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PCAC AND CHIRAL ANOMALIES*

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Abstract

We discuss means of testing the validity of weak and strong PCAC when applied to the theory of chiral anomalies. A factorization property (abstracted from a model due to Drell) is proposed and applied to $\eta \to \gamma\gamma$, $\eta \to \pi\pi\gamma$, $\gamma \to \pi\pi\pi$, and $\gamma\gamma \to \pi\pi\pi$.

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I. Introduction

It is still an open question whether the low energy theorems obtained from the anomalous Ward identities of SU3 $\times$ SU3[1] correctly and usefully describe the on-mass-shell quantities which are measured in the laboratory. The question has considerable interest, because if the answer is affirmative, we stand to learn a great deal about the relevance of renormalized perturbation theory to hadronic physics,[2] about the short distance singularities of products of currents,[3,4] and about the constituents of hadronic currents. The central problem is the reliability of the PCAC hypothesis, which must be used to compare the low energy theorems with experiment. Because of this problem, it is not trivial to decide whether the theorems (which are supposed to be exact for unphysical values of the momenta) are correct or not, and it is possible that they may be correct but not be useful. PCAC in its most naive versions,[5] "strong PCAC," has been systematically criticized by Brandt and Preparata,[6] and, most recently, Drell[7] has suggested that the application of strong PCAC to the low energy theorem for $\pi_0 \longrightarrow \gamma\gamma$ may lead to an underestimate of the decay rate by one order of magnitude.

In this note, we discuss ways to test the applicability of strong PCAC and Drell's version of weak PCAC to the low energy theorems obtained from chiral Ward identity anomalies. We

(A) propose a "factorization" scheme, abstracted from the model of Drell, which relates the application of PCAC in the many low energy theorems which follow from anomalous Ward identities, and

(B) call attention to experimental tests which distinguish dramatically between strong PCAC and Drell's version of weak PCAC.

Specifically, we find from a study of the decays $\eta \longrightarrow \gamma\gamma$ and $\eta \longrightarrow \pi_+ \pi_- \gamma$ results which are compatible with strong PCAC for the pion but not for the eta (and which therefore suggest an hypothesis such as three triplets of quarks[8,9] to explain the rate for $\pi_0 \longrightarrow \gamma\gamma$). However, these results are not conclusive since they suffer from uncertainties common to all eta low energy theorems—the η-η' mixing problem and the largeness of the η mass. In particular, we have assumed that the amplitudes which are presumably represented exactly by the anomaly at $p_\eta^2 = 0$ are still dominated by the anomaly at $p_\eta^2 = m_\eta^2$; in view of the large value of m_η, this is a very speculative assumption. We therefore consider the low energy theorems for $\gamma \longrightarrow \pi\pi\pi$ and $\gamma\gamma \longrightarrow \pi\pi\pi$, which do not suffer from the uncertainties of the eta calculation. The predictions of weak and strong PCAC for these processes differ by orders of magnitude in the rate.

II. Factorization

We begin by discussing the factorization scheme, based on an assumption abstracted from the weak PCAC model of Drell.[7] An anomalous chiral Ward identity implies an exact low energy theorem for $\mathscr{F}_\pi(0)$, defined by

$$\frac{m_\pi^2 - q^2}{\frac{1}{\sqrt{2}} F_\pi m_\pi^2} \langle \gamma(k_1, \epsilon_1)\gamma(k_2, \epsilon_2)|D_3|\Omega\rangle$$

$$= \epsilon_{\mu\nu\sigma\tau}\, k_1^\mu k_2^\nu \epsilon_1^\sigma \epsilon_2^\tau \mathscr{F}_\pi(q^2) \tag{1}$$

where $F_\pi = .96\, m_\pi$ is the π_+ decay constant, $q = k_1 + k_2$ is the sum of the photon momenta, and $D_3 = \partial_\mu A_3^\mu$ is the divergence of the neutral, $I = 1$ axial current. The low energy theorem states that

$$\mathscr{F}_\pi(0) = -\frac{2\alpha}{\pi}\frac{\sqrt{2}}{F_\pi} S \tag{2}$$

and in renormalized perturbation theory to any finite order,[2] S is determined by the bare constituents which circulate in the loop of Fig. 1. For the single triplet quark model,[10] S = 1/6; for three-triplet models,[8,9] S = 1/2. The strong PCAC hypothesis asserts that

$$\mathscr{F}_\pi(0) \cong \mathscr{F}_\pi\left(m_\pi^2\right) \tag{3}$$

in which case S = 1/2 but not S = 1/6 is consistent with the experimental rate[11] for $\pi_0 \rightarrow \gamma\gamma$.

Drell presents a semiquantitative argument, according to which (3) might be replaced by

$$\mathscr{F}_\pi(0) \cong \frac{1}{3}\, \mathscr{F}_\pi\left(m_\pi^2\right) \tag{4}$$

so that S = 1/6 would be compatible with $\Gamma(\pi_0 \rightarrow \gamma\gamma)$. Equation (4) is derived by assuming a dispersion relation in q^2 for the quark-quark-divergence vertex of Fig. 1. Off mass-shell effects of the quarks are neglected, so the discussion is similar to the dispersive treatment of the Goldberger-Treiman relation.[12] Qualitatively, the idea is that unlike the nucleon Goldberger-Treiman relation, in this quark Goldberger-Treiman relation there are no hadronic form factors which suppress the contribution of high mass states in the π_0 channel. Therefore, <u>if</u> important high mass states (resonant or continuum) are indeed present, they may make an important contribution, significantly changing the prediction of strong PCAC. Pursuing a semiquantitative argument based on a model of hadrons as bound states, Drell concludes that there are important contributions from high mass states, and he arrives at Eq. (4).

Our attitude in this note is (following Drell) to adopt the hypothesis that it is not seriously wrong to assume a dispersion relation for the quark-quark-divergence vertex, in which off mass-shell effects of the quarks are neglected. We present some simple consequences of this hypothesis. We do not commit ourselves as to whether there are important high mass phenomena in the π_0 channel and we do not assume a bound state model of the hadrons.

We define a pion extrapolation factor, E_π:

$$\mathscr{F}_\pi\left(m_\pi^2\right) \equiv E_\pi \; \mathscr{F}_\pi(0) \tag{5}$$

Let $H(q^2)$ be the quark-quark-divergence vertex function (dependence on quark momenta is suppressed). By hypothesis

$$H(q^2) = \frac{1}{\sqrt{2}} \; \frac{F_\pi m_\pi^2 \, g_Q}{m_\pi^2 - q^2 - i\epsilon} + \frac{1}{\pi} \int_{9m_\pi^2}^{\infty} \frac{\rho(\mu^2)\, d\mu^2}{\mu^2 - q^2 - i\epsilon} \tag{6}$$

where g_Q is the coupling of the π_0 to the quark and $\rho(\mu^2)$ is the spectral weight function. Then, as in ref. 7, E_π is determined to be

$$E_\pi = \left\{ 1 + \frac{\sqrt{2}}{F_\pi g_Q \pi} \int_{9\,m_\pi^2}^{\infty} \frac{\rho(\mu^2)\, d\mu^2}{\mu^2} \right\}^{-1} . \tag{7}$$

Extrapolation factors E_η and E_K are defined[13] in analogy with Eq. (5), and their values are fixed by dispersion relations like Eq. (7).

In addition to the triangle diagram anomalies, there are also anomalies in chiral Ward identities of four and five currents.[14] All the chiral anomalies are interrelated[15] by SU3 × SU3, which is presumed to be an exact symmetry of the leading short distance singularities[3] and hence of the anomalies. Thus the value of the V-V-A triangle anomaly determines the values of all the other SU3 × SU3 anomalies.

Consider a four-point function, consisting of a photon and three axial current divergences,

$$\frac{\left(m_a^2-p^2\right)\left(m_b^2-q^2\right)\left(m_c^2-r^2\right)}{\left(F_a F_b F_c\right)\left(m_a^2 m_b^2 m_c^2\right)} \int_{x,y,z} e^{i(px+qy+rz)} \langle T^* D_a(x) D_b(y) D_c(z) V_{em}^{\mu}(0) \rangle_{\Omega}$$

$$\equiv \epsilon^{\mu\alpha\beta\gamma} p_\alpha q_\beta r_\gamma \mathscr{F}_{abc}(p^2, q^2, r^2, w^2, \ldots) \tag{8}$$

where $w = p + q + r$ is the momentum of the photon.

$\mathscr{F}_{abc}(0,0,0,0,\ldots)$ is determined by anomalous Ward identities[16-18] (we give an example below). Furthermore, to any finite order in a renormalizeable theory of quarks, $\mathscr{F}_{abc}(0,0,0,0,\ldots)$ is given by the square diagram of Fig. 2. By hypothesis, we may write dispersion relations at each of the three quark-quark-divergence vertices, so that the on-mass-shell four-point function is

$$\mathscr{F}_{abc}\left(m_a^2, m_b^2, m_c^2, 0, \ldots\right) = E_a E_b E_c \; \mathscr{F}_{abc}(0,0,0,0,\ldots) \tag{9}$$

where E_a, E_b, E_c are the extrapolation factors defined in (5) and below. Equation (9) is an example of what we call the PCAC factorization property. If it is correct, then the extrapolation to the mass shell for all low energy theorems obtained from chiral anomalies is determined by just three numbers, E_π, E_k, and E_η. We now proceed to consider some consequences of (9).

III. $\eta \rightarrow \gamma\gamma$ and $\eta \rightarrow \pi^+\pi^-\gamma$

We define $\mathscr{F}_\eta(p^2)$ and $\mathscr{F}_{\eta\pi_+\pi_-}\left(p^2, k_+^2, k_-^2, w^2, \ldots\right)$ as in Eq. (1) and (8). Then anomalous Ward identities imply the following low energy theorems:

$$\mathscr{F}_\eta(0) = -\frac{1}{\sqrt{3}} \frac{2\alpha}{\pi} \frac{1}{F_\eta} S \tag{10}$$

$$\mathscr{F}_{\eta\pi_+\pi_-}(0,0,0,0,\ldots) = -\frac{1}{\sqrt{3}} \frac{e}{\pi^2} \frac{1}{F_\pi^2 F_\eta} S, \tag{11}$$

where $e = \sqrt{4\pi\alpha}$ is the electron charge and S is the model dependent quantity defined in (2). The derivation of (10) is identical to the derivation of (2).[1] Equation (11) is contained in the effective Lagrangian of Wess and Zumino[15] and Aviv and Zee.[18] It is a consequence of the Ward identity.

$$p_\mu k_{+\nu} k_{-\sigma} \int_{x,y,z} e^{i(px-k_+y-k_-z)} \langle T^* A_8^\mu(x) A_+^\nu(y) A_-^\sigma(z) V_3^\lambda(0) \rangle_\Omega$$

$$= \frac{S}{2\sqrt{3}\,\pi^2} \epsilon^{\mu\nu\sigma\lambda} p_\mu k_{+\nu} k_{-\sigma}$$

$$+ 2k_{-\sigma} \int_{x,y} e^{ipx-i(k_+ + k_-)y} \langle T^* D_8(x) V_3^\sigma(y) V_3^\lambda(0) \rangle_\Omega$$

$$-i \int_{x,y,z} e^{i(px-k_+y-k_-z)} \langle T^* D_8(x) D_+(y) D_-(z) V_3^\lambda(0) \rangle_\Omega \qquad (12)$$

The first term on the right-hand side is the four-point anomaly. In the low-energy limit, the second term contributes because of the triangle anomaly. Using gauge invariance, the left-hand side is shown to be of fifth order in momentum, hence negligible in the low-energy limit. Including the contribution of the triangle anomaly and taking the low-energy limit, (12) is found to imply (11).

Now according to the factorization property, the physical amplitudes are given by

$$\mathscr{F}_\eta\left(m_\eta^2\right) = E_\eta \mathscr{F}_\eta(0) \qquad (13)$$

and

$$\mathscr{F}_{\eta\pi_+\pi_-}\left(m_\eta^2, m_\pi^2, m_\pi^2, 0, \ldots\right) = E_\pi^2 E_\eta \mathscr{F}_{\eta\pi_+\pi_-}(0,0,0,0,\ldots) \qquad (14)$$

Performing the phase space integration, we then find from (10), (11), (13), and (14) that

$$\frac{\Gamma(\eta \rightarrow \pi_+ \pi_- \gamma)}{\Gamma(\eta \rightarrow \gamma\gamma)} = E_\pi^4 \frac{4}{e^2 F_\pi^4} \left(\frac{m_\eta^4}{96\pi^2}\right) (7.48 \times 10^{-3}) \tag{15}$$

Notice that (15) does not depend on E_η and S (or on the poorly known F_η). Experimentally[11] (15) is known to be $\sim .13 \pm .01$, from which we calculate that

$$E_\pi \cong 1.1 \ . \tag{16}$$

Equation (16) is consistent with strong PCAC for the pion, (3), but not with (4), which would require the branching ratio in (15) to be enhanced by a factor of $\sim 10^2$.

According to Eq. (14), the physical amplitude $\mathscr{F}_{\eta\pi_+\pi_-}$ should be constant, independent of the pion and photon momenta in the laboratory. This means that the Dalitz plot should be determined essentially by the phase space. At the 10 to 15% level, this is indeed the case experimentally.[19]

If we accept (16), then the experimental rate for $\pi_0 \rightarrow \gamma\gamma$ implies $S \cong 1/2$, as in the three triplet models.[8,9] If we further make the SU3 assumption that $F_\eta \cong \frac{1}{\sqrt{2}} F_\pi$, then we may calculate E_η from the experimentally[11] determined rate for $\eta \rightarrow \gamma\gamma$. The result is

$$E_\eta \cong 2.5 \ . \tag{17}$$

Thus we would conclude that strong PCAC is valid for the pion but not for the eta. In contrast to Drell, we would say that there are no important contributions to the spectral integral in (6).

Equations (16) and (17) are amusing and plausible results, but they should be regarded with an attitude of healthy skepticism. In addition to the hypothesis necessary to derive the factorization property (9), the analysis just presented suffers from at least two other sources of uncertainty:

(A) η - η' mixing: Our analysis assumes that the mixing angle vanishes, $\theta = 0$. The branching ratios for $\eta' \to \gamma\gamma$ and $\eta' \to \pi_+\pi_-\gamma$ are rather well known experimentally, but the total width of the η' is not. From the Gell-Mann-Okubo mass formula, $|\theta| \sim 11^\circ$,[20] and in some models[21] $|\theta|$ is even an order of magnitude smaller. If $\Gamma(\eta' \to \gamma\gamma)$ and $\Gamma(\eta' \to \pi_+\pi_-\gamma)$ are not much greater than $\Gamma(\eta \to \gamma\gamma)$ and $\Gamma(\eta \to \pi_+\pi_-\gamma)$, respectively, and if $|\theta| \lesssim 11^\circ$, then our analysis is not substantially affected.

(B) $m_\eta = 548.8 \pm .6$ MeV[11]: We know from the theory of the anomalies that at the zero energy points, the amplitudes are given by the diagrams of Fig. 1 and 2, and we have discussed how to extrapolate these diagrams to the physical points. But the physical amplitudes also receive contributions from other terms, which vanish at the zero energy point. Strong PCAC would lead us to expect these corrections to be of order m_η^2/m_H^2 with m_H a typical hadron mass, at worst the rho mass. According to weak PCAC, there may also be correction terms which are enhanced by the same extrapolation factors as the leading terms; in perturbation theory, these are exemplified by quark loop diagrams with internal gluon lines[22] (see Fig. 3). Schematically, the ratio of amplitudes may be represented by

$$\frac{\mathcal{M}(\eta \to \pi_+\pi_-\gamma)}{\mathcal{M}(\eta \to \gamma\gamma)} \sim \frac{E_\eta E_\pi^2 (1 + A) + B}{E_\eta (1 + A') + B'} \sim 1$$

where the value one on the right-hand side is determined by experiment. Here A, A', B, and B' are $0\left(m_\eta^2/m_H^2\right)$, A and A' being the enhanced corrections exemplified by Fig. 3. The solution presented above, $E_\pi \sim 1$, was obtained by neglecting the correction terms, A, A', B, and B'. However, one can imagine plausible values of A, A', B, B', and E_η which are consistent with $E_\pi \sim 3$ or, indeed, with a wide range of values of E_π. Thus the soft eta theorem is inherently inconclusive.

We, therefore, proceed to consider the soft pion theorems of the next section, which are free of the two difficulties just discussed.

IV. $\gamma \to \pi\pi\pi$ and $\gamma\gamma \to \pi\pi\pi$

Anomalous Ward identities imply a low energy theorem[16-18] for $\gamma \to \pi_+\pi_-\pi_0$, analogous to the theorem for $\eta \to \pi_+\pi_-\gamma$ discussed in the previous section. The low energy theorem is

$$\mathscr{F}_{\pi_+\pi_-\pi_0}(0,0,0,0,\ldots) = -\frac{e}{\pi^2}\,\frac{\sqrt{2}\,S}{F_\pi^3} \quad . \tag{18}$$

We then have the predictions for the physical amplitudes

$$\frac{\mathscr{F}_{\pi_+\pi_-\pi_0}\left(m_\pi^2,\ m_\pi^2,\ m_\pi^2,\ 0,\ \ldots\right)}{\mathscr{F}_\pi\left(m_\pi^2\right)} = \frac{2\,E_\pi^2}{e\,F_\pi^2} \tag{19}$$

With $E_\pi^2 = 1$ or 9, the contrast between the predictions is quite dramatic. Equation (19) is free of problems (A) and (B) enumerated in the previous section, but its verification presents a considerable challenge to experimental technique. The experimental possibilities have already been discussed in some detail by Aviv and Zee[18] and by Zee.[23] Here we shall restrict ourselves to a few comments, particularly with regard to how the situation is modified by the weak PCAC hypothesis.

Before we discuss the experimental possibilities, let us just mention a set of assumptions which allow one to evaluate $\mathscr{F}_{\pi_+\pi_-\pi_0}\left(m_\pi^2,\ m_\pi^2,\ m_\pi^2,\ 0,\ \ldots\right)$ from already available experimental data. These are (1) that $\mathscr{F}_{\pi_+\pi_-\pi_0}\left(m_\pi^2, m_\pi^2, m_\pi^2, w^2,\ldots\right)$

satisfies an unsubtracted dispersion relation in w^2, which (2) may be dominated by its vector meson poles. Using the experimental data,[24] we then find

$$\frac{\mathscr{F}_{\pi_+\pi_-\pi_0}\left(m_\pi^2, m_\pi^2, m_\pi^2, 0, \ldots\right)}{\mathscr{F}_\pi\left(m_\pi^2\right)} \cong \frac{7}{e F_\pi^2} \tag{20}$$

where the principal contribution is from the ω, the contribution of the ϕ being smaller by an order of magnitude. The result (20) ($E_\pi \sim 2$) is neatly lodged between the predictions of strong and weak PCAC, suggesting that a plague on both houses may not be out of order. Of course, we have little feeling for the reliability of assumptions (1) and (2), so we proceed to discuss how $\mathscr{F}_{\pi_+\pi_-\pi_0}$ might be determined more directly from experiment.

More conservatively, we follow the proposal of reference 23 to extract $\mathscr{F}_{\pi_+\pi_-\pi_0}\left(m_\pi^2, m_\pi^2, m_\pi^2, 0, \ldots\right)$ from the data for $\sigma\left(e^+e^- \to \gamma \to \pi_+\pi_-\pi_0\right)$ by assuming a once subtracted dispersion relation for $\mathscr{F}_{\pi_+\pi_-\pi_0}\left(m_\pi^2, m_\pi^2, m_\pi^2, w^2, \ldots\right)$ with the subtraction fixed at $w^2 = 0$ by (19). The value of the subtracted dispersion integral is estimated by taking the contribution of the ω pole. We then find that the cross section near threshold is given by[25]

$$\sigma(w^2) \cong 4.6\times 10^{-11}\, m_\pi^{-2} \left(\frac{\sqrt{w^2} - 3m_\pi}{m_\pi}\right)^4 \cdot \left| E_\pi^2 + c_\omega \frac{w^2}{w^2 - m_\omega^2} \right|^2 . \tag{21}$$

We have checked this threshold approximation by calculating the phase space integral numerically: at $\sqrt{w^2} = 4m_\pi$, the right-hand side of (21) is too small by a factor of 2.

In agreement with reference 23, we find from $\Gamma(\omega \to e^+e^-)$ that $|c_\omega| \sim 4$. The experimental problem is then seen to be whether $\sigma(w^2)$ can be measured with sufficient accuracy and the "background," represented here by $c_\omega \frac{w^2}{w^2 - m_\omega^2}$, known reliably enough so that the value of E_π can be extracted. A further uncertainty is due to the unknown phase[26] of c_ω: when the phase is $\pi/2$, E_π^4 must be extracted from

$$E_\pi^4 + \left| c_\omega \frac{w^2}{w^2 - m_\omega^2} \right|^2 .$$

At $\sqrt{w^2} = 4m_\pi$, the background factor is ~ 4. We conclude that it may be possible to extract $\mathscr{F}_{\pi_+\pi_-\pi_0}\left(m_\pi^2, m_\pi^2, m_\pi^2, 0, \ldots\right)$ from $\sigma(e^+e^- \rightarrow \gamma \rightarrow \pi_+\pi_-\pi_0)$ if $E_\pi = 3$ but probably not if $E_\pi = 1$.

Another possible method for measuring $\mathscr{F}_{\pi_+\pi_-\pi_0}$ is Primakoff double pion production. This method is discussed in references 17 and 23. We have nothing to add except the obvious comment that the separation of the low energy term from the background will be greatly facilitated if $E_\pi = 3$.

Finally we discuss the low energy theorems[16,17] for $\gamma\gamma \rightarrow \pi_+\pi_-\pi_0$ and $\gamma\gamma \rightarrow \pi_0\pi_0\pi_0$ in the context of weak PCAC. The theorems relate these amplitudes to $\mathscr{F}_\pi(0)$ and $\mathscr{F}_{\pi_+\pi_-\pi_0}(0,0,0,0,\ldots)$. According to strong PCAC, the physical amplitudes are assumed to be approximately equal to the amplitudes at the low energy point. We wish to remark that the predictions of weak PCAC for these processes are ambiguous.

We shall consider $\gamma\gamma \rightarrow \pi_0\pi_0\pi_0$; similar remarks apply to $\gamma\gamma \rightarrow \pi_+\pi_-\pi_0$. The low energy theorem is obtained by adding to the pole diagram of Fig. 4 terms which satisfy the requirements of current algebra and gauge invariance. The result is proportional to $\mathscr{F}_\pi(0)$ because of the $\gamma\gamma \rightarrow (\pi_0)_{\text{virtual}}$ vertex, and the question is how $\mathscr{F}_\pi(0)$ should be extrapolated when the external pions and photons have physical momenta. In the weak PCAC ansatz, the only important variable is the invariant mass at the pion leg, which we call q^2. For convenience, we assume that the continuum in (7) may be represented by a pole, which Drell[7] calls the π' and assigns a mass $m' \cong 1.6$ GeV. Then it is straightforward to show that

$$\frac{\mathscr{F}_\pi(q^2)}{\mathscr{F}_\pi(0)} = E_\pi + (E_\pi - 1)\frac{m'^2\left(q^2 - m_\pi^2\right)}{m_\pi^2\left(m'^2 - q^2\right)} \tag{22}$$

The ambiguity arises in choosing the appropriate value of q^2 (a similar ambiguity arises in a proposal to determine the sign of $\mathscr{F}_\pi$ from proton Compton scattering[1,27]). If we regard Fig. 4 as a Feynman diagram, then we have $q^2 = s \geq 9\,m_\pi^2$, and using $m' \sim 1.6$ GeV, $s \sim 16\,m_\pi^2$, and $E_\pi \sim 3$, (22) yields an extrapolation factor of ~ 35 in the amplitude. On the other hand, a dispersive approach would suggest that we choose $q^2 = m_\pi^2$, so that (22) yields a factor of $E_\pi \sim 3$. Of course, in either case the weak PCAC prediction for $\sigma(\gamma\gamma \rightarrow \pi_0\pi_0\pi_0)$ is greatly enhanced (by one or three orders of magnitude) over the prediction of strong PCAC.

Concluding Remarks

Our principal assumption, abstracted from the model of Drell, is that the quark-quark-divergence vertex satisfies an unsubtracted dispersion relation in which off mass shell effects of the quarks can be neglected. This hypothesis implies the factorization property (9). Application of (9) to $\eta \rightarrow \pi_+\pi_-\gamma$ and $\eta \rightarrow \gamma\gamma$ suggests that strong PCAC may be valid for the pion but not for the eta. Further applications of (9) to $\gamma \rightarrow \pi\pi\pi$ and $\gamma\gamma \rightarrow \pi\pi\pi$ provide dramatic though experimentally difficult tests of weak and strong PCAC and of the theory of the anomalies.

Another possible application of (9) is to the contribution of the anomaly to the low energy theorem for $K\ell_4$ decay. The results reported[28,29] are consistent[30] with $E_\pi^2\,E_k = 1$ if $S = 1/2$ or $E_\pi^2\,E_k = 3$ if $S = 1/6$.

Apart from the original application to $\pi_0 \rightarrow \gamma\gamma$, we still have little indication that the elegant theory of anomalies is actually relevant to Ward identities of hadronic currents. More evidence would enable us to resolve the uncertainties due to PCAC, to confirm the relevance of the theory to hadrons, and, then, in the last stage, to acquire valuable information on the structure of the currents. At the present stage of confronting the theory with testable predictions, we look forward eagerly to data for $\sigma(\gamma \longrightarrow \pi\pi\pi)$ and $\sigma(\gamma\gamma \longrightarrow \pi\pi\pi)$, and on the values of Crewther's constants,[4] K and R. It is an important and challenging problem to develop additional proposals for confronting the theory with experiment.

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30. Of course, any attempt to use soft K theorems to test the anomaly will suffer uncertainties from the large value of m_K, analogous to the uncertainties due to m_η which were discussed in Section III.

Figure Captions

Fig. 1. The triangle diagram for $\langle \Omega | D_3 | \gamma\gamma \rangle$.

Fig. 2. The square diagram for $\mathscr{F}_{abc}$.

Fig. 3. Quark loop diagram with internal gluon lines.

Fig. 4. Pole diagram for $\gamma\gamma \longrightarrow \pi_0 \pi_0 \pi_0$.

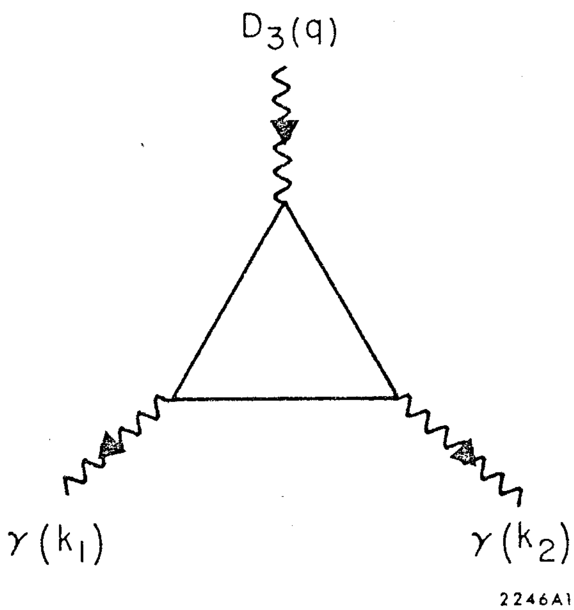

Fig. 1

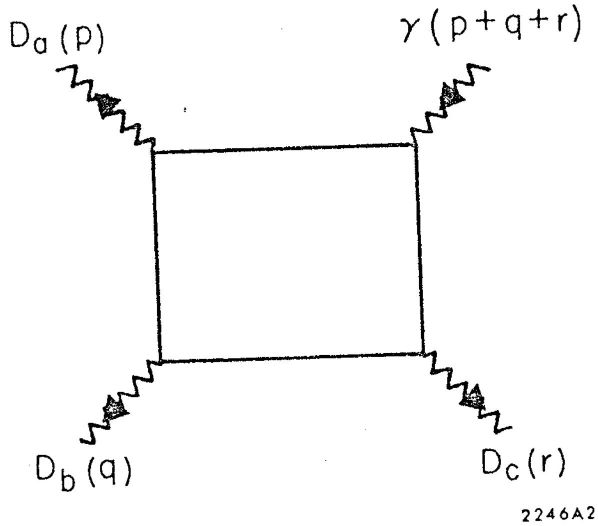

Fig. 2

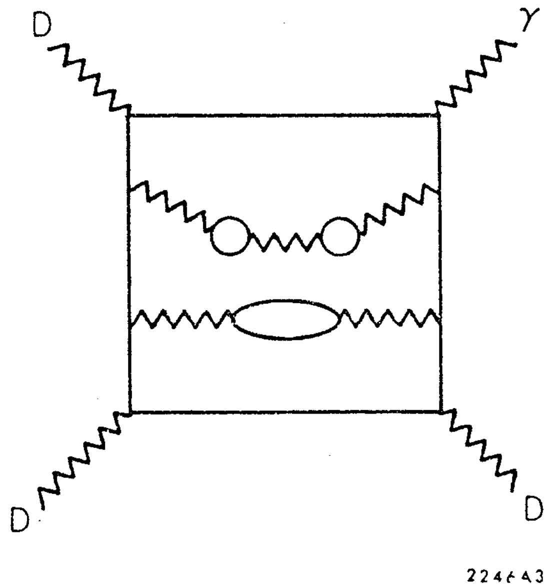

Fig. 3

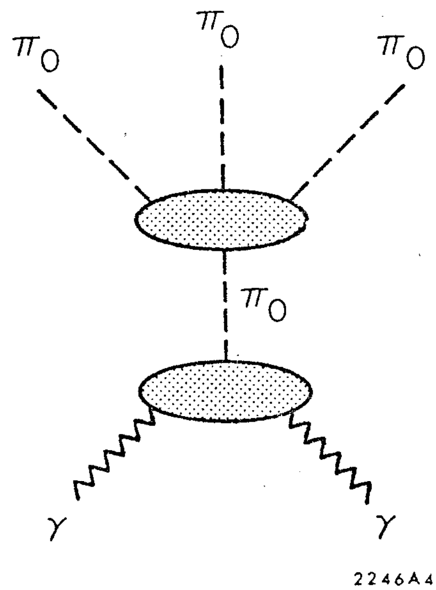

Fig. 4