the conserved currents of a Higgs-Kibble-Weinberg[2] type theory allowed one to obtain interesting results without resorting to a specific Lagrangian formalism. This paper extends the discussion presented in Ref. 1 and provides examples of the application of these ideas to the discussion of two non-abelian schemes: the first being Weinberg's original model of leptons,[3] and the second being the coupling of this model to hadrons.[4] These examples show that the abstract language we shall introduce is easily applied to the discussion of specific models, and that its use leads to a simplification of some arguments.

Besides the fact that it is nice to have a language for discussing gauge theories in the absence of specific Lagrangian models, we believe the approach to be described has the following additional advantages:

(1) It clarifies the connection between gauge theories, Gell-Mann current algebra and the Goldstone boson interpretation of the partially conserved axial-vector current hypothesis (PCAC).

(2) It avoids the issue of whether or not hadrons (including the Goldstone bosons) are composite particles by avoiding the use of Lagrangians.

(3) It provides another view of the way these theories control fermion masses and mass-differences, and provides a simple way of discussing the distinction between theories in which fermion mass-differences are calculable and those in which these mass-differences are controlled but not calculable.

(4) It provides a unified language for the discussion of normal, Goldstone and Higgs-type symmetry schemes and clarifies their relationship to one another.

(5) <u>It suggests a possibly interesting way of using low energy data for purely hadronic processes in order to rule out general classes of models for weak, electromagnetic and strong interactions.</u>

(6) <u>It serves as a useful framework for simplifying the discussion of the renormalization of specific Lagrangian models.</u>

Points (1) through (5) will be discussed in this paper; point (6) will be discussed in a forthcoming paper.

Since much of our discussion requires an appreciation of the differences—from a current algebra point of view—between normal, Goldstone and Higgs-type symmetry schemes, we include Appendix I, which reviews the major differences between a normal and a Goldstone symmetry. The appendix is pedagogical in nature and the reader familiar with these ideas should refer to it only to clarify matters of notation. In Section 2, we define a general Higgs-type symmetry. This section contains the essentially new elements of our formalism. The remainder of this paper is devoted to developing some of the obvious consequences of this approach.

In the belief that simple examples are often more instructive than general statements, Section 3 provides a discussion, from the current algebra point of view, of a generalized version of Weinberg's original model of leptons, and a generalized version of the coupling of this scheme to a σ-like model for hadrons. The discussion of this second model is included because it provides a non-abelian example of the ideas discussed in general terms in Ref. 1. Finally, in Section 4 we discuss some general points including the procedure we alluded to in point (5).

2. THE DEFINITION OF A HIGGS-TYPE SYMMETRY IN TERMS OF CURRENTS

The definition, to be given in this section, of what we choose to call a general symmetry of Higgs-type will be stated entirely in terms of the structure of currents.

We shall make no explicit reference to Lagrangian field theories; however, our assumptions H1, H2, and H3 are an abstraction of properties which can be shown to be true for renormalized perturbation theory.

A. One-Current Higgs-Symmetry Definition

For pedagogical reasons, we start our discussion by treating the case of a single conserved current and progress to the general case.

As in the case of a normal or Goldstone symmetry, our first assumption is that there exists a single conserved vector current $j^{\mu}(x)$. The general Higgs-type theory is defined by the following three assumptions:

H1. There exists a massive vector meson, W^{μ}, of mass M–such that

$$\langle W^{\mu} | j^{\nu}(0) | 0 \rangle = \frac{i}{g}\left(-M^2 g^{\mu\nu} + k^{\mu}k^{\nu}\right) \tag{1}$$

The kinematic factor $(-M^2 g^{\mu\nu} + k^{\mu}k^{\nu})$ says that there are only three polarization states for the massive vector meson, and the g^{-1} can be understood as a formal way of stating the fact that in a Lagrangian model, it is $g \int d^3x\, j^o(\underline{x}, t)$ which is the charge. Note the fact that the conserved current $j^{\mu}(x)$ has non-vanishing matrix element between the single vector meson state and vacuum is a characteristic of theories in which the Higgs mechanism gives the vector meson its mass; it is not true, for example, for the current discussed in the usual formulation of massive quantum electrodynamics.

H2. The vector meson mass vanishes in the limit $g \longrightarrow 0$ and is the only physical particle mass to do so. And

H3. All form factors from which the single vector meson contribution has been removed pass smoothly, in the limit $g \longrightarrow 0$, to the corresponding (Goldstone boson free) form factors of a Goldstone theory which possesses a single conserved current, $j^{\mu}(x)$, and a Goldstone boson, $|\phi\rangle$, such that

$$\langle\phi|j^{\mu}(0)|0\rangle = -iq^{\mu}f_{\phi} \tag{2}$$

The meaning of H1, H2, and H3 will be made more clear in the discussion of the vector meson mass formula which follows.

B. Vector Meson Mass Formula

We obtain an interesting formula for the behavior of the vector meson mass as a function of "g" by considering the vacuum expectation value of the time-ordered product of two currents

$$0|T\left(j^{\mu}(q)\,j^{\nu}(-q)\right)|0\rangle = i\left[\frac{M^2}{g^2}\,\frac{\left(-M^2g^{\mu\nu}+q^{\mu}q^{\nu}\right)}{q^2-M^2}+C^{(g)}(q^2)\,g^{\mu\nu}+D^{(g)}(q^2)q^{\mu}q^{\nu}\right] \tag{3}$$

The r.h.s. of Eq. (3) is the most general form allowed by Poincare invariance, and we have explicitly displayed the vector meson contribution.

Taking the divergence of $j^{\mu}(q)$ in Eq. (3) and using current conservation immediately yields

$$\frac{M^2}{g^2} + C^{(g)}(0) = 0 \tag{4}$$

If we were in the Goldstone world, the most general form of the time-ordered product in Eq. (3) would be

$$\langle 0|T\,j^{\mu}(q)\,j^{\nu}(-q)|0\rangle = i\left[\frac{q^{\mu}q^{\nu}}{q^2}\,f_{\phi}^2 + C^{(0)}(q^2)\,g^{\mu\nu} + D^{(0)}(q^2)\,q^{\mu}q^{\nu}\right] \tag{5}$$

and (see Appendix I) current conservation applied to Eq. (5) yields:

$$C^{(0)}(0) = -f_{\phi}^2 \tag{6}$$

The assumption, H3, is to be interpreted here as $\lim_{g \to 0} C^{(g)}(0) = C^{(0)}(0)$; hence, Eq. (6) tells us that

$$M^2(g) = g^2 f_\phi^2 + O(g^4) , \tag{7}$$

a familiar property of Higgs models.

C. Remarks about Ward Identities and Lagrangian Models

The connection between the defining assumptions, H1, H2 and H3, and Lagrangian approaches to the discussion of a general U(1)-Higgs model can be clarified by investigating general properties of matrix elements of $j^\mu(x)$ taken between physical particle states. In this section we present one such discussion in order to derive a Ward identity which is essentially equivalent to the one derived by B. W. Lee[5] in his treatment of the U(1)-Higgs model. We also use this result to discuss some features of the $g \neq 0$, $f_\phi \longrightarrow 0$ limit of the relations which we derive in order to see in what sense this limit corresponds to massless quantum electrodynamics. (Note: Eq. (7) tells us that $f_\phi \longrightarrow 0$ is just another way of letting $M \longrightarrow 0$.)

The matrix element we shall consider is $j^\mu(q)$ taken between scalar particle states (identical arguments work for spin 1/2). Consider

$$\langle A(p')|j^\mu(q)|B(p)\rangle = \left[(p'+p)^\mu \bar{g}^{(g)}_{AB}(q^2) + q^\mu \bar{h}^{(g)}_{AB}(q^2) + \left(-\frac{1}{g}\right)\frac{\left(M^2 g^{\mu\nu} - q^\mu q^\nu\right)}{q^2 - M^2} H^{(g)}_{\nu AB}(q^2)\right] \tag{8}$$

where $q = (p' - p)$ and in line with H3 we have assumed that the function $H^{(g)}_{\nu AB}(q^2)$ takes care of the single vector meson contribution to this matrix element and that the functions $\bar{g}^{(g)}_{AB}(q^2)$ and $\bar{h}^{(g)}_{AB}(q^2)$ tend smoothly to their corresponding Goldstone forms. Clearly, unitarity requires, at $q^2 = M^2$, that the function $\left(+M^2 g^{\mu\nu} - q^\mu q^\nu\right)H^{(g)}_{\nu AB}(q^2)$ is the sum of the three on-shell amplitudes for the coupling of the three polarizations of $|W^\mu\rangle$ to the particles $|A\rangle$ and $|B\rangle$. Invoking current conservation, Eq. (8) yields

$$\left(m_A^2 - m_B^2\right) g_{AB}^{(g)}(0) + \frac{1}{g}\left(q^\nu H_{\nu AB}^{(g)}(0)\right) = 0 . \tag{9}$$

If, instead of being in the Higgs case, we were in the Goldstone case, Eq. (8) would read

$$\langle A(p')|j^\mu(q)|B(p)\rangle = \left[(p'+p)^\mu g_{AB}^{(0)}(q^2) + q^\mu \bar{h}_{AB}^{(0)}(q^2) - \frac{q^\mu}{q^2} f_\phi G_{\phi AB}^{(0)}\right] \tag{8'}$$

and current conservation would require

$$\left(m_A^2 - m_B^2\right) g_{AB}^{(0)}(0) = f_\phi G_{\phi AB}^{(0)} \tag{9'}$$

Hence, H3 tells us that

$$\lim_{g \to 0} \left(\frac{1}{g} q^\nu H_{\nu AB}^{(0)}(q^2{=}0)\right) + f_\phi G_{\phi AB} = 0 \tag{10}$$

Since we have already argued that gf $\cong$ M, those familiar with B. W. Lee's treatment of the abelian Higgs model will see that this identity is the $g = 0$ limit of the Ward identity so vital to a successful renormalization of that scheme.

One other point work making is that the assumption of a smooth limit corresponding to $g \neq 0$, $f_\phi \longrightarrow 0$ for equations like Eq. (10) (which are free of ambiguous kinematic factors) is consistent with the idea that such a limit is equivalent to massless electrodynamics. In fact, in that limit, Eq. (10) simply goes over to the usual result for massless photon amplitudes which follows from gauge invariance. This fact will prove useful in our discussion of more general models, such as the Weinberg models of leptons, in which one would like to have one massless vector meson. Note that the $f_\phi \longrightarrow 0$ limit is "smooth" in the aforementioned sense amounts to the statement that $g \longrightarrow 0$ limit corresponds to a normal symmetry.

D. The Simplest Non-Abelian Case—Definition

The simplest non-trivial extension of these ideas to the case in which one has more than one conserved current, whose equal time commutators of charge densities close into some generalized Gell-Mann current algebra, will be discussed next. This is the case for which one assumes the existence of n-conserved currents, $j^{\mu}_{\alpha}(x)$, satisfying the commutation relations[6]

$$\left[j^{0}_{\alpha}(\vec{x}, t), j^{\mu}_{\beta}(\vec{y}, t)\right] = i f_{\alpha\beta\gamma}\, j^{\mu}_{\gamma}(\vec{x}, t)\, \delta^{3}(\vec{x}-\vec{y}) + \text{S.T.} \tag{11}$$

where $f_{\alpha\beta\gamma}$ are the structure constants of some semi-simple Lie algebra G (i.e., G would be SU(2), SU(3), O(4), etc.). Paralleling the case of a single current Higgs model, we assume the existence of n-massive vectors mesons $|W^{\mu}_{\alpha}\rangle$ such that

$$\langle W^{\mu}_{\alpha}\, |\, j^{\nu}_{\beta}(0)\,|\,0\rangle = \frac{i}{g} X_{\alpha\beta} \left(-M^{2}_{\alpha}\, g^{\mu\nu} + q^{\mu} q^{\nu}\right) \tag{12}$$

where $X_{\alpha\beta}$ is assumed to be some non-singular matrix. The next step is to assume the obvious generalizations of H1, H2, and H3, and specify the Goldstone limit of this scheme to correspond to a world possessing n-conserved currents $j^{\mu}_{\alpha}(x)$ having the same equal time algebra, G, and n-Goldstone bosons such that

$$\langle \phi_{\alpha}\, |\, j^{\mu}_{\beta}\,|\,0\rangle = -i q^{\mu} f_{\alpha\beta} \tag{13}$$

Here, $f_{\alpha\beta}$ is also assumed to be a non-singular matrix (although at a later point we shall—in line with the comments made about the $f_{\phi} \longrightarrow 0$ limit of the abelian case—discuss what happens if one relaxes this condition).

To obtain a mass formula for the mesons $|W^{\mu}_{\alpha}\rangle$ we consider the T-product

$$\langle 0|T\, j^{\mu}_{\alpha}(q) j^{\nu}_{\beta}(-q)\; 0\rangle = \left[\sum_{\gamma}\left[\frac{X^{*}_{\gamma\alpha}}{g} M^{2}_{\gamma} \frac{\left(-M^{2}_{\gamma}\, g^{\mu\nu} + q^{\mu} q^{\nu}\right)}{\left(q^{2} - M^{2}_{\gamma}\right)} \frac{X_{\gamma\beta}}{g}\right] + C^{(g)}_{\alpha\beta}(q^{2})\, g^{\mu\nu} + D^{(g)}_{\alpha\beta}(q^{2})\, q^{\mu} q^{\nu}\right] \tag{14}$$

Current conservation then implies

$$\sum_{\gamma} X^{*}_{\gamma\alpha}\, M^{2}_{\gamma}\, X_{\gamma\beta} + g^{2} C^{(g)}_{\alpha\beta}(0) = 0 \tag{15}$$

If we let X^{+} be the matrix such that $(X^{+})_{\alpha\beta} = X^{*}_{\beta\alpha}$, we can rewrite this in the suggestive form

$$\sum_{\gamma} X^{+}_{\alpha\gamma}\, M^{2}_{\gamma} X_{\gamma\beta} + g^{2} C^{(g)}_{\alpha\beta}(0) = 0 \ . \tag{15'}$$

Two facts are immediately clear: first that $C^{(g)}_{\alpha\beta}(0)$ is a hermitian matrix, and second that $(X^{-1})_{\alpha\beta}$ is a matrix s.t. $(X^{-1})^{+}\, C^{(g)}\, (X^{-1})$ is a diagonal matrix.

If we now, as in the one current case, consider the same T-product as in Eq. (14), but for the corresponding Goldstone world, we obtain

$$\langle 0 | T\left(j^{\mu}_{\alpha}(q)\, j^{\nu}_{\beta}(-q)\right) | 0 \rangle \doteq \left[\sum_{\gamma} f^{+}_{\alpha\gamma} f_{\gamma\beta} \frac{q^{\mu} q^{\nu}}{q^{2}} + C^{(0)}_{\alpha\beta}(q^{2})\, g^{\mu\nu} + D^{(0)}(q^{2})\, q^{\mu} q^{\nu}\right] \tag{16}$$

(where $f^{+}_{\alpha\gamma} = f^{*}_{\gamma\alpha}$) and so, invoking current conservation, we obtain

$$\sum_{\gamma} f^{+}_{\alpha\gamma} f_{\gamma\beta} = -C^{(0)}_{\alpha\beta}(0) \ . \tag{17}$$

The obvious generalization of H3 tells us that

$$\sum_{\gamma} X^{+}_{\alpha\gamma}\, M^{2}_{\gamma}\, X_{\gamma\beta} = g^{2} \sum_{\gamma} f^{+}_{\alpha\gamma}\, f_{\gamma\beta} + O(g^{4}) \tag{18}$$

which is the general form of the mass-matrix obtained for a Lagrangian model of this sort. Under the simplest possible hypothesis the matrix $X^{-1}_{\alpha\beta}$ can be taken to be the unitary matrix which diagonalizes the hermitian matrix, $C^{(g)}_{\alpha\beta}(0)$.

At this point we should note that having diagonalized $C^{(g)}_{\alpha\beta}$, one could take the limit $g \neq 0$, and let some eigenvalue of $C^{(g)}_{\alpha\beta}$ go to zero. In general, we will only be interested in cases in which only one such eigenvalue is set equal to zero. What this will amount to is having fewer Goldstone bosons than conserved current Goldstone limit which—in the sense of H3—corresponds to the limit $g \longrightarrow 0$. It is not hard to see that in such a situation once the diagonalization has been carried out, one obtains a theory with (n-1)-massive vector mesons and a massless vector meson which plays the role of the ordinary photon. An explicit discussion of this point will be given in our discussion of the Weinberg model of leptons in Section 3.

E. More Complicated Schemes

Even more interesting than the previous case is a world possessing many conserved currents whose equal time algebra closes to some product of the form $G_1 \otimes$, $G_2 \otimes, \ldots \otimes G_m$, where the G_i are semi-simple Lie algebras. This is the kind of scheme we have to define if we wish to consider the most general models for coupling chiral $SU(2) \otimes SU(2)$ or $SU(3) \otimes SU(3)$ worlds of hadrons (possessing exact low energy theorems) to leptons, and requiring the final symmetry to be of Higgs-type. There are many reasons why one might be interested in schemes of this type and the reasons we find most interesting have been discussed in detail in Ref. 1. We shall not, at this time, say more about this other than to note that even the Weinberg model of leptons is of this form and so having the fully general formalism corresponding to these cases is necessary.

The generalizations of the previous discussions of a single semi-simple algebra are totally straightforward. We assume that we have a large number of conserved currents which we will denote $j^{(G_j)}_{\alpha}(x)$. ($j = 1, \ldots m$). The superscript (G_j) denotes the particular subalgebra of $G_1 \otimes, \ldots \otimes G_m$ that the current belongs to and the index "α" runs from 1 to the dimension, d_j, of G_j.

Next, we assume the existence of as many massive vector mesons as there are conserved currents, such that

$$\langle W^{\mu}_{\rho} \left| j^{\nu(G_j)}_{\alpha}(0) \right| 0 \rangle = \frac{+i}{g_j} X^{(G_j)}_{\rho\alpha} \left(-M^2_{\rho} g^{\mu\nu} + q^{\mu} q^{\nu} \right) \tag{19}$$

where the g_j ($j = 1, \ldots, n$) can be different from each (G_j). (The introduction of these g_j's corresponds to the freedom in gauge models of coupling commuting sets of currents with different coupling constants.) The Goldstone limit of these theories are assumed to have as many Goldstone bosons, $|\phi_{\rho}\rangle$, as conserved currents so that

$$\langle \phi_{\rho} \left| j^{(G_j)\mu}_{\alpha}(0) \right| 0 \rangle = -iq^{\mu} f^{(G_j)}_{\rho\alpha} \tag{20}$$

Repeating the arguments of the preceding section, we arrive at the formula

$$\sum_{\gamma} X^{(G_j)^+}_{\alpha\gamma} M^2_{\gamma} X^{(G_\ell)}_{\gamma\beta} = \sum_{\gamma} g_j f^{(G_j)^+}_{\alpha\gamma} f^{(G_\ell)}_{\gamma\beta} g_\ell + O(g_j g_\ell) \tag{21}$$

which relates the vector meson masses to the parameters defining the Goldstone case. The notation in Eq. (21) is not as complicated as it seems at first glance, and its meaning is readily understood within the context of any reasonably simple model, such as the Weinberg model of leptons discussed in the next section.

F. Some Observations

Before going on to discuss specific examples, there are two general points we should make at this stage of our discussion. The first has to do with the hermitian matrix $C^{(g)}_{\alpha\beta}(0)$. It is entirely possible that the Goldstone limit, $C^{(0)}_{\alpha\beta}(0) = -f^{+}_{\alpha\beta} f_{\gamma\beta}$, could have some degenerate eigenvalues. In that event, unless there is a symmetry which forces this degeneracy, one would expect the degeneracy to be lifted as we go away from $g = 0$.

This would be the general case if, for example, the Goldstone world possessed a larger symmetry structure (i.e., there were actually more conserved currents around) than the corresponding model of Higgs-type. Clearly, in this event one should take care, when it is relevant, that one is working with those linear combinations of vector meson states which are smooth limits of eigenstates of $C^{(g)}_{\alpha\beta}(0)$.

The next point has to do with classification of Higgs-type symmetries in terms of the mass formulae they permit. As we discuss in Appendix I, the presence of Goldstone bosons in vector currents gives rise to formulae for mass differences of particles in terms of their couplings to the Goldstone bosons. Moreover, this persists to lowest order in "g" when we go over to the corresponding theory of Higgs type (i.e., we have a zeroth order mass relations of the sort discussed by Weinberg and Georgi and Glashow[7]). Therefore, we have two possible classes of theories: First, there are those theories for which, for example, fermion mass differences persist in the limit $g \longrightarrow 0$, due to the fact that in that limit there is a Goldstone boson coupled to the fermions in question. These are theories in which we shall call the mass differences <u>controlled</u> but not calculable. The second class of theories, the ones in which for symmetry reasons the Goldstone couplings in question vanish in the theory corresponding (in the sense of H3) to the limit $g \longrightarrow 0$, shall be the kind of theories in which we call mass differences calculable. Obviously, depending upon the particle in question, both sets of possibilities can exist in the same theory.

3. WEINBERG MODELS

This section is devoted to discussing Weinberg's original SU(2) ⊗ U(1) model of leptons, and his SU(2) ⊗ U(1) model of leptons coupled to a generalized σ-model for hadrons, from the current algebra point of view. Before proceeding, however, we should point out that we always assume we are discussing schemes which are anomaly free. When one gets down to using this language to describe possibly realistic schemes interesting distinctions can be made based upon the nature of the anomaly cancelling scheme being used; however, for the present we will not bother with these details and shall assume that all current algebra manipulations are anomaly free without specifying how this is accomplished.

A. Model of Leptons

The generalized version of the original Weinberg model of leptons is completely specified by the following statements about the currents:

W1. There exist three currents $j_i^\mu(x)$ $(i = 1, 2, 3)$ (corresponding to the (V - A) currents of his scheme) and a fourth current $j_4^\mu(x)$ satisfying the following equal time algebra:

$$\left[j_i^0(\vec{x}, t), j^\mu{}_j(\vec{y}, t)\right] = i\epsilon_{ijk}\; j_k^\mu(\vec{x}, t)\, \delta^3(\vec{x} - \vec{y}) + \text{S.T.} \tag{22}$$

$$\left[j_i^0(\vec{x}, t), j_4^\mu(\vec{y}, t)\right] = 0 \tag{23}$$

(In other words, we have an SU(2) ⊗ U(1) of conserved currents.)

W2. There exist four vector mesons $|W_i^\mu\rangle$ $(i = 1, 2, 3)$ and $|W_4^\mu\rangle$ such that

$$\langle W_i^\mu | j_j^\nu(0) | 0\rangle = \frac{i}{g} X_{ij}\left(-M_i^2 g^{\mu\nu} + k^\mu k^\nu\right) \tag{24}$$

$$\langle W_i^\mu | j_4^\nu(0) | 0\rangle = \frac{i}{g'} X_{i4}\left(-M_i^2 g^{\mu\nu} + k^\mu k^\nu\right) \tag{25}$$

$$\langle W_4^{\mu} | j_i^{\nu}(0) | 0 \rangle = \frac{i}{g} X_{4i} \left(-M_4^2 g^{\mu\nu} + k^{\mu} k^{\nu} \right) \tag{26}$$

$$\langle W_4^{\mu} | j_4^{\nu}(0) | 0 \rangle = \frac{i}{g'} X_{44} \left(-M_4^2 g^{\mu\nu} + k^{\mu} k^{\nu} \right) \tag{27}$$

W3. The Goldstone limit, $(g,g') \longrightarrow 0$, of this scheme corresponds to having four vector mesons $|\phi_1\rangle$, $|\phi_2\rangle$, $|\phi_3\rangle$ and $|\phi_4\rangle$ such that

$$\langle \phi_i | j_j^{\mu}(0) | 0 \rangle = -i\, q^{\mu} f_{\phi} \delta_{ij} \tag{28}$$

$$\langle \phi_i | j_4^{\mu}(0) | 0 \rangle = +i\, q^{\mu} f_{\phi} \delta_{i3} \tag{29}$$

$$\langle \phi_4 | j_j^{\mu}(0) | 0 \rangle = 0 \tag{30}$$

$$\langle \phi_4 | j_4^{\mu}(0) | 0 \rangle = -i\, q^{\mu} f'_{\phi} \tag{31}$$

[Equations (29) and (31) are chosen so that in the $f'_{\phi} = 0$ limit the current $(j_{3+} j_{4-})$ which we shall identify with the electromagnetic current a la the Gell-Mann-Nishijima relation-couples to no Goldstone boson.]

Given W1 - W3, Eq. (21) becomes the following matrix equation:

$$\begin{pmatrix} X_{11}^* & X_{21}^* & X_{31}^* & X_{41}^* \\ X_{12}^* & X_{22}^* & X_{32}^* & X_{42}^* \\ X_{13}^* & X_{23}^* & X_{33}^* & X_{43}^* \\ X_{14}^* & X_{24}^* & X_{34}^* & X_{44}^* \end{pmatrix} \begin{pmatrix} M_1^2 & 0 & 0 & 0 \\ 0 & M_2^2 & 0 & 0 \\ 0 & 0 & M_3^2 & 0 \\ 0 & 0 & 0 & M_4^2 \end{pmatrix} \begin{pmatrix} X_{11} & X_{12} & X_{13} & X_{14} \\ X_{21} & X_{22} & X_{23} & X_{24} \\ X_{31} & X_{32} & X_{33} & X_{34} \\ X_{41} & X_{42} & X_{43} & X_{44} \end{pmatrix} =$$

$$\begin{pmatrix} g^2 f_{\phi}^2 & 0 & 0 & 0 \\ 0 & g^2 f_{\phi}^2 & 0 & 0 \\ 0 & 0 & g^2 f_{\phi}^2 & -gg' f_{\phi}^2 \\ 0 & 0 & -gg' f_{\phi}^2 & g'^2 \left(f_{\phi}^2 + f_{\phi'}^2 \right) \end{pmatrix} + O\left(g^4, g'^4, g^2 g'^2, \ldots\right) \tag{32}$$

Making the simplest assumption that (X^{-1}) is the matrix which diagonalizes the r.h.s., we see that the masses of the four-vector mesons are given by the eigenvalues of the matrix on the r.h.s. of Eq. (32). Therefore, we see that there are two vector mesons, coupled to the currents j_1^μ and j_2^μ, of mass $M_1^2 = M_2^2 = g^2 f_\phi^2 + O(g^4)$ one vector meson of mass

$$M_3^2 = \frac{f_\phi^2 (g^2+g'^2) + f'^2_\phi g'^2 + \sqrt{\left(f_\phi^2(g^2+g'^2)\right)^2 + f'^4_\phi g'^4 + 2f'^2_\phi f_\phi^2 g'^2 (g'^2 - g^2)}}{2} \tag{33}$$

and a vector meson of mass

$$M_4^2 = \frac{f_\phi^2 (g^2+g'^2) + f'^2_\phi g'^2 - \sqrt{\left(f_\phi^2(g^2+g'^2)\right)^2 + f'^4_\phi g'^4 + 2f'^2_\phi f_\phi^2 g'^2 (g'^2 - g^2)}}{2} \tag{34}$$

The matrix (X^{-1}) is obviously the rotation matrix which takes this to diagonal form, and the limit $f'_\phi = 0$ corresponds to the usual Weinberg case which has $M_3^2 = f_\phi^2 (g^2 + g'^2)$ and $M_4^2 = 0$. We have gone through this argument only to show how the general point about the $f'_\phi \longrightarrow 0$ g', $g \neq 0$ limit works out as expected. It should be obvious that we can proceed from the outset setting $f'_\phi = 0$ ignoring the fact that this implies that Eq. (19) does not really make sense, so long as we understand that only the formulae relating the various form factors make sense.

With this in mind, we complete our discussion of the Weinberg model for $f'_\phi = 0$. In this case we see that the matrix (X) has the simple form

$$X = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\theta & -\sin\theta \\ 0 & 0 & \sin\theta & \cos\theta \end{pmatrix} \tag{35}$$

where $\cos\theta = g/\sqrt{g^2+g'^2}$ and $\sin\theta = g'/\sqrt{g^2+g'^2}$ in order that the mass-squared matrix have the form

$$M^2 = \begin{pmatrix} g^2 f_\phi^2 & 0 & 0 & 0 \\ 0 & g^2 f_\phi^2 & 0 & 0 \\ 0 & 0 & f_\phi^2 (g^2+g'^2) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{36}$$

It is then a trivial consequence of inverting Eq. (24) - (27) by multiplying their matrix form by $(X^{-1})\,ij$, that $|W_4^\mu\rangle$ couples solely to the current

$$j^\mu_{em} = \left(\sin\theta\, g\, j_3^\mu + \cos\theta\, g'\, j_4^\mu\right) = \frac{g g'}{\sqrt{g^2+g'^2}} \left(j_3^\mu + j_4^\mu\right), \tag{37}$$

and $|W_3^\mu\rangle$ couples to the combination

$$j_z^\mu = \cos\theta\, g\, j_3^\mu - \sin\theta\, g'\, j_4^\mu \tag{38}$$

(Note: The combination $j_3^\mu + j_4^\mu$ is precisely the combination of currents which in the limit $f'_\phi = 0$ has no coupling to the Goldstone boson and this is a general result.) Equation (37) can be immediately interpreted as a formula for 'e', namely, $e = g g'/\sqrt{g^2+g'^2}$ which is the usual result of the Weinberg model. We shall see that this result is compatible with the discussion—which follows next—of the restrictions our formalism places upon electron-neutrino form factors.

Although the structure of the vector meson mass matrix is one of the most interesting features of the Weinberg model of leptons, it is very amusing to see how much of the Lagrangian model is reproduced when one goes on to consider the couplings of electrons and neutrinos. To see this, consider a general fermion matrix element of the form

$$\langle \ell'(p')|j^\mu_\alpha(q)|\ell(p)\rangle = i\bar{u}_{\ell'}(p')\Big[\gamma^\mu\ \bar{g}^\alpha_{\ell'\ell}(q^2) + q^\mu \bar{h}^\alpha_{\ell'\ell}(q^2) + q_\nu\sigma^{\mu\nu}\bar{s}^\alpha_{\ell'\ell}(q^2)$$

$$+\gamma^\mu\gamma^5\bar{g}^{5\alpha}_{\ell'\ell}(q^2) + q^\mu\gamma^5\bar{h}^{5\alpha}_{\ell'\ell}(q^2) + q_\nu\sigma^{\mu\nu}\gamma^5\bar{s}^{5\alpha}_{\ell'\ell}(q^2) \tag{39}$$

$$+\sum_\gamma \frac{X^+_{\alpha\gamma}}{g_\alpha}\frac{\left(-M^2_\gamma g^{\mu\nu} + q^\mu q^\nu\right)}{q^2 - M^2_\gamma}\left(H^\gamma_{\nu\ell'\ell}(q^2) + H^{5\gamma}_{\nu\ell'\ell}(q^2)\gamma^5\right)\Big]u_\ell(p)$$

where we have explicitly exhibited the vector meson contribution and adopted an obvious notation in which the Greek letters $\alpha, \gamma = 1, 2, 3, 4$ and $g_\alpha = g$ for $\alpha = 1, 2, 3$ and g' for $\alpha = 4$. The remainder of the notation in Eq. (39) is self-explanatory.

Taking the divergence of Eq. (39) yields:

$$0 = \bar{u}_\ell(p)\Big[(m_{\ell'} - m_\ell)\bar{g}^\alpha_{\ell'\ell}(0) + (m_{\ell'} + m_\ell)\gamma^5\bar{g}^{5\alpha}_{\ell'\ell}(0)$$

$$+\sum_\gamma \frac{X^+_{\alpha\gamma}}{g_\alpha} q^\nu\left(H^\gamma_{\nu\ell'\ell}(0) + H^{5\gamma}_{\nu\ell'\ell}(0)\gamma^5\right)\Big]u_\ell(p) \tag{40}$$

The barred form factors are assumed, in the limit $g \to 0$, to go smoothly to their Goldstone counterparts, which appear in the analogous equation to Eq. (40)—namely,

$$\langle \ell'(p')|j^\mu_\alpha(q)|\ell(p)\rangle = i\bar{u}_{\ell'}(p')\Big[\gamma^\mu\bar{g}^\alpha_{\ell'\ell}(q^2) + q^\mu\ \bar{h}^\alpha_{\ell'\ell}(q^2) + q_\nu\sigma^{\mu\nu}\bar{s}^\alpha_{\ell'\ell}(q^2)$$

$$+\left(\gamma^\mu\bar{g}^{5\alpha}_{\ell'\ell}(q^2) + q^\mu\bar{h}^{5\alpha}_{\ell'\ell}(q^2) + q_\nu\sigma^{\mu\nu}\bar{s}^{5\alpha}_{\ell'\ell}(q^2)\right)\gamma^5 \tag{39'}$$

$$-\frac{q^\mu}{q^2}\sum_\gamma f^+_{\alpha\gamma}\left(G_{\phi_\gamma\ell'\ell} + G^5_{\phi_\gamma\ell'\ell}\gamma^5\right)\Big]u_\ell(p)$$

In the Goldstone case, current conservation yields

$$O = \bar{u}_{\ell'}(p)\Big[(m'_\ell - m_\ell)\, \bar{g}^{\alpha}_{\ell'\ell}(0) + (m'_\ell + m_\ell)\, \bar{g}^{5\alpha}_{\ell'\ell}(0)\,\gamma^5$$

$$- \sum_\gamma f^+_{\alpha\gamma}\left(G_{\phi_\gamma \ell'\ell} + G^5_{\phi_\gamma \ell'\ell}\gamma^5\right)\Big] u_\ell(p) \qquad (40')$$

which, using Eq. (28) - (31), yields (setting $f'_\phi = 0$):

$$f_\phi G_{\phi_i ee} = -f_\phi G_{\phi_3 ee} = 0 \qquad (i = 1, 2, 3) \qquad (41)$$

$$(m_e - m_\nu)\, g^i_{e\nu}(0) = f_\phi G_{\phi_i e\nu}$$
$$(m_e - m_\nu)\, g^4_{e\nu}(0) = -f_\phi G_{\phi_3 e\nu} \qquad (42)$$

$$f_\phi G_{\phi_i \nu\nu} = -f_\phi G_{\phi_3 \nu\nu} = 0 \qquad (43)$$

$$2 m_e g^{5i}_{ee} = f_\phi G^5_{\phi_i ee}$$
$$2 m_e g^{54}_{ee} = -f_\phi G^5_{\phi_3 ee} \qquad (44)$$

$$(m_e + m_\nu)\, g^{5i}_{e\nu} = f_\phi G^5_{\phi_i e\nu}$$
$$(m_e + m_\nu)\, g^{54}_{e\nu} = -f_\phi G^5_{\phi_3 e\nu} \qquad (45)$$

$$2 m_\nu g^{5i}_{\nu\nu} = f_\phi G^5_{\phi_i \nu\nu}$$
$$2 m_\nu g^{54}_{\nu\nu} = -f_\phi G^5_{\phi_3 \nu\nu} \qquad (46)$$

Equations (41) - (46) tell us that certain Goldstone boson couplings must vanish and that in general $g^3_{e\nu}(0) = -g^4_{e\nu}(0)$, and $g^{53}_{\ell'\ell} = -g^{54}_{\ell'\ell}$. Moreover, H3 implies these relations also hold for the Higgs case to lowest order in "g". Forcing $m_\nu = 0$ places further, non-trivial, constraints upon these couplings. It is easy to check that this essentially fixes the parameters of the Weinberg theory.

Another amusing point worth making before concluding this discussion has to do with our identification of the electromagnetic current with the combination

$$e\, j^\mu_{em} = g \sin\theta\, j^\mu_3 + g' \cos\theta\, j^\mu_4 \,.$$

Taking this between electron states at rest, we obtain

$$e \langle e | j^\mu_{em} | e \rangle = \frac{g g'}{\sqrt{g^2 + g'^2}}\; \bar{u}_e(p) \Big[\gamma^\mu \left(g^3_{ee}(0) + g^4_{ee}(0) \right) + \gamma^\mu \gamma^5 \left(g^{53}_{ee}(0) + g^{54}_{ee}(0) \right) \Big] u_e(p) \tag{47}$$

Now Eq. (41) tells us this reduces to

$$e \langle e | j^\mu_{em} | e \rangle = \frac{g g'}{\sqrt{g^2 + g'^2}}\; \bar{u}_e(p) \Big[\gamma^\mu \left(g^3_{ee}(0) + g^4_{ee}(0) \right) \Big] u_e(p) \tag{41'}$$

and if, as in the Weinberg model, we let $g^3_{\ell\ell}(0) = g^4_{\ell\ell}(0) = -1/2$, then our identification of $e = gg'/\sqrt{g^2 + g'^2}$ is completely consistent.

B. Weinberg Model of Hadrons and Leptons

In this section, we present a discussion of a general model for the coupling of the Weinberg SU(2)⊗ U(1) scheme for leptons, to a chiral SU(2) ⊗ SU(2) ⊗ U(1) scheme for hadrons which possesses exact Goldberger-Treiman relations, g_V/g_A - sum rules, Adler self-consistency conditions, etc. The principal reason we bother to go through this discussion is it provides us with an instructive example in a non-abelian case of the general effect we discussed in Ref. 1, namely, the strong feedback of spontaneous leptonic symmetry breaking into the hadron world which occurs when the Goldstone limit of a gauge theory has a larger Goldstone symmetry than its Higgs-type analogue.

A second reason for considering this example is that it provides a simple theory in which one could have hadronic fermion mass differences which are either controlled or calculable.[8]

The definition of the coupled hadron-lepton $SU(2) \otimes U(1)$ scheme proceeds in exactly the same way as in part A of this section insofar as the discussion of the number of conserved currents and vector mesons is concerned. It is only in the discussion of the $g \rightarrow 0$ limit of this theory that interesting new features arise; hence, we shall assume that Eq. (22) - (27) carry over intact to this case.

There are two importantly different cases of the $g \rightarrow 0$ limit which we shall discuss. The first case is described by saying that the $g \rightarrow 0$ limit corresponds to a Goldstone world possessing only an $SU(2) \otimes U(1)$ of conserved currents and three Goldstone bosons, $|\chi_i\rangle$ ($i = 1, 2, 3$), satisfying equations of exactly the same from as Eq. (28) and (29). (Note that, for example, the vector meson mass matrix is then given by the appropriate form of Eq. (36).)

In this case, one would imagine that these Goldstone bosons couple to both leptons and hadrons, and so—in this Goldstone world—baryons and leptons satisfy mass formulae of the sort

$$(m_{B'} - m_B)\, g^{\alpha}_{B'B}(0) = f_{\chi_i}\, G_{\chi_i B'B} \tag{48}$$

(This follows from arguments identical to those presented in order to derive Eq. (40').) Hence, this theory has the property that, for example, the neutron-proton mass difference can survive the passage $g \rightarrow 0$ and although it is controlled by a sum rule relating it to Goldstone boson coupling constants, it is not calculable solely as a function of "g" unless, of course, the Goldstone boson couplings $G_{\chi_i np}$

happen to be zero. In that event, the smoothness of the passage to the Goldstone limit will force the mass differences to start out in order "g^2". One would then call them calculable. It is obvious from models which have been discussed in the literature[9] that both cases are realizable within the context of renormalizable field theories. The second case usually corresponds to have in the $g = 0$ limit a much larger number of conserved currents and Goldstone bosons.

We shall devote the remainder of this section to a brief discussion of how, within the context of this scheme, one sees that even very weak couplings of leptons to hadrons can generally be used to produce large hadronic symmetry violations. The ideas being discussed here have been more completely discussed in Rev. 1, and we content ourselves with only touching upon those points which we find particularly interesting.

For purely pedagogical reasons, we limit our discussion to the case in which the $g \to 0$ limit corresponds to an $SU(2) \otimes U(1)$ Goldstone world. We shall then assume that there exists a limited set of small coupling constants $\{\epsilon_i\}$ such that in the limit $\{\epsilon_i\} \longrightarrow 0$ all lepton hadron couplings vanish. Our basic assumption will be that this world of uncoupled hadrons and leptons possesses many more conserved currents and Goldstone bosons than is the case of $\{\epsilon_i\} \neq 0$.[10]

To be specific, we will assume that we have an $SU(2) \otimes SU(2) \otimes U(1)$ equal-time algebra of conserved hadronic currents and an $SU(2) \otimes U(1)$ of leptons. The totally decoupled world of leptons will be assumed to have the Goldstone boson structure of the Weinberg model, and so requires no further description. The hadronic world, in this limit, is assumed to have the symmetry structure of a generalized σ-model, and we now list its properties.

First, we assume the existence of seven conserved currents $V^{\mu}_{H,i}(x)$ $(i=1,2,3)$, $A^{\mu}_{H,i}(x)$ $(i = 1,2,3)$ and $Y^{\mu}_{H}(x)$ whose equal-time algebra closes, in the usual way, to

$SU(2) \otimes SU(2) \otimes U(1)$. To be precise, we assume that the combinations

$$j^{\pm,\mu}_{H,i}(x) = \frac{1}{2}\left[V^{\mu}_{H,i}(x) \pm A^{\mu}_{H,i}(x)\right] \tag{49}$$

satisfy the commutation relations.

$$\left[j^{\pm,0}_{H,i}(\vec{x},t), j^{\pm,\mu}_{H,j}(\vec{y},t)\right] = i\epsilon_{ijk} j^{\pm,\mu}_{H,k}(\vec{x},t)\,\delta^3(\vec{x}-\vec{y}) + \text{S.T.} \tag{50}$$

$$\left[j^{\pm,0}_{H,i}(x,t), j^{\mp\mu}_{H,j}(y,t)\right] = 0 \tag{51}$$

$$\left[j^{\pm,0}_{H,i}(x,t), Y^{\mu}_{H}(y,t)\right] = 0 \tag{52}$$

The second assumption is that there exist three pseudo-scalar Goldstone bosons $|\pi'_i\rangle$ such that

$$\langle\pi'_i | j^{\mu}_{H,j}(0)|0\rangle = + i q^{\mu} f_{\pi'}\delta_{ij} \tag{53}$$

$$\langle\pi'_i | Y^{\mu}_{H}(0)|0\rangle = 0 \tag{54}$$

or, in other words, $|\pi'_i\rangle$ are coupled to vacuum solely by the axial-vector currents.

This second assumption says that the limit $\{g, g', \epsilon_i\} = 0$ corresponds to a world of leptons of Weinberg-type, and a hadronic world possessing a normal SU(2)-isospin symmetry—generated by the three vector currents, $V^{\mu}_i(x)$—and three massless pseudo-scalar mesons which satisfy exact low-energy theorems.

The third defining assumption is that the $SU(2) \otimes U(1)$ of conserved hadronic plus leptonic currents, $j^{\mu(g)}_{H+L,\alpha}$ $(\alpha = 1,2,3,4)$, corresponds in the limit $\{g, g', \epsilon_i\} = 0$, to the SU(2) of currents $j^{\mu(0)}_{H+L,i} = j^{\mu}_{H,i} + j^{\mu}_{L,i}$, and the U(1) current, $j^{\mu(0)}_{H+L,4} = j^{+(\mu)}_{H,3} + Y^{\mu}_{H} + j^{\mu}_{L,4}$. (Note that this U(1) combination guarantees the existence of a conserved electromagnetic current.)

This third assumption specifies a great deal about the nature of the limit $\{\epsilon_i\} \longrightarrow 0$. The reason for this lies in the fact that in the limit $\{\epsilon_i\} = 0$ we have six Goldstone bosons: three hadronic Goldstone bosons $|\pi'_i\rangle$ and three leptonic Goldstone bosons $|\phi_i\rangle$. All six of these bosons are forced to be massless since they couple non-trivially to the six conserved currents $A^{\mu}_{Hi}(x)$ and $j^{\mu}_{Li}(x)$. Now, as we go away from $\{\epsilon_i\} = 0$, the separate leptonic and hadronic currents are no longer conserved, but only the four currents corresponding to the combined lepton hadron SU(2) $\otimes$ U(1) (generated by the currents $j^{\mu}_{H+L,i}(x)$ and $j^{\mu}_{H+L,i,4}(x)$). This tells us that there are at most four boson states that will be kept massless as a consequence of current conservation; in fact, we shall see that our assumptions tell us there are only three states, $|\chi_i\rangle$, which pass smoothly to the particular linear combinations of hadron and lepton Goldstone bosons which owe their masslessness to the conservation of the combined lepton and hadron currents. To be more specific, consider the combinations

$$\begin{aligned} |\pi_i^{(0)}\rangle &= \cos\theta\,|\pi'_i\rangle + \sin\theta\,|\phi_i\rangle \\ |\chi_i^{(0)}\rangle &= \cos\theta\,|\pi'_i\rangle - \sin\theta\,|\phi_i\rangle \end{aligned} \tag{55}$$

If, in the limit $\{\epsilon_i\} = 0$, we choose $\cos\theta = f_\phi/\sqrt{f^2_{\pi'} + f^2_\phi}$,

$$\langle\pi_i^{(0)}|j^{-\mu}_{H,j} + j^{\mu}_{L,j}|0\rangle = \frac{-iq^\mu}{\sqrt{f^2_\phi + f^2_{\pi'}}}\left[-f_\phi f_{\pi'} + f_{\pi'} f_\phi\right]\delta_{ij} = 0 \tag{56}$$

$$\langle\pi_i^{(0)}|j^{+,\mu}_{H,3} + Y^{\mu}_{H,i} + j^{4}_{L,4}|0\rangle = \frac{-iq^\mu}{\sqrt{f^2_\phi + f^2_{\pi'}}}\left[f_\phi f'_\pi - f_{\pi'} f_\phi\right] = 0 \tag{57}$$

This shows us that the three states, $|\pi_i^{(0)}\rangle$, are Goldstone bosons which do not couple to the sum of the hadronic and leptonic currents; hence, they owe their masslessness to the fact that the limit $\{\epsilon_i\} \to 0$ has a much larger symmetry group $\left((SU(2) \otimes SU(2) \otimes U(1))_H \otimes (SU(2) \otimes U(1))_L\right)$ than the group $(SU(2) \otimes U(1))_{H+L}$ of the coupled theory. The states, $|\chi_i^{(0)}\rangle$, are coupled by the sum of hadronic and leptonic currents to vacuum in the usual way and the $\{\epsilon_i\} \to 0$ limit of

$$\begin{aligned} \langle \chi_i^{(\epsilon)} | j^{\mu}_{H+L,j} | 0 \rangle &= -iq^{\mu} f(\epsilon) \\ \langle \chi_i^{(\epsilon)} | j^{\mu}_{H+L,4} | 0 \rangle &= +iq^{\mu} f(\epsilon) \end{aligned} \tag{58}$$

must correspond to

$$\begin{aligned} \langle \chi_i^{(0)} | j^{\mu}_{H+L,j} | 0 \rangle &= +iq^{\mu} \sqrt{f_{\phi}^2 + f_{\pi'}^2} \\ \langle \chi_i^{(0)} | j^{\mu}_{H+L,4} | 0 \rangle &= -iq^{\mu} \sqrt{f_{\phi}^2 + f_{\pi'}^2} \end{aligned} \tag{59}$$

It is clear that only these states can be smoothly related to the massless Goldstone states of the $\{\epsilon_i\} \neq 0$ coupled theory of leptons and hadrons. The states, $|\chi_i\rangle$, are the ones which are replaced by massive vector mesons in the Higgs-type theory and the $|\pi_i\rangle$ will show up as low-mass meson states which remember, to some degree, their Goldstone nature due to the very simple form of the mixing formula, Eq. (55).

It is important to note that Eq. (55) is determined primarily by the structure of currents in the totally symmetric theory and not by the details of the interactions which couple leptons and hadrons—so long as they are "weak."

As explained in Ref. 1, these mesons—which we would identify with the pions of the real world—would be expected to show 10% violations of PCAC-identities even for couplings whose strength is consistent with their being due to second-order weak interactions. Moreover, the presence of Goldstone bosons in the vector currents of the fully coupled theory leads to leptonically induced hadronic violations of isospin. In the case of isospin, this is not a very interesting possibility, but as we shall remark in the next section, it is more interesting when one considers models based upon SU(3) ⊗ SU(3)-Goldstone schemes for hadrons.

Other interesting general features of this model can be obtained by pursuing arguments of the sort given throughout this discussion; however, they are better discussed at another time.

4. SOME GENERAL REMARKS

In the previous sections of this paper, we have discussed general properties of Higgs-type theories from the current algebra point of view. Moreover, we have deliberately limited our discussion of models to simple but unrealistic theories. One could proceed to translate many other possibly more sensible Lagrangian schemes into this general language and also, using the current algebra framework, one could develop all sorts of models which can be contrived to fit any given preconceived notion of what constitutes and interesting result. Instead, however, we would like to make a few comments about why we do not think this is necessarily the most fruitful approach which can be taken and suggest an alternative which we believe deserves more attention than it has received to date.

The first point we would like to make is that from the current algebra point of view many of the features of renormalizable theories, such as formulae for vector meson masses, sum rules for fermion mass-differences, low energy scattering theorems, etc., really amount to nothing more than a kind of spectroscopy: that is, they are equivalent to the naive perturbation theory arguments one makes when discussing ordinary symmetry breaking. There are subtle differences, of course, in that we are discussing theories with abnormal symmetry limits, and that we are not necessarily discussing symmetry breaking as much as the transition between different kinds of symmetries. Nevertheless, the kinds of results we have discussed and the nature of the most general assumptions which lead to these results represent a discussion of dynamics at the crudest level. In the same sense that the success of the Gell-Mann-Okubo mass formula gives us no really detailed information about the structure of hadronic interactions, we do not believe these kinds of statements, even if true, should be taken to be a revelation of the details of lepton-hadron dynamics. Instead, they should be thought of in the same way one usually things about SU(3) results, namely, as a way of using experimental information to place limits upon the structure of "symmetry breaking terms in the Hamiltonian" (we use this term in a very vague sense).

The second point we would like to make is that even if one does believe that one will be able to give a reasonably convincing argument for a specific Lagrangian model of the world, not enough attention has been paid to the general way in which symmetry properties of the lepton world feed back automatically into the hadronic world and are capable of generating large symmetry breaking effects. In the kind of theories outlined in Section 3(B) in which one supposes all of hadronic SU(3) and even SU(3) ⊗ SU(3) breaking to come from the coupling of hadrons to leptons, there must exist non-trivial relations which one can establish between the structure of the violations of SU(3), PCAC low energy theorems and the SU(3) ⊗ SU(3) transformation properties of the leptonic Goldstone bosons. For example, as we showed in Ref. 1, derivations of the generalized Goldberger-Treiman relations for the octet of pseudo-scalar mesons are related to constants of the type f_{ϕ_i}, $G^5_{\phi_i BB'}$, etc., where $|\phi_i\rangle$ are the Goldstone bosons of the lepton world. There must also be relations for baryon mass differences, violations of SU(3) in the scattering of π's, K's, and η's off baryons, violations of PCAC-low energy theorems in meson-baryon scattering processes and the same set of parameters. Obviously, within the framework of any specific assumptions about the symmetry structure of the lepton world, one must then have sum rules among parameters which measure deviations from symmetry predictions for hadronic processes. The success or failure of such relations should provide one with another way of getting a handle on what a correct theory of leptons must look like, at least insofar as its symmetry structure is concerned.

We wish to emphasize that this exploration of relations between hadron symmetry breaking and lepton structure rests upon very general assumptions which must be true in any Lagrangian theory for which perturbation theory is assumed to have any relevance. They will exist whether one believes one is doing spectroscopy and that there are underlying "partons" or "quarks" which bind strongly to produce the Goldstone phenomenon—an effect which one could not treat easily within the framework of Lagrangian perturbation theory—or one believes he is discovering a Lagrangian which is close to that of the real world. We wish to point out that the search for relations of this sort, which depend only upon the most general aspects of the Lagrangian formalism, merits a great deal more attention, since it opens up the possibility of distinguishing between different models of leptons on the basis of low-energy hadronic data.

APPENDIX I

Normal vs Goldstone Symmetries

In order to make our review of the differences between normal and Goldstone symmetries as simple as possible, we divide the discussion into two parts. First, we discuss the case of a single conserved current (i.e., a U(1) symmetry) and show the differences between a normal and Goldstone version of this theory. Second, we give a brief discussion of the important features of what happens if we have several conserved currents whose equal time commutators close to a Lie algebra, G.

A. One Current-Normal Symmetry

We begin by assuming the existence of a conserved vector current $j^u(x)$ (or axial current $j^{5\mu}(0)$), such that

$$\partial_\mu j^u(x) = 0 \tag{A.1}$$

$$\partial_\mu j^{5\mu}(x) = 0 \tag{A.1'}$$

To extract useful information from Eq. (1) for Eq. (1'), consider $j^u(x)$ taken between fermion states of the same intrinsic parity. The most general expression for this matrix element is:

$$\langle A|j^u(0)|B\rangle = \bar{u}_A(p')\left[\gamma^\mu g_{AB}(q^2) + q^\mu h_{AB}(q^2) + q_\nu \sigma^{\mu\nu} s_{AB}(q^2)\right] u_B(p) \tag{A.2}$$

where

$$q^\mu = (p' - p)^\mu \quad .$$

Taking the divergence of Eq. (2) and using Eq. (1), we obtain

$$\bar{u}_A(p')\left[\left(m_A - m_B\right) g_{AB}(q^2) + q^2 h_{AB}(q^2)\right] u_B(p) \tag{A.3}$$

The assumption that the conserved current, $j^{u}(x)$, corresponds to a normal symmetry is seen to be equivalent to the statement

$$\lim_{q^2 \to 0} q^2 h_{AB}(q^2) = 0 \tag{A.4}$$

since then Eq. (A.3) yields

$$\left(m_A - m_B\right) g_{AB}(0) = 0 \tag{A.5}$$

Therefore, either $m_A = m_B$ <u>or</u> $g_{AB}(0) = 0$. As it is conventional to call $g_{AB}(0)$ the matrix element of the vector charge taken between fermion states $|A\rangle$ and $|B\rangle$, one way to paraphrase the content, Eq. (A.5), is to note that it is equivalent to the usual statement one makes about conserved charges, namely, "a conserved vector charge can have non-vanishing matrix elements only between states of the same mass."

The same argument applied to the discussion of an axial-vector current (if Eq. (4) holds) yields[10]

$$\left(m_A + m_B\right) g^5_{AB}(0) = 0 \tag{A.6}$$

We have, therefore, obtained the result that the axial charge has no matrix elements between massive states of the same intrinsic parity.

B. <u>One Current-Goldstone Symmetry</u>

In order to escape the conclusions embodied in Eq. (A.5), one relaxes Eq. (A.4) by allowing $\lim_{q^2 \to 0} q^2 h_{AB}(q^2) \neq 0$. To make this consistent with naive dispersive or field theoretic ideas, we assume the existence of a zero mass scalar (pseudo-scalar) boson coupled by the conserved vector (axial-vector) current to vacuum. In other words, we assume there exists a single scalar particle state $|\phi^S\rangle$ (or pseudo-scalar $|\phi^{PS}\rangle$ such that

$$\langle\phi^{S}|j^{\mu}(0)|0\rangle = -iq^{\mu}f_{\phi} \tag{A.7}$$

or

$$\langle\phi^{PS}|j^{5\mu}(0)|0\rangle = -iq^{\mu}f^{5}_{\phi} \tag{A.7'}$$

The masslessness of $|\phi^{S}\rangle$ (or $|\phi^{PS}\rangle$) is a direct consequence of Eq. (A.7) (Eq. A.7')) and so does not amount to an extra assumption. Equation (A.7) leads in the usual way to replacement of Eq. (A.4) by

$$h_{AB}(q^2) = -\frac{f_{\phi}\,G_{\phi AB}}{q^2} + \bar{h}_{AB}(q^2) \tag{A.4'}$$

where $\lim_{q^2\to 0} \overline{q^2 h_{AB}}(q^2) = 0$ and $G_{\phi AB}$ stands for the usual (ϕ, A, B) - coupling constant (we shall use $G^5_{\phi AB}$ to stand for the ϕ^{PS}, A, B coupling constant).

Substitution of Eq. (A.4') into Eq. (A.3) yields

$$\bar{u}_A(p')\left[\left(m_A - m_B\right)g_{AB}(q^2) - fG_{\phi AB} + q^2\bar{h}_{AB}(q^2)\right]u_B(p) = 0 \tag{A.3'}$$

which implies

$$\left(m_A - m_B\right)g_{AB}(0) = f_{\phi}\,G_{\phi AB} \tag{A.5'}$$

In the case of an axial vector current, this becomes

$$\left(m_A + m_B\right)g^5_{AB}(0) = f^5_{\phi}\,G^5_{\phi AB} \ . \tag{A.6'}$$

Equations (A.5') and (A.6') exhibit the great difference between a Goldstone and normal symmetry, namely, in the Goldstone case vector current conservation relates low energy Goldstone boson amplitudes to mass differences, while axial-vector current conservation gives formulae analogous to the famous Goldberger-Treiman relation. (Note: these "low-energy theorems" replace conventional statements about the existence of multiplets which are degenerate in mass.)

C. Non-Abelian Algebra of Currents—Normal Symmetry

One can readily extend this discussion to the situation in which we have a set of conserved vector currents $j^{\mu}_{\alpha}(x)$ $(\alpha = 1, \ldots, n)$ and a corresponding number of axial-vector currents $j^{5u}_{\alpha}(x)$. Following Gell-Mann, we could assume that these currents satisfy the following equal-time algebra:

$$\left[j^{0}_{\alpha}(\vec{x},t),\ j^{\mu}_{\beta}(\vec{y},t)\right] = i f_{\alpha\beta\gamma}\, j^{\mu}_{\gamma}(\vec{x},t)\ \ {}^{3}(\vec{x}-\vec{y}) + \text{S.T.} \tag{A.8}$$

$$\left[j^{0}_{\alpha}(\vec{x},t),\ j^{5\mu}_{\beta}(\vec{y},t)\right] = i f_{\alpha\beta\gamma}\, j^{5\mu}_{\gamma}(\vec{x},t)\ \ {}^{3}(\vec{x}-\vec{y}) + \text{S.T.} \tag{A.9}$$

$$\left[j^{50}_{\alpha}(\vec{x},t),\ j^{5\mu}_{\beta}(\vec{x},t)\right] = i f_{\alpha\beta\ \gamma}\, j^{\mu}_{\gamma}(\vec{x},t)\ \ {}^{3}(\vec{x}-\vec{y}) + \text{S.T.} \tag{A.10}$$

where the constants $f_{\alpha\beta\gamma}$ are the structure constants of some Lie algebra G. These assumptions amount to defining a very general kind of chiral symmetry scheme corresponding to having a chiral algebra $G \otimes G$. The easiest way to see this is to form the conventional even-and-odd parity combinations of currents

$$j^{\pm,\mu}_{\alpha}(x) = \frac{1}{2}\left[j^{\mu}_{\alpha}(x) \pm j^{5\mu}_{\alpha}(x)\right] . \tag{A.11}$$

It is then simple to see that

$$\left[j^{+,0}_{\alpha}(\vec{x},t),\ j^{-\mu}_{\beta}(\vec{y},t)\right] = 0 \tag{A.12}$$

and

$$\left[j^{\pm,0}_{\alpha}(\vec{x},t),\ j^{\pm,0}_{\beta}(\vec{y},t)\right] = +i f_{\alpha\beta\gamma}\, j^{\pm,0}_{\gamma}(\vec{x},t)\ \ {}^{3}(\vec{x}-\vec{y}) + \text{S.T.} \tag{A.13}$$

As noted, these currents close to two commuting algebras G and so they generate the product algebra $G \otimes G$.

Paralleling our discussion of the one-current case, we begin by considering the single fermion matrix element

$$\langle A|j^{\mu}_{\alpha}(0)|B\rangle = \bar{u}_{A}(p')\left[\gamma^{\mu} g^{\alpha}_{AB}(q^2) + q^{\mu} h^{\alpha}_{AB}(q^2) + q_{\nu}\sigma^{\mu\nu} s^{\alpha}_{AB}(q^2)\right] u_{B}(p) \tag{A.14}$$

Taking the divergence of this expression yields

$$\bar{u}_A(p')\left[\left(m_A - m_B\right) g^{\alpha}_{AB}(q^2) + q^2 h^{\alpha}_{AB}(q^2)\right] u_B(p) = 0 \tag{A.15}$$

and setting $q^2 = 0$, implies for the normal symmetry case (i.e., $\lim_{q^2 \to 0} q^2 h^{\alpha}_{AB}(q^2) = 0$)

$$\left(m_A - m_B\right) g^{\alpha}_{AB}(0) = 0 \tag{A.16}$$

What this says, of course, is that in the case of a normal symmetry the charge matrices $g^{\alpha}_{AB}(0)$ vanish except between states of equal mass. The corresponding equations for axial-vector currents between states of the same intrinsic parity are

$$\left(m_A + m_B\right) g^{5\alpha}_{AB}(0) = 0 \tag{A.17}$$

In order to present a simple derivation—for the case of a normal symmetry—that particle states belong to irreducible representations of the group (or algebra) in question, we shall restrict ourselves to the case of a set of vector currents closing to a semi-simple Lie algebra G. This amounts to ignoring the axial-vector currents in the $G \otimes G$ algebra defined in Eq. (A.8), (A.9), (A.10); however, the results are easily extendable.

We begin by studying the time-ordered product

$$\langle A | T\left(j^{\mu}_{\alpha}(q)\, j^{\nu}_{\beta}(-k)\right) | B \rangle \equiv \int d^4x\, d^4y\, e^{+iq\cdot x - ik\cdot y} \langle A \left| T\left(j^{\mu}_{\alpha}(x)\, j^{\nu}_{\beta}(y)\right) \right| B \rangle \tag{A.18}$$

The most general form this expression can take is where we have explicitly separated in covariant form the contribution of single fermion intermediate states:

$$\langle A \left| T\left(j^{\mu}_{\alpha}(q)\, j^{\nu}_{\beta}(-k)\right) \right| B \rangle =$$

$$\bar{u}_A(p') \left[\sum_c \left\{ \gamma^{\mu} g^{\alpha}_{AC}(q^2) + q^{\mu} h^{\alpha}_{AC}(q^2) + q_{\nu}\sigma^{\mu\nu} S^{\alpha}_{AC}(q^2) \right\} \frac{(\not{p}' + \not{q} + m_c)}{\left(p'^2 + q^2 + 2p'\cdot \mathring{q} - m_c^2\right)} \times \right.$$

$$\left\{\gamma^{\nu} g^{\beta}_{CB}(k^2) - k^{\nu} h^{\beta}_{CB}(k^2) - k_{\nu}\sigma^{\mu\nu} s^{\beta}_{CB}(k^2)\right\} +$$

$$\sum_{c} \left\{\gamma^{\nu} g^{\beta}_{AC}(k^2) - k^{\nu} h^{\beta}_{AC}(k^2) - k_{\nu}\sigma^{\mu\nu} s^{\beta}_{AC}(k^2)\right\} \frac{\not p - \not q + m_c}{\left(p^2 + q^2 - 2p\cdot q - m_c^2\right)} \times \tag{A.19}$$

$$\left.\left\{\gamma^{\mu} g^{\alpha}_{CB}(q^2) + q^{\mu} h^{\alpha}_{CB}(q^2) + q_{\nu}\sigma^{\mu\nu} s^{\alpha}_{CB}(q^2)\right\}\right] u_B(p) + X^{\mu\nu}_{\alpha\beta}(p', q; k, p)$$

where $X^{\mu\nu}_{\alpha\beta}(p', q; k, p)$, defined in the obvious manner, amounts to the continuum contribution. Taking the divergence on μ, we have—in the usual way—

$$\partial_{\mu} \langle A' | T\left(j^{\mu}_{\alpha}(q)\, j^{\nu}_{\beta}(-k)\right) | B\rangle = i f_{\alpha\beta\gamma} \langle A | j^{\nu}_{\gamma}(q-k) | B\rangle \ . \tag{A.20}$$

If we substitute in Eq. (A.20), Eq. (A.14) and (A.19), set $q^2 = 0$, and use the fact that at $q^2 = 0$ (assuming no parity doubling of fermions) $g^{\alpha}_{AC}(0)$ is non-vanishing if and only if $m_A = m_C$, we finally obtain for $q^{\mu} = 0 = k^{\nu}$

$$\bar{u}_A(p')\Big[\gamma^{\nu}\left(g^{\alpha}_{AC}(0)\, g^{\beta}_{CB}(k^2) - g^{\beta}_{AC}(k^2)\, g^{\alpha}_{CB}(0)\right) - k^{\nu}\left(g^{\alpha}_{AC}(0)\, h^{\beta}_{CB}(k^2)\right.$$

$$\left. - h^{\beta}_{AC}(k^2)\, g^{\alpha}_{CB}(0)\right) - k_{\lambda}\sigma^{\nu\lambda}\left(g^{\alpha}_{AC}(0)\, s^{\beta}_{CB}(k^2) - s^{\beta}_{AC}(k^2)\, g^{\alpha}_{CB}(0)\right)\Big] u_B(p)$$

$$+ \lim_{q^{\mu}\to 0} q_{\lambda} X^{\lambda\nu}_{\alpha\beta(AB)} = i f_{\alpha\beta\gamma}\, \bar{u}_A(p')\Big[\gamma^{\nu} g^{\gamma}_{AB}(k^2) - k^{\nu} h^{\gamma}_{AB}(k^2)$$

$$- k_{\lambda}\sigma^{\nu\lambda} s^{\gamma}_{AB}(k^2)\Big] u_B(p) \tag{A.21}$$

where $p' = p + k$. If, in addition, one takes $k^{\nu} = 0$, we see that if $\lim_{q^{\nu}\to 0} q_{\lambda} X^{\lambda\nu}_{\alpha\beta(AB)} = 0$ (or, in other words, if one-particle states saturate the commutator), then

$$\left[g^{\alpha}_{AC}(0),\ g^{\beta}_{CB}(0)\right] = i f_{\alpha\beta\gamma}\, g^{\gamma}_{AB}(0) \tag{A.22}$$

and so the charge matrices link only single-particle states which belong to the same irreducible representation of the charge algebra G.

Summarizing, we have seen that the non-trivial normal symmetry case is equivalent to assumptions of the general form $\lim_{q^2 \to 0} q^2 h^{\alpha}_{AB}(q^2) = 0$ and $\lim_{q^{\mu} \to 0} q_{\lambda} X^{\lambda\nu}_{\alpha\beta,(AB)} = 0$. Obviously similar arguments can be carried through for boson states with similar results.

D. Non-Abelian Current Algebra—Goldstone Symmetry

It is easy to see why the statement that particles fall into degenerate multiplets corresponding irreducible representations of G must fail in the Goldstone case. If, as before, we define the Goldstone case by the assumption[11] that there exist scalar mesons $|\phi^{S}_{\alpha}\rangle$ (or pseudo-scalar mesons $|\phi^{PS}_{\alpha}\rangle$) such that

$$\langle\phi^{S}_{\alpha}, j^{\mu}_{\beta}(0)|0\rangle = -iq^{\mu} f_{\alpha} \delta_{\alpha\beta} \quad , \tag{A.23}$$

Eq. (A.14) and Eq. (23) imply

$$\left(m_A - m_B\right) g^{\alpha}_{AB}(0) = f_{\alpha} G_{\phi_{\alpha} AB} \quad . \tag{A.24}$$

(Similar results involving sums of masses follow for the axial-vector currents.)

The appropriate versions of Eq. (A.19), (A.20), (A.21), and (A.24), plus the assumption that the fermions of our theory have non-zero mass, tell us that one cannot require the charge matrices $g^{\alpha}_{AB}(0)$ to satisfy the commutation relations given in Eq. (A.22). Hence, we have another important way in which the Goldstone case is quite different from the normal case.

Obviously, in the general case for which a subalgebra $H \subset G$ is of normal symmetry type and the remainder is of Goldstone type, then the usual symmetry statements hold only for H.

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5. B. W. Lee, Phys. Rev. D5, 823 (1972) and for an earlier discussion of this model see G. T. Hooft, Nucl. Phys. B33, 173 (1971); ibid., B35, 167 (1971).

6. The S. T. stands for Schwinger terms and in all of the derivations to follow if one understands time ordered products to really be covariant T*-products these terms can be ignored.

7. S. Weinberg, Phys. Rev. Letters 29, 388 (1972);
H. Georgi and S. L. Glashow, Phys. Rev. D6, 2977 (1972).

8. To establish the relation of this terminology to that of Weinberg, Georgi and Glashow we note: a controlled but not computable mass difference would be a non-vanishing quantity whose value in zero loop approximation is fixed by Goldstone boson coupling constants in a Lagrangian containing all renormalizable terms consistent with the symmetry principles adopted; a computable mass difference is one whose value in the zero loop approximation vanishes.

9. For example see the paper of T. Hagimore and B. W. Lee (to be published in Phys. Rev. D5) for an example of the controlled but non-calculable case and for a discussion of the second type of mass formula see S. Weinberg, Phys. Rev. Letters 29, 388 (1972).

10. If we had assumed $|A\rangle$ and $|B\rangle$ were fermion states having opposite intrinsic parity we would have obtained

$$\langle A | j^{\mu}(0) | B \rangle = \bar{u}_A(p') \left[\left\{ \gamma^{\mu} g^5_{AB}(q^2) + q^{\mu} h^5_{AB}(q^2) + q_{\nu} \sigma^{\mu\nu} s^5_{AB}(q^2) \right\} \gamma^5 \right] u_B(p)$$

Taking the divergence of this expression yields $(m_A + m_B)\, g^5_{AB}(0) = 0$; hence, the conclusions for the vector and axial-vector current are simply interchanged.

11. Actually, we could assume a more complicated structure for the f's (i.e., a general matrix $f_{\alpha\beta}$), however, for the purposes of this brief argument this would add nothing to the point we wish to make.