

AN INVESTIGATION
OF THE
VERY INCOMPLETE MEISSNER EFFECT*

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Since the discovery by Meissner and Ochsenfeld¹ that a magnetic field is expelled from the bulk of a superconductor in the transition from the normal to the superconducting state, this effect (named the Meissner effect) has been considered a crucial property of superconductivity. Small deviations from complete flux exclusion have generally been observed, starting with Meissner and Ochsenfeld¹ who noted that up to 10% of the flux remained frozen into their 3mm diameter Pb and Sn cylinders.

Several explanations for existence of the trapped flux (incomplete Meissner effect) have been proposed. In the first one, Mendelssohn² postulated inhomogeneity in the form of a multi-connected system of thin filaments having critical fields above that of the majority of material within the superconductor. The high critical fields of these connected filaments, known as the Mendelssohn sponge, can be caused by strains, impurity concentration gradients, or lattice imperfections. If such a specimen is placed in a magnetic field sufficient to make it entirely normal, and the field is subsequently reduced, the anomalous regions will become superconducting first, trapping flux by virtue of their

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connectivity. A second explanation can be made analogously by assuming a distribution of transition temperatures. A third explanation valid even for completely homogeneous superconductors has been proposed by Rabinowitz³, who pointed out that cooling which proceeds from the outside, coupled with low bulk thermal conductivity initiates the superconducting transition at the outside. The superconductor is thereafter multiply-connected, which prevents flux in the internal macroscopic normal regions from escaping as these regions shrink to microscopic size. This mechanism should be valid from the smallest external fields to fields on the order of the critical field; near the latter, flux pinning, viscosity and flux flow will play important roles. A fourth, and most general, explanation is valid even for homogeneous superconductors cooled slowly and uniformly. For any magnetic field below the value of the critical field at a given bath temperature, the specimen must enter the intermediate state (due to magnetic field gradient) for Type I or the mixed state for Type II as the superconducting critical fields increase from zero at the transition temperature T_c to their final values at the bath temperature. The slow and uniform cooling ensures nearly thermodynamic equilibrium, resulting in an almost uniform lattice of normal regions containing flux trapped within a network of multiply-connected superconductor. Similarly, when a superconductor is held below T_c in a field above the critical magnetic field, as the external field is reduced Type II superconductors must pass through the mixed state while Type I superconductors pass through the intermediate state. Flux trapping takes place in both cases because the superconductor is multiply connected, and there is no necessity to invoke the Mendelssohn sponge.

It is clear that several mechanisms exist which theoretically allow significant flux trapping even in superconducting geometries which are initially simply-connected. This paper reports experimental evidence for stable trapping of dipole, quadrupole, and sextupole fields transverse to the axes of superconducting cylinders of Pb, Nb, and Nb₃Sn. Not only are large amounts of flux trapped, but the field magnitudes and shapes are preserved to reasonable accuracy. The magnetic fields sensed by Hall probes (2.5 x 3.2 x 0.5 mm) on the outside surfaces of all samples, and inside the hollow samples indicate that centrally there is less than 5% (occasionally up to 10%) flux exclusion, i. e., virtually no Meissner effect in these materials from low field levels to fields $\sim H_c$ for Pb and Nb. Our samples are fairly large, with characteristic outside diameters of several cm. The hollow samples have wall thicknesses of a few mm. No attempts were made to anneal the materials after machining.

Although the work of Campbell, et al⁴, as well as that of many prior investigators indicated that highly hysteretic magnetization curves are possible, previous research⁵⁻⁷ in the trapping of magnetic fields was primarily directed at the trapping of solenoidal magnetic fields parallel to the axis of a hollow cylinder. To our knowledge no one before us has stored, or suggested trapping, a multipole magnetic field transversely to the axis of a hollow superconducting cylinder.⁸ We have stored fields transverse to the cylinder axis by two methods: cooling the superconductor to 4.2°K with the multipole field initially on; or by starting in the superconducting state at 4.2°K and subsequently establishing the external multipole field at a high enough value to drive the field into the superconductor by exceeding the critical field. It should be possible to store any multipole magnetic field configuration by either method. By the latter method,

exceeding $H_c(\text{Pb})$ or H_{c2} provided a quick way to find the largest field we could store. In either case we found that upon removal of the external field, the stored field replicated the original field with reasonable fidelity, better than our 5% limit of measurement for the sextupole as seen in Fig. 1. Distortion was noted in the case of the quadrupole field as shown in Fig. 2 for the worst case.

An illuminating result was obtained with a Nb cylinder of 6.2 cm O.D. x 5.1 cm long. It had a rectangular coaxial cavity of 2.6 cm width x 0.7 cm breadth. The maximum field stored in the cavity when the sample was aligned with the 2.6 cm sides perpendicular to the field was 25 to 50% greater than the maximum field trapped with the sides parallel to the field. We interpret this as an illustration of the effect of flux flow and pinning in determining the maximum storable field. The total pinning force per fluxoid which enters the cavity is greater in the former case in which it passes through a thicker portion of the specimen, retarding the flux motion and acting to give a larger flux density for this direction.

In the case of a solid (6.2 cm diameter, 2.5 cm thick) Nb disk in a uniform dipole field perpendicular to the disk faces, the stored field decreased near the disk edges as expected. A 50% larger maximum field could generally be stored parallel to the disk faces; only occasionally could as large a perpendicular field be stored. When the disk with perpendicularly stored field was rotated in a uniform field of 2300 Oe, we measured a maximum torque of about 4.5×10^7 dyne-cm for the interaction of the trapped dipole moment with the external field. This implies that the magnetic dipole moment is roughly $2.5 \times 10^5 \text{ G-cm}^3$, and that the mean trapped field is approximately 3100 Oe, in fair agreement with the Hall probe measurements of 2700 Oe, since no correction was made for friction.

At higher levels of applied field, there is evidence for significant rotation of the trapped flux.

Transverse dipole fields were also stored in hollow Nb cylinders 4.4 cm I.D. x 4.9 cm O.D. x 13.5 cm long. A 2600 Oe field was only metastably trapped, decaying with large flux jumps. Fields of approximately 2000 Oe were stably stored. Longitudinal dipole fields of 2500 Oe were stably stored.

Sextupole fields of various magnitudes were stored in the hollow Nb cylinders. The maximum (limited by our sextupole rather than the Nb) was 170 Oe at a 2.0 cm radius just inside the Nb cylinder, and radial dependence $H=43r^2$ Oe/cm². There was no detectable change in the magnitude and spatial distribution of the stored sextupole field with respect to the original field. The measurement error of $\pm 5\%$ in this case is due largely to slight displacement of the Hall probes during rotation. Fig. 1 shows the output voltage from one of these probes as it was rotated around the inside periphery of the Nb cylinder.

The maximum quadrupole field stably stored in the Nb cylinders was 1800 Oe with a gradient of 830 Oe/cm. In this case there is a slight distortion of the field with pipping at the peaks as shown for the worst case in Fig. 2. The distortion vanishes at smaller radii as shown. The inner radius signal is larger because this Hall probe is more sensitive. Pipping was not observed in all the Nb cylinders nor was it observed in the Pb or Nb₃Sn cylinders. The maximum storable field in a Nb₃Sn cylinder of 1.67 cm I.D. could not be reached by our quadrupole. The Nb₃Sn easily stored a quadrupole field of 2300 Oe/cm gradient.

In the case of Pb, both hollow and solid cylinders were used. The stored fields were stable, with a maximum field perpendicular to the cylinder axis of 290 Oe, a maximum field parallel to the axis of 500 Oe. The dipole field trapped

transversely to the cylinder axis appears to be similar for the hollow and solid cylinders as judged by the external fields of both. The Pb and Nb cylinders were made from greater than 99.9% purity stock material.

The practical consequences of utilizing the very incomplete Meissner effect for the replication and storage of multipole magnetic fields are manifold.⁹ Numerous applications to devices in which high magnetic fields are required, but which are limited by stringent space and/or weight limitations, is inviting. Application to high energy particle beams at high-energy accelerators is apparent. After detailed studies of field fidelity and stability are conducted, the technologies of electron microscopy and nuclear magnetic resonance could well benefit. In addition the ability to replicate a complex field pattern in a simply machined or vapor deposited superconducting shape implies the need to construct only one expensive high precision magnet which could then be used to charge the inexpensive replicating devices in the analogous way that replica gratings are produced from one master optical grating. Obviously extensive investigations of replicating fidelity are still to be carried out.

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References

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Figure Captions

- Fig. 1. Radial component of magnetic field inside Nb cylinder versus azimuthal angle showing original and stored sextupole field.
- Fig. 2. Radial component of magnetic field inside Nb cylinder versus azimuthal angle showing original and stored quadrupole field for the case of largest observed distortion.