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ON ASYMPTOTIC CONSERVATION OF HELICITY FOR VERTEX FUNCTION INVOLVING ARBITRARY SPIN*

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ABSTRACT

We examine the matrix element of a current with arbitrary number s of four vector indices between one particle states of definite helicity. The conservation of the angular momentum in the brick wall reference frame was used to derive a set of linear relations between the matrix elements. Requiring the helicities of the particles to be conserved asymptotically, we derive a restriction on the spins of the particles. Specifically we show that the helicity cannot be conserved if the larger of the spins of the initial and the final particles are greater than s for massive particles. For the vector current, this means that the matrix element can conserve the helicity asymptotically only between the states of spin less than or equal to 1.

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I. INTRODUCTION

It is of some theoretical interest to classify a general particle-current-particle vertex in terms of the conservation of the helicity of the particles because of the following:

(1) Minimality Principle. The standard formulation of the principle of the minimal interactions, which one often quotes in deriving the vertex of the spinor electrodynamics, is ambiguous for the case of spin 1 boson.[1] Requirement of the helicity conservation across the vertex at high energies,[2] however, fixes the coupling uniquely either for the spinor electrodynamics yielding the usual γ_μ vertex or for the spin 1 particle, in which case we obtain the coupling of the Yang-Mills type occurring in the unified theory of weak and electromagnetic interaction.[3]

(2) Helicity Conserving Diffraction Scattering. It was recently conjectured[4] and confirmed in the elastic scatterings involving particles with low spins within some uncertainties[5] that the helicity is conserved in all the diffraction scattering at high energy in the strong interaction. If the Pomeron exchange can be considered similar to the usual Regge exchanges with their factorization properties and the particle contents, then the particle-current-particle vertex is again relevant here.

Can one generalize the above features to the particles with arbitrarily higher spins? It is with this question in mind that we study the matrix element of the current operator $\Gamma_{\mu_1 \cdots \mu_s}(0)$ between one particle states of arbitrary spin and definite helicity. Of course, the conservation of the angular momentum forbids the helicity to be conserved at large scattering angles, therefore we must restrict ourselves to the case where the initial and the final particles have large energy and are almost parallel to each other. The problem is then whether the conservation of the angular momentum would also impose any conditions for this restricted situation. By going to the brick wall reference frame, where it is especially simple to impose the conservation of the angular momentum, we find that the vertex for the particles of higher spins can not in general conserve the helicity. Specifically, we shall show that the helicity can not be conserved if the larger of the spins of the initial and the final particles are greater than s for the massive particles.[6] This condition is different for massless particles because of their different properties under the Lorentz transformations.

In Section II, we write down the relation between our matrix elements defined in the laboratory reference frame and those defined in the brick wall reference frame. Conservation of the angular momentum in the brick wall reference frame then makes these relations into a set of linear equations relating the matrix elements in the laboratory system among themselves. If the larger of the spins of the initial and the final particles is greater than s, these equations can be solved to express all the matrix elements diagonal in the helicity in terms of off diagonal elements. In Section III, we first define the precise meaning of our asymptotic helicity conservation. The results of Section II is then applied to get the condition mentioned above. For current with only one Lorentz index, s=1, all the vertex which conserve the helicity are given. The differences between massive and massless cases are discussed. Finally, Section IV contains some concluding remarks.

II. RELATIONS BETWEEN LABORATORY AND BRICK WALL FRAME

We consider the matrix element

$$M^{j'j}_{\lambda'\lambda} = \langle \underset{\sim}{p}' j' \lambda' | \Gamma(0) | \underset{\sim}{p}, j, \lambda \rangle \quad , \tag{1}$$

where j, λ and j', λ' are the spin and the helicity of the initial and the final particles respectively and

$$p = (E,\ -Q,\ 0,\ P)$$

and (2)

$$p' = (E',\ Q',\ 0,\ P')$$

are the corresponding momenta (see Fig. 1). In this section, we shall assume that the masses m and m' do not vanish and choose $Q \gtrsim 0$ and $Q' > 0$. Also, we shall assume that $j' \geq j$ without loss of generality. To avoid lengthy writings, the tensor indices of the operator Γ, as well as of the matrix element $M^{j'j}_{\lambda'\lambda}$ are suppressed:

$$\Gamma \rightarrow \Gamma_{\mu_1 \ldots \mu_s} \quad , \qquad M^{j'j}_{\lambda'\lambda} \rightarrow M^{j'j}_{\lambda'\lambda(\mu_1 \ldots \mu_s)}$$

and

$$\Lambda\Gamma \rightarrow \Lambda^{\nu_1}_{\mu_1} \Lambda^{\nu_2}_{\mu_2} \ldots \Lambda^{\nu_s}_{\mu_s} \Gamma_{\nu_1 \ldots \nu_s} \quad . \tag{3}$$

With the Lorentz transformation $U(\Lambda) = e^{i\omega K_3} e^{-i\theta J_2}$, where

$$\tan\theta = \left(\frac{P'}{E'} - \frac{P}{E}\right) \Big/ \left(\frac{Q}{E} + \frac{Q'}{E'}\right) \quad , \tag{4}$$

and

$$\cosh\omega = \frac{1}{\sqrt{1-\beta^2}} \quad , \qquad \beta = \frac{P\cos\theta + Q\sin\theta}{E} = \frac{P'\cos\theta - Q'\sin\theta}{E'} \quad , \tag{5}$$

we obtain the brick wall reference frame:

$$p \rightarrow (\eta,\ -\Delta,\ 0,\ 0)$$

$$p' \rightarrow (\eta',\ \Delta',\ 0,\ 0)$$

where

$$-\Delta = -Q\cos\theta + P\sin\theta \quad ,$$

$$\Delta' = Q'\cos\theta + P'\sin\theta \quad ,$$

and

$$\eta = \sqrt{m^2+\Delta^2} \quad , \qquad \eta' = \sqrt{m'^2+\Delta'^2} \quad .$$

Equation (1) can therefore be written as follows:

$$M^{j'j}_{\lambda'\lambda} = \langle \underset{\sim}{p}'j'\lambda' | U^{-1}(\Lambda)\ U(\Lambda)\ \Gamma U^{-1}(\Lambda)\ U(\Lambda) | \underset{\sim}{p} j\lambda \rangle$$

$$= \sum_{n,n'} D^{j'}_{\lambda'n'}(R'^{-1})\ \langle \underset{\sim}{\Delta}'j'n' | (\Lambda\Gamma) | -\underset{\sim}{\Delta} jn \rangle\ D^{j}_{n\lambda}(R) \quad , \tag{6}$$

where the D's are the rotation matrix and R and R' are given by

$$R = e^{-\frac{\pi}{2}iJ_2} R_w R_{\hat{p}} \quad , \qquad R_w = B^{-1}(-\underset{\sim}{\Delta})\ U(\Lambda)\ B(\underset{\sim}{p})$$

and (7)

$$R' = e^{\frac{\pi}{2}iJ_2} R'_w R_{\hat{p}'} \quad , \qquad R'_w = B^{-1}(\underset{\sim}{\Delta}')\ U(\Lambda)\ B(\underset{\sim}{p}') \quad .$$

Here R_w denotes the Wigner rotation, $B(\underset{\sim}{p}) = e^{-i\underset{\sim}{\xi}\cdot\underset{\sim}{K}}$ boosts the particle at rest to a state with momentum $\underset{\sim}{p}$, and $R_{\hat{p}}$ rotates the z-axis to the direction of $\underset{\sim}{p}$. It can be shown that R_w and R'_w are of the form

$$R_w = e^{iJ_2\alpha} \quad , \qquad R'_w = e^{-iJ_2\alpha'} \quad . \tag{8}$$

In general, α and α' are complicated functions of momenta and masses. But in the limit P, $P' \to \infty$ and Q, Q' finite they are simple:

$$\alpha = \tan^{-1}(\Delta/m) \quad , \qquad \alpha' = \tan^{-1}(\Delta'/m') \quad . \tag{9}$$

From (7) and (8), we have

$$R = e^{i\phi J_2} \quad , \qquad R' = e^{-i\phi' J_2} \quad , \tag{10}$$

where

$$\phi = -\frac{\pi}{2} + \alpha - \theta_p \quad , \qquad \theta_p = \sin^{-1}(Q/P) \quad ,$$

and

$$\phi' = -\frac{\pi}{2} + \alpha' - \theta_{p'} \quad , \qquad \theta_{p'} = \sin^{-1}(Q'/P') \quad .$$

Therefore one can replace the rotation matrices D's in Eq. (6) by the d-functions. Inverting these equations, we get our desired relation:

$$\sum_{\lambda'\lambda} M^{j'j}_{\lambda'\lambda}\, d^{j'}_{n'\lambda'}(\phi')\, d^{j}_{\lambda n}(\phi) = \langle \underset{\sim}{\Delta}' j' n' \,|\, (\Lambda\Gamma) \,|\, -\underset{\sim}{\Delta} j n \rangle \quad . \tag{11}$$

Now the l.h.s. of Eq. (11) is the helicity amplitude in the brick wall reference frame. Since the largest angular momentum carried by Γ is s, this must vanish if $|n+n'| > s$, in which case

$$\sum M^{j'j}_{\lambda\lambda}\, d^{j'}_{n'\lambda}(\phi') d^{j}_{\lambda n}(\phi) = -\sum_{\lambda\neq\lambda'} M^{j'j}_{\lambda'\lambda}\, d^{j'}_{n'\lambda'}(\phi')\, d^{j}_{\lambda n}(\phi) \quad , \qquad |n+n'| > s \quad . \tag{12}$$

Let us consider a set of pairs (n, n') for which (12) is satisfied. Consider the case

$$j' > s \quad . \tag{13}$$

Then we choose $n_i = i$ and $n'_i = j'$ for $i \geq 0$ and $n'_i = -j'$ for $i < 0$ with $i = j, j-1, \ldots, -j$, Eq. (12) now becomes a set of 2j+1 linear equations which expresses the 2j+1 diagonal elements $M_{\lambda\lambda}$ in terms of the off-diagonal elements $M_{\lambda'\lambda}$, $\lambda'\neq\lambda$. We write

$$\sum_k A_{ik}\, x_k = y_i \quad , \qquad i, k = j, j-1, \ldots, -j \quad , \tag{14}$$

where

$$A_{ik} = d^{j}_{ki}(\phi)\, d^{j'}_{j'k}(\phi') \qquad \text{for } i \geq 0$$

$$= d^{j}_{ki}(\phi)\, d^{j'}_{-j'k}(\phi') \qquad \text{for } i < 0 \quad ,$$

$$x_k = M^{j'j}_{kk}$$

and

$$y_i = -\sum_{\lambda'\neq\lambda} M^{j'j}_{\lambda'\lambda}\, d^j_{\lambda_i}(\phi)\, d^{j'}_{j'\lambda'}(\phi') \qquad \text{for } i \geq 0$$

$$= -\sum_{\lambda'\neq\lambda} M^{j'j}_{\lambda'\lambda}\, d^j_{\lambda_i}(\phi)\, d^{j'}_{-j'\lambda'}(\phi') \qquad \text{for } i < 0 \quad .$$

It is easy to convince oneself that the determinant of the matrix A does not vanish for arbitrary ϕ and ϕ', therefore Eq. (14) has a solution. Thus we have expressed all the diagonal elements $M_{\lambda\lambda}$ in terms of the off-diagonal elements $M_{\lambda',\lambda}$ when the condition, Eq. (14), is met. We observe that the discrete symmetries, i.e., P, C, or T invariances were not used in our derivation. If these symmetries exist, then not all of the $M_{\lambda\lambda}$ are independent, which should of course be consistent with Eq. (14).

III. CONDITION FOR THE HELICITY CONSERVATION

We now use the results of the previous section to derive the condition for the helicity conservation for the vertex (1). As was noticed earlier, the helicity can not in general always be conserved in view of the angular momentum conservation (e.g., the scattering by 180 degrees). Accordingly, we must define the precise meaning of our asymptotic helicity conservation. For this purpose, we first consider the high energy behavior of the matrix element (1). From the Lorentz invariance, i.e., Eq. (6), we have[8]

$$M^{j'j}_{\lambda'\lambda}(00\ldots0,\ 33\ldots3) \rightarrow \left(\frac{P}{m}\right)^s L^{j'j}_{\lambda'\lambda} + 0\,(P^{s-1})$$

as

$$P \rightarrow \infty, \qquad P-P',\ Q,\ Q' \text{ finite} \quad , \tag{15}$$

where

$$L^{j'j}_{\lambda'\lambda} = d^{j'}_{\lambda'n'}(-\phi')\, d^j_{n\lambda}(-\phi) < \underset{\sim}{\Delta}'j'n' \,|\, \Gamma_{++\ldots+} \,|\, \underset{\sim}{\Delta}jn> \quad , \tag{16}$$

$$(+\ \equiv\ 0+3) \quad .$$

In (15), the number of 0 or 3 indices is arbitrary (their sum is, of course, s). Notice that when there are ℓ transverse indices t_i=1 or 2 in the current, then the leading behavior of the matrix element will be $P^{s-\ell}$. We shall say that the helicity is conserved asymptotically if

$$L^{j'j}_{\lambda'\lambda} \propto \delta_{\lambda'\lambda} \quad . \tag{17}$$

Notice that (17) is always true when Q=Q'=0, so we must consider Q≠0, Q'≠0 cases to obtain nontrivial restriction. The cases with massive and massless particles will now be considered separately.

A. Massive Particles (m≠0, m'≠0)

For this case, the results of the previous section tells us that when j'>s, we can express all the diagonal elements $M_{\lambda\lambda}$ in terms of the off-diagonal elements $M_{\lambda'\lambda}$ and the d-functions $d^j_{\lambda\mu}(\phi)$ and $d^{j'}_{\lambda\mu}(\phi')$. Furthermore the angles ϕ and ϕ' are finite in the above limit in view of Eq. (9). Therefore if we require (17), $L^{j'j}_{\lambda'\lambda}$ must identically vanish, which in turn means that $\langle \underset{\sim}{\Delta}'j'n'|\Gamma_{++\ldots+}|-\underset{\sim}{\Delta}jn\rangle = 0$ for all n and n'. From the rotational properties of Γ, it then follows that Γ=0. Excluding this as the trivial case, we have therefore shown that <u>if j'>s, where j' is the larger of the two spins j and j', then the helicity can not be conserved asymptotically</u>.

When j'≤ s, the helicity conservation is in general possible. As an important example, we consider the case s=1. The only possible combinations of (j, j') which satisfy the helicity conservation are $(\frac{1}{2}, \frac{1}{2})$ (0, 1) and (1. 1) (the case (0, 0) is trivial). When $\partial_\mu \Gamma_\mu = 0$, the corresponding vertices are:

$$(\tfrac{1}{2}, \tfrac{1}{2}): \quad V_\mu = \bar{\psi}\gamma_\mu\psi \quad ,$$

$$(1, 0): \quad V_{\mu\alpha} = -q^2 g_{\mu\alpha} + q_\mu q_\alpha \quad ,$$

and

$$(1, 1): \quad V_{\mu\alpha\beta} = (p+p')_\mu\, g_{\alpha\beta} - (p'+q)_\alpha\, g_{\beta\mu} + (q-p)_\beta\, g_{\alpha\mu} \quad . \tag{18}$$

In (18), the indices α and β are to be contracted with the polarization vectors of the particle p and p' respectively. We emphasize that the case listed and the vertices given in (18) are complete. As mentioned in the introduction, we have obtained the usual γ_μ coupling for $(\frac{1}{2}, \frac{1}{2})$ and the Weinberg coupling for (1, 1).

B. The Case with Massless Particle

When one or both of the particles in (1) are massless, our treatment should be modified because they belong to different little group and also because there are only two helicity states j and -j for massless particle of spin j.[9] Under the Lorentz transformation one particle state of a massless particle changes simply as

$$U(L)\,|p,\lambda> = e^{-i\lambda\Phi(L)}\,|Lp,\lambda> \quad , \qquad \lambda = \pm j$$

where $\Phi(L)$ is a number determined by L. Therefore derivation for the present case is simpler but different from the massive case. We shall state only the results here: When one or both of the particles in (1) are massless, the helicity can not be conserved if $j+j' > s$. Notice that this is more stringent than the case A. For example when s=1, the helicity conservation for the present case is possible only when both of the initial and the final particles have spins equal to 0 or $\frac{1}{2}$. It should be pointed out that $V_{\mu\alpha\beta}$ in (18) does not give the correct coupling in the limit $m \to 0$ because the gauge invariance is not satisfied, i.e., $V_{\mu\alpha\beta}p_\alpha \neq 0$. When both particles are massless, we must of course require $j=j'$ in order to have the helicity conservation.

IV. CONCLUDING REMARKS

We have shown that the requirement of the asymptotic conservation of the helicity on the vertex functions puts a strict condition on the spins of the particles. One can also study the consequences of the asymptotic helicity conservation of the four point functions in the diffraction scattering to get restrictions on the t-channel exchanges.[10] With the absence of the experiment which measures the cross sections involving particles with spin greater than 1, and also with our present ignorance on the nature of the Pomeron coupling, the implication of these studies seem uncertain.

For our discussions of the vertex functions, one might consider that the current with s=1, e.g., the electromagnetic or the Gell-Mann currents, are more fundamental than those with larger spins. The requirement of the asymptotic helicity conservation then limits the spins of the particles to be 0, $\frac{1}{2}$ or 1. For this case, it is known[11] that the vertices also satisfies the Drell-Hearn sum rule.[12] Since the latter sum rule is essentially the statement of a good high energy behavior of the cross sections, our restriction might well be related to the well known fact that the field theories of higher spins are not renormalizable.

Finally it is amusing to note that the theories with spin greater than 1 encounter difficulties even in the free field level due to the mass instability.[13] It seems therefore that our work puts more pessimism on the already troubled theories of higher spin.

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FOOTNOTES AND REFERENCES

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5. If A_2 is considered as resonance, this rule seems to be violated in experiments. For a survey of experimental situation, see F. J. Gilman, invited talk presented at 3rd International Conference on Experimental Meson Spectroscopy, University of Pennsylvania, Philadelphia, 1972.

6. After the main parts of this manuscript was completed, the author learned that the same conclusion for certain special cases was already appeared in the literature. See M. L. Blackmon, M. J. King and K. C. Wali, Phys. Letters 35B, 44 (1971). Also by same authors, Phys. Rev. D4, 559 (1971). Their argument was based on the explicit examples of the vertex functions. Our method is general and shows clearly the connection of our restriction with the angular momentum conservation. I thank F. J. Gilman for calling attention to the above papers.

7. For formula, see F. R. Halpern, Special Relativity and Quantum Mechanics (Prentice Hall, Englewood Cliff, New Jersey, 1968), Appendix 3.

8. We restore the Lorentz indices neglected in the previous section. See Eq. (3).

9. One can of course read the Wigner's paper, or S. Gasiorowicz, Elementary Particle Physics (John Wiley and Sons, Inc., New York, 1966), Chapter IV.

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FIGURE CAPTIONS

1. Diagram for Eq. (1).

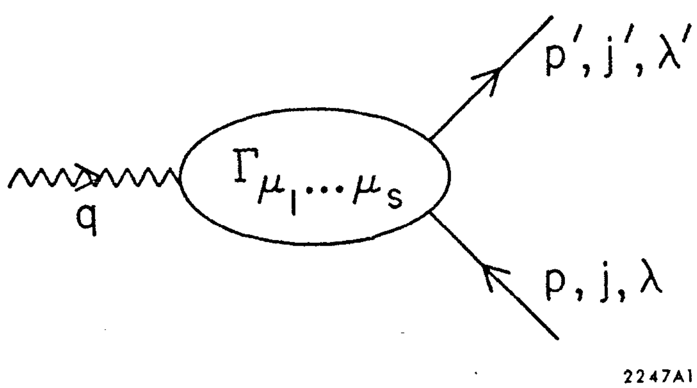

Fig. 1