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TWO-PHOTON CROSS SECTION FOR W-PAIR PRODUCTION BY COLLIDING BEAMS*

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ABSTRACT

We calculate the total cross section for $e^+e^- \to e^+e^-W^+W^-$, a process which proceeds via two virtual photons. If the intermediate boson ($W^{\pm}$) has no anomalous magnetic moment and pointlike vertices, this process can yield a larger cross section than the one-photon process $e^+e^- \to W^+W^-$ at sufficiently high energies. Otherwise, the one-photon mechanism is dominant. Numerical results for several values of m_W and the magnetic moment are presented. The effect of the Weinberg theory is shown to be negligible in these results.

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I. INTRODUCTION

In this note we consider the problem of colliding electron beam production of intermediate boson ($W^{\pm}$) pairs if such bosons exist. This is an example of a fundamental process which can be studied for the first time by colliding beam machines with high energies and luminosities that now exist or are under construction.[1] The lowest-order process which proceeds via annihilation into one virtual photon has been well studied ($e^+e^- \to W^+W^-$).[2] Here we examine the higher-order mechanism ($ee \to \gamma^*\gamma^* ee \to eeW^+W^-$) which employs two virtual photons. Several groups[3-5] have studied such two-photon processes in other cases (e.g., pion pair production) and found that their cross sections exceeded the one-photon processes at reasonable colliding beam energies (at $E \sim 1$ GeV for pions). It is reasonable to ask whether such a circumstance of a large ratio of the two-photon to the one-photon process also happens in the case of vector bosons. Our result is that it can if the W boson has no anomalous magnetic moment, but not for nonzero moment. The W boson is assumed to have point-like form factors. The effects of the Weinberg theory[6] of weak and electromagnetic interactions on this process are shown to be negligible. In Section II the two-photon cross section is calculated and the results are displayed numerically for different values of m_W and the anomalous magnetic moment and are compared to the one-photon process. The modifications to the calculation due to the Weinberg theory are discussed in Section III.

II. TWO-PHOTON CROSS SECTION FOR W-PAIR PRODUCTION

In Fig. 1 are shown the diagrams which contribute to $ee \to eeW^+W^-$ to order α^4 in the cross section. One contrasts them with the lowest order diagrams for $e^+e^- \to W^+W^-$, shown in Fig. 2, which proceeds via one virtual photon. Very simple considerations reveal the striking difference between these two processes. In Fig. 2 the photon has $k^2 = 4E^2 = 4W^2 > 0$, which is large and timelike; however, in Fig. 1 for either photon

$$-k^2 \cong 2EE' (1 - \cos\theta') + m_e^2 (E-E')^2/EE' \quad ,$$

where θ' is the angle between the initial and final electrons, and E (E') is the initial (final) electron energy in the lab. If the electrons are detected very close to the forward direction, $\cos\theta' \cong 1$ and $k^2 \cong -m_e^2$, which is small and spacelike. In fact, the photons are essentially real ($k^2 \cong 0$) and one can consider the calculation of these diagrams in two parts: a) study the spectrum of "almost real" photons emitted by the electrons, b) calculate the process $\gamma\gamma \rightarrow W^+W^-$. This general approach, clearly the Weizsäcker-Williams[7] approximation in the context of relativistic quantum theory, has been investigated by many people.[3-5] In particular, Brodsky, Kinoshita, and Terazawa[3] have given an exhaustive discussion of the general two-photon process $ee \rightarrow ee\gamma^*\gamma^* \rightarrow eeX$, and have compared exact calculations with calculations in this "equivalent photon" approximation (e.g., for $ee \rightarrow ee\pi^0$). They have shown that the approximation is a very good one (erring by the order of 10%) and becomes better for a more massive final state X. This is reasonable since the equivalent photon approach is, roughly speaking, an expansion in $k^2/m_X^2 \cong m_e^2/m_X^2$.

Here we simply draw upon the general results of Brodsky et al. applied to the situation at hand. Their central result is that

$$\sigma_{ee\rightarrow eeX}(E) \cong 2\left(\frac{\alpha}{\pi}\right)^2 (\ln E/m_e)^2 \int_{s_{th}}^{4E^2} \frac{ds}{s}\, \sigma_{\gamma\gamma\rightarrow X}(s)\, f\left(\frac{\sqrt{s}}{2E}\right) \tag{1}$$

where s_{th} is the threshold value of s and

$$f(x) = (2+x^2)^2 \ln 1/x - (1-x^2)(3+x^2) \quad . \tag{2}$$

Our problem, then, is to determine $\sigma_{\gamma\gamma \to W^+W^-}(s)$. We assume that the W boson obeys the standard quantum electrodynamics of massive vector bosons as given by Lee and Yang[8] and has no strong interactions (i.e., pointlike form factors). The W boson has a magnetic moment $\mathcal{M} = 1+\kappa$ in units of $e/2m_W$; the quadrupole moment is here not arbitrary but given by $Q = -e\kappa/m_W^2$. Figure 3 gives the relevant diagrams and notation.[9] Note that with the bosons of momenta k_1, k_2, q_1, q_2 are associated polarization vectors $\epsilon_1^\mu, \epsilon_2^\nu, \eta_1^\alpha, \eta_2^\beta$ respectively. The photon-photon center-of-mass frame is chosen for convenience; as usual, $s = (k_1+k_2)^2 = (q_1+q_2)^2$, and θ is the angle between q_1 and k_1 in this frame. E is the colliding beam energy $(p_1^o = p_2^o = E)$ in the lab.

The S-matrix element is then

$$S_{fi} = \frac{-ie^2}{(2k_1^o 2k_2^o 2q_1^o 2q_2^o)^{1/2}} (2\pi)^4 \delta^{(4)}(q_1+q_2-k_1-k_2) \epsilon_1^\mu \epsilon_2^\nu \eta_1^\alpha \eta_2^\beta M_{\mu\nu\alpha\beta} \tag{3}$$

where

$$M_{\mu\nu\alpha\beta} = M^d_{\mu\nu\alpha\beta} + M^c_{\mu\nu\alpha\beta} + M^{sg}_{\mu\nu\alpha\beta}$$

and

$$\begin{aligned} M^d_{\mu\nu\alpha\beta} = \Big\{ & g_{\alpha\rho}(q_1+k_2-q_2)_\mu - g_{\alpha\mu}\left[q_1 \mathcal{M} - (\mathcal{M}-1)(k_2-q_2)\right]_\rho \\ & -g_{\rho\mu}\left[(k_2-q_2)\mathcal{M} - (\mathcal{M}-1)q_1\right]_\alpha \Big\} \frac{-g^{\rho\sigma} + \dfrac{(k_2-q_2)^\rho (k_2-q_2)^\sigma}{m_W^2}}{(k_2-q_2)^2 - m_W^2} \Big\{ g_{\beta\sigma}(k_2-2q_2)_\nu \\ & -g_{\sigma\nu}\left[(k_2-q_2)\mathcal{M} + (\mathcal{M}-1)q_2\right]_\beta + g_{\beta\nu}\left[q_2\mathcal{M} + (\mathcal{M}-1)(k_2-q_2)\right]_\sigma \Big\} \end{aligned} \tag{4}$$

is the direct term (Fig. 3a). Also,

$$M^c_{\mu\nu\alpha\beta} = M^d_{\mu\nu\alpha\beta}(q_1 \longleftrightarrow q_2,\ \alpha \longleftrightarrow \beta)$$

is the crossed term (Fig. 3b) and

$$M^{sg}_{\mu\nu\alpha\beta} = 2\, g_{\mu\nu} g_{\alpha\beta} - g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha}$$

is the seagull contribution (Fig. 3c) required by Bose statistics and gauge invariance, $k_1^{\mu} M_{\mu\nu\alpha\beta} = k_2^{\nu} M_{\mu\nu\alpha\beta} = 0$. This requirement is explicitly satisfied by the above tensor.

Proceeding to the cross section in the standard way, one gets

$$\sigma_{\gamma\gamma \to W^+W^-}(s) = \frac{\pi\alpha^2}{2s} \left(1 - \frac{4m_W^2}{s}\right)^{1/2} \int_{-1}^{1} d\cos\theta\, |\overline{\mathfrak{M}}_{fi}|^2 \tag{5}$$

where

$$|\overline{\mathfrak{M}}_{fi}|^2 = \frac{1}{4} \sum_{\text{spins}} \epsilon_1^{\mu} \epsilon_2^{\nu} \eta_1^{\alpha} \eta_2^{\beta} \epsilon_1^{\mu'} \epsilon_2^{\nu'} \eta_1^{\alpha'} \eta_2^{\beta'} M_{\mu\nu\alpha\beta} M_{\mu'\nu'\alpha'\beta'} \tag{6}$$

has been summed over final spins and averaged over initial ones. This can be expressed as

$$|\overline{\mathfrak{M}}_{fi}|^2 = \frac{1}{4} \left(g^{\alpha\alpha'} - q_1^{\alpha} q_1^{\alpha'}/m_W^2\right)\left(g^{\beta\beta'} - q_2^{\beta} q_2^{\beta'}/m_W^2\right)\left(\widetilde{M}^{\mu\nu}_{\alpha\beta} \widetilde{M}_{\mu\nu\alpha'\beta'} - 8 g_{\alpha\beta} g_{\alpha'\beta'}\right) \tag{7}$$

where $\widetilde{M}$ denotes that the supplementary conditions $q_1 \cdot \eta_1 = q_2 \cdot \eta_2 = 0$ have been used to set terms proportional to q_1^{α} or q_2^{β} to zero, and also, for the photons, $k_1 \cdot \epsilon_1 = k_2 \cdot \epsilon_2 = 0$ have been used to set terms with k_1^{μ} and k_2^{ν} to zero. There is a subtlety here responsible for the extra term which is explained further in the Appendix. Essentially, the condition $k \cdot \epsilon = 0$ cannot be naively applied to both photons in a two-photon process without some care being taken to obtain a correct and truly gauge-invariant result.

To obtain an explicit expression for $|\overline{\mathfrak{M}}_{fi}|^2$, the algebraic computer program "Reduce" by A. C. Hearn[10] was used. The necessity of this is evident from the complexity of Eq. (4). The reader is referred to the work of

Kim and Tsai[5] for the complete expression for $\sigma_{\gamma\gamma \to W^+W^-}$. Here it suffices to note that

$$\sigma_{\gamma\gamma \to W^+W^-} \sim \frac{\alpha^2 s}{m_W^2} \qquad (\kappa \neq 1) \;, \qquad \sigma_{\gamma\gamma \to W^+W^-} \sim \frac{\alpha^2}{m_W^2} \qquad (\kappa = 1) \qquad (8)$$

to illustrate the behavior of the cross section. The full result will be used in the numerical integration required to derive the accurate cross sections.

Now the expression for $\sigma_{\gamma\gamma \to W^+W^-}(s)$, Eq. (5), can be put into Eq. (1) to give the complete expression for the cross section of $ee \to eeW^+W^-$. We would like this answer in the lab frame for convenience. This is not difficult since $\sigma_{\gamma\gamma \to W^+W^-}(s)$, even though calculated in the photon-photon center-of-mass frame, is a function only of s and is therefore Lorentz invariant. Thus

$$\sigma_{ee \to eeW^+W^-}(E) \cong \frac{\alpha^4}{\pi} (\ln E/m_e)^2 \int_{4m_W^2}^{4E^2} \frac{ds}{s^2}\, f\left(\frac{\sqrt{s}}{2E}\right) \left(1 - \frac{4m_W^2}{s}\right)^{1/2} \int_{-1}^{1} d\cos\theta \; |\mathfrak{M}_{fi}|^2 \qquad (9)$$

where f(x) was given in Eq. (2). This is the desired result. To investigate its behavior, it is necessary to integrate out the complicated expression for $|\mathfrak{M}_{fi}|^2$ obtained earlier. However, to get a qualitative idea, we use the leading dependence given in Eq. (8). This yields $(\kappa \neq 1)$

$$\sigma_{2\gamma}(E) \sim \alpha^4 (\ln E/m_e)^2 \frac{E^2}{m_W^4} I(m_W/E) + \text{terms of lower order in } E \qquad (10)$$

where

$$I(y) = \int_y^1 x\,(1 - y^2/x^2)^{1/2} f(x)\, dx \qquad (11)$$

If $E \gg m_W$, $I(m_W/E) \cong I(0) \cong 0.4$. Note that, except for a factor of $(\ln E/m_e)^2/m_W^2$, this is a function only of E/m_W. This should be compared with the one-photon prediction of

$$\sigma_{ee\to W^+W^-}(E) = \left(\pi\alpha^2\beta^3/3\gamma^2 m_W^2\right)\left[\gamma^4\kappa^2 + (\kappa^2+3\kappa+1)\gamma^2+3/4\right] \tag{12}$$

as given by Tsai and Hearn,[2] where $\gamma \equiv E/m_W$, $\beta \equiv (1-\gamma^{-2})^{1/2}$. Note that, if $\kappa = 0$, $\sigma_{1\gamma} \sim \pi\alpha^2/3m_W^2$, a constant at high energy, whereas if $\kappa \neq 0$, $\sigma_{1\gamma} \sim \pi\alpha^2 E^2\kappa^2/3m_W^4$. From Eq. (10) it seems likely that $\sigma_{2\gamma}$ would have no chance to overtake $\sigma_{1\gamma}$ unless $\kappa = 0$. This surmise must be examined quantitatively, of course. The integrations were done using the multidimensional Monte Carlo integration routine "Shep" by G. C. Sheppey. The results are shown in Figs. 4 - 6 for various values of m_W and κ. Note that, if $\kappa = 0$ and $m_W \sim 2$ GeV, then $\sigma_{2\gamma}$ overtakes $\sigma_{1\gamma}$ at E $\sim$ 30 GeV and is a significant fraction at lower energies. But for nonzero κ, $\sigma_{2\gamma}$ is always a couple of orders of magnitude lower than σ_1 .

III. EFFECT OF WEINBERG THEORY

In Fig. 7 are shown some of the diagrams contributed in lowest order, in addition to those of Fig. 1, by the Weinberg theory.[6] This is a gauge theory with spontaneously broken symmetries which renders the weak interactions finite and unifies them with electromagnetic interactions, but at the price of additional massive neutral vector (Z) and scalar (ϕ) fields. Here only the Z bosons contribute since the coupling of ϕ to the leptons is proportional to the lepton mass. A detailed calculation of the cross section is not our purpose here, but simply to show that these corrections are negligible.

The point can be made with any part of the Weinberg corrections, say, Fig. 7a which contributes a term proportional to

$$\frac{e^2g^2}{2}\left[\bar{u}(p_1')\gamma_\mu u(p_1)\right]\left[\bar{u}(p_2')\gamma_\nu\,(dP_- + cP_+)\,u(p_2)\right]\frac{g^{\mu\mu'}}{k_1^2}\,\frac{g^{\nu\nu'}-k_2^\nu k_2^{\nu'}/m_Z^2}{k_2^2-m_Z^2}\,M_{\mu'\nu'\alpha\beta}\eta_1^\alpha\eta_2^\beta \tag{13}$$

where $g^2/8m_W = G/\sqrt{2}$, $P_\pm = (1\pm\gamma_5)/2$, and c, d are constants ($c=2(1-R)$, $d=1-2R$; $R=m_W^2/m_Z^2$). Evidently, if $k_2^2/m_Z^2 \ll 1$, this term is of order $1/m_Z^2$ compared to purely electromagnetic ones. A simple estimate of the ratio of differential cross sections of scattered electrons forward versus at large angle shows that nearly all events involve electron scattering forward and so $|k_2^2| \cong m_e^2$; also $m_Z > m_W \sim 40$ GeV in the Weinberg theory. Thus these additional terms do not contribute significantly.

The tensor $M_{\mu\nu\alpha\beta}$ has the same form for ZW^+W^- and γW^+W^- vertices, except in the Weinberg theory κ is constrained to be 1, that is, a Yang-Mills[11] type vertex. It should be noted that, at infinitely high energies, the additional diagrams will probably prevent the cross section from violating the unitarity bound since the ZW^+W^- and γW^+W^- vertices have opposite signs. In the one-photon case, Weinberg showed[6] that $\sigma_{1\gamma} \propto 1/E^2$ eventually. To resolve this question in the present case will not be essayed here.

IV. CONCLUSION

In the experimental quest for the elusive intermediate boson the two-photon process here discussed may not be without significance. For the special case of $\kappa = 0$, we have noted its role. In this case, a luminosity of 10^{32} cm^{-2} sec^{-1} would give $\sim 10^{-1}$ counts sec^{-1} of these events at $E \sim 20$ GeV and $m_W = 2$ GeV. Admittedly, for a particle with such a large mass (if, that is, it exists) not to have an anomalous moment is hard to believe. Indeed, as Weinberg and Kim and Tsai have remarked,[6,13] $\kappa = 1$ would assure that the W Compton scattering would satisfy a Drell-Hearn[12] sum rule. This leads to an interesting point. If the W indeed does have strong interactions and is described by form factors that decrease rapidly with q^2, the two-photon process might indeed dominate the one-photon by virtue of its soft photons. On the other hand, one would expect such W's to be, for instance, electroproduced off protons; Kogut[14] has shown that one could probe in this way up to $m_W \sim 5$ GeV at SLAC energies. If these W's are more massive still, the two-photon process would play a useful role in searching for them and in setting limits on their mass and strong interactions.

ACKNOWLEDGEMENTS

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APPENDIX

Consider, for simplicity, the case of the production of a pair of spin 0 bosons.[15] The amplitude is proportional to $\epsilon_1^\mu \epsilon_2^\nu M_{\mu\nu}$, where

$$M_{\mu\nu} = -\frac{(2q_1-k_1)_\mu (k_2-2q_2)_\nu}{2k_2\cdot q_2} - \frac{(2q_1-k_2)_\nu (k_1-2q_2)_\mu}{2k_1\cdot q_2} - 2g_{\mu\nu} \tag{A. 1}$$

which satisfies $k_1^\mu M_{\mu\nu} = k_2^\nu M_{\mu\nu} = 0$ explicitly. Now apply the subsidiary conditions $k_1\cdot\epsilon_1 = k_2\cdot\epsilon_2 = 0$; then (dropping an overall factor of 2)

$$\widetilde{M}_{\mu\nu} = \frac{q_{1\mu} q_{2\nu}}{k_2\cdot q_2} + \frac{q_{1\nu} q_{2\mu}}{k_1\cdot q_2} - g_{\mu\nu} \tag{A. 2}$$

and

$$M_{\mu\nu} = \widetilde{M}_{\mu\nu} - \frac{1}{2} k_{2\nu}\left(\frac{q_{1\mu}}{k_2\cdot q_2} + \frac{q_{2\mu}}{k_1\cdot q_2}\right) - \frac{1}{2} k_{1\mu}\left(\frac{q_{1\nu}}{k_1\cdot q_2} + \frac{q_{2\nu}}{k_2\cdot q_2}\right)$$
$$+ \frac{1}{4} k_{1\mu} k_{2\nu}\left(\frac{1}{k_2\cdot q_2} + \frac{1}{k_1\cdot q_2}\right) \tag{A. 3}$$

so that

$$k_1^\mu \widetilde{M}_{\mu\nu} = k_{2\nu}\,, \qquad k_2^\nu \widetilde{M}_{\mu\nu} = k_{1\mu}\,, \tag{A. 4}$$

which is, by itself, "pseudo-gauge-invariant" even though $\epsilon_1^\mu k_2^\nu \widetilde{M}_{\mu\nu} = \epsilon_2^\nu k_1^\mu \widetilde{M}_{\mu\nu} = 0$. Now $M_{\mu\nu} M^{\mu\nu} = \widetilde{M}_{\mu\nu} \widetilde{M}^{\mu\nu} - 2$, so dropping <u>both</u> $k_{1\mu}$ and $k_{2\nu}$ (i.e., using <u>both</u> subsidiary conditions) results in an error in the cross section. It is easily verified that setting <u>either</u> $k_{1\mu}$ <u>or</u> $k_{2\nu}$ to zero will give the correct answer.

Alternatively, use the full expression for the summation

$$\sum_{\text{spins}} \epsilon_1^{\mu}\epsilon_1^{\mu'}\epsilon_2^{\nu}\epsilon_2^{\nu'} M_{\mu\nu}M_{\mu'\nu'} = \left(-g^{\mu\mu'} - \frac{k_1^{\mu}k_1^{\mu'}}{(k_1\cdot\eta)^2} + \frac{k_1^{\mu}\eta^{\mu'} + k_1^{\mu'}\eta^{\mu}}{k_1\cdot\eta}\right)$$

$$\times\left(-g^{\nu\nu'} - \frac{k_2^{\nu}k_2^{\nu'}}{(k_2\cdot\eta)^2} + \frac{k_2^{\nu}\eta^{\nu'} + k_2^{\nu'}\eta^{\nu}}{k_2\cdot\eta}\right) M_{\mu\nu}M_{\mu'\nu'}$$

$$= M_{\mu\nu}M^{\mu\nu} \tag{A.5}$$

where $\eta = (1,0,0,0)$ is a unit timelike vector. On the rhs, tedious calculation verifies that

$$M^{\mu\nu}M_{\mu\nu} = \left(-g^{\mu\mu'} - \frac{k_1^{\mu}k_1^{\mu'}}{(k_1\cdot\eta)^2} + \frac{k_1^{\mu}\eta^{\mu'} + k_1^{\mu'}\eta^{\mu}}{k_1\cdot\eta}\right)\left(-g^{\nu\nu'} - \frac{k_2^{\nu}k_2^{\nu'}}{(k_2\cdot\eta)^2} + \frac{k_2^{\nu}\eta^{\nu'} + k_2^{\nu'}\eta^{\nu}}{k_2\cdot\eta}\right)\widetilde{M}_{\mu\nu}\widetilde{M}_{\mu'\nu'} \tag{A.6}$$

so that the gauge terms in $\Sigma\epsilon^{\mu}\epsilon^{\mu'}$ "compensate" for the "gauge terms" omitted in $\widetilde{M}$. It is precisely the conservation of the current that enforces this. No such problem exists for the massive vector field since the gauge freedom has been removed. Precisely analogous results to the above are found for $M_{\mu\nu\alpha\beta}$, where now $k_1^{\mu}\widetilde{M}_{\mu\nu\alpha\beta} = -2k_{2\nu}g_{\alpha\beta}$ and $k_2^{\nu}\widetilde{M}_{\mu\nu\alpha\beta} = -2k_{1\mu}g_{\alpha\beta}$ are the analogous of the "pseudo-gauge-invariance" statements in Eq. (A.4). Then follows

$$\sum_{\text{spins}} \epsilon_1^{\mu}\epsilon_1^{\mu'}\epsilon_2^{\nu}\epsilon_2^{\nu'} M_{\mu\nu\alpha\beta}M_{\mu'\nu'\alpha'\beta'} = \widetilde{M}^{\mu\nu}_{\alpha\beta}\widetilde{M}_{\mu\nu\alpha'\beta'} - 8\,g_{\alpha\beta}g_{\alpha'\beta'} \quad .$$

FOOTNOTES

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15. For some discussion of this point, see J. Schwinger, Particles, Sources and Fields (Addison-Wesley Co., Reading, Mass., 1970), pp. 292ff.

FIGURE CAPTIONS

1. Diagrams for the two-photon process $ee \to eeW^+W^-$: (a) direct, (b) crossed, (c) seagull.
2. Diagram for the one-photon process $e^+e^- \to W^+W^-$.
3. Diagrams for the process $\gamma\gamma \to W^+W^-$: (a) direct, (b) crossed, (c) seagull.
4. Total cross sections for $\sigma_{1\gamma}$ and $\sigma_{2\gamma}$ when $\kappa = 0$: (a) $m_W = 2$ GeV, (b) $m_W = 5$ GeV, (c) $m_W = 10$ GeV. Note: in (c) the scale on the left refers to $\sigma_{1\gamma}$ and the scale on the right refers to $\sigma_{2\gamma}$.
5. Total cross sections for $\sigma_{1\gamma}$ and $\sigma_{2\gamma}$ when $\kappa = 1$: (a) $m_W = 2$ GeV, (b) $m_W = 5$ GeV, (c) $m_W = 10$ GeV. Note: the scale on the left refers to $\sigma_{1\gamma}$ and the scale on the right refers to $\sigma_{2\gamma}$.
6. Total cross sections for $\sigma_{2\gamma}$ when $m_W = 2$ GeV and $\kappa = 0, 1, 2$.
7. Some of the diagrams contributed to the two-photon process by the Weinberg theory.

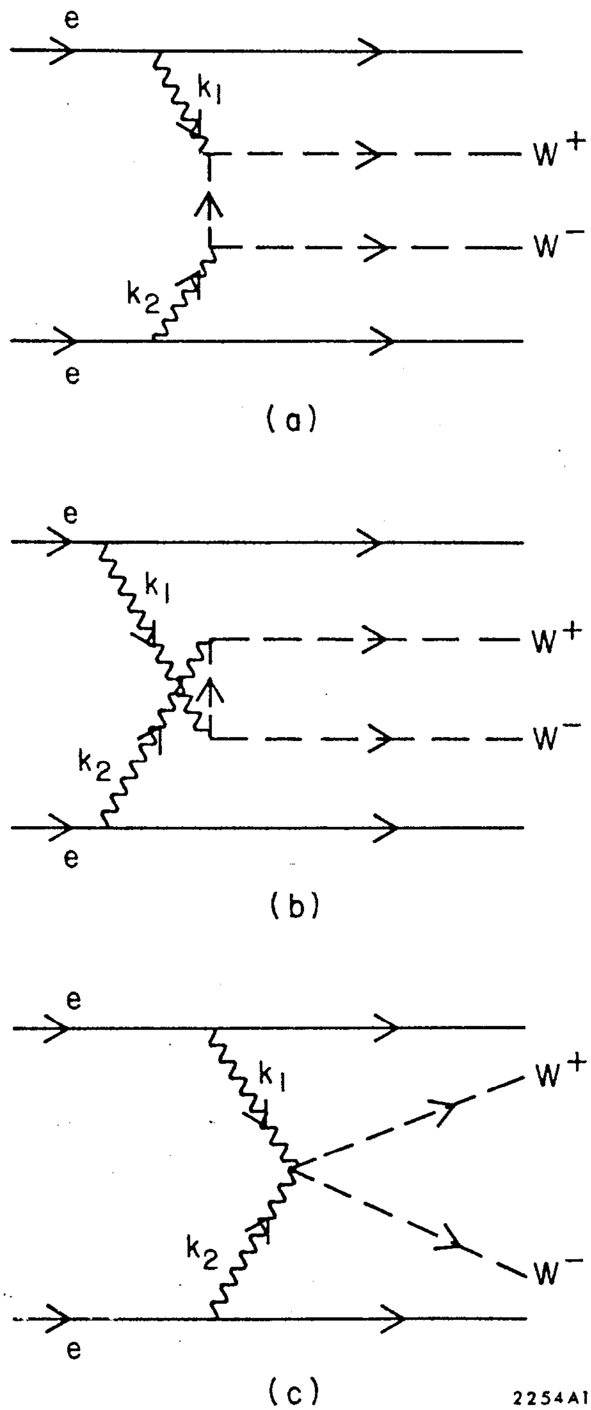


Fig. 1

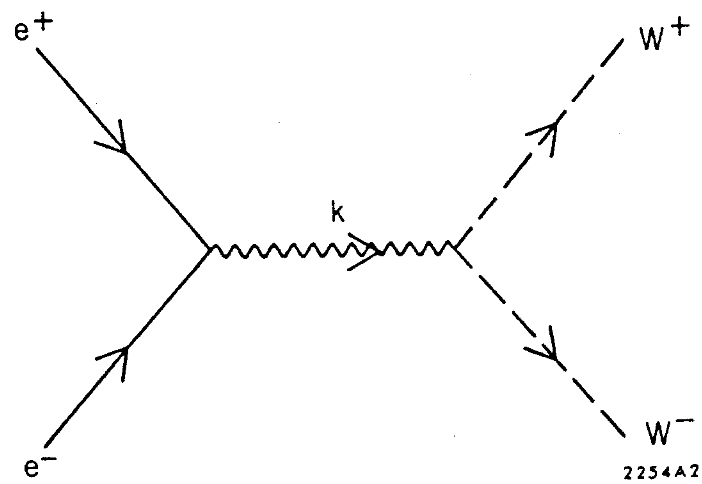
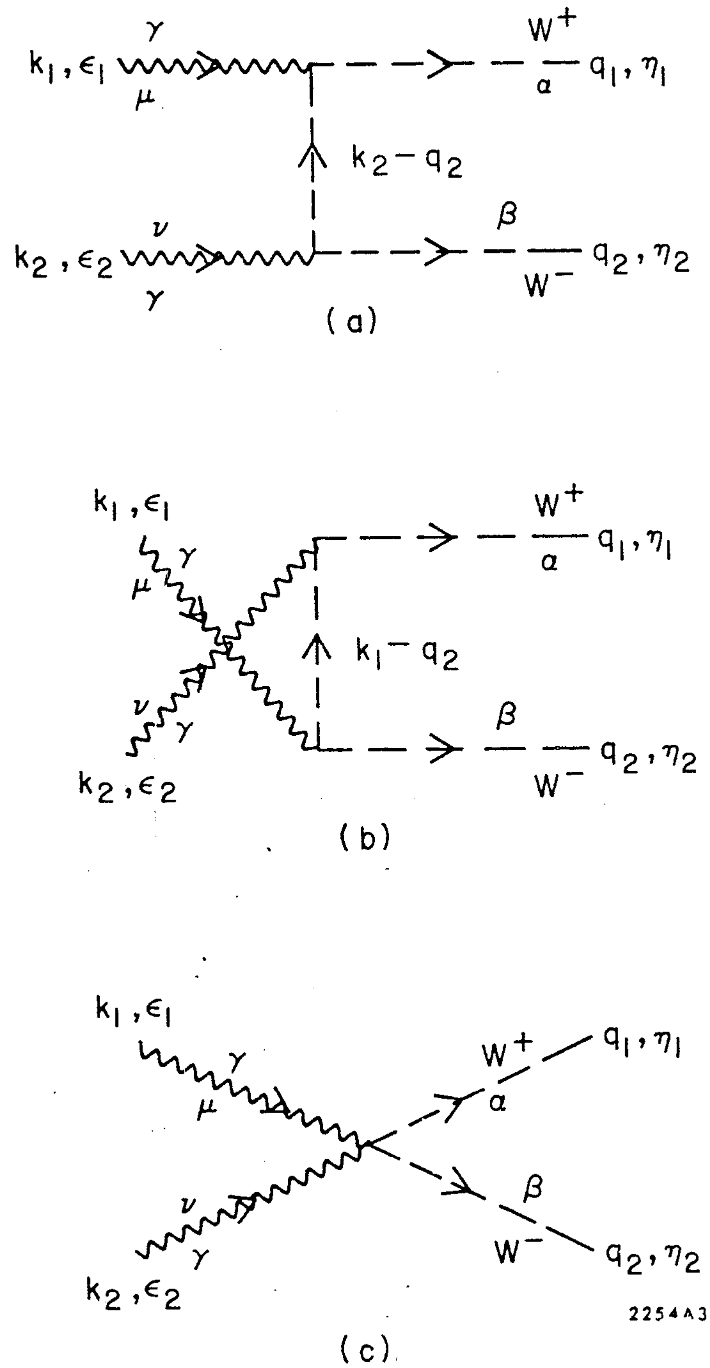
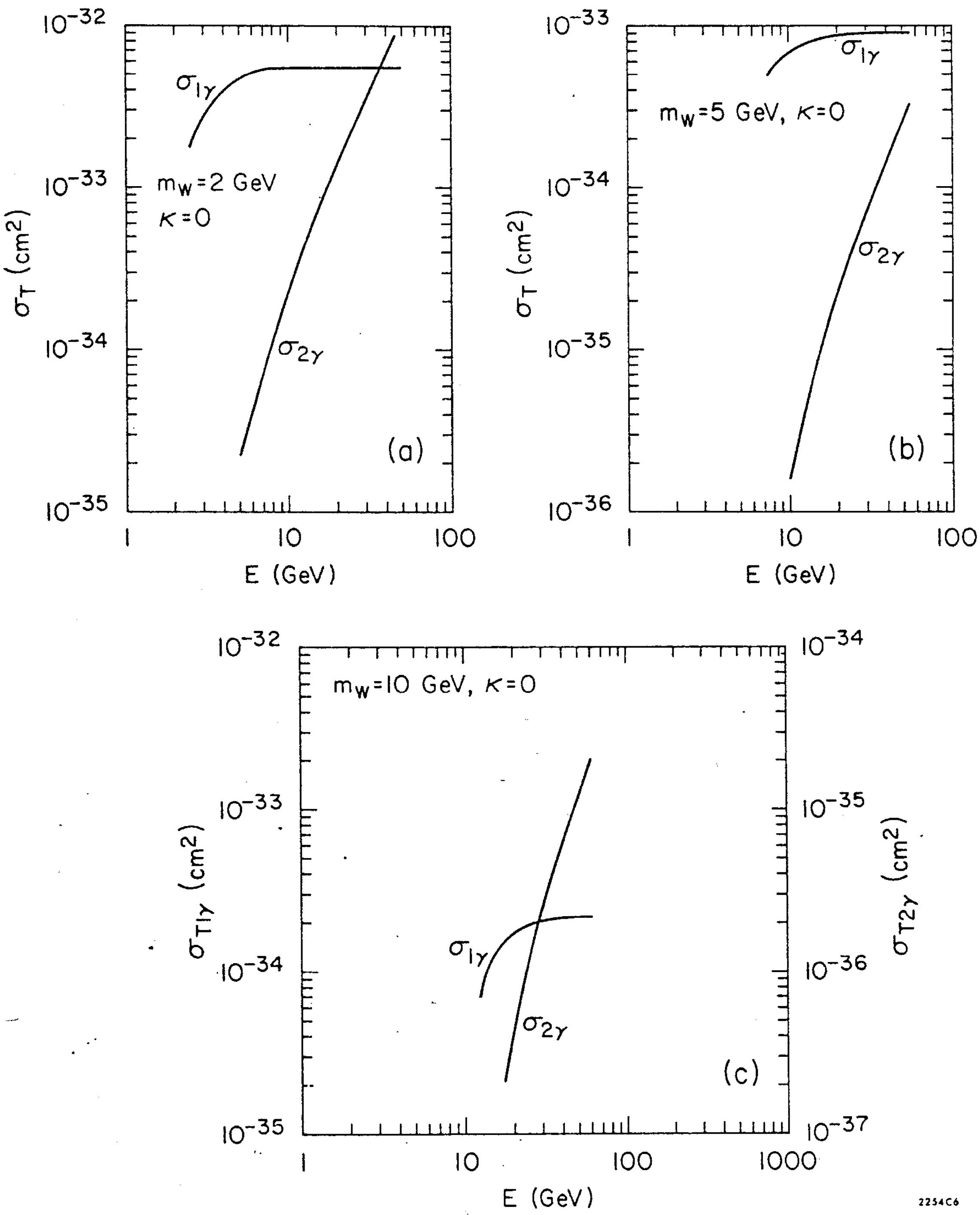


Fig. 2



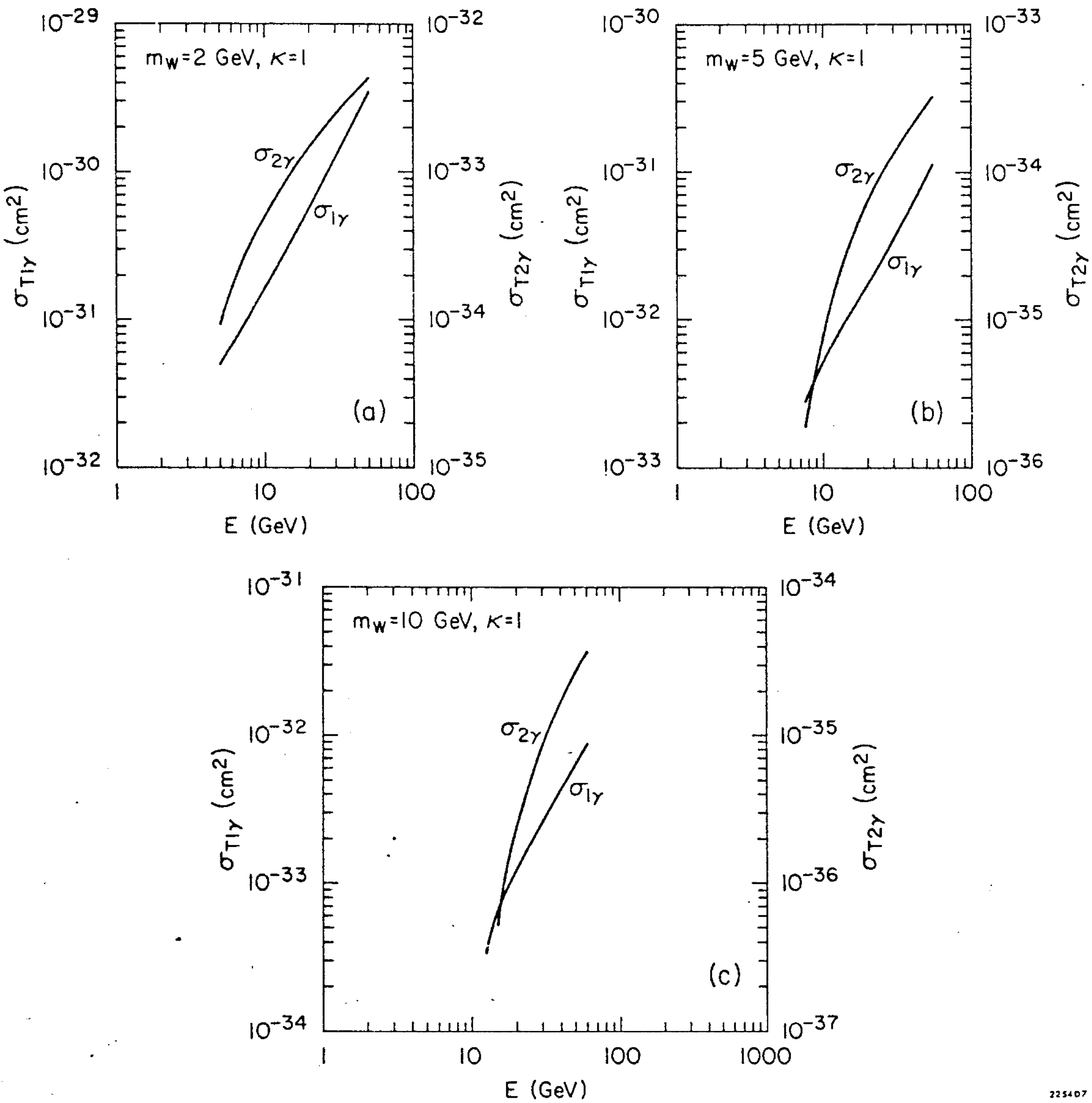
k_1, ϵ_1
γ
μ
W^+
α
q_1, η_1
$k_2 - q_2$
k_2, ϵ_2
ν
γ
β
W^-
q_2, η_2
(a)
$k_1 - q_2$
(b)
2254A3
(c)

Fig. 3



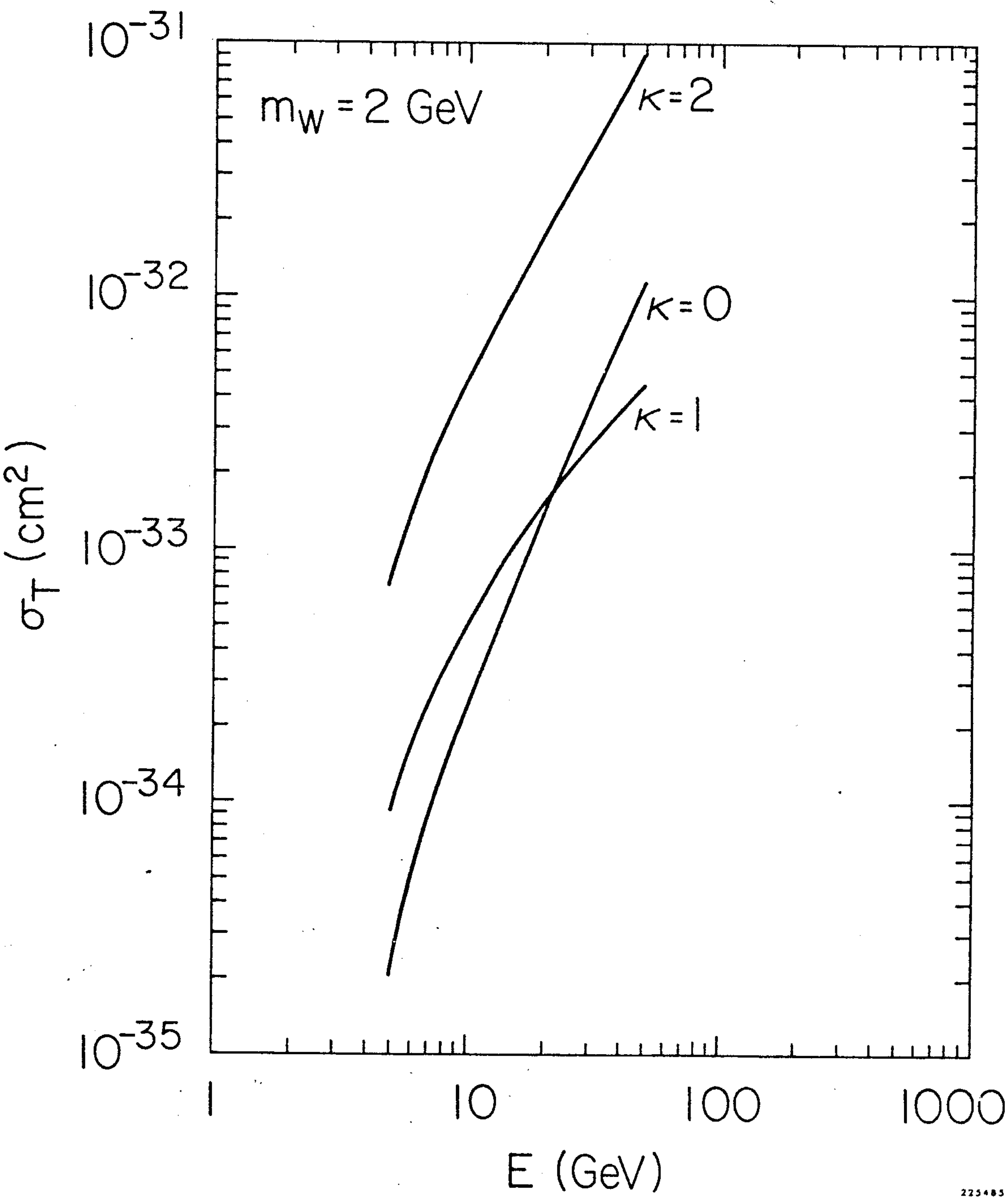
10^{-32}
10^{-33}
10^{-34}
10^{-35}
σ_T (cm^2)
$\sigma_{1\gamma}$
$\sigma_{2\gamma}$
m_W=2 GeV
κ=0
(a)
1
10
100
E (GeV)
10^{-33}
10^{-34}
10^{-35}
10^{-36}
σ_T (cm^2)
$\sigma_{1\gamma}$
m_W=5 GeV, κ=0
$\sigma_{2\gamma}$
(b)
1
10
100
E (GeV)
10^{-32}
10^{-33}
10^{-34}
10^{-35}
$\sigma_{T1\gamma}$ (cm^2)
m_W=10 GeV, κ=0
$\sigma_{1\gamma}$
$\sigma_{2\gamma}$
(c)
10^{-34}
10^{-35}
10^{-36}
10^{-37}
$\sigma_{T2\gamma}$ (cm^2)
1
10
100
1000
E (GeV)
2254C6

Fig. 4



m_W=2 GeV, κ=1
$\sigma_{2\gamma}$
$\sigma_{1\gamma}$
(a)
$\sigma_{T1\gamma}$ (cm^2)
$\sigma_{T2\gamma}$ (cm^2)
10^{-29}
10^{-30}
10^{-31}
10^{-32}
10^{-32}
10^{-33}
10^{-34}
10^{-35}
1
10
100
E (GeV)
m_W=5 GeV, κ=1
$\sigma_{2\gamma}$
$\sigma_{1\gamma}$
(b)
$\sigma_{T1\gamma}$ (cm^2)
$\sigma_{T2\gamma}$ (cm^2)
10^{-30}
10^{-31}
10^{-32}
10^{-33}
10^{-33}
10^{-34}
10^{-35}
10^{-36}
1
10
100
E (GeV)
m_W=10 GeV, κ=1
$\sigma_{2\gamma}$
$\sigma_{1\gamma}$
(c)
$\sigma_{T1\gamma}$ (cm^2)
$\sigma_{T2\gamma}$ (cm^2)
10^{-31}
10^{-32}
10^{-33}
10^{-34}
10^{-34}
10^{-35}
10^{-36}
10^{-37}
1
10
100
1000
E (GeV)

Fig. 5



$m_W = 2$ GeV
$\kappa = 2$
$\kappa = 0$
$\kappa = 1$
10^{-31}
10^{-32}
10^{-33}
10^{-34}
10^{-35}
σ_T (cm^2)
1
10
100
1000
E (GeV)
225485

Fig. 6

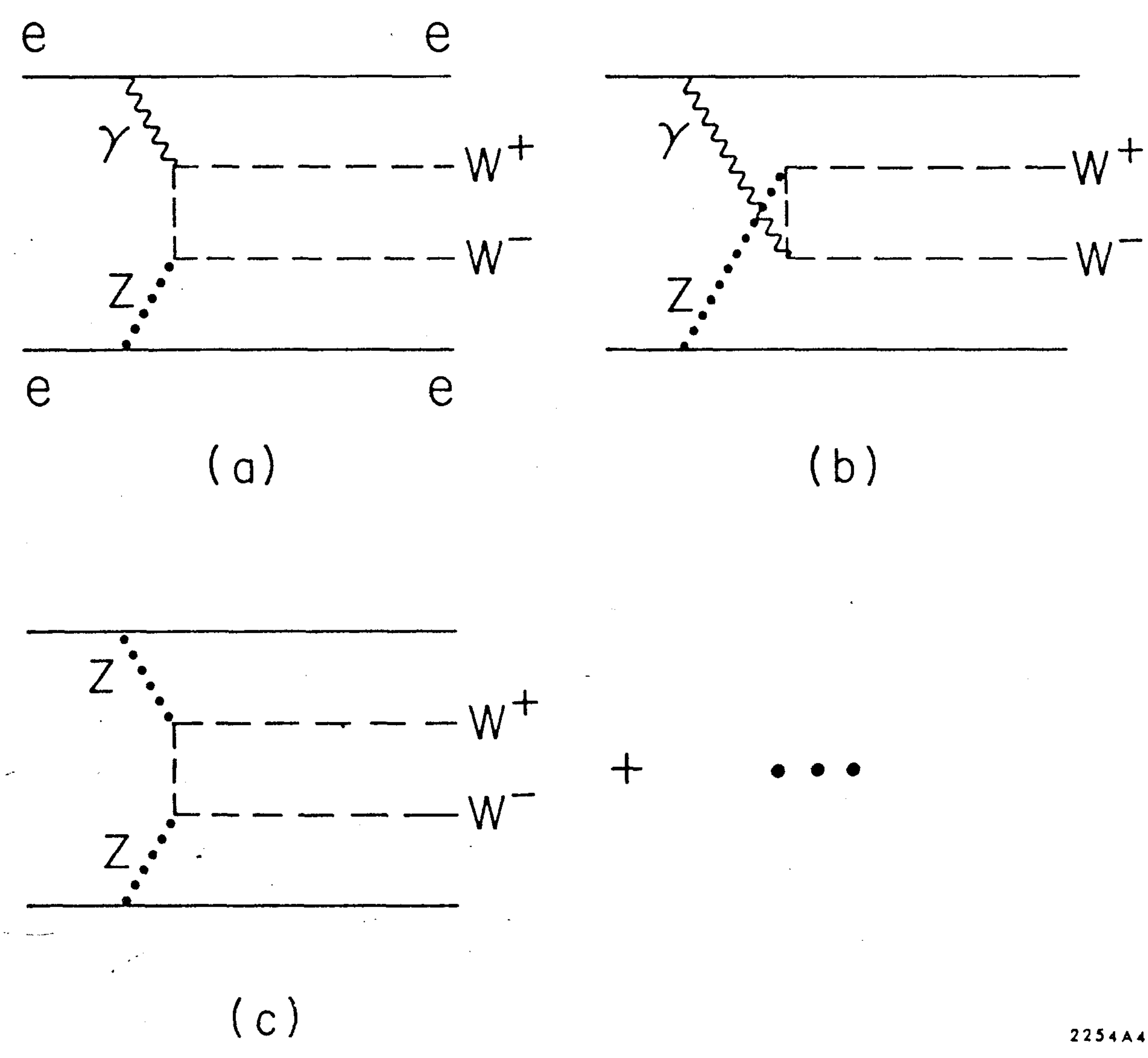


Fig. 7