SLAC-PUB-1221
(A)
March 1973

KLYSTRONS FOR ACCELERATOR IMPROVEMENTS*

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A theory of relativistic large signal rf interaction phenomenon in klystrons has been developed by Tore Wessel-Berg. Based on that theory the SLACKLY computer program was written and is used successfully in the design of improved klystrons for use at SLAC. Good correlation between the predictions of the SLACKLY program and the experimental results has been obtained.

Introduction

High energy physics demand for new accelerator systems and improvements has put considerable emphasis on increasing the operating efficiency of tubes used for accelerator power sources. The klystron has played a very important role in most linear accelerators due to its capability to provide efficient and stable microwave power. Intrinsic properties of the klystron due to the separation of the various functions (beam generation, rf input, interaction region, rf output, and beam collection) make it ideally suited to applications where high phase stability, low harmonic content and long life are desired. At the Stanford Linear Accelerator Center several accelerator improvement programs all requiring new klystron designs (RLA, SPEAR)-or significant efficiency improvements to the present designs are under way.

Since the beginning of operation in 1966, the 245, 21 MW klystrons originally installed on the accelerator have been gradually improved. By 1967 the efficiency had been increased by minor modifications in drift distances, and it was planned to upgrade the accelerator energy by increasing the rf power output at each station to 30 MW.[1] This increase in power output was made possible by a combination of the efficiency improvement (from 35% to 40% approximately) and of an increase of 8% in the klystron operating beam voltage.

Any further increase in rf power output and accelerator beam energy beyond that point cannot be accomplished without a major redesign of the modulators and power distribution system, unless a further substantial improvement in klystron efficiency is realized. To determine the feasibility of such an improvement, a theoretical investigation of relativistic large signal rf interaction phenomena in the klystron was undertaken with the help of Tore Wessel-Berg, of the Norwegian Institute of Technology, Trondheim, Norway. With support originally from MIT Lincoln Laboratories, and then from SLAC, he undertook, in 1967, to work out a theory of the interaction region.

The results of the theoretical work of Wessel-Berg were then analyzed at SLAC, and a computer program was designed and written based on his algorithm. This program has been used successfully in the redesign of the high power klystrons now in operation at SLAC as well as in the design of other klystrons contemplated by SLAC for other accelerator improvements (i.e., RLA, SPEAR II).

This paper will briefly discuss the theory, the computer program and some experimental results.

Description of the Algorithm

Polarization variables were chosen to formulate the analysis in preference to Eulerian or Lagrangian variables in order to separate and reduce the non-linearities. In polarization variables, the position of an element is represented by the sum of a vector $\bar{r}_0(t)$ representing the undisplaced position and a vector $\bar{s}(t)$ representing the displaced position of the element.

*Work supported by the U. S. Atomic Energy Commission.

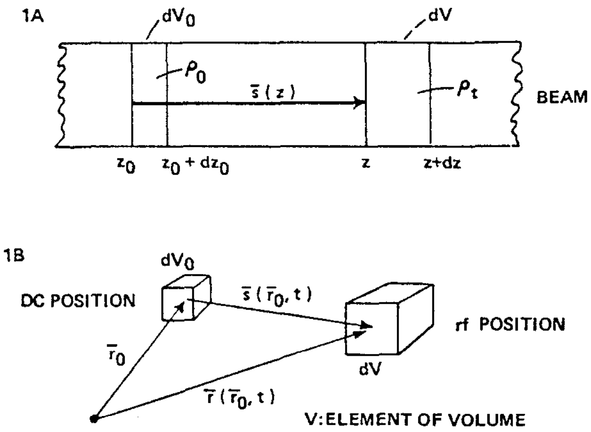

FIG. 1--Sketch showing the undisplaced and displaced volume elements in the one-dimensional and two-dimensional cases.

In the one-dimension case the ratio of the new charge density $\rho(t)$ to the old charge density (ρ_0) is related to displacement $\bar{s}(z)$ by

$$\rho_t/\rho_s = [(1 + \partial\bar{s}(z)/\partial(z)]^{-1} \qquad (1)$$

where the rf charge density $(\rho) = \rho_t - \rho_0$. $\bar{s}(t)$ can be shown to be the difference between simple small signal analysis and the complicated large signal case. In fact if $\bar{s} = 0$ the formulation is exactly small signal. The basic elements to extend the theory to the second dimension is indicated in Fig. 1b.

Because of the scope of this paper, the major equation to be solved in this analysis is given without proof:[2,3]

$$\frac{d\underset{\sim}{A}(z)}{dz} = -\beta_p \alpha^{3/2} \underset{\sim}{N}(z,\bar{s})\, \underset{\sim}{V}(z)\, \underset{\sim}{A}(z) - \frac{j\eta\alpha_0^{3/2}}{4\nu_0^2\beta_p} e^{j\underset{\sim}{D}z} \underset{\sim}{F}^{-1}\left[\underset{\sim}{K}V_1 + \underset{\sim}{K}_{+2}V_1^*\right] f(z,\bar{s}) \qquad (2)$$

$\underset{\sim}{A}(z)$ is an infinite state vector representing the normalized modes of a klystron at a point z on the axis. The theory discussed in this paper is one-dimensional and therefore neglects transverse effects.

$$\underset{\sim}{A}(z) = \begin{bmatrix} \underset{\sim}{A}_n \\ \vdots \\ \underset{\sim}{A}_2 \\ \underset{\sim}{A}_1 \\ \underset{\sim}{A}_0 \\ \underset{\sim}{A}_{-1} \\ \vdots \end{bmatrix}$$

where $\underset{\sim}{A}_n$ contains the nth harmonic component of the fundamental. $\underset{\sim}{A}_0$ contains the dc beam information. Each $\underset{\sim}{A}_n$ matrix contains two elements. The component of displacement $s_n(z)$ defined by

$$s(z,t) = \sum_{-\infty \le n \le \infty} S_n(z)\, e^{jn\omega t}$$

and the nth component of the polarization velocity defined by the beam velocity, $(u_n(z)/\nu_0)$, where $u_n(z)$ and ν_0 are the harmonic rf velocity and dc beam velocity respectively. These two elements are related to A_n by

$$\begin{bmatrix} u_n(z)/\nu_0 \\ s_n(z) \end{bmatrix} = \underset{\sim}{S}_n e^{-j\underset{\sim}{D}_n z} \underset{\sim}{A}_n(z)$$

The matrices are normalized by beam propagation and reduction factors appearing in the $\underset{\sim}{S}_n$ and $\underset{\sim}{D}_n$ matrices and thus the state vector $\underset{\sim}{A}$ in differential form represents the complete state of the klystron at any point. The state vector is stepped through the interaction region starting at the input cavity, through the first drift section, second cavity and so on through the output cavity. In the absence of any electric fields such as in the drift sections the interaction of the harmonic components among themselves is calculated by the first term in the differential equation $(d\underset{\sim}{A}(z)/dz)$.

β_p is the plasma propagation constant and α is the inverse of the relativistic mass correction factor.

$\underset{\sim}{N}(z, \bar{s})$ is the large signal space charge coupling matrix. Under small signal conditions this term vanishes because $\bar{s} = 0$ and therefore, the first term in the differential equation vanishes.

$\underset{\sim}{V}$ is a partitioned diagonal matrix consisting of 2×2 matrices:

$$\underset{\sim}{V} = \begin{bmatrix} \ddots & & & & \\ & \underset{\sim}{U} & & & \\ & & \underset{\sim}{U} & & \\ & & & \underset{\sim}{U}_0(z) & \\ & & & & \underset{\sim}{U} \\ & & & & & \ddots \end{bmatrix}$$

Where

$$\underset{\sim}{U} = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \quad \text{and} \quad \underset{\sim}{U}_0(z) = \begin{bmatrix} 1 & 1 \\ -\beta_p & -\beta_p \end{bmatrix}$$

The final term in this equation represents the effects of the gap voltage being impressed on the beam and all other terms are propagation and reduction factors except for $f(z, \bar{s})$ which represents the distribution of the gap voltage (V_1) to the various harmonics. The term f_q is the component of $f(z, \bar{s})$ obtained from

$$f(z, \bar{s}) = \sum_{-\infty \leq q \leq \infty} f_q e^{jq\omega t}$$

Here f(z) is a function of the static voltage distribution across the gap and

$$\int_{gap} f(z) = 1$$

The potential at any point (z) in the gap is $V_1 f(z)$. $\underset{\sim}{F}$ is a diagonal matrix specifying the various plasma reduction factors. $\underset{\sim}{K}$ and $\underset{\sim}{K}_{+2}$ are partitioned diagonal matrices whose elements have the form

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

and where the elements in $\underset{\sim}{K}_{+2}$ are displaced two places to the left of $\underset{\sim}{K}$. The voltage (V_1) induced across a gap is

$$V_1 = \frac{(Y_c^* + Y_e^*) A - Y_{e,2} A^*}{(Y_c + Y_e)(Y_c^* + Y_e^*) - Y_{e,2} Y_{e,2}^*}$$

where the terms $Y_e, Y_{e,2}$ and A are defined by various integrals over the gap

$$A = 2j\beta_e I_0 \left\{ \left[\int_{z_0}^{z_1} \bar{f}^*(z, \hat{s}) e^{-j\underset{\sim}{D}z} dz \right] \underset{\sim}{n} \underset{\sim}{A}(z_0) - \beta_p \alpha_0^{3/2} \int_{z_0}^{z_1} \bar{f}^*(z, \hat{s}) e^{-j\underset{\sim}{D}z} \int_{z_0}^{z_1} \underset{\sim}{n}\underset{\sim}{N}(x, \hat{s}) \underset{\sim}{V}(x) \underset{\sim}{A}(x)\, dx\, dz \right\}$$

$$Y_e = -\frac{\beta_e I_0}{2\beta_p} \frac{\eta}{\nu_0^2} \alpha_0^{3/2} \int_{z_0}^{z_1} \bar{f}(z, \hat{s}) e^{-j\underset{\sim}{D}z} \int_{z_0}^{z} e^{j\underset{\sim}{D}x} \underset{\sim}{F}^{-1} \underset{\sim}{n}\underset{\sim}{K}\underset{\sim}{f}(x, \hat{s})\, dx\, dz$$

$$Y_{e,2} = -\frac{\beta_e I_0}{2\beta_p} \frac{\eta}{\nu_0^2} \alpha_0^{3/2} \int_{z_0}^{z_1} \bar{f}^*(z, \hat{s}) e^{-j\underset{\sim}{D}z} \int_{z_0}^{z} e^{j\underset{\sim}{D}x} \underset{\sim}{F}^{-1} \underset{\sim}{n}\underset{\sim}{K}_{+2} f(x, \hat{s})\, dx\, dz$$

Y_e determines the beam loading admittance, $Y_{e,2}$ is a small correction to the beam loading occurring only in the large signal case, A is the state vector at z. Y_c is the cavity admittance. The integrals above depend on the value of $f(z, \bar{s})$ which is a function of $\hat{s}(t)$. The displacement $\hat{s}(t)$ depends on the state vector A(z) and the state vector depends on the voltage (V_1) across the gap. Therefore we have a circular definition in which V_1 determines V_1. Upon entering a gap a calculation of the small signal values of Y_e, $Y_{e,2}$ and V_1 are made from

$$Y_e = \frac{I_0 \beta_e}{2\beta_p F_1} \frac{\eta}{\nu_0^2} \alpha_0^{3/2} \int_{z_0}^{z_1} \int_{z_0}^{z} f_0^*(z) f_0(x) [1, 1] e^{-j\underset{\sim}{D}(z-x)} \begin{bmatrix} 1 \\ -1 \end{bmatrix} dx\, dz$$

$$V_1 = \frac{2j\beta_e I_0}{Y_c + Y_e} \int_{z_0}^{z_1} f_0^*(z) [1, 1] e^{-j\underset{\sim}{D}z} dz\, \underset{\sim}{A}_1(z_0)$$

Knowing V_1, the differential equation is stepped through the gap and A(z) is determined across the gap by integrating the differential equation. Large signal values for Y_e, $Y_{e,2}$ and V_1 are thus calculated. Upon convergence of V_1 the state vector at the end of the gap is used to continue the problem through the following drift space. The problem is continued to the output gap where once the voltage has converged a calculation of power output is made

$$\text{Power output } (P_0) = \frac{1}{2} \left| Y_e + Y_c \right| V_1 V_1^*$$

This theory has several nice features. The main advantage is the inclusion of non-linear harmonic interactions characteristic of large signal, high efficiency klystrons. Using polarization variables additionally provides two other advantages. Electron overtaking is automatically taken care of because of the relation of the displaced and undisplaced charge elements. Analysis is made using displaced elements but all the summations are taken over undisplaced elements. Secondly, the same variables are used for both drift and gap regions making it not necessary to change from time based variables in the drift sections to space oriented variables in the gap.

SLACKLY, SLAC Klystron Interaction Program

SLACKLY, the code name for the Center's large signal relativisitic klystron program was designed and written in 1970 by D. L. Russell and R. L. Stringall of SLAC. The program is written in Fortran IV and makes use of five other subprograms.

The displaced position referenced to the undisplaced position for one phase angle of the rf cycle is graphically demonstrated in Fig. 2. The 45° line indicates the small signal case where $\bar{s}(t) = 0$ or $\bar{r}_0(t) = \bar{r}_0(t) + \bar{s}(t)$ and the dashed lines represent the undisplaced location of the 4th and 5th cavities, in this klystron design. The 5th cavity is the output. Ideally to have maximum efficiency for every phase angle of the rf cycle the plot of $\bar{r}_0(t) + \bar{s}(t)$ should have a slope of zero in the output, i.e., for 100% efficiency no electrons should leave this gap region.

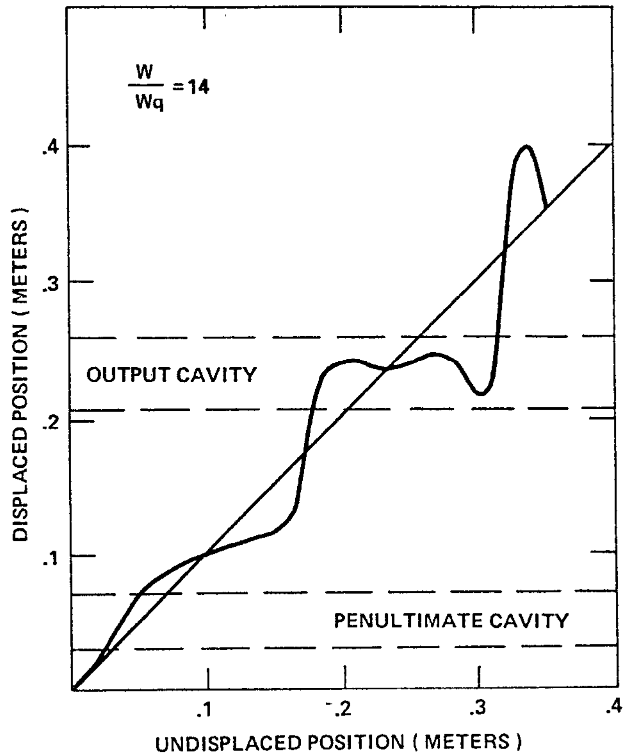

FIG. 2--Displacement curve (single phase angle).

This plotting feature of the program is extremely valuable for establishing the output gap when designing a klystron.

Figure 3 gives a typical phase diagram of elements at the center of the output gap. The two plots demonstrate the effect of beam loading in the output in a slightly overdriven case. The plot where $V_5 = 1.2\ V_0$ is determined by the normal output impedance and when $V_5 = .1\ V_0$ the output cavity Q was decreased to 5% of normal giving essentially no interaction of gap voltage with the beam.

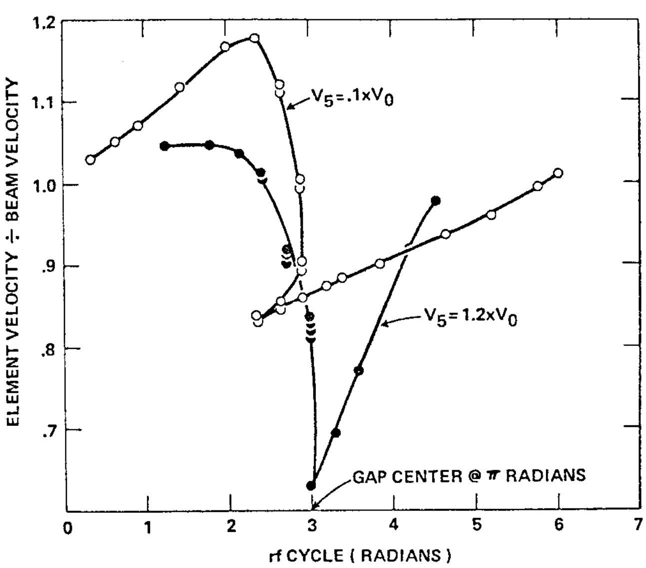

FIG. 3--Element phase diagram.

Comparison of Theory and Experimental Results

Excellent correlation has been established between calculated and fabricated designs using the same parameters for the interaction region up to voltages of 300 kV with perveances of $2.0 \times 10^{-6} A/V^{3/2}$. Calculated results at voltages greater than 400 kV appear to be questionable due to the relativistic correction approximations used and are being studied for improvement.

Efficiency curves are shown in Fig. 4 for both calculated and measured results on the SLAC 3431 klystron.[4] These values are plotted against the drive power. A maximum efficiency of 52% was calculated and mean of 48.6% was measured in test on a sample of 12 klystrons run in electromagnets. This tube operates at a frequency of 2856 MHz, a beam voltage of 270 kV, a microperveance of $2.02 \times 10^{-6}\ A/V^{3/2}$. 37 MW of rf power output was obtained on an average.

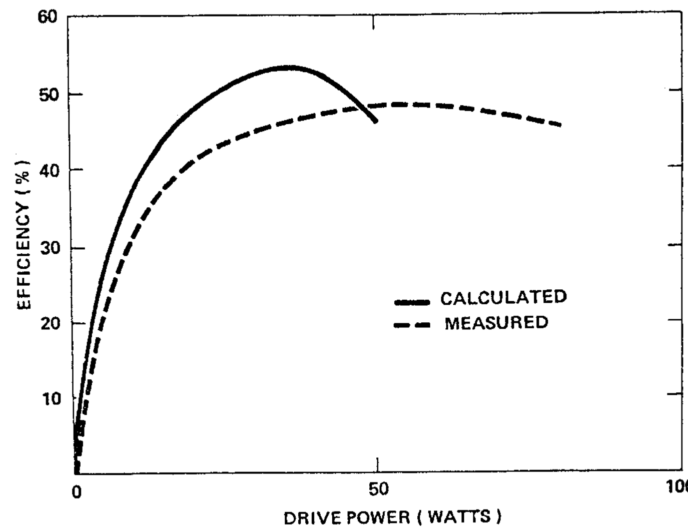

FIG. 4--Computer comparative results.

It was unfortunately not possible to take full advantage of the computed and measured efficiency of the 3431 SLAC klystron because marginal stability was obtained at the output gap. Excessive gradients at the gap noses caused occasional rf breakdown. Decreased rf fields gradients at the output gap eliminated the breakdown problem, but also reduced the coupling coefficient, resulting in a loss in efficiency to approximately 45% in electromagnet.

The use of the SLACKLY program in the redesign of the klystron indicated that the theoretical efficiency could be increased to between 58 and 60% by the use of a much longer interaction region. Based on these results an experimental program was undertaken to demonstrate the feasibility of obtaining better than 40 MW peak power at 270 kV, and approximately 60 MW at 300 kV. Several tubes have been built and tested to verify the accuracy of the program's predictions of these higher powers. To date, we have measured powers in excess of 42 MW at 270 kV and in excess of 55 MW at 300 kV, at efficiencies of between 52 and 54%.

These results were obtained by using low rf gradients at the output gap and no instabilities were observed. A study of materials which would allow us to increase the voltage gradient and the coupling coefficient is now under way in the hope of achieving further efficiency increases.

Good correlation has been also obtained in the application of this computer program to the design of recirculation linear accelerator klystrons.

Conclusion

A relativistic theory including large-signal nonlinear harmonic interactions, electron overtaking, and the same variable system for both drift and gap regions has been developed. A computer program, SLACKLY, based on that theory has proved a most useful tool in the design and understanding of klystron interaction problems.

Confirmation of the validity of the SLACKLY program was first obtained with the analysis of the SLAC 30 MW klystrons now in use in the accelerator. On the basis of the correlation observed a klystron redesign was undertaken to achieve efficiencies well in excess of 50% at a microperveance of 2. Experimental verification of the accuracy of the computer program has again been obtained through the tests of the first 3 sample tubes built following the SLACKLY program design.

It is expected that SLACKLY will give us still better understanding of harmonic interactions by the use of the plotting features which can be produced on-line or as the output of the computer program. We are planning further klystron efficiency improvements through SLACKLY, through the improvement of materials to give us better rf breakdown voltage characteristics and through further improvements of the beam characteristics which can be detained using the SLAC electron gun design computer program.[5]

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