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UNIFIED MUELLER PICTURE OF DEEP INELASTIC INCLUSIVE LEPTO-PRODUCTION*

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ABSTRACT

The techniques of Mueller analysis are applied to deep inelastic processes such as $e^-p \to e^-hX$ and $\nu p \to \mu^- hX$ where h is a specified hadron and X is unobserved. In the combined Bjorken-Regge limit ($\omega = (2M\nu/Q^2)$ fixed and large) there are three domains: the target fragmentation region, the central plateau, and the current fragmentation region. Only the current fragmentation region provides information not accessible in hadronic collisions. For this region we find an essentially unique Mueller description consistent with assumed Bjorken scaling for inclusive processes. A conclusion of the analyses is that the current fragmentation region should contain three segments including a plateau of length $\sim \log (Q^2/M^2)$. The Mueller description is compared with parton, multiperipheral and other models.

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1. INTRODUCTION

Soon after the proposal of Bjorken[1] that deep inelastic electro-(or neutrino-) production should scale as a function of $\omega = 2M\nu/Q^2$, Regge concepts were applied to the domain of large ω by assuming that the Regge and scaling limits could be interchanged.[2] In this paper we propose a unified treatment of single particle inclusive lepto-production in the Bjorken limit at large ω using Regge concepts as applied by Mueller[3] to inclusive hadronic reactions.

For the total cross-section of virtual transverse photons on nucleons,[4] the Regge expansion for large ω in the deep inelastic scaling region has been shown to be:[2]

$$\nu\sigma^T = \sum_j \tilde{\beta}^j_{\gamma_T}\, \omega^{\alpha_j} \beta^j_N \tag{1. 1}$$

where $\tilde{\beta}^j_{\gamma_T}$ is the virtual transverse photon-Regge vertex (with the Q^2 dependence removed), β^j_N is the Regge coupling to the target, and α_j is the Regge intercept. (A similar expansion holds for $\nu(\sigma^T + \sigma^L)$.) Similarly for the single particle inclusive cross-section describing target fragmentation, the Regge expansion has been given as:[5]

$$\nu \left\langle E\frac{d\sigma^T}{d^3p}\right\rangle_{\substack{\text{azimuthal}\\ \text{average}}} = \sum_j \tilde{\beta}^j_{\gamma_T}\, \omega^{\alpha_j} F_j^{N\to h}(p_\perp\, ,\ p\cdot P) \tag{1. 2}$$

where $p_\perp$ is the transverse component of the four-momentum p of the outgoing hadron, h, and P is the four-momentum of the target. The inclusive Regge vertex, $F_j^{N\to h}$, is the same as that appearing in purely hadronic inclusive reactions. (Again, a similar expression holds for $\nu E\left(d(\sigma^T + \sigma^L)/d^3p\right)$.)

For the fragmentation of the current, we find that a generalization of Bjorken scaling and the Mueller picture lead to the prediction that, for large ω,

$$\nu \left\langle E \frac{d\sigma^T}{d^3p} \right\rangle_{\substack{\text{azimuthal}\\ \text{average}}} = \sum_j \beta_N^j \quad \omega^{\alpha_j} \tilde{F}_j^{\gamma_T \to h} \quad (\omega_1, p_\perp) \qquad (1.\ 3)$$

where $\omega_1 = p \cdot q/(-q \cdot q)$ and q is the four-momentum of the current. Equation (1. 3) is our fundamental result. It yields a description of final state hadrons similar to that proposed by Bjorken[6] on the basis of a parton model, but from a completely different point of view. In particular, we find that the current fragmentation region contains three distinct subregions: two fragmentation regions (i.e. regions of finite length in rapidity) and one plateau of length $\sim \ln(Q^2/M^2)$. Together with the earlier results (Eqs. (1. 1) and (1. 2)), this affords a complete Mueller picture of inclusive lepto-production in the combined Bjorken-Regge domain.

The outline of the paper is as follows: In Section 2 we establish the kinematics. In Section 3 we review the application of the Mueller-Regge picture to the kinematical regions of target fragmentation and the central plateau. In Section 4 we combine the Mueller-Regge expansions with a generalized Bjorken scaling to derive our primary result for the current fragmentation region. The consequences of these expressions are explored in Section 5 and comparison is made with other models for inclusive lepto-production in Section 6.

2. KINEMATICS

Let the momenta of the virtual photon, the target, and the observed hadron be q, P, and p respectively. We then define

$$\begin{aligned} M\nu &= P \cdot q, \qquad \mu \nu_1 = p \cdot q, \qquad M\kappa = P \cdot p \\ Q^2 &= -q^2, \qquad \omega = 2M\nu/Q^2, \qquad \omega_1 = 2\mu\nu_1/Q^2 \end{aligned} \qquad (2.\ 1)$$

where M is the target mass and μ is the mass of the observed hadron. In any frame moving parallel to the virtual photon we can parameterize the momenta as:

$$
\begin{aligned}
q &= Q\,(\sinh y_1,\ 0,\ 0,\ -\cosh y_1) \\
P &= M\,(\cosh y_2,\ 0,\ 0,\ \sinh y_2) \\
p &= (\mu_\perp \cosh y,\ p_x,\ p_y,\ -\mu_\perp \sinh y)
\end{aligned}
\tag{2. 2}
$$

where $\mu_\perp^2 = \mu^2 + p_\perp^2 = \mu^2 + p_x^2 + p_y^2$. Then we have

$$
\begin{aligned}
M\nu &= MQ \sinh (y_1 + y_2) \\
\mu\nu_1 &= \mu_\perp Q \sinh (y_1 - y) \\
M\kappa &= \mu_\perp M \cosh (y_2 + y)
\end{aligned}
\tag{2. 3}
$$

In the laboratory frame, $y_2 = 0$ and $Q \sinh y_1 = \nu$ or $y_1 \sim \ln 2\nu/Q$. The outgoing hadron is confined to a region of phase space given (in the high energy limit) by:

$$
\ln \frac{\mu_\perp}{M} \le y \le \ln \frac{s}{\mu_\perp^2} + \ln \frac{\mu_\perp}{M}
\tag{2. 4}
$$

where $s = (\omega - 1)Q^2 + M^2$. Anticipating our results with the Mueller-Regge analysis, we shall divide this rapidity interval into five regions when both ω and Q^2 are large. The regions are defined as:

$$
\begin{aligned}
&\text{(I)} && y - y_{min} = \zeta, \text{ where } y_{min} = \ln\left(\frac{\mu_\perp}{M}\right) \text{ and } \zeta \text{ is finite [target fragmentation]} \\
&\text{(II)} && y - y_{min} = \beta \ln\omega,\ 0<\beta<1 \text{ [target plateau]} \\
&\text{(III)} && y - y_{min} = \ln\omega + \zeta,\ \zeta \text{ finite} \\
&\text{(IV)} && y - y_{min} = \ln\omega + \beta\ln(Q^2/\mu_\perp^2),\ 0<\beta<1 \\
&\text{(V)} && y - y_{max} = -\zeta, \text{ where } y_{max} = \ln(s/\mu_\perp M) \text{ and } \zeta \text{ is finite.}
\end{aligned}
\tag{2. 5}
$$

Using equation (2. 3) we find for the values of the invariants $2\mu\nu_1$ and $2M\kappa$ the following:

$$
\begin{aligned}
&\text{(I)} && 2\mu\nu_1 \sim \omega Q^2 e^{-\zeta} \\
& && 2M\kappa \sim \mu_\perp^2 e^{\zeta} + M^2 e^{-\zeta}
\end{aligned}
$$

$$\text{(II)} \quad 2\mu\nu_1 \sim Q^2\omega^{1-\beta}$$

$$2M\kappa \sim \mu_\perp{}^2\omega^\beta$$

$$\text{(III)} \quad 2\mu\nu_1 \sim Q^2 e^{-\zeta}$$

$$2M\kappa \sim \mu_\perp{}^2\omega e^{+\zeta}$$

$$\text{(IV)} \quad 2\mu\nu_1 \sim \mu_\perp Q\left[(Q/\mu_\perp)^{1-2\beta} - (Q/\mu_\perp)^{2\beta-1}\right] \tag{2. 6}$$

$$2M\kappa \sim \mu_\perp^2\omega(Q/\mu_\perp)^{2\beta}$$

$$\text{(V)} \quad 2\mu\nu_1 \sim -Q^2 e^{-\zeta}$$

$$2M\kappa \sim \omega Q^2 e^{-\zeta}$$

These five domains cover the entire final particle phase space.

The experimentally measured cross-section, integrated over the azimuthal angle (associated with the outgoing hadron), is related to the inclusive structure functions of Drell and Yan[7] by

$$\frac{d^4\sigma}{dQ^2 d\nu\, d\kappa\, d\nu_1} = \frac{4\pi\alpha^2}{Q^4}\,\frac{E'}{E}\left[2\sin^2\left(\frac{\theta}{2}\right)\mathcal{W}_1(\nu, Q^2, \nu_1, \kappa) + \cos^2\left(\frac{\theta}{2}\right)\mathcal{W}_2(\nu, Q^2, \nu_1, \kappa)\right] \tag{2. 7}$$

where E and E' are the initial and final electron momenta and θ is the scattering angle in the laboratory. The integration over the azimuthal angle reduces the number of invariant amplitudes to two. The cross section integrated over the hadronic variables is given in terms of the standard structure functions W_1 and W_2 by the familiar expression:

$$\frac{d^2\sigma}{dQ^2 d\nu} = \frac{4\pi\alpha^2}{Q^4}\,\frac{E'}{E}\left[2\sin^2\left(\frac{\theta}{2}\right)W_1(\nu, Q^2) + \cos^2\left(\frac{\theta}{2}\right)W_2(\nu, Q^2)\right] \tag{2. 8}$$

In terms of virtual photon-hadron scattering, the inclusive and total cross-sections for longitudinal and transverse photons are as follows:

$$\frac{d^2\sigma^T}{d\nu_1 d\kappa} = \frac{8\pi^2 M\alpha}{2M\nu - Q^2} \mathscr{W}_1(\nu, Q^2, \nu_1, \kappa)$$

$$\frac{d^2\sigma^L}{d\nu_1 d\kappa} = \frac{8\pi^2 M\alpha}{2M\nu - Q^2}\left[-\mathscr{W}_1(\nu, Q^2, \nu_1, \kappa) + \frac{\nu^2 + Q^2}{Q^2} \mathscr{W}_2(\nu, Q^2, \nu_1, \kappa)\right]$$

$$\sigma^T = \frac{8\pi^2 M\alpha}{2M\nu - Q^2} W_1(\nu, Q^2)$$

$$\sigma^L = \frac{8\pi^2 M\alpha}{2M\nu - Q^2}\left[-W_1(\nu, Q^2) + \frac{\nu^2 + Q^2}{Q^2} W_2(\nu, Q^2)\right]$$

The inclusive cross sections may be cast in a more convenient form using the relation:

$$\frac{d^3p}{E} = \frac{\mu}{(\nu^2 + Q^2)^{\frac{1}{2}}} d\nu_1 d\kappa\, d\phi \qquad (2.\ 11)$$

Together with Eqs. (2. 9) and (2. 10) this gives

$$\left\langle \frac{E}{\sigma^T} \frac{d\sigma^T}{d^3p} \right\rangle_{\text{az. av.}} = \frac{1}{2\pi} \frac{E d\sigma^T}{\sigma^T p_\perp dp_\perp dp_z} = \frac{1}{2\pi} \frac{(\nu^2 + Q^2)^{\frac{1}{2}}}{\mu} \frac{\mathscr{W}_1(\nu, Q^2, \nu_1, \kappa)}{W_1(\nu, Q^2)}$$

$$\left\langle \frac{E}{\sigma^L} \frac{d\sigma^L}{d^3p} \right\rangle_{\text{az. av.}} = \frac{1}{2\pi} \frac{E d\sigma^L}{\sigma^L p_\perp dp_\perp dp_z} = \frac{1}{2\pi} \frac{(\nu^2 + Q^2)^{\frac{1}{2}}}{\mu}$$

$$\times \frac{\left[-\mathscr{W}_1(\nu, Q^2, \nu_1, \kappa) + \frac{\nu^2 + Q^2}{Q^2} \mathscr{W}_2(\nu, Q^2, \nu_1, \kappa)\right]}{\left[-W_1(\nu, Q^2) + \frac{\nu^2 + Q^2}{Q^2} W_2(\nu, Q^2)\right]} . \qquad (2.\ 12)$$

The notation "az. av." means an average over the azimuthal angle associated with the outgoing hadron.

3. MUELLER-REGGE ANALYSIS FOR TARGET FRAGMENTATION AND THE CENTRAL PLATEAU

The application of Muellerism to the regions of target fragmentation and the central plateau in virtual photon-hadron scattering is quite direct. We recapitulate the arguments here for the sake of completeness.

For fixed Q^2 and $\nu\to\infty$, we can write a Regge expansion for the transverse and longitudinal total cross-sections (see Fig. 1):

$$\nu\sigma^{T,L}(\nu, Q^2) = \sum_i \beta^i_{\gamma_{T,L}}(Q^2)\,\nu^{\alpha_i}\beta^i_N \qquad (3.1)$$

where $\beta^i_{\gamma_{T,L}}(Q^2)$ and β^i_N are the couplings of the virtual photon and the nucleon, respectively, to the i^{th} Regge pole with intercept α_i. Bjorken scaling requires W_1 and νW_2 to become functions of ω only as Q^2, $\nu\to\infty$ with ω fixed. If the scaling and the Regge regions overlap for ω and Q^2 large, then in this region the Regge expansions for $\nu\sigma^T$ and $\nu(\sigma^T+\sigma^L)$ must become functions of ω only. Consequently,

$$\beta^i_{\gamma_T}(Q^2) \to (Q^2/2M)^{-\alpha_i}\,\tilde{\beta}^i_{\gamma_T} \qquad (3.2)$$

and Equation (3.1) becomes[2]

$$\nu\sigma^T(\nu, Q^2) \underset{Bj}{\to} \sum \tilde{\beta}^i_{\gamma_T}\,\omega^{\alpha_i}\beta^i_N \;. \qquad (3.3)$$

A similar expression holds for $\nu(\sigma^T+\sigma^L)$.

For the single particle inclusive cross-section, the standard Mueller technique can easily be applied to kinematical region (I), which is known as target fragmentation (see Fig. 2). The Regge expansion for fixed Q^2 is:

$$\nu \left\langle E \frac{d\sigma^{T,L}}{d^3p} \right\rangle_{\text{az. av.}} = \sum \beta^i_{\gamma_{T,L}} (Q^2)\, \nu^{\alpha_i} F_i^{N\to h}(p_\perp, \kappa) \tag{3. 4}$$

where $F_i^{N\to h}(p_\perp, \kappa)$ is the same inclusive Regge vertex describing fragmentation of N into h in purely hadronic reactions. In the scaling region at large ω, we can remove the Q^2 dependence from the virtual photon coupling via Eq. (3. 2) which yields[5]:

$$\nu \left\langle E \frac{d\sigma^{T}}{d^3p} \right\rangle_{\text{az. av}} = \sum \tilde{\beta}^i_{\gamma_T}\, \omega^{\alpha_i} F_i^{N\to h}\, (p_\perp, \kappa) \tag{3. 5}$$

Thus, the factorization of Regge singularities implies a generalized Bjorken scaling for target fragmentation. Both the scaling functions and the scaling variables are specified. Moreover, Eq. (3. 5) implies that:

$$\frac{1}{\sigma^T} \left\langle E \frac{d\sigma^{T}}{d^3p} \right\rangle_{\text{az. av.}} \xrightarrow[\text{Regge}]{\text{Bj}} \mathscr{F}_T(\omega, \kappa, p_\perp) \xrightarrow[\omega\to\infty]{} \mathscr{F}_T(\kappa, p_\perp)$$

$$= \frac{F_{\mathbb{P}}^{N\to h}(p_\perp, \kappa)}{\beta^N_{\mathbb{P}}} \tag{3. 6}$$

Again, similar expressions hold for the sum of the longitudinal and transverse cross-sections. In terms of the structure functions $\mathscr{W}_1$ and $\mathscr{W}_2$ defined by

Drell and Yan, Eq. (3. 5) may be re-written as:

$$\begin{aligned} \nu \mathcal{W}_1(\nu, Q^2, \kappa, p_\perp) &\xrightarrow[\text{Regge}]{\text{Bj}} \mathcal{F}_1(\omega, \kappa, p_\perp) \\ \nu^2 \mathcal{W}_2(\nu, Q^2, \kappa, p_\perp) &\xrightarrow[\text{Regge}]{\text{Bj}} \mathcal{F}_2(\omega, \kappa, p_\perp) \end{aligned} \tag{3. 7}$$

These relations for the target fragmentation region have also been derived from the parton model[7] and a light cone analysis[8].

The Mueller-Regge analysis can also be applied to kinematic region (II), which is known as the central hadronic plateau (see Fig. 3). The Regge expansion is:

$$\nu \left\langle E \frac{d\sigma^{T,L}}{d^3p} \right\rangle_{\text{az. av.}} = \sum_{i,j} \beta^i_{\gamma_{T,L}} (Q^2) \nu_1^{\alpha_i} f_h^{ij}(p_\perp) \kappa^{\alpha_j} \beta^j_N \tag{3. 8}$$

for fixed Q^2 with ν_1 and κ large compared to the masses. In the scaling region, we can again use Eq. (3. 2) which yields[5]:

$$\nu \left\langle E \frac{d\sigma^{T}}{d^3p} \right\rangle_{\text{az. av}} = \sum_{i,j} \tilde{\beta}^i_{\gamma_T} \omega_1^{\alpha_i} f_h^{ij}(p_\perp) \kappa^{\alpha_j} \beta^j_N (M/\mu)^{\alpha_i} \tag{3. 9}$$

From Eq. (3. 9) we see that for large Q^2 the Regge expansion is valid only if $\omega_1 = 2\mu\nu_1/Q^2$ is large (i.e. $2\mu\nu_1/Q^2 \gg 1$). As a result, the kinematical region satisfying this restriction, which we have called region (II), extends a distance in rapidity of order $\ln \omega$ from the target fragmentation boundary. Keeping only the Pomeranchuk contributions and using $(2\mu\nu_1)(2M\kappa)/2M\nu \sim \mu_\perp^2$ (valid in this region), we may re-write Eq. (3. 8) as:

$$\frac{1}{\sigma^T} \left\langle E \frac{d\sigma^T}{d^3p} \right\rangle_{\text{az. av.}} \xrightarrow[\omega \to \infty]{} \mathcal{F}_T(p_\perp) \tag{3. 10}$$

which implies a flat plateau of length log ω in rapidity and height equal to that of the plateau in hadronic reactions.

4. MUELLER-REGGE ANALYSIS OF CURRENT FRAGMENTATION

Using the framework of the Mueller-Regge analysis, we seek to extend the generalized Bjorken scaling found in regions (I) and (II) to regions (III) - (V), which are known as current fragmentation. The aim is to describe the inclusive cross-section in terms of ω and finite variables associated with the outgoing particle and the virtual photon.

We first turn to the problem of determining which structure functions (i.e. what power of ν times $\mathscr{W}_1$ and $\mathscr{W}_2$) should exhibit scaling. As reviewed in Section 3, the Mueller-Regge picture with the assumption of scaling for the total cross-section implies that $\nu\mathscr{W}_1$ and $\nu^2\mathscr{W}_2$, or equivalently $(1/\sigma^T)\langle E\, d\sigma^T/d^3p\rangle_{az.\, av.}$ and $(1/\sigma^T + \sigma^L)\langle E\, d(\sigma^T + \sigma^L)/d^3p\rangle_{az.\, av.}$ scale in regions (I) and (II) as functions of ω and finite hadron variables. Moreover, $(1/\sigma^T)\langle E\, d\sigma^T/d^3p\rangle_{az.\, av.}$ is finite as $\omega \to \infty$. We shall use the energy-momentum sum rule to make plausible the same scaling behavior, i.e. $\nu\mathscr{W}_1$ and $\nu^2\mathscr{W}_2$, in region (V). Consider the inclusive energy sum rule[9]:

$$\sum_i \int E_i \left(\frac{E_i}{\sigma} \frac{d\sigma}{d^3p_i}\right) \frac{d^3p_i}{E_i} = E_{tot} \tag{4. 1}$$

where the sum is over the types of outgoing particles. The energy of a particle in Region (V) is $0(\nu)$ while in region (IV) it is $0(\nu/Q^{2-2\beta})$ with $0<\beta<1$. Since the length of region (IV) in rapidity is $\sim \ln Q^2$, there is no asymptotic contribution to the sum rule from this region unless $E_i/\sigma\, (d\sigma/d^3p_i)$ grows as a power of Q at fixed ω (of course no faster than Q). If $E_i/\sigma\,(d\sigma/d^3p_i)$ is bounded in region (IV) as $Q^2 \rightarrow \infty$ for fixed large ω, then the sum rule is dominated by region (V) and becomes:

$$\sum_i \int_{\text{region (V)}} \frac{E_i}{\nu} \left(\frac{E_i}{\sigma} \frac{d\sigma}{d^3p_i} \right) \frac{d^3p_i}{E_i} = 1 \tag{4. 2}$$

Since $E_i \sim 0(\nu)$ and the integration is over a finite length in rapidity, the sum rule is satisfied most simply if $E_i/\sigma\,(d\sigma/d^3p_i)$ is independent of Q^2, i.e., the invariant cross-section scales as a function of ω. This scaling behavior in region (V), $\nu\mathscr{W}_1$ and $\nu^2\mathscr{W}_2$ functions of ω rather than ν and Q^2, is motivated by the energy sum rule and also implied by the parton model. <u>We shall henceforth assume that $\nu\mathscr{W}_1$ and $\nu^2\mathscr{W}_2$ become independent of Q^2 in regions (III), (IV), and (V) as well as in regions (I) and (II).</u> Our assumption is inconsistent with any model in which the multiplicity grows faster than log Q^2 at fixed ω.

We now write a Mueller-Regge expansion for the current fragmentation region. For fixed Q^2 and $\nu \rightarrow \infty$, the following expansion holds in regions (III) - (V) (see Fig. 4).

$$\nu \left\langle E \frac{d\sigma^{T,L}}{d^3p} \right\rangle_{\text{az. av.}} = \sum \beta_N^i \, \nu^{\alpha_i} \, F_i^{\gamma_{T,L} \rightarrow h} (Q^2, \nu_1, p_\perp) \tag{4. 3}$$

For generalized Bjorken scaling to hold (what we assumed above) $\nu \langle E\, d\sigma^T/d^3p \rangle_{\text{az. av.}}$ must become in the Bjorken limit a function of $\omega = 2M\nu/Q^2$ and <u>finite variables</u> describing the outgoing hadron. We conclude that:

$$\lim_{Bj} F_i^{\gamma_T \rightarrow h}(Q^2, \nu_1, p_\perp) = (Q^2/2M)^{-\alpha_i} \, \tilde{F}_i^{\gamma_T \rightarrow h} (\omega_1, p_\perp) \tag{4. 4}$$

where $\omega_1 = (2\mu\nu_1/Q^2) = (p\cdot q/-q\cdot q)$. All Q^2 dependence in the invariant cross-section is removed through Eq. (4. 4) for the residue. The choice of ω_1 as the scaling variable is dictated by the considerations shown below.

The variable ω_1 appears naturally in the adjacent region (II) where it furnishes one of the large quantities for the double Regge expansion. At the boundary between the current fragmentation and the double Regge regions, ω_1 becomes finite and is, as we shall see, the appropriate variable for connecting the two regions. The requirement that the double Regge expansion of Eq. (3. 9) merge continuously with Eq. (4. 3) in the transition zone implies the following for fixed and reasonably large ω_1:

$$\lim_{Bj} F_i^{\gamma_T \to h}(Q^2, \nu, p_\perp) = (Q^2/2M)^{-\alpha_i} \sum_j (\omega_1)^{\alpha_j - \alpha_i} f_h^{ij}(p_\perp) \, (\mu_\perp^2/2M)^{\alpha_j} \, (M/\mu)^{\alpha_i} \tag{4. 5}$$

The factor $(Q^2)^{-\alpha_i}$ serves, as before, to guarantee Bjorken scaling. The remaining function depends on ω_1 and $p_\perp$ only which forces Eq. (4. 4) at one end of current fragmentation (region III). In the rest of the current fragmentation region, ω_1 remains a finite variable -- non-zero in the finite rapidity regions (III) and (V) and zero in the log (Q^2/M^2) rapidity region (IV). At the phase space boundary, i.e. the edge of region (V), a longitudinal scaling variable (necessarily ω_1) is certainly required, which again implies Eq. (4. 4). Thus, if regions (III), (IV), and (V) are to be described simultaneously as a single current fragmentation region in a Mueller description, it must be in the form of Eq. (4. 4). The relation of the variables ω_1 and κ in the various regions is as follows: in the proton fragmentation region, ω_1 is large and κ finite; in the central plateau ω_1 and κ are both large; and in the current fragmentation region ω_1 is finite and

and κ large. Also, the variable $\omega_1 = p\cdot q/-q\cdot q$, like $\omega = P\cdot q/-q\cdot q$, is dimensionless and contains no arbitrary mass factors. Of course, one could suppose that in addition to $p_\perp$ and ω_1, the residue is a function of some other variable like ν_1/Q or $\nu_1/Q^{3/2}$. Such variables would be non-trivial only over a finite rapidity interval in region (IV) where ω_1 vanishes. The arbitrariness of such dimensionless variables (e.g. defined with arbitrary mass factors, etc.), which are not required elsewhere and which would destroy the unique Mueller description of the current fragmentation region in terms of two finite variables, induces us not to provide for such dependence in the residue. Consequently, our expression for the Mueller-Regge expansion in the current fragmentation region (regions (III) - (V)) becomes:

$$\nu \left\langle E \frac{d\sigma^T}{d^3p} \right\rangle_{\text{az. av.}} = \sum \beta_N^i \, \omega^{\alpha_i} \, \tilde{F}_i^{\gamma_T \to h} (\omega_1, p_\perp), \tag{4. 6}$$

and similarly for $\sigma^T + \sigma^L$. The implications of Eq. (4.6) are examined in the next section.

5. CONSEQUENCES

Since regions (I) and (II) have already been treated in the literature, we focus on regions (III), (IV), and (V) -- the current fragmentation region -- for which we have a single expression given by Eq. (4. 6) or equivalently:

$$\lim_{\text{Bj}} \frac{1}{\sigma^T} \left\langle E \frac{d\sigma^T}{d^3p} \right\rangle_{\text{az. av.}} = \mathscr{F}_T(\omega, \omega_1, p_\perp) \quad \xrightarrow[\omega \to \infty]{} \quad \mathscr{F}_T(\omega_1, p_\perp) \tag{5. 1}$$

From Eq. (2. 6) we see that in the current fragmentation region ω_1 takes on the values:

Region (III): $\omega_1 = e^{-\zeta}$ where $y - y_{min} = \ln\omega + \zeta$, ζ finite

Region (IV): $\omega_1 = (Q/\mu_\perp)^{-2\beta} - (Q/\mu_\perp)^{2\beta-2}$ where $y - y_{min} = \ln\omega + \beta\ln(Q^2/\mu_\perp^2)$, $0<\beta<1$

Region (V): $\omega_1 = -e^{-\zeta}$ where $y - y_{max} = -\zeta$, ζ finite

Note that ω_1 changes sign in region (IV) and is asymptotically zero. Thus in the Bjorken limit the longitudinal dependence of the structure functions vanishes in region (IV). Consequently, a flat plateau of unknown height (possibly zero) and length log (Q^2/M^2) develops between the two finite fragmentation regions (III) and (V). This prediction for current fragmentation was first arrived at by Bjorken[6] using the parton model. We have arrived at the same result using generalized Bjorken scaling and Mueller analysis without the introduction of partons. The resulting distribution of final state hadrons is represented in Fig. 5.

We can exploit our Regge expansion to draw some phenomenological conclusions. Consider the junction of Regions (II) and (III) (ω_1 large and fixed) for the production of $\pi^\pm$. Region (II) is dominated by diagrams with Pomerons in in the $\gamma\gamma$ and $N\bar{N}$ channels. As we move into into Region (III) non-leading terms in the $\gamma\gamma$ channel become important. If there is a $\mathbb{P}$ in the $N\bar{N}$ channel, only $\mathbb{P}' = f$ is permitted in the $\gamma\gamma$ channel. The residue $\tilde{\beta}_f^{\gamma_T}$ appears to be positive in that νW_2 seems to be falling at large ω.[10] Furthermore ISR data on $pp \to \pi^\pm X$ indicate that $f_{\pi^\pm}^{\mathbb{P}f}$ is negative.[11] If these surmises are correct, then the plateaus for $\pi^\pm$ should turn downward as one moves from region (II) to region (III). Of course this may not persist throughout region (III) and we cannot draw any conclusions about the relative heights of the plateaus in regions (II) and (IV).

The energy conservation sum rule places a constraint on inclusive Regge residues.[9] In particular, working in the laboratory frame, we have

$$\sum_i \int^{y_{max}} \mu_{\perp i} \cosh y_i \; E_i \frac{d\sigma^T}{d^3 p_i} \; d^2 p_{\perp i} dy_i = E\sigma^T \tag{5.3}$$

or

$$\sum_i \int^{y_{max}} \mu_{\perp i} \cosh y_i \; \frac{1}{\nu} \sum_j \beta_N^j \, \omega^{\alpha_j} \, \tilde{F}^{\gamma_T \to i} (\omega_1, p_\perp) \, dy \, d^2 p_\perp$$

$$= \sum_j \beta_N^j \, \omega^{\alpha_j} \tilde{\beta}^j_{\gamma_T} \tag{5.4}$$

Using Eq. (2.6),

$$\sum_i \int_0^\infty d\zeta e^{-\zeta} \int d^2 p_\perp \sum_j \beta_N^j \, \omega^{\alpha_j} \tilde{F}_j^{\gamma_T \to i} (\omega_1 = -e^{-\zeta}, p_\perp^2) = \sum \beta_N^j \omega^{\alpha_j} \tilde{\beta}^j_{\gamma_T} \tag{5.5}$$

or

$$\sum_i \int_{-1}^0 d\omega_1 \int d^2 p_\perp \; \tilde{F}_j^{\gamma_T \to i} \; (\omega_1, p_\perp^2) = \tilde{\beta}^j_{\gamma_T} \tag{5.6}$$

and similarly for $\sigma^T + \sigma^L$. Of course only Region (V) contributes to the energy $\sigma^T + \sigma^L$ sum rule in our framework as mentioned above. Eq. (5.6) is analogous to the sum rules for purely hadronic residues which are of the form

$$\sum_i \int_0^1 dx \int d^2 p_\perp F_j^{a \to c} (x, p_\perp^2) = \beta_j^a \tag{5.7}$$

The usual arguments for internal symmetries[12] can be applied to the inclusive photon fragmentation vertices. Thus $\widetilde{F}_{\mathbb{P}}^{\gamma_{T,L}\to h} = \widetilde{F}_{\mathbb{P}}^{\gamma_{T,L}\to \bar{h}}$ where $\bar{h}$ is the anti-particle of h. Similarly $\widetilde{F}_{\rho,\omega}^{\gamma_{T,L}\to h} = -\widetilde{F}_{\rho,\omega}^{\gamma_{T,L}\to \bar{h}}$ while $\widetilde{F}_{f,A_2}^{\gamma_{T,L}\to h} = \widetilde{F}_{f,A_2}^{\gamma_{T,L}\to \bar{h}}$. Thus in the large ω limit π^+ and π^- should occur equally in regions (III), (IV) and (V), with corrections of order $\omega^{-\frac{1}{2}}$.

Our development applies analogously to neutrino production, where we consider cross-sections for "W bosons" with helicity ±1 or 0. CP relates $W^+_{\lambda=1}$ to $W^-_{\lambda=-1}$ etc. In this way we obtain

$$\begin{aligned} \widetilde{F}_{\mathbb{P},f,A_2}^{W^+_\lambda\to h} &= \widetilde{F}_{\mathbb{P},f,A_2}^{W^-_{-\lambda}\to \bar{h}} \\ \widetilde{F}_{\rho,\omega}^{W^+_\lambda\to h} &= -\widetilde{F}_{\rho,\omega}^{W^-_{-\lambda}\to \bar{h}} \end{aligned} \qquad (5.8)$$

where $\bar{h}$ = CPh. From isospin invariance

$$\begin{aligned} \widetilde{F}_{\mathbb{P},f,\omega}^{W^+_\lambda\to h} &= \widetilde{F}_{\mathbb{P},f,\omega}^{W^-_{\lambda}\to h'} \\ \widetilde{F}_{\rho,A_2}^{W^+_\lambda\to h} &= -\widetilde{F}_{\rho,A_2}^{W^-_{\lambda}\to h'} \end{aligned} \qquad (5.9)$$

where $h' = e^{-i\pi I_2} h$. As a consequence

$$\begin{aligned} \widetilde{F}_{\mathbb{P},f,\rho}^{W^+_\lambda\to \pi^+} &= \widetilde{F}_{\mathbb{P},f,\rho}^{W^+_{-\lambda}\to \pi^+} \\ \widetilde{F}_{\omega,A_2}^{W^+_\lambda\to \pi^+} &= -\widetilde{F}_{\omega,A_2}^{W^+_{-\lambda}\to \pi^+}, \quad \text{etc.} \end{aligned} \qquad (5.10)$$

Since the value of $\widetilde{F}_j^{\gamma_{T,L}\to h}$ (or $\widetilde{F}_j^{W^{\pm}_{\lambda}\to h}$) at $\omega_1 \sim 0$ controls the distribution over a rapidity interval of length $\sim \log Q^2$, the contribution to the additive quantum numbers from region (IV) must total zero. This is most easily implemented by having

$$\widetilde{F}_j^{\gamma_{T,L}\to h}(\omega_1 = 0, p_\perp) = \widetilde{F}_j^{\gamma_{T,L}\to \bar{h}}(\omega_1 = 0, p_\perp)$$

implying a zero in the residue for the ρ and ω trajectories.

6. COMPARISON WITH OTHER MODELS

A similar prediction for the current fragmentation region occurs in the parton model,[6,13] which is of course much more specific than the general Mueller-Regge analysis. (Since the parton model obeys generalized Bjorken scaling, the results of our analysis apply.) The current fragmentation region in the parton framework consists of two finite fragmentation regions and a plateau of length $\log(Q^2/M^2)$. The height of the current plateau is equal to that of the proposed plateau in $e^+e^- \to hX$ and the fragmentation regions correspond to the parton (region (V)) and the "hole" (region (III)) where the parton was knocked out by the current. This description of the current fragmentation region (which is valid at large Q^2) is independent of the size of ω unlike the Mueller-Regge analysis where ω must be large.

The multipheripheral[14] and "cut-off" field theory[15] models also yield results which are a special case of the general Mueller-Regge analysis. The multiplicity in these models goes as $\log \omega$ at all Q^2 which implies a current plateau height of zero. The current fragmentation region consists only of the particle or parton knocked out by the current and its "hole"[16] -- scaling cannot be obtained otherwise in these models.[17]

Also, of course, models exist which are incompatible with the assumption of generalized Bjorken scaling or short range order and, hence, exhibit a different structure for the current fragmentation region. These models include those in which the multiplicity from the current fragmentation region increases as a power of Q^2.[18]

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FIGURE CAPTIONS

1. Regge diagram for virtual photon-nucleon total cross section.

2. Mueller diagram for virtual photon + nucleon $\rightarrow$ hadron (h) + anything in the target fragmentation region (I).

3. Mueller diagram for virtual photon + nucleon $\rightarrow$ hadron (h) + anything in the central plateau region (II).

4. Mueller diagram for virtual photon + nucleon $\rightarrow$ hadron (h) + anything in the current fragmentation region (III, IV, V).

5. Schematic representation of the invariant cross-section $\frac{1}{\sigma} E \frac{d\sigma}{d^3p}$ versus rapidity y for single particle inclusive electro- or neutrino-production at large ω and Q^2.

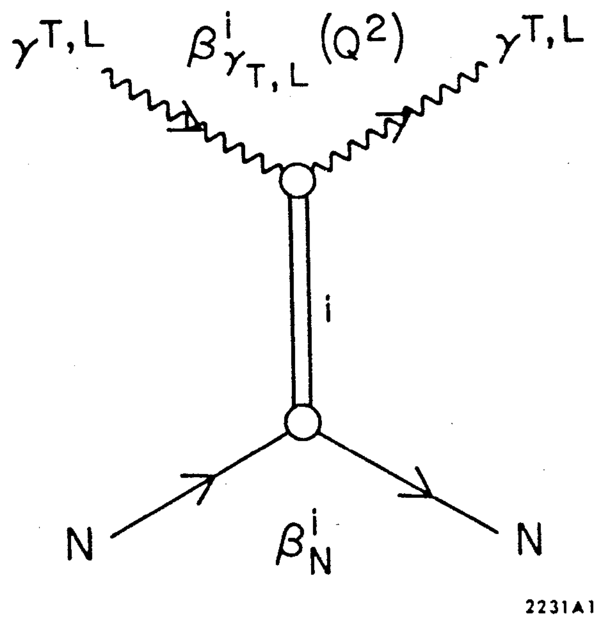


Fig. 1

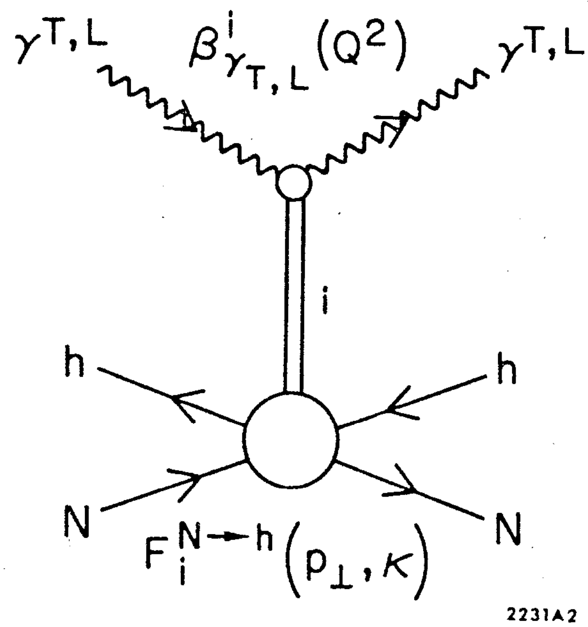
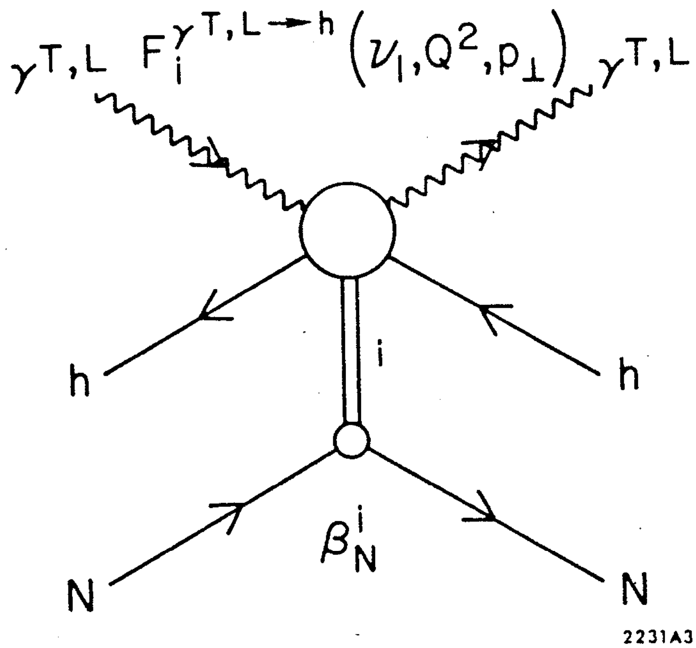

$\gamma^{T,L}$
$\beta^{i}_{\gamma_{T,L}}(Q^2)$
$\gamma^{T,L}$
i
h
h
N
N
$F_i^{N\to h}(p_\perp,\kappa)$
2231A2

Fig. 2



$\gamma^{T,L}$
$F_i^{\gamma^{T,L}\to h}(\nu_1, Q^2, p_\perp)$
$\gamma^{T,L}$
h
i
h
β_N^i
N
N
2231A3

Fig. 3

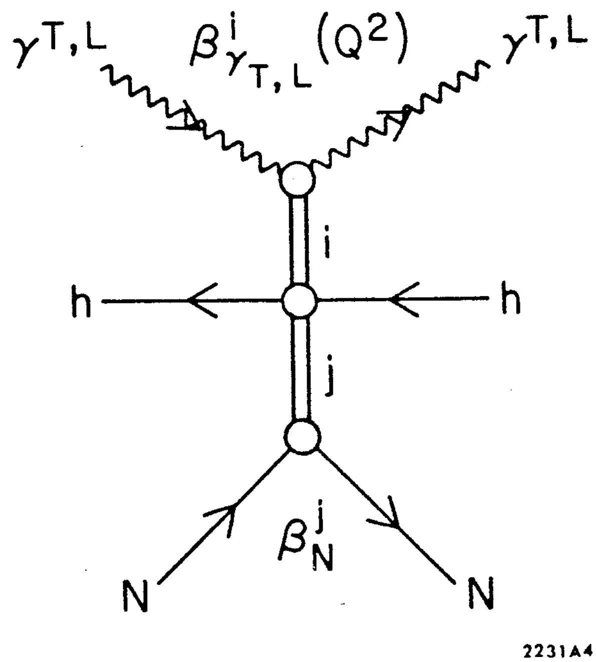
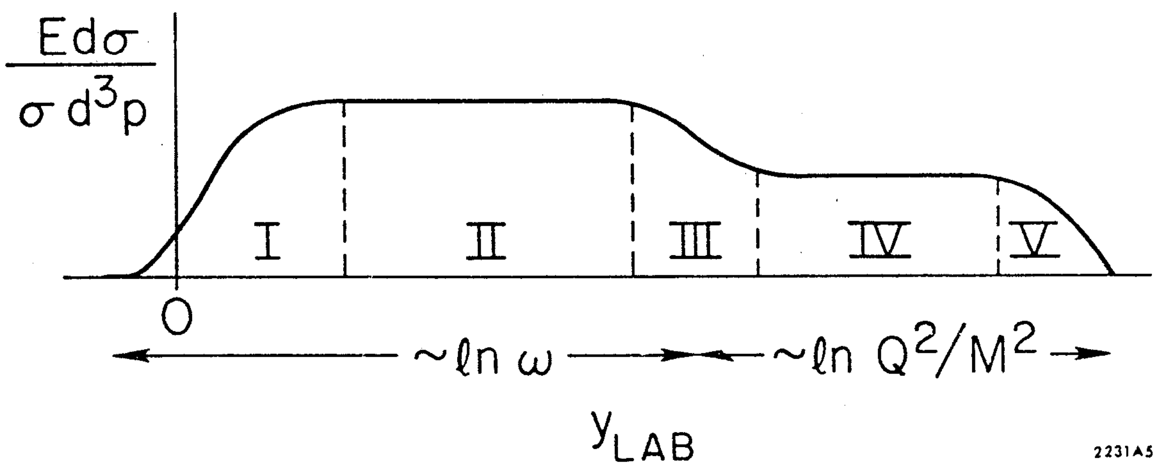


Fig. 4



$\frac{E d\sigma}{\sigma d^3p}$
I
II
III
IV
V
0
$\sim \ell n\, \omega$
$\sim \ell n\, Q^2/M^2$
y_{LAB}
2231A5

Fig. 5