# A MODEL FOR LOW ENERGY PION-NUCLEON SCATTERING\*

W. A. Ross, † G. Wrighton, and D.W.G.S. Leith

Stanford Linear Accelerator Center Stanford University, Stanford, California 94305

### ABSTRACT

Assuming the Harari-Freund conjecture on duality we make use of a model consisting of s-channel resonances plus a diffraction, or Pomeron, term to fit  $\pi$  p elastic differential cross sections and polarization data from 500 MeV/c to 1300 MeV/c. A fit is made to the observed data by varying as parameters the masses, widths, and elasticities of the resonances as well as the parameters associated with the Pomeron amplitude. A good fit to the data is obtained and the resulting parameters are presented. Important properties of the Pomeron such as its spin structure and energy dependence are extracted. Our results indicate a preference for spin conservation over helicity conservation in either the s- or the t-channel of the Pomeron amplitude.

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\*Work supported by the U. S. Atomic Energy Commission. †Present address: Physics Department, McGill University, Montreal, Canada.

### 1. Introduction

Recently a model incorporating both resonant and diffractive effects has been used to fit  $\pi p$  and K p elastic differential cross sections<sup>1, 2)</sup> around 1 GeV/c. In this paper we extend a previously unpublished attempt to describe  $\pi p$  scattering in the energy range 500 MeV/c to 1300 MeV/c. With this model we can extract information about the Pomeron amplitude at these energies by making the Harari-Freund conjecture<sup>3)</sup> associating the Pomeron amplitude with the diffractive, non-resonant background. Using this approach we can investigate the s and t dependence as well as the spin structure of the Pomeron term by fitting both the differential cross sections and the polarization data for  $\pi p$ elastic scattering.

In section 2 we present some motivation for our model. In section 3 we give our parameterization of the transition amplitudes. In sections 4 and 5 we give results and conclusions of this work.

### 2. Motivation for the model

In an attempt to construct a model of the pion nucleon scattering amplitude, we shall take for our basic assumption the Harari-Freund duality conjecture<sup>3)</sup>. Thus the amplitude is taken to be composed of the crossed channel Pomeranchuk singularity (equivalent in the direct channel to an isospin independent nonresonating background) and a sum of direct channel resonances (equivalent to the "ordinary" Regge trajectories in the crossed channel). This conjecture has received some measure of support from studies of  $\pi N$  and KN data as represented by phase shift analysis<sup>4,5)</sup>. While these authors have established the usefulness of this decomposition of the amplitude, there are quantitative differences between their conclusions. Specifically, while Harari's<sup>4)</sup> analysis of

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 $\pi N$  amplitudes favours s-channel helicity conservation for the Pomeron, the work of Meyers and Salin<sup>5)</sup> (KN) seem to favour spin conservation.

From these studies then, we conclude that the "Pomeron plus resonances" model is likely to be a constructive approach but that the details of the energy dependence and spin structure of the Pomeron amplitude at these energies are still unresolved. With this in mind, we have attempted to fit the  $\pi$  p elastic differential cross section and polarization data at all angles with the amplitude:

 $A = A_{RES} + A_{POMERON}$ .

### 3. Transition amplitudes

3.1 General conventions

First we shall define the transition amplitudes quite generally. For a given isospin state the scattering amplitude matrix is given by:

$$\mathbf{A}^{\mathbf{I}}(\mathbf{k}, \theta) = \mathbf{f}^{\mathbf{I}}(\mathbf{k}, \theta) + \mathbf{i} \boldsymbol{\sigma} \cdot \hat{\mathbf{n}} \mathbf{g}^{\mathbf{I}}(\mathbf{k}, \theta)$$

Here k and  $\theta$  are the center-of-mass momentum and scattering angle. The terms  $f^{I}$  and  $g^{I}$  are respectively the spin-non-flip and spin-flip amplitudes for isospin I, and  $\hat{n}$  is the normal to the scattering plane.

For a particular process the physical scattering amplitude is

$$A(k, \theta) = \sum_{i} C_{I} A^{I}(k, \theta) ,$$

where  $C_{I}$  is the appropriate isospin coupling factor.

Next, writing

$$A(k, \theta) = f(k, \theta) + i\sigma \cdot \hat{n} g(k, \theta)$$

one can easily get

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\mathbf{k},\theta) = \left| \mathbf{f}(\mathbf{k},\theta) \right|^2 + \left| \mathbf{g}(\mathbf{k},\theta) \right|^2$$

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and

$$P(k, \theta) \frac{d\sigma}{d\Omega}(k, \theta) = 2 \operatorname{Im} |f(k, \theta) g^{*}(k, \theta)|$$

Here P is the polarization and  $d\sigma/d\Omega$  the differential cross section. We can then perform the usual partial wave decomposition to get

where

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$$P_{\ell}^{1}(x) = \sqrt{1-x^{2}} \quad \frac{dP_{\ell}(x)}{dx}$$

3.2 Resonant amplitude

The resonant contributions will of course appear only in the specific resonance channels whereas the diffraction term will spill over into all  $\ell$  values.

The resonant term will be given by:

$$\begin{split} \mathbf{f}_{\mathrm{RES}}^{\mathrm{I}}(\mathbf{k},\theta) &= \frac{1}{\mathbf{k}} \sum_{\boldsymbol{\ell} = \boldsymbol{\ell}_{\mathrm{RES}}} \left\{ (\boldsymbol{\ell}+1) \; \mathbf{a}_{\boldsymbol{\ell}+}^{\mathrm{IRES}} + \boldsymbol{\ell} \; \mathbf{a}_{\boldsymbol{\ell}-}^{\mathrm{IRES}} \right\} \mathbf{P}_{\boldsymbol{\ell}}(\cos \theta) \quad , \\ \mathbf{g}_{\mathrm{RES}}^{\mathrm{I}}(\mathbf{k},\theta) &= \frac{1}{\mathbf{k}} \; \sum_{\boldsymbol{\ell} = \boldsymbol{\ell}_{\mathrm{RES}}} \left\{ \mathbf{a}_{\boldsymbol{\ell}+}^{\mathrm{IRES}} - \mathbf{a}_{\boldsymbol{\ell}-}^{\mathrm{IRES}} \right\} \mathbf{P}_{\boldsymbol{\ell}}(\cos \theta) \quad , \end{split}$$

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where the sum is over all resonant partial waves. Such resonant partial wave amplitudes can be taken to have a Breit-Wigner form

$$a^{\text{RES}} = \frac{x}{\epsilon - i}$$

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where  $x = \Gamma_{elastic}(k)/\Gamma(k)$  is the elasticity of the resonance.

$$\epsilon = 2 (E_{RES} - E) / \Gamma(k)$$
,

 $E_{\rm RES}$  and  $\Gamma(k)$  are the energy and (energy-dependent) width of the resonance.

For our purposes we take

$$\Gamma(k) = \frac{k v_{\ell}(kR)}{k_{RES} v_{\ell}(k_{RES}R)} \Gamma_{R} :$$

Here R is the interaction radius and  $v_{\ell}(x)$  is the appropriate barrier penetration factor given by Blatt and Weisskopf<sup>6)</sup> as:

$$\mathbf{v}_{\boldsymbol{\ell}}(\mathbf{x}) = \left\{ \left[ \mathbf{x} \mathbf{j}_{\boldsymbol{\ell}}(\mathbf{x}) \right]^2 + \left[ \mathbf{x} \mathbf{n}_{\boldsymbol{\ell}}(\mathbf{x}) \right]^2 \right\}^{-1}$$

with  $j_{\ell}(x)$  and  $n_{\ell}(x)$  the spherical Bessel functions.

# 3.3 Diffraction amplitude

We choose to determine the form of the diffraction amplitude phenomenologically, using the data on the near-forward scattering differential cross section.

For our purposes we fit the near-forward cross sections empirically by the form

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\theta) = \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \bigg|_{\theta=0} \, \mathrm{e}^{\mathrm{bt}} \quad ,$$

where  $t = -2k^2(1 - \cos \theta)$  is the invariant momentum transfer.

While this dependence can be explained by various models, we shall take it as an empirical fit. Next, working specifically with  $\pi$  p elastic data we can examine the energy dependence of b. This is shown in fig. 1. In doing so we observe that b has some structure showing peaks at particular values of k. A closer examination shows a distinct correlation between these peaks and the positions of  $\pi N$  resonances.

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In addition b may well level out, approaching a constant at high energies above the region where resonances are dominant.

Thus, we have chosen to use an angular dependence of the form  $\exp(\frac{1}{2}bt)$  for our Pomeron amplitude. Moreover, we shall assume that the energy dependence of b is quite smooth — in fact we will take b to be a constant throughout our energy range<sup>7</sup>. So, the Pomeron amplitude will be of the form iA(k) exp( $\frac{1}{2}bt$ ).

Next, in order to determine A(k), we can make use of the optical theorem and the total cross section data readily available in the literature<sup>8)</sup>. Taking the diffraction amplitude to be purely imaginary one can readily show that  $\frac{4\pi}{k}A(k)$  is the contribution of the diffraction term to the total cross section. Using this relationship we can now fit the total cross section in our energy range by using accepted resonance parameters<sup>9)</sup> and a smooth function (in fact a polynomial in k) for the background contribution,  $\frac{4\pi}{k}A(k)$ . The resulting A(k) is a function which is very small in the lower portion of the energy range and which grows quite rapidly in the upper energy region. This is shown in Fig. 2, where the fit to the total cross section data. The algebraic expression used in this fitting procedure (with the best fit coefficients), is

$$\sigma_{\text{tot}}^{\text{P}}(\mathbf{k}) = 4\pi \left( 0.689 - 2.93\mathbf{k} + 1.38\mathbf{k}^2 + 3.36\mathbf{k}^3 + 3.65\mathbf{k}^4 \right) ,$$

where k is in GeV/c and  $\sigma_{tot}^{P}$  is in millibarns. Thus, we choose our diffractive amplitude to be

$$A_{\text{POMERON}} = i CA (k) \exp(\frac{1}{2}bt)$$

where A(k) is fixed by the fit described above. C and b are two constant parameters which will be varied to fit the experimental data.

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Next, in order to write down the Pomeron amplitudes explicitly, we must determine the spin structure. For our purpose we shall make fits assuming spin conservation and assuming helicity conservation. For the case of spin conservation, the Pomeron amplitudes are given by:

$$f_{DIFF}(k, \theta) = A_{POMERON}(k, \theta)$$
,  
 $g_{DIFF}(k, \theta) = 0$ .

For the case of s-channel helicity conservation the Pomeron amplitudes are given by:

$$f_{\text{DIFF}}(k,\theta) = \frac{(1+\cos\theta)}{2} A_{\text{POMERON}}(k,\theta)$$
$$g_{\text{DIFF}}(k,\theta) = \frac{-\sin\theta}{2} A_{\text{POMERON}}(k,\theta) ,$$

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which corresponds to the s-channel helicity amplitudes being given by:

$$f_{++}^{(s)}(k,\theta) = \cos \frac{\theta}{2} A_{\text{POMERON}}(k,\theta)$$
$$f_{+-}^{(s)}(k,\theta) = 0 \quad .$$

And for the case of t-channel helicity conservation the Pomeron amplitudes are given by:

$$f_{\text{DIFF}}(\mathbf{k}, \theta) = \frac{\mathbf{E} + \mathbf{M} - \cos \theta (\mathbf{E} - \mathbf{M})}{2\mathbf{M}} \text{ i CA } (\mathbf{k}) \exp(\frac{1}{2}\mathbf{b}t)$$
$$g_{\text{DIFF}}(\mathbf{k}, \theta) = -\frac{(\mathbf{E} - \mathbf{M})}{2\mathbf{M}} \sin \theta \text{ i CA } (\mathbf{k}) \exp(\frac{1}{2}\mathbf{b}t) ,$$

where  $E = \sqrt{M^2 + k^2}$ , which corresponds to the t-channel helicity amplitudes being given by:

$$f_{++}^{(t)}(k,\theta) = - \frac{\sqrt{\frac{1}{4}t - M^2}}{\sqrt{s}} A_{\text{POMERON}}(k,\theta)$$
$$f_{+-}^{(t)}(k,\theta) = 0 \quad .$$

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Note that we have applied physical constraints to impose the  $\cos \theta/2$  factor on the s-channel helicity non-flip amplitude. Also, in keeping with its diffractive origin, we have chosen the Pomeron exchange term to have no isospin dependence.

# 4. Application to $\pi^{-}p$ elastic scattering

The model was tested by fitting  $\pi^- p \rightarrow \pi^- p$  differential cross sections and polarization data from 500 MeV/c to 1300 MeV/c (1369 MeV to 1831 MeV in c.m. energy). This included 61 differential cross sections and 45 polarizations for a total of 2083 data points, and comprised most of the world's data in this momentum range<sup>10)</sup>.

The amplitudes were taken to be

$$f(k, \theta) = f_{DIFF}(k, \theta) + 2/3 [I = 1/2 \text{ resonance terms}] + 1/3 [I = 3/2 \text{ resonance terms}]$$
$$g(k, \theta) = g_{DIFF}(k, \theta) + 2/3 [I = 1/2 \text{ resonance terms}]$$

+ 1/3 [I=3/2 resonance terms]

The initial parameters of 17 prominant resonances were taken from ref. 9. The best fit was obtained by minimizing  $\chi^2$  while varying the resonance width, resonance energy, the resonance coupling, and the diffraction parameters b and C. The parameters of five of the resonances, at the borders of our energy region, were held fixed.

Below are some of the characteristics of the best fits.

No.	of data points	=	2083
No.	of fitting parameters	Ξ	39
No.	of degrees of freedom	=	2044
No.	of resonances used	=	12 + 5 fixed

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$$\chi^{2} = \sum_{\text{data points}} \left(\frac{\sigma_{\text{fit}} - \sigma_{\text{exp}}}{\Delta \sigma_{\text{exp}}}\right)^{2} = \begin{cases} 5322 & \text{spin conservation} \\ 6044 & \text{s-channel helicity} \\ \text{conservation} \\ 5656 & \text{t-channel helicity} \\ \text{conservation} \end{cases}$$

The first comment we would like to make is about the quality of the data. In particular, the experiment errors used in the fitting procedure were those quoted by the authors and the normalization of the experiments was not allowed to vary. While the compatibility of the different experiments is questionable we chose to treat the data in this way in order to keep the number of fitting parameters to a minimum. While the confidence level resulting from our fit (taken at face value) is not very high, in view of the above-mentioned problems we feel the  $\chi^2$  is quite satisfactory. In any case the qualitative nature of the fit is extremely good as can be seen in figs. 3 and 4. These figures show a sample of the fits to differential cross sections and to polarizations at several energies. In addition, in fig. 5 we show the differential cross section at a fixed angle ( $\theta = 0^\circ$ ,  $90^\circ$ ,  $180^\circ$ ) as a function of energy. The amount of scatter in this figure is indicative of the relative compatibility of the various experiments.

The best fit resonance parameters are shown in table 1 compared with the average parameters of ref. 9. Note that some of these resonances  $\left[P_{33}(1236), F_{37}(1950), F_{17}(1990), D_{13}(2040), G_{17}(2180)\right]$  had their parameters fixed since their resonant energies are outside the range of our fit. The parameters of the other resonances were all free parameters in the fit. It is therefore quite significant that they agree so well with the parameters of ref. 9.

In order to ascertain the necessity for the individual resonances, each resonance was removed and the data was refitted. With few exceptions this resulted in a rise in  $\chi^2$  of from a few hundred to several thousand units.

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For example, absence of the very prominent  $D_{15}(1672)$  yielded a best fit  $\chi^2$  of ~30 000. On the other hand, a few resonances [specifically the  $S_{31}(1650)$ ,  $D_{33}(1674)$ ,  $F_{31}(1952)$ ,  $F_{35}(1972)$ , and  $G_{17}(2180)$ ] yielded  $\chi^2$  increases of ~100 units. Note however that these resonances are those which contribute minimally to the amplitude. They occur in low partial waves, are only weakly coupled to the  $\pi^-p$  system, or are outside the energy range considered.

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The uniqueness of these resonant parameters can be summarized as follows. The prominent resonances  $\left[ \text{particularly the D}_{15}(1672) \text{ and F}_{15}(1688) \right]$ are very well determined. Others, such as the P<sub>11</sub>(1470) and the D<sub>13</sub>(1520) are reasonably well determined, but as there are other resonances with the same spin-parity quantum numbers (recall that with only  $\pi^-p$  data we cannot distinguish isospins), there could be some mixing of different resonant amplitudes. However, we feel that the results are unlikely to deviate much from those presented. The s-wave resonance parameters are not that well determined by this fitting procedure, a feature shared with many models of  $\pi N$ scattering. It is also important to note that for much of our energy range the Pomeron term is mostly s-wave. We have also tried adding extra resonances, one at a time, in each possible spin parity state. This resulted in no improvement in any case and all such resonances "went away", i.e., their resonant parameters varied in such a way as to make no contribution to the amplitude in the region of study.

The parameters of the Pomeron term are also provided in table 1. The results for the diffraction slope b should be somewhat less than the observed values (since these are primarily due to forward resonance peaks). The

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results b = 1.74 and  $1.82 (GeV/c)^{-2}$  for the s-channel and t-channel helicitynon-flip fit and  $b = 1.39 (GeV/c)^{-2}$  for the spin non-flip fit are consistent with this expectation. This latter value is shown along with the experimentally observed slopes in fig. 1. Also note that for all our fits b' is rather large  $(b' \approx 18 (GeV/c)^{-2}/GeV)$  which corresponds to b rising from 1.4  $(GeV/c)^{-2}$ at 1.24 GeV/c to 2.0  $(GeV/c)^{-2}$  at 1.3 GeV/c. This seems to imply that the slope wants to grow at higher energies.

The parameter C provides the normalization of the Pomeron amplitude, with C=1 corresponding to our fit of the total cross section data. The value for C obtained from the fits to the differential cross sections and polarization is:

C = 0.86 for the spin non-flip fit, and

C = 0.74 and 0.69 for the s- and t-channel helicity-non-flip fits. Our earlier experience (ref. 1) suggests that C is entirely real, i.e., the diffraction term is purely imaginary. The curve shown in fig. 2, the fit to the total cross section data, uses C = 0.86 and seems to give a good description of the data.

The interaction radius was found to be R = 0.95 fm, which is a reasonable value. An additional property of our fits is the interesting feature that the spin non-flip parameterization of the Pomeron amplitude provides a better fit than the helicity-non-flip parameterizations ( $\chi^2 = 5322$  vs  $\chi^2 = 6004$ , 5656). Thus we feel compelled to conclude that this model for the amplitude seems to favour a Pomeron amplitude which is spin non-flip over one which is helicity non-flip.

The s-channel helicity conservation hypothesis suggested in ref. 4 is seen to provide a  $\chi^2$  significantly higher than the best fit which corresponds to spin conservation. The fact that the t-channel helicity conservation hypothesis,

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as proposed for reactions<sup>12, 13)</sup> such as  $\pi^- p \to A_1^- p$  or  $K^- p \to Q^- p$ , provides an intermediate value of  $\chi^2$  need not be taken as support for that mechanism. This result may be due to the kinematic fact that, for these energies, E is not much larger than M, so that for the t-channel helicity conservation hypothesis, the f and g amplitudes deviate only slightly from f and g for spin conservation, which is seen to provide the best  $\chi^2$ .

In addition, in order to demonstrate the necessity of our Pomeron term, we have made fits with C=0, i.e., only resonances in the amplitude. The best fit for this amplitude has a  $\chi^2$  over 8400 units — a value which clearly implies the need for a Pomeron term. Moreover, the resonant parameters were drastically altered in order to get even the poor fit which was obtained.

### 5. Conclusions

We find it very satisfying that such a simple model is able to provide a good representation of the data over a wide range of energies. Moreover, it is quite encouraging that the resonance parameters obtained in our best fit are in such good agreement with those of ref. 9.

We regard the quality of this fit as indicating good support for the Harari-Freund hypothesis. In particular, since the hypothesis seems to work quite well, we believe this work has allowed us to investigate the structure of the Pomeron amplitude at these energies. In this way we have concluded that the amplitude is predominantly a spin non-flip term which is purely imaginary. Its energy and angular dependence are described above.

A further point is that this model can be a successful means of extending elastic phase shift analyses to higher energies, in order to provide useful information about higher energy resonances. The only problem with this

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approach can come from uncertainties about the nature of the Pomeron amplitude at the energies concerned, while the advantages come from the parameterizing the amplitude in a physically interesting manner.

### FOOTNOTES AND REFERENCES

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$$b = b_0 + b$$
 (E<sub>cm</sub> - 1.8)

where  $\mathbf{b}_0$  is the value used for smaller energies and  $\mathbf{b}$  is a free parameter

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# Table 1

Best fit parameters (see text). Units of energy and width are GeV. For Pomeron parameters b is in units of  $(GeV/c)^{-2}$  and b is in units of  $(GeV/c)^{-2}/GeV$ . Other parameters are dimensionless.

Resonance	Spin Conserva	tion	s–Channel Helicity Conservation	t-Channel Helicity Conservation	Particle Data Group Param- eters (ref. 9)		
P <sub>33</sub>	$\begin{array}{llllllllllllllllllllllllllllllllllll$	00 236 120					
F <sub>17</sub>	0. 1. 0.	109 989 238					
G <sub>17</sub>	0. 2. 0.	350 180 299	These resonant parameters were fixed at these values in all fits (see text).				
F <sub>37</sub>	0. 1. 0.	520 952 202					
D <sub>13</sub>	0. 2. 0.	300 039 274					
Р <sub>13</sub>	0. 2. 0.	675 059 175	0.84 2.027 0.150	0.406 2.090 0.238	0.25 1.908' 0.325		
D <sub>33</sub>	0. 1. 0.	76 547 120	0.78 1.561 0.129	0.158 1.581 0.156	0.13 1.674 0.240		
F <sub>35</sub>	0. 1. 0.	295 848 099	0.159 1.821 0.046	0.157 1.828 0.055	0.17 1.885 0.273		
s <sub>11</sub>	0. 1. 0.	$36 \\ 510 \\ 143$	0.357 1.495 0.159	$0.366 \\ 1.507 \\ 0.138$	0.39 1.535 0.118	-	
$s_{11}$	0. 1. 0.	472 691 140	$0.585 \\ 1.727 \\ 0.282$	0.480 1.684 0.142	0.69 1.706 0.250		

Table 1 (cont'd)

Resonance	Spin Conservation	s-Channel Helicity Conservation	t-Channel Helicity Conservation	Particle Data Group Param- eters (ref. 9)
P <sub>11</sub>	0.690 1.476 0.274	0.715 1.492 0.278	0.740 1.487 0.294	0.61 1.468 0.240
Р <sub>11</sub>	$0.135 \\ 1.636 \\ 0.307$	0.013 1.653 0.371	0.016 -1.706 0.376	0.34 1.783 0.350
Р <sub>13</sub>	0.232 2.124 0.203	0.185 2.045 0.210	0.094 1.838 0.297	0.27 1.864 0.335
D <sub>13</sub>	$\begin{array}{c} 0.394 \\ 1.510 \\ 0.075 \end{array}$	0.414 1.511 0.066	0.645 1.521 0.099	0.53 1.520 0.120
D <sub>15</sub>	0.390 1.672 0.145	$0.395 \\ 1.674 \\ 0.138$	$0.384 \\ 1.672 \\ 0.154$	0.42 1.672 0.142
F <sub>15</sub>	0.715 1.681 0.130	0.730 1.684 0.129	0.730 1.679 0.133	0.62 1.688 0.127
s <sub>31</sub>	0.27 1.667 0.114	0.39 1.670 0.074	0.25 1.672 0.139	0.27 1.650 0.151
POMERON	(C=) 0.861 (b=) 1.39 (b'=) 16.78	$0.74 \\ 1.74 \\ 20.89$	$0.693 \\ 1.82 \\ 18.63$	
$\chi^2$	5322	6004	5656	

# FIGURE CAPTIONS

1) Behaviour of the diffraction slope b as a function of energy. The parameter, b, is obtained from fits to the near forward differential cross section of the form

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\theta) = \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\theta=0) \,\,\mathrm{e}^{\mathrm{bt}}$$

The solid line is the value of b used in the Pomeron amplitude.

- π<sup>-</sup>p total cross section as a function of energy. The data points are from ref. 8; the solid curve is the result of our fit; the dashed line is the Pomeron contribution.
- π p differential cross sections. The experimental points are taken from ref. 10; the solid curves represent the best fit of our model.
- π p polarization. The experimental points are taken from ref. 10; the solid curves represent the best fit of our model.
- 5)  $\pi$  p differential cross sections at fixed angle vs energy. The experimental points are taken from ref. 10; the solid curves represent the best fit of our model.



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Fig. 4

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