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## Introduction

The position of the closed orbit in SPEAR is affected by the field errors and the alignment errors in the magnets. Deviations of the beam orbit from the ideal position are detected by the 20 position monitors located at approximately every quarter betatron wavelength along the orbit. The closed orbit errors as measured by these monitors can be corrected by using appropriate localized beam bumps at some of the magnets chosen as correcting elements.

A computer program MOVQ has been written to find an "optimum" set of magnets to be used as correctors and to calculate the strength of the beam bump for each corrector. The method of corrector selection is the same as that used by Autin and Bryant ${ }^{1}$ for the ISR orbit correction at CERN. The values of the field strength for the correctors are chosen such that the sum of the squares of the orbit deviations is minimized. This program has been used successfully for SPEAR orbit corrections. The purpose of the report is to give a brief description of the computational procedure used in MOVQ and to present some results. Other methods of corrector selection can be found in the literatures. $2,3,4$

## Theory

Tife orbit correction vector $\vec{\Delta}_{\mathrm{C}}$ at all the monitors is related to the beam bump $\vec{\theta}$ at all the correctors by:

$$
\begin{equation*}
\vec{\Delta}_{\mathrm{C}}=\mathrm{T} \vec{\theta} \tag{1}
\end{equation*}
$$

where $T$ is a $m \times n$ matrix with $m$ the number of monitors and $n$ the number of correctors; the elements of $T$ are ${ }^{5}$

$$
\begin{equation*}
\mathrm{T}_{\mathrm{ij}}=\frac{\sqrt{\beta_{\mathrm{i}} \beta_{j}}}{2 \sin \pi \nu} \quad \cos \nu\left( \pm \pi+\phi_{\mathrm{i}}-\phi_{j}\right) \tag{2}
\end{equation*}
$$

where $\nu$ is the betatron wave number, $\beta$ is the betatron function, $\nu \phi$ is the betatron phase. The plus sign in front of $\pi$ is used for the case of $\phi_{i}<\phi_{j}$ and the minus sign for $\phi_{i}>\phi_{j}$. The orbit deviation is given by:

$$
\begin{equation*}
\vec{\Delta}=\vec{\Delta}_{0}+\vec{\Delta}_{c} \tag{3}
\end{equation*}
$$

where $\vec{\Delta}_{0}$ is the orbit deviation measured before correction. The sum of the squares of the orbit deviation

$$
\begin{equation*}
S=\sum_{i=1}^{n}\left(\Delta_{0 i}+\Delta_{c i}\right)^{2} \tag{4}
\end{equation*}
$$

We wish to find a vector $\vec{\theta}_{c}$ which minimizes the value of $S$, i.e., $\partial S / \partial \theta_{j}=0$ for $j=1,2, \ldots, n$. Since $\partial \Delta_{c i} / \partial \theta_{j}=T_{i j}$ the condition for $S$ to be a minimum becomes

$$
\begin{equation*}
\widetilde{\mathrm{T}}\left({\overrightarrow{\Delta_{0}}}^{+}+\vec{\Delta}_{\mathrm{c}}\right)=0 \tag{5}
\end{equation*}
$$

where $\widetilde{T}$ is the transpose of $T$.
Thus, the desired beam bump strength vector is given by

$$
\begin{equation*}
\vec{\theta}_{c}=-(\tilde{T} T)^{-1} \widetilde{T} \vec{\Delta}_{0} \tag{6}
\end{equation*}
$$

with the minimum value of $S$ given by substituting this vector into Eqs. (1) and (4).
Computation

For a given operating configuration we calculate the matrix elements of $T$ using the $\beta$ and $\phi$ values at the middle of each magnet. The dimension of $T$ is $20 \times n$ where $n$ is the maximum number of magnets to be used as correctors. The computational procedure in MOVQ is as follows:

1. Scan the $n$ possible correcting magnets and calculate for each magnet the value of S . Remove from the list of correcting magnets the magnet $c_{1}$ which has the least value of $S$, say $S_{1}$.
2. Scan the remaining ( $n-1$ ) correcting magnets and calculate the value of $S$ for each magnet when paired with the magnet $c_{1}$. In addition to $c_{1}$, remove from the list the magnet $c_{2}$ which gives the least value of $S$, say $S_{2}$.
3. etc.

By repeating this procedure $k$ times we can find $k$ magnets $\left(c_{1}, c_{2}, \ldots, c_{k}\right)$ to be used as correctors. Since $S_{n}+1 \leq S_{n}$, this procedure may be terminated when the value of $S_{k}$ is less than some acceptable value $S_{a}$.

## Results

Figure 1a shows the values of horizontal orbit deviations measured at the 20 monitors before orbit correction. The maximum orbit deviation is 13.6 mm and the rms deviation is 7.1 mm .

Figure 2 shows a plot of the predicted rms orbit deviation $\Delta_{\mathrm{rms}}$ and the maximum orbit deviation $\Delta_{\max }$ for six successive scans. It can be seen that the value of $S$ decreases as the number of correcting elements increases. When three or more correctors are used, the value of $\Delta_{\mathrm{rms}}$ becomes comparable to or less than 1.5 mm , the accuracy of the position monitors. Orbit deviations measured by the position monitor for the case of using three bending magnets as correctors are shown in Fig. 1b. It has been found that the differences between the measured and predicted values are within the accuracy of the position monitors.

Similar results have been found for the vertical orbit corrections. For SPEAR operation the value of $\beta$ at the interaction region, $\beta_{\mathrm{y}}{ }^{*}$, varies over a large range. Larger orbit deviations are usually found for lower values of $\beta_{y}{ }^{*}$. Since for low $\beta_{y}{ }^{*}$ configurations $\beta_{y}$ becomes large at the quadrupoles near the interaction region, the use of these magnets for corrections is very effective.

Based on these observations, SPEAR orbit corrections up to date have been made by powering the trim windings on three bending magnets in the SPEAR cells for horizontal orbit corrections and the four radially focussing quadrupole magnets near the interaction regions for both horizontal and vertical corrections. By using these correctors, peak orbit distortions can be reduced to $\pm 3 \mathrm{~mm}$ over a wide range of operating configurations.

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## Acknowledgement

We Wish to thank Drs. L. J. Laslett, G. Lambertson and A. A. Garren of LBL and Professor G. Golub of Stanford University for their many useful discussions. As a result of these discussions, the problem was solved in a much shorter time than would have been possible otherwise.

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FIG. 1--Predicted orbit deviations for different scans.


FIG. 2a--Horizontal orbit deviations before correction (cm).


FIG. 2b--Measured horizontal orbit deviations after corrections using the trim windings on three magnets as predicted by the program (cm).


[^0]:    *Work supported by the U. S. Atomic Energy Commission.
    (To be published in the Proceedings of the 1973 Particle Accelerator Conference, San Francisco, California, March 5-7, 1973)

