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SCALING IN DEEP INELASTIC PION ELECTROPRODUCTION*

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ABSTRACT

An attempt is made to apply a light cone dominance technique to the deep inelastic electroproduction of pions. A scaling law is derived which is in agreement with preliminary data.

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The structure of current commutators near the light cone has provided an illuminating basis for the description of the scaling phenomena observed in deep inelastic electron-nucleon scattering experiments.[1] The present paper is devoted to an attempt to apply such techniques to the special case where pions are electroproduced from nucleons.[2] Such a study is rather timely since preliminary data have already been reported and it is likely that more extensive data on this process will be forthcoming during the coming year.[3] Although this recent data is rather scant, we shall show that a naive application of light cone techniques gives a good description of the scaling phenomena thus far observed. Although it is not obvious that the structure of commutators near the light cone dominates such processes, we feel that it is still worth entertaining and investigating such a possibility in view of the rather limited application of such techniques to realistic experimental situations. We shall say more of this below.

We begin by discussing the exclusive process where the final hadronic state consists of a pion and a nucleon only. The generalization to the inclusive case where the pion alone is detected is straightforward. In order to state our results in a form easily accessible to experiment we must first discuss the general structure of the cross section. In the center-of-mass (CM) system of the hadrons, the differential cross section can be expressed in terms of an equivalent virtual photoproduction cross section[4]

$$\frac{d\sigma}{d\Omega} = \frac{M^2 \, |\underline{k}|_{CM}}{16\pi^2 W^2 \, |q|_{CM}} \Big[\tfrac{1}{2} (T_{xx} + T_{yy}) + \tfrac{1}{2} \epsilon (T_{xx} - T_{yy}) - \frac{q^2}{\nu^2} \epsilon \, T_{zz} + \left\{ - \frac{2q^2}{\nu^2} \epsilon (1 + \epsilon) \right\}^{\frac{1}{2}} T_{xz} \Big] \tag{1}$$

where k_μ and q_μ are, respectively, the pion and photon four-momenta and W is the total CM energy; M is the nucleon mass and ϵ represents the polarization of the virtual photon:

$$\epsilon = \left[1 - \frac{2(\nu^2 - q^2)}{q^2} \tan^2 \tfrac{1}{2}\theta_e\right]^{-1} \tag{2}$$

ν and θ_e are the laboratory (LAB) frame values of the electron energy loss and scattering angle, respectively; the tensor $T_{\mu\nu}$ defined by $T_{\mu\nu} = M_\mu M_\nu^*$ where $M_\mu \equiv < \pi p' \mid j_\mu \mid p >$ and j_μ is the electromagnetic current operator, contains the interesting hadron-dynamics, and is to be expressed in the LAB system for use in Eq. (1). The z-axis is defined to be coincident with the direction of $\underset{\sim}{q}$ whilst the electrons define the xy plane. Thus all of the ϕ (azimuthal) dependence is contained in $(T_{xx} - T_{yy}) \sim \cos 2\phi$ and $T_{xz} \sim \cos\phi$. In what follows we shall limit ourselves to the case where the particle spins are unobserved. In terms of $d\sigma/d\Omega$, the measured electron cross section is

$$\frac{d^3\sigma}{dE' d\Omega' d\Omega} = \frac{\alpha}{2\pi^2 q^2} \frac{E'}{E} \frac{(\nu^2 - q^2)^{\frac{1}{2}}}{1 - \epsilon} \frac{d\sigma}{d\Omega} . \tag{3}$$

The main result of this paper can be summarized as follows: in the deep inelastic limit, i.e. when $q^2 \rightarrow -\infty$ with $\omega \equiv -2M\nu/q^2$ fixed, the $T_{\mu\nu}$ become functions of ω and $t \equiv (k-q)^2$, only. Hence in this limit,

$$W^2 \frac{d\sigma}{d\Omega} \simeq W^4 \frac{d\sigma}{dt} \rightarrow F(\omega, t) . \tag{4}$$

In other words, asymptotically, the combination $W^4 \frac{d\sigma}{dt}$ loses all dependence upon q^2, ϕ and ϵ, and becomes predominantly transverse in character. We now attempt to motivate these assertions.

Using the standard LSZ reduction formalism together with PCAC allows one to express M_μ in the following way

$$f_\pi M_\mu = k^\nu C_{\mu\nu} + E_\mu \tag{5}$$

where

$$C_{\mu\nu} \equiv i\int d^4x\, e^{iq\cdot x} < p' \mid \theta(x_0)\left[j_\mu(x), A_\nu(0)\right] \mid p > \tag{6}$$

and

$$E_\mu \equiv \int d^4x\, e^{iq\cdot x} < p' \mid \delta(x_0)\left[j_\mu(x),\ A_0(0)\right] \mid p > \tag{7}$$

Here, A_μ represents the weak axial vector current, and f_π the pion decay constant, and we have set $k^2 = 0$. We now use the null-plane version of the Bjorken "theorem" [5]; to this end we introduce light cone variables which, for an arbitrary vector a^μ, are defined as follows: $a_\pm = \frac{1}{\sqrt{2}}(a_0 \pm a_z)$ and $\underset{\sim}{a}_\perp = (a_x, a_y)$. The theorem then states that, in the limit $q_- \to \infty$ with q_+, $q_\perp$, p and p' fixed,

$$C_{\mu\nu} \to -\frac{1}{q_-}\int d^4x\, e^{iq\cdot x} < p' \mid \delta(x_+)\left[j_\mu(x), A_\nu(0)\right] \mid p > + 0(1/q_-)^2 . \tag{8}$$

We shall refer to the commutator that governs this limit as a null-plane commutator since causality requires that $\underline{x}_\perp = \underline{0}$ where $x_+ = 0$ (leaving x_- arbitrary in general). This theorem can easily be conjectured by observing that causality allows the step function $\theta(x_0)$ occurring in Eq. (6) to be replaced by $\theta(x_+)$. A straightforward application of Fourier transform techniques then leads to the series in Eq. (8).

If we work in the LAB (where $q_0 = \nu$ and $\underset{\sim}{p} = \underset{\sim}{0}$) then the limit implicit in (8) corresponds precisely to the usual Bjorken limit since $q_- \to \sqrt{2}\nu \to \infty$ and $q_+ \to \sqrt{2}/\omega$ which is to be kept fixed. Furthermore with p and p' fixed (i.e. fixed t)

$$k^{\nu} C_{\mu\nu} \to q_- C_{\mu+} \to -L_{\mu} \equiv \int d^4x\, e^{iq\cdot x} < p' \mid \delta(x_+)[j_{\mu}(x), A_+(0)] \mid p > \tag{9}$$

Thus, in this limit

$$f_{\pi} M_{\mu} \to E_{\mu} - L_{\mu} \,. \tag{10}$$

Now, apart from Schwinger terms, causality requires the commutators to have the following form

$$[j_{\mu}(x), A_0(0)]\, \delta(x_0) = \tilde{A}_{\mu}(0)\, \delta^{(4)}(x) \tag{11}$$

$$[j_{\mu}(x), A_+(0)]\, \delta(x_+) = \tilde{\mathscr{A}}_{\mu}(x_-)\, \delta(x_+) \delta^2(x_{\perp}) \,. \tag{12}$$

In the Gell-Mann current algebra $\tilde{A}_{\mu}$ is essentially model independent and is simply identified with A_{μ}; on the other hand, although $\tilde{\mathscr{A}}_{\mu}(x_0)$ does depend upon the model, it can generally be decomposed in the following manner

$$\tilde{\mathscr{A}}_{\mu}(x_-) = A_{\mu}(0)\, \delta(x_-) + B_{\mu}(x_-) \,. \tag{13}$$

This is true, for example, in the quark model where certain components of the psuedovector B_{μ} have the bilocal form $\sim m\bar{\psi}(x_-)\gamma_5\psi(0)$ where $\psi(x)$ is the quark field and m its mass; using the notion of PCAC this operator can be thought of as the bilocal generalization of the pion field.

Using (11), (12) and (13) in Eq. (10) then leads to the asymptotic form

$$f_{\pi} M_{\mu} \to \int_{-\infty}^{\infty} dx_-\, e^{iq_+ x_-} < p' \mid B_{\mu}(x_-) \mid p > \,. \tag{14}$$

From this we immediately deduce that the scalar invariants occurring in M_{μ} depend asymptotically only upon the variables ω and t. Note that the conserved nature of j_μ must be reflected in the commutation relations (11) and (12) and these automatically ensure that (14) will be consistent with the conservation requirement that $M_+ \to 0$.[6] From this equation we can construct $T_{\mu\nu}$ and deduce the result claimed above; namely, that the relevant components are functions of ω and t only. More explicitly we expect (e.g. in a quark model)

$$\tfrac{1}{2}(T_{xx} + T_{yy}) \to F_1(\omega, t) - F_2(\omega, t)(k_x^2 + k_y^2) \tag{15a}$$

$$\tfrac{1}{2}(T_{xx} - T_{yy}) \to \tfrac{1}{2} F_2(\omega, t)(k_x^2 - k_y^2) \tag{15b}$$

$$T_{zz} \to F_1(\omega, t) - F_2(\omega, t)\Delta_z^2 \tag{15c}$$

and

$$T_{xz} \to -F_2(\omega, t)k_x\Delta_z \tag{15d}$$

where $\Delta_z \equiv k_z - q_z$ (as expressed in the LAB) $\simeq -(\frac{1}{2}t + M\omega)$ and the F_i are Lorentz scalars.

Some remarks concerning this "derivation" are in order:

i) We have thus far neglected the presence of possible Schwinger terms in the commutation relations (11) and (12). If such terms are present in (11) then we make the usual assumption that they are cancelled by the presence of possible seagull terms in $C_{\mu\nu}$. If they are present in (12), as they are for example, even in a simple quark model, they can only introduce terms proportional to q_+ and $q_\perp$ so that the general form of the result remains unchanged.

ii) The extension to the inclusive case is straightforward since the state $| p' >$ is now replaced by an arbitrary state $| N >$ and (14) becomes

$$f_\pi M_\mu^{(N)} \to < N | \int_{-\infty}^{\infty} dx_- \, e^{iq_+x_-} B_\mu(x_-) | p > .$$

The corresponding $T_{\mu\nu}$ ($\equiv M_{\mu}^{(N)} M_{\nu}^{(N)*}$) thus retain their scaling character, the only difference being that they now also depend upon the invariant mass (W') of the state $|N>$ and the cross section is differential in this variable. In other words, (4) is to be replaced by

$$W^4 \frac{d^2\sigma}{dW'^2 dt} \to F(\omega, t, W'^2) . \tag{16}$$

There is recent data from Cornell[3] which gives $d^2\sigma/dW'^2dt$ "averaged" over angles $\theta \leq 9.6^{\circ}$ for two values of W (2.14 and 2.66 GeV) at the same value of ω (4.1). Eq. (16) says that the spectra should be identical (even including the "elastic" peak) except for normalizations which should be in the ratio of $\simeq (2.66/2.14)^4 \approx 2.41$. The experiment finds that this is indeed the case and that the ratio of normalizations is ~ 2.34. Obviously the separation of the various terms in Eq. (1) would be desirable and we look forward to comparing more detailed aspects of the data with our predictions.

iii) Although this agreement with experiment is encouraging there are some serious defects to the "derivation". First of all, the limit required in the asymptotic expansion forces $k^2 \to \infty$ unless $q_+ + p_+ - p_+ \simeq 0$; hence the formal limit must be taken rather delicately for we must always demand that $(p-p')_+$ be fixed at q_+. If this is done, $k^2 \to -k_\perp^2$ and by choice of frame we can set $k_\perp = 0$ (this does not affect the general form of our results). It would thus appear that setting $k^2 = 0$ (as we did in Eq. (5)) is consistent and probably necessary for the technique to work. A related problem to this is the problem of the pion pole[7], i.e. we know that M_μ must in general contain such a pole at $k^2 = \mu^2$ yet it appears to be missing in the asymptotic form, Eq. (14). We have attempted to resolve this problem by appealing to the infamous DGS representation and have been able to show that, at least in the massless case, Eq. (8) reproduces the correct asymptotic expansion of the matrix element which does contain the pole. An extension to the massive case may involve an assumption which requires that the light cone singularity dominates some distance into the light cone characterized by the particle mass. We shall expound upon these ideas in a later paper.

iv) Finally we present a somewhat different way of seeing the result contained in Eq. (10). Suppose we again use causality to replace $\theta(x_0)$ in Eq. (6) by $\theta(x_+)$ then we can derive the following result analogous to Eq. (5)

$$f_\pi M'_\mu = k^\nu C_{\mu\nu} + L_\mu \tag{17}$$

where

$$M'_\mu \equiv \int d^4x\, e^{iq\cdot x} < p \,|\, \theta(x_+)\left[j_\mu(x), \phi(0)\right] | \, p > \tag{18}$$

M'_μ differs from M_μ only by the presence of a possible seagull term σ_μ. Subtracting (17) from (5) we obtain

$$\sigma_\mu = E_\mu - L_\mu . \tag{19}$$

If we assume that in the asymptotic region only the seagull survives (i.e. $M'_\mu \to 0$), then our previous result, Eq. (10), immediately follows.

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References

1. See, e.g., R. Jackiw, R. van Royen, and G. B. West, Phys. Rev. D2 (1970) 2473.

2. An attempt in this direction has recently been given by N. Dombey and R. J. Shann, Phys. Lett. 42B (1972) 486.

3. "Preliminary Report on a Study of Scaling in the Inclusive Electroproduction Reactions $e^- + p \rightarrow e^- + \pi^\pm + X^*$" by C. J. Bebek et al., Cyclotron Laboratory, Harvard.

4. See, e.g., C. W. Akerlof et al., Phys. Rev. 163 (1967) 1482, Appendix 1.

5. J. M. Cornwall and R. Jackiw, Phys. Rev. D4 (1971) 367.

6. It should be noted that with $k^2 = 0$, $q^\mu M_\mu = g$ the conventional pion nucleon coupling constant. Although this differs from the standard mass shell condition ($q^\mu M_\mu = 0$) the requirement that $M_+ \rightarrow 0$ in the asymptotic region remains intact.

7. J. Sucher and C. H. Woo, Phys. Rev. Letters 27 (1971) 696.