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MEASUREMENTS OF INELASTIC ELECTRON SCATTERING CROSS SECTIONS NEAR ONE-PION THRESHOLD *

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ABSTRACT

The cross sections for single-arm inelastic electron-proton scattering were measured near one-pion threshold for six values of q^2 between 0.23 and 1.84 GeV^2. The cross sections for non-resonant pion production were determined and, using a particular formulation of soft-pion theory, the normalized axial vector form factor of the nucleon, $G_A(q^2)/G_A(0)$, was extracted.

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A connection between the slope of the electroproduction cross section at pion threshold and the axial vector form factor of the nucleon has been discussed in various papers.[1] In the early works, a general form for the cross section of any process with a soft pion was related to the cross section for the same process without the pion emission. These considerations are similar in spirit to soft photon emission in electromagnetic bremsstrahlung. Using PCAC, the pseudoscalar nature of the pion leads to a connection between the form factor, G_A, of the axial vector part of the nucleon current and the cross section $ep \rightarrow e\pi^+n$ near threshold. Using a particular theoretical formulation we can determine G_A from the slope of the cross section at threshold. Of course, we could view such a determination of G_A as a means of testing the soft-pion and PCAC assumptions that are used in the theory. When sufficiently accurate measurements of G_A are made with neutrino experiments, electroproduction results will be a strong test of the ideas underlying soft-pion calculations.

During an experiment at SLAC to measure the inelastic electron scattering inclusive cross section, $d^2\sigma/d\Omega dE'$, at an angle of 4°, we took the opportunity to do a high statistics measurement in the vicinity of one-pion threshold. A careful study of the systematic errors involved in this type of measurement was undertaken in order to understand the inherent difficulties with this approach and to obtain a realistic estimate of the experimental uncertainty in extracting the slope. In this kind of experiment the contribution from the $\pi^o p$ final state, typically of the order of 10%, must be accounted for by the theory. Direct measurements of the $\pi^+ n$ final state[2] ease the demands on theory.

Our experiment was done at several different incident energies, E_o, as shown in Table I. The values of q^2, the square of the virtual photon four-momentum, at the $\pi^+ n$ threshold are also given. The incident electron energy resolution was $\pm$ .05%, and the beam intensity was monitored by toroids[3] with an accuracy better than $\pm$ 1%. The beam was incident on a 7 cm liquid hydrogen target, and the positions and angles of the scattered electron after passing through the SLAC 20-GeV spectrometer were measured in proportional wire chambers[4] with resolution of $\pm$ 0.05% in momentum and $\pm$ 0.05 mrad in both projected angles. Electrons were clearly identified with a lead-lucite shower counter. In addition to data taken with the hydrogen target, cross sections were measured with a dummy target of aluminum with approximately the same thickenss in radiation lengths as the full target. In order to minimize the effects of any bin-to-bin nonuniformities, the technique of "scanning" was employed in which the central momentum of the spectrometer was changed in small steps, approximately corresponding to the wire chamber resolution, until the entire region of interest had been scanned. The data from each of these momentum settings were combined to yield the final spectrum. Thus, each point in the spectrum is measured by many wires in the system.

After the cross section from the dummy target was subtracted, the data were radiatively corrected[5-6] to yield the final values for the analysis. Most of the radiative correction to the cross section near threshold is the contribution from the "elastic tail" process $ep \rightarrow ep\gamma$ which was calculated allowing small adjustments within our resolution to obtain agreement with the data just below threshold. Fig. 1 shows the measured cross section and the elastic radiative tail calculation for the spectrum at 13 GeV incident energy.

In addition to the s-wave pion production, there is a contribution to the observed cross section due to production of the $\Delta(1238)$. In order to take this into account, the entire spectrum at each energy was fit to the sum of a smooth background plus resonance shapes for each of the prominent nucleon resonances. The cross section used for the $\Delta(1238)$ was written with explicit p-wave threshold behavior.[7] From these fits, values of amplitude, mass, and width of the resonances were determined for each incident energy.

Using this resonance information the cross sections above one-pion threshold were then fit to the form

$$\frac{d^2\sigma}{d\Omega\, dE'} = A\, \sigma_M\, q^* + \left(\frac{d^2\sigma}{d\Omega d\, E'}\right)_{Res} \qquad (1)$$

where σ_M is the Mott cross section, $\left[\alpha \cos(\theta/2)/2\, E_0 \sin^2(\theta/2)\right]^2$ q^* is the pion momentum in the π^+n center of mass, and $(d^2\sigma/d\Omega dE')_{Res}$ is the sum of the previously determined resonance cross sections.

In this fit there were two adjustable parameters: A, the slope of the cross section at pion threshold, and ΔW, a possible shift in missing mass, reflecting the experimental uncertainty in the position of pion threshold. The results for A and ΔW are listed in Table I. The data and fits are shown in Fig. 2 for the spectrum at 13 GeV.

The errors shown in Table I arise from several sources. Firstly, the statistical error from the fitting procedure was determined by finding the value of A that caused an increase in χ^2 of 1, allowing ΔW to vary freely. This is the standard correlated fit error. Secondly, the value of ΔW was arbitrarily changed by $\pm$ 0.05% of the incident energy, A alone was refit, and the changes in A with respect to the previous best fit values were determined. The larger of these two estimates of the error in A was chosen as reflecting the uncertainty in the missing mass scale. Thirdly, in order to determine the sensitivity of the result to the elastic tail and resonance subtractions, the calculated cross sections for these two processes were changed by $\pm$ 5% and the data fit again. The change in A induced by these causes was used as the error corresponding to a reasonable uncertainty in the calculations. Finally, the error in A arising from the uncertainty in the missing mass scale and the errors from elastic tail and resonance were then combined in quadrature, yielding the errors shown in Table I. The errors shown in parentheses are the statistical errors assuming that ΔW is a known quantity. By far the major source of the quoted error is the uncertainty in ΔW which we have taken equal to the energy resolution. We believe that this procedure yields the best estimate of the errors in the experimental measurement of the slope, A. We note that the present experiment has energy resolution improved by a factor of 2 and statistical errors 3-4 times smaller than the previous measurements of the SLAC-MIT collaboration[8] which were used in the analysis of Nambu and Yoshimura.[1]

In order to evaluate the normalized form factor, $G_A(q^2)/G_A(0)$, A, we used Eqs. 1 and 2 of Nambu and Yoshimura[2,9]. The calculated values of $G_A(q^2)/G_A(0)$ are listed in Table I and shown in Fig. 3. These values are insensitive to the exact values of the vector nuclear electromagnetic form factors. In particular they change very slightly if G_{En} is set equal to zero, in contrast with some analyses in which the pion electroproduction cross section near threshold is used to determine G_{En} assuming a parameterization for G_A.[10]

We have fit the axial form factor by the traditional dipole form similar to that used for the vector form factors:

$$G_A(q^2)/G_A(0) = B/(1 + q^2/M_A^2)^2 \qquad (2)$$

With B set equal to unity, as assumed in the theory, we find $M_A = (1.521 \pm 0.064)$ GeV. If both B and M_A are fit, we find $B = 1.052 \pm 0.065$ and $M_A = (1.449 \pm 0.102)$ GeV.

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9. Note that A is proportional to the strong interaction coupling constant and depends on the vector electromagnetic form factors of the neutron and proton as well as on $G_A(q^2)/G_A(0)$. We used $G_{Mp}/\mu_p = \sqrt{f(q)}/\,(1 + q^2/.71)^2$ where f(q), given in Ref. 5, takes account of the known deviations from the simple dipole expression.

We also used $G_{Mn}/\mu_n = G_{Mp}/\mu_p$, $G_{Ep} = (1-.06q^2)\ G_{Mp}/\mu_p$ and $G_{En} = -\mu_n\ G_{Ep}\ \tau/(1 + 5.6\tau)$ with $\tau = q^2/4M_n^2$. For a discussion of the form factors, see R. Wilson, Elastic Scattering and Resonant Electroproduction, 1971 International Symposium on Electron and Photon Interactions at High Energies, Cornell.

10. See, for example, Y. I. Titov, *et. al.*, Sov. J. Nuc. Phys. **15**, 273 (1972), and A. M. Gleeson, M. G. Grundzik, and J. G. Kuriyan, Phys. Rev. **173**, 1708 (1968).

TABLE I

E_o (GeV)	q^2 (GeV^2)	A (GeV^{-2})	ΔW (MeV)	$G_A(q^2)/G_A(0)$
7	0.23	$1.100 \pm 0.181 (0.031)$	0.2	$0.831 \pm 0.067 (0.011)$
7	0.23	$1.190 \pm 0.187 (0.035)$	2.9	$0.862 \pm 0.067 (0.011)$
7	0.23	$1.236 \pm 0.191 (0.035)$	2.0	$0.877 \pm 0.066 (0.012)$
10	0.47	$0.695 \pm 0.176 (0.024)$	5.4	$0.694 \pm 0.086 (0.011)$
10	0.47	$0.680 \pm 0.173 (0.013)$	3.6	$0.687 \pm 0.086 (0.006)$
13	0.79	$0.464 \pm 0.132 (0.009)$	- 2.0	$0.582 \pm 0.081 (0.005)$
16	1.19	$0.203 \pm 0.089 (0.010)$	3.3	$0.399 \pm 0.092 (0.009)$
16	1.19	$0.162 \pm 0.078 (0.004)$	6.1	$0.361 \pm 0.096 (0.004)$
18	1.50	$0.198 \pm 0.052 (0.010)$	-14.7	$0.388 \pm 0.048 (0.008)$
20	1.84	$0.108 \pm 0.032 (0.005)$	- 8.8	$0.292 \pm 0.041 (0.006)$

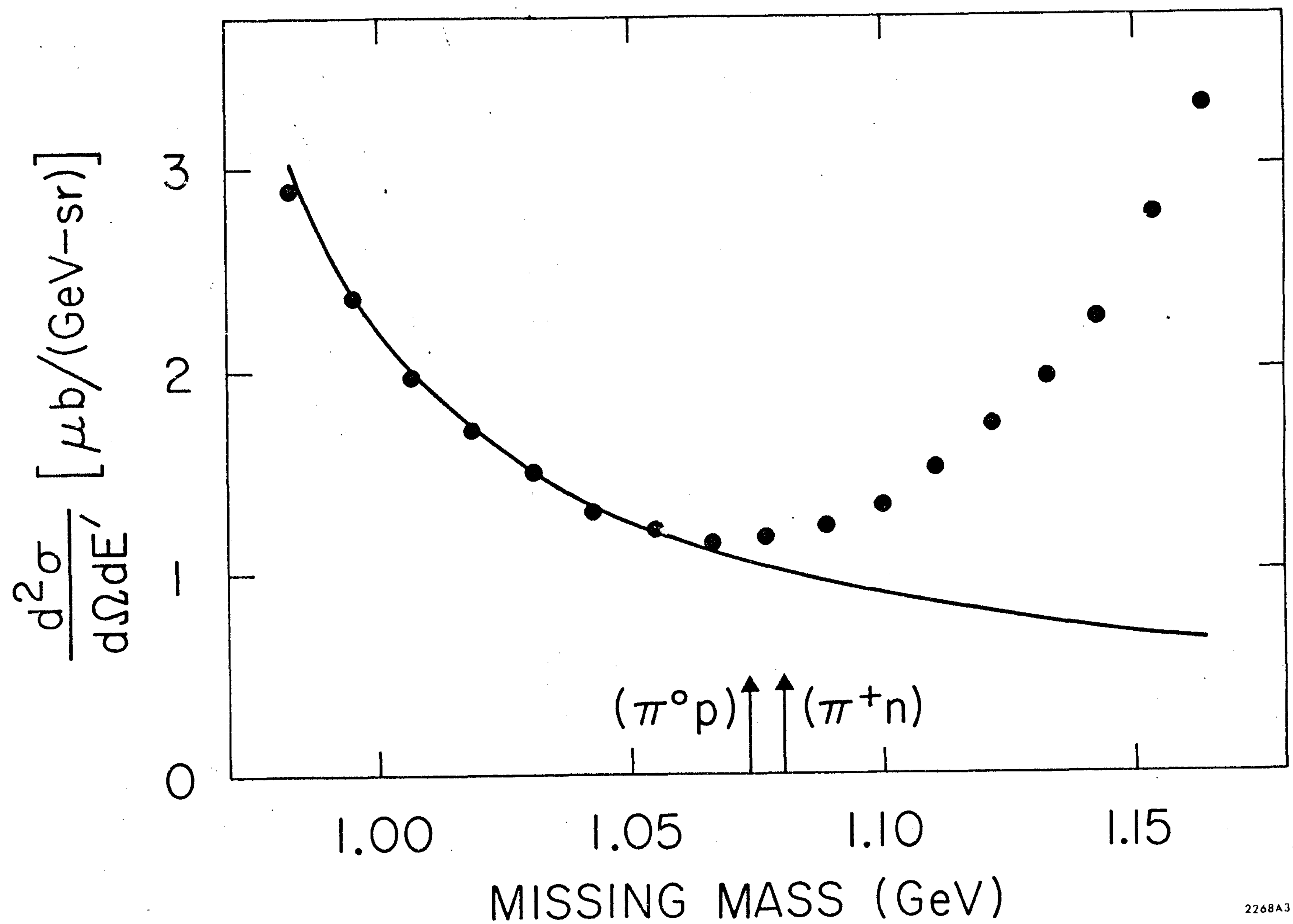
E_o is the incident electron energy, q^2 the four-momentum transfer squared to the nucleon, A is the fitted slope, and ΔW the fitted threshold shift. The errors are discussed in detail in the text. $G_A(q^2)/G_A(0)$ is derived from A using the expressions of Nambu and Yoshimura.

FIGURES

1. Experimental values of the electroproduction cross section, $\frac{d^2\sigma}{d\Omega dE'}$, vs. missing mass of the unobserved final state, for the 13 GeV data. The line is the calculated elastic tail contribution. The errors on the cross section points are smaller than the symbols.

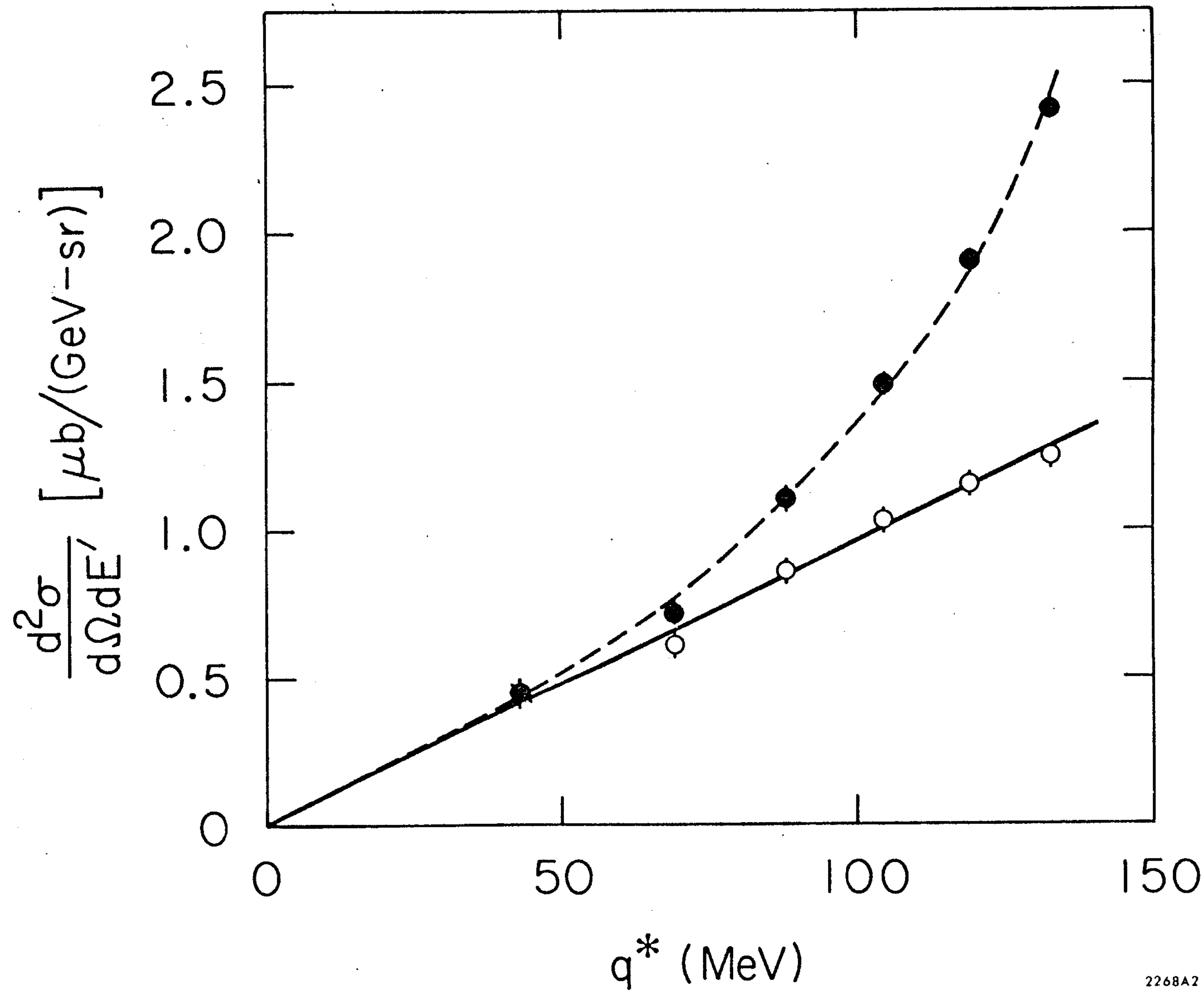
2. Cross sections after radiative corrections for the 13 GeV data vs. q^*, the pion momentum. Closed circles are the measured cross section; open circles are values after the resonances have been subtracted. The lines are the fits to the data. The dashed line is the full fit using Eq. 1, and the solid line is the linear term only.

3. The normalized axial vector form factor of the nucleon, $G_A(q^2)/G_A(0)$, using the Nambu-Shrauner theory, plotted as a function of q^2. The line is a fit to the data using Eq. 2, with $B = 1.052 \pm 0.065$, $M_A = (1.449 \pm 0.102)$ GeV. The errors shown are the ones without parentheses in Table I (see text).



$\frac{d^2\sigma}{d\Omega dE'}$ [μb/(GeV-sr)]
3
2
1
0
(π°p)
(π^+n)
1.00
1.05
1.10
1.15
MISSING MASS (GeV)
2268A3

Fig. 1



$\frac{d^2\sigma}{d\Omega dE'}$ [μb/(GeV-sr)]
2.5
2.0
1.5
1.0
0.5
0
0
50
100
150
q* (MeV)
2268A2

Fig. 2

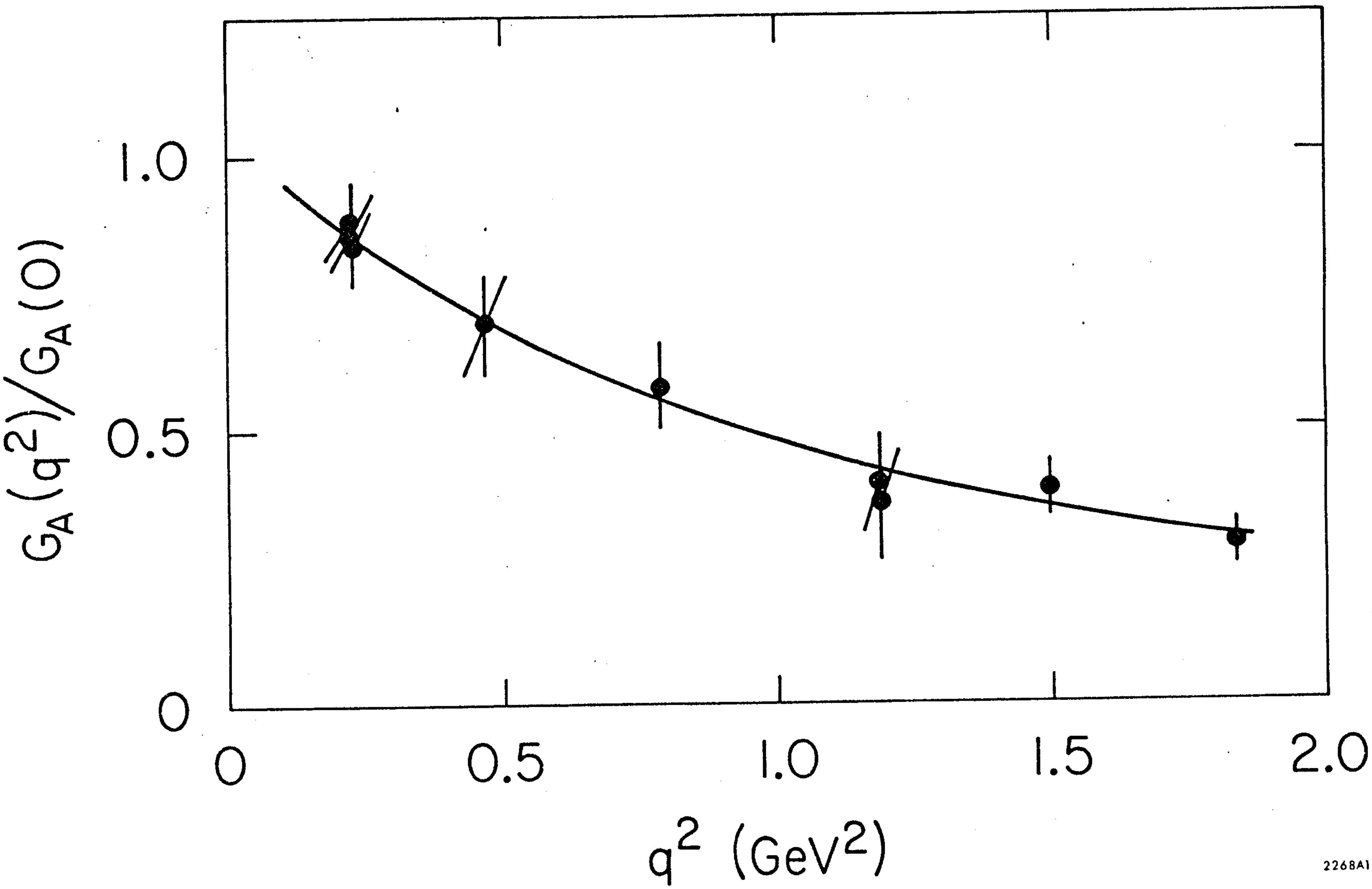


Fig. 3