SLAC-PUB-1171
(TH) (EXP)
December 1972

DUALITY AND PROTON-PROTON SCATTERING AT ALL ANGLES*

D. D. Coon

Stanford Linear Accelerator Center, Stanford, California 94305, and
Department of Physics, University of Pittsburgh, Pittsburgh,
Pennsylvania 15213

and

Uday P. Sukhatme

Department of Physics, University of Washington, Seattle, Washington 98195

and

J. Tran Thanh Van

Stanford Linear Accelerator Center, Stanford, California 94305, and
Laboratoire de Physique Théorique et Particules Elementaires,**
Université de Paris Sud, Bât. 211, 91 Orsay, France.

(Submitted for publication)

* Work supported in part by the U.S. Atomic Energy Commission.

** Laboratoire associé au Centre National de la Recherche Scientifique.

Abstract :

We obtain a very good description of all available large angle pp elastic scattering data using a dual model with logarithmic trajectories. Our fit strongly suggests that the dual amplitude is responsible for filling in a pronounced dip produced by the Pomeranchuk singularity. This new explanation of the shoulder in pp data survives the test of an enormous extrapolation from $s \simeq 20\ (\mathrm{GeV}/c)^2$ to $s \simeq 2600\ (\mathrm{GeV}/c)^2$ and succesfully predicts the depth of the dip at $t \simeq -1.2\ (\mathrm{GeV}/c)^2$ in the recent ISR data. The addition of an eikonalized Pomeranchukon exchange term to our dual amplitude results in an excellent description of pp scattering data at all angles.

We present a simple description of all available high energy pp elastic scattering data[1,2] in terms of dual and nondual contributions to the amplitude. Although some details of our picture might change, the tests provided by the vast range of pp data are so stringent, that the fits presented here constitute strong evidence for the validity of our general picture. A striking new feature is that the range of applicability and usefulness of duality and Reggeism is extended to high momentum transfers far beyond the normal range.

Our amplitude has the form[3]

$$A^{pp}(s,t) = P + (\text{dual contribution}). \tag{1}$$

P, the nondual component, is the contribution of the Pomeranchuk singularity. It is responsible for the forward diffraction peak, and becomes unimportant at large values of $|t|$. The differential cross section at large $|t|$ is governed by the dual contribution.

The flat behavior, at large negative values of t, of the effective Regge trajectory in pp scattering cannot be explained by the linear trajectories of the Veneziano model, for any reasonable value of the slope α'. Consequently, the dual model which we use is a generalization of the Veneziano model, which allows logarithmic trajectories. More precisely, we use a modified, tachyon-free version of a dual amplitude proposed by Coon and Baker.[4,5]

$$D = C \frac{G\left(\frac{q}{\tau\omega}\right)}{G\left(\frac{q}{\tau}\right) G\left(\frac{q}{\omega}\right)} q^{\alpha(t)\alpha(u)} , \tag{2}$$

where $\tau = b - at$, $\omega = b - au$, $\alpha(t) = \ln \tau / \ln q$, $\alpha(u) = \ln \omega / \ln q$. a, b, and the "base" q $(0 < q < 1)$ are parameters of the theory and C is the normalization constant. The infinite product $G(x)$ is defined by

$$G(x) \equiv \prod_{l=0}^{\infty} (1 - xq^{l}) \quad . \qquad (3)$$

This function has been extensively studied by mathematicians.[6] Roughly speaking, $G(q^{\alpha})$ is analogous to the inverse gamma function $\Gamma^{-1}(\alpha)$. The poles in any channel (for example the t-channel) are located at $\tau = q^{n}$ $(n = 1, 2, 3, \ldots)$. The Regge trajectory $\alpha(t)$ is logarithmic, and goes to infinity at $t = b/a$, which is an accumulation point of poles, reminiscent of the threshold point in Coulomb scattering. The amplitude also has a branch point here, due to the factor $q^{\alpha(t)\alpha(u)}$. When $q \rightarrow 1$, the Regge trajectory becomes linear and the amplitude reduces to the familiar Lovelace amplitude.[7]

In nucleon-nucleon scattering, the Harari-Freund hypothesis together with the absence of exotics, implies exchange degeneracy for both the $(\rho - A_2)$ and $(\omega - f)$ contributions.[8] Therefore, the dual amplitude, which is real, must contain two terms D_{ρ} and D_{ω} having the form given in Eq. (2). For simplicity, we neglect both the spin and isospin of the external protons. Since we must have $\alpha_{\rho}(m_{\rho}^2) = \alpha_{\omega}(m_{\omega}^2) = 1$, D_{ρ} and D_{ω} have only three free parameters each. However, factorization properties of the amplitude require that $q_{\rho} = q_{\omega}$.[9] Also, if we constrain the amplitude to describe both the s dependence and normalization of $\sigma_{tot}(\bar{p}p) - \sigma_{tot}(pp)$, then all parameters in D_{ω} are practically fixed. Therefore, the description of all available large $|t|$ $\left[-t \lesssim 5\ (\mathrm{GeV}/c)^2\right]$ pp scattering at all energies $p_L \gg 5$ GeV/c, must be accomplished with only three adjustable parameters, namely the ones which characterize the ρ contribution. It is indeed remarkable that a very satisfactory fit can be obtained with $q_\rho = q_\omega = .795$, $\alpha_\rho(0) = .80$, $\alpha_\omega(0) = .37$, $C_\rho = .007$, $C_\omega = .42$.

Fig. 1 (drawn for $p_L = 19.2$ GeV/c) shows all the typical features of our dual fit. The comparison between theory and experiment at 90° is shown in Fig. 2. For $p_L \gtrsim 8$ GeV/c, the ρ contribution is dominant. The ω contribution, which we fixed from total cross section data (i.e. the amplitude at 0°), fills the difference between the experimental 90° data and the ρ contribution for $p_L \lesssim 8$ GeV/c. This shows that our dual amplitude has good extrapolation properties in the whole angular range. Our model gives a natural explanation to the observation of Barger, Geer and Halzen[10] concerning the existence of two types of contributions in large angle pp scattering.

Some consequences of our fit are :

a) $\alpha_\rho(0) = .80$. This value is somewhat higher than the intercept deduced from the Serpukhov measurements[11] of $\sigma_{tot}(\pi^+ p) - \sigma_{tot}(\pi^- p)$ which give $\alpha_\rho(0) = .69$.[12]

b) Universality of ω and ρ couplings turned out to be approximately true in the range 5 GeV/c $\lesssim p_L \lesssim$ 25 GeV/c. We found the ω/ρ contribution ratio at $t = 0$ to be about 10 for $p_L = 12$ GeV/c, as compared with the expected theoretical value of 9.[13]

c) It is possible to extract the Pomeranchukon contribution P from the data. If we assume that P is pure imaginary, then since $D_\rho + D_\omega$ is real, there is no interference, and we have

$$\left(\frac{d\sigma}{dt}\right)_P = \left(\frac{d\sigma}{dt}\right)_{exp} - \left(\frac{d\sigma}{dt}\right)_{dual} \qquad (4)$$

The "pure" Pomeranchukon contribution at $p_L = 19.2$ GeV/c is shown in Fig. 1 . Observe the presence of a pronounced dip at $t \simeq -1.2$ $(\mathrm{GeV/c})^2$ in this curve, whereas only a shoulder is observed in the experimental cross section. In this region of t, the Pomeranchuk singularity dominates. Consequently, it is reasonable to expect that as s increases, the dip coming from the "pure" Pomeranchukon will begin to show up as a dip at $t \simeq -1.2$ $(\mathrm{GeV/c})^2$ in the experimental data. Recent ISR data confirms this expectation very clearly (see Fig. 3).[2]

d) The reason for the dip in $\left(\frac{d\sigma}{dt}\right)_P$ at $p_L = 19.2$ GeV/c is that the dual contribution passes just below the shoulder in the data at $t \simeq -1.2$ $(\mathrm{GeV/c})^2$. When extrapolated to ISR energies, it is remarkable that the dual amplitude again passes just below the dip (Fig. 3). This strongly suggests that the dual contribution can be thought of as a form of background, which, when subtracted out, reveals the true shape, zeros, and diffractive nature of the "pure" Pomeranchukon.

One possible description of P is provided by the eikonal formalism with the Pomeranchukon Born term taken to be proportional to the square of the proton's electromagnetic form factor.[14] This description has a dip at $t \simeq -1.2$ $(\mathrm{GeV/c})^2$.

$$P^{\mathrm{Born}} = K\left[G(t)\right]^2 \left(\frac{E_L}{E_o}\right)^{1+\alpha_P' t}, \qquad G(t) = \frac{1}{(1-t)^2} \,. \tag{5}$$

where, E_L = laboratory energy of the incident proton, and α_P' = slope of the Pomeranchuk trajectory (taken to be 0.1). The addition of this Pomeranchukon contribution to our previous dual amplitude leads to an excellent fit of pp scattering data at all angles. The results are shown in Fig. 4. The theoretical curves are almost as good as smooth lines drawn by hand through the experimental data points !

Like all dual amplitudes, Eq. (2) has the deficiency of not being unitary. Its n-point generalization is not yet known, although a closely related theory does have a good n-point function and full factorization properties.[5] The physics underlying logarithmic trajectories which go to infinity producing a finite resonance region, followed by a smooth Regge region are not clear. However, it is physically acceptable to view logarithmic trajectories as resulting from a simple approximation in which $\text{Im}\,\alpha$ = constant on the right hand cut.[4]

Our successful fits to all available elastic pp data over big energy and angular ranges, are better fits, with fewer parameters, than any other previous work. Besides, each term we have used has a definite theoretical significance in terms of Regge pole exchanges. Therefore, a dual theory with nonlinear trajectories is not only a phenomenological success, but may quite possibly contain many significant features of a complete theory of strong interactions.

A detailed account of our fitting procedure, a comparison with other previous attempts to explain pp data,[15] an extension of this work to include polarization, an application to $\bar{p}p$ scattering via crossing, etc. will be given at length in another forthcoming article.

We are grateful to M. Baker and S. Yu for numerous enlightening discussions and for their active participation in the initial stages of this work. We also wish to thank S. Brodsky, R. Blankenbecler, J. Gunion and M. Arik for many helpful conversations. Two of the authors (D.C. and J.T.T.V.) gratefully acknowledge the hospitality of the theory group at SLAC, where a major part of their work was done.

REFERENCES AND FOOTNOTES

1. D. Harting, et al., Nuovo Cimento 38, 60 (1965); G. Cocconi, et al., Phys. Rev. 138, B 165 (1965); K. J. Foley, et al., Phys. Rev. Letters 15, 45 (1965); C. W. Akerlof, et al., Phys. Rev. 159, 1138 (1967); C. M. Ankenbrandt, et al., Phys. Rev. 170, 1223 (1968); J. V. Allaby, et al., Phys. Letters 23, 389 (1966); 25B, 156 (1967); 27B, 49 (1968); 28B, 67 (1968); 34B, 431 (1971); R. A. Carrigan Jr., et al., Phys. Rev. Letters 24, 683 (1970); G. G. Beznogikh, et al., Phys. Letters 39B, 411 (1972).

2. G. Barbiellini, et al., Phys. Letters 39B, 663 (1972); Data from the Rubbia group, presented at the 16th International Conference on High Energy Physics, Chicago, Illinois (1972).

3. This is in accordance with the Harari-Freund hypothesis. P. G. O. Freund, Phys. Rev. Letters 20, 235 (1968); H. Harari, Phys. Rev. Letters 20, 1395 (1968).

4. D. D. Coon and M. Baker (to be published). We have kept only the (t,u) term, since the s-channel is exotic.

5. There exists a factorizable, dual resonance model with logarithmic trajectories which is closely related to the dual amplitude of Eq. (2). Relevant papers on this related model are: D. Coon, Phys. Letters 29B, 669 (1969); M. Baker and D. D. Coon, Phys. Rev. D 2, 2349 (1970) and D 4, 1234 (1971); D. D. Coon, S. Yu, and M. Baker, Phys. Rev. D 5, 1429 (1972).

6. See, for example, L. J. Slater, Generalized Hypergeometric Functions, Cambridge University Press (1966).

7. C. Lovelace, Phys. Letters 28B, 264 (1968).

8. The ρ and ω trajectories are non-degenerate in our theory.

9. It is known that factorization of the multi-Veneziano amplitude with different trajectory species is possible only if they have a universal slope. The same factorization requirement forces all q-values to be the same in dual models with logarithmic trajectories, for example the model described in Ref. 5.

10. V. Barger, K. Geer, and F. Halzen, University of Wisconsin preprint (1972).

11. S. P. Denisov, et al., Phys. Letters 36B, 415 (1971); 36B, 528 (1971).

12. Recall that pion-nucleon charge exchange scattering (in which the helicity flip amplitude is dominant) gives $\alpha_\rho(0) = .5$. The discrepancy could possibly be explained by Regge cuts.

13. See, for example, V. Barger and D. Cline, Phenomenological Theories of High Energy Scattering, W. A. Benjamin, Inc., New York (1969).

14. L. Durand, III and R. Lipes, Phys. Rev. Letters 20, 637 (1968); S. Frautschi and B. Margolis, Nuovo Cimento 56A, 1153 (1968); C. B. Chiu and J. Finkelstein, Nuovo Cimento 59A, 92 (1969); A. Capella, J. Kaplan, A. Krzywicki, and D. Schiff, Nuovo Cimento 63A, 141 (1969). Note that the description given in these papers does not explain the

change in slope near $t = -.1\ (\mathrm{GeV}/c)^2$ at ISR energies.

15. H. D. I. Abarbanel, S. D. Drell, and F. J. Gilman, Phys. Rev. 177, 2458 (1969); T. Y. Cheng, S. Y. Chu, and A. W. Hendry, Indiana University preprint (1972); J. F. Gunion, S. J. Brodsky, and R. Blankenbecler, Phys. Letters 39B, 649 (1972); A. D. Krisch, Phys. Rev. Letters 19, 1149 (1967).

FIGURE CAPTIONS

Fig. 1. Differential cross section for pp elastic scattering at $p_L \simeq 19.2$ GeV/c. The solid and dashed lines are the dual and "pure" Pomeranchukon contributions respectively.

Fig. 2. Comparison of experimental points with theoretical dual model curve for 90° pp scattering.

Fig. 3. pp data at ISR energy $s = 2016\ (\mathrm{GeV/c})^2$. The dual contribution (solid line) passes just below the dip at $t \simeq -1.2\ (\mathrm{GeV/c})^2$. Subtraction of the dual contribution from the experimental data [Eq. (4)] yields the dashed "pure" Pomeranchukon curve.

Fig. 4. Comparison of theoretical curves with high energy pp elastic scattering data.

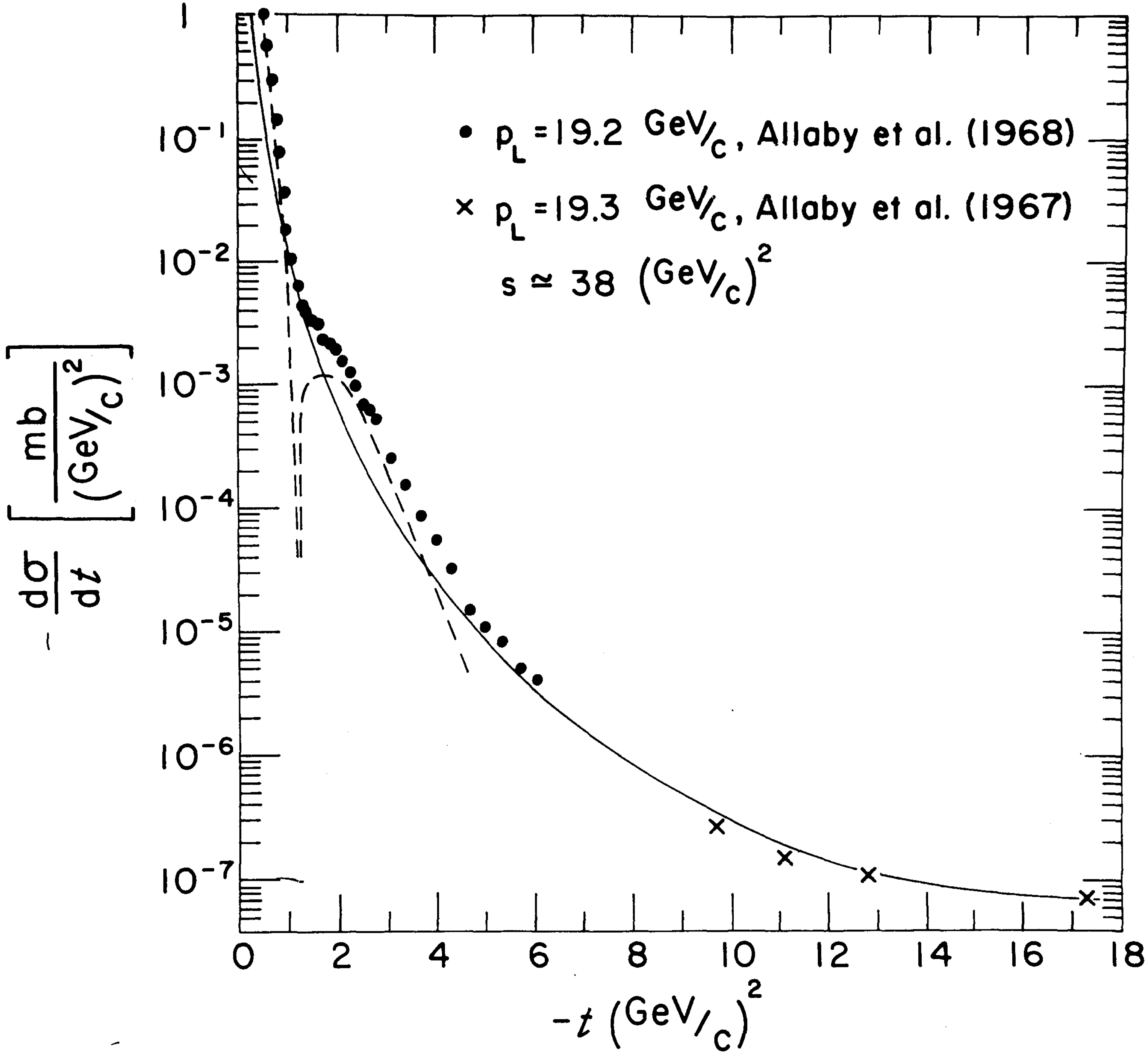
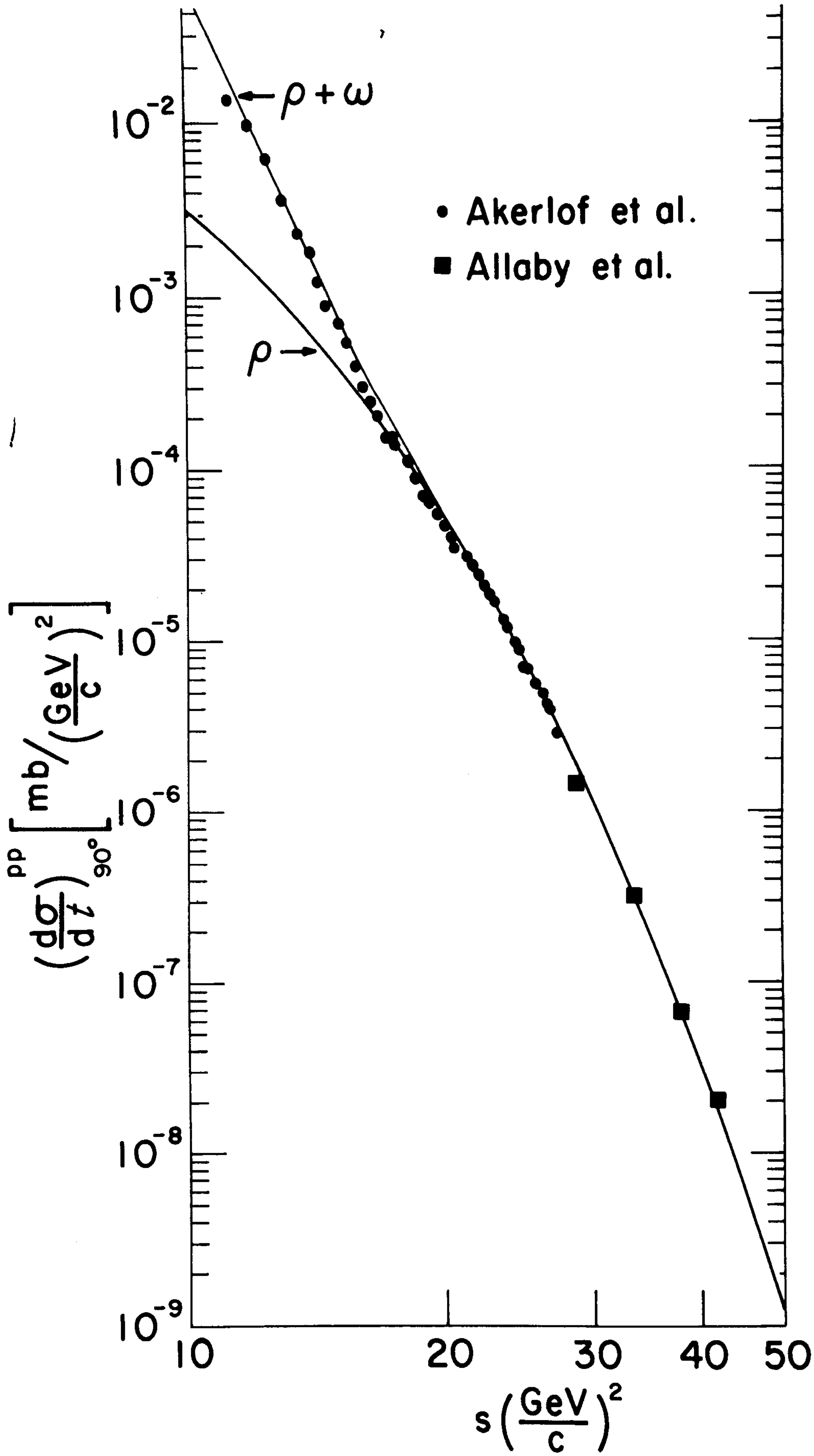

• $p_L = 19.2$ GeV/c, Allaby et al. (1968)
× $p_L = 19.3$ GeV/c, Allaby et al. (1967)
$s \simeq 38$ (GeV/c)^2
$-\frac{d\sigma}{dt}$ $\left[\frac{mb}{(GeV/c)^2}\right]$
$-t$ (GeV/c)^2
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
2
4
6
8
10
12
14
16
18

Fig. 1



ρ + ω
ρ
• Akerlof et al.
■ Allaby et al.
$\left(\frac{d\sigma}{dt}\right)^{pp}_{90°} \left[mb / \left(\frac{GeV}{c}\right)^2 \right]$
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10
20
30
40
50
$s \left(\frac{GeV}{c}\right)^2$

Fig. 2

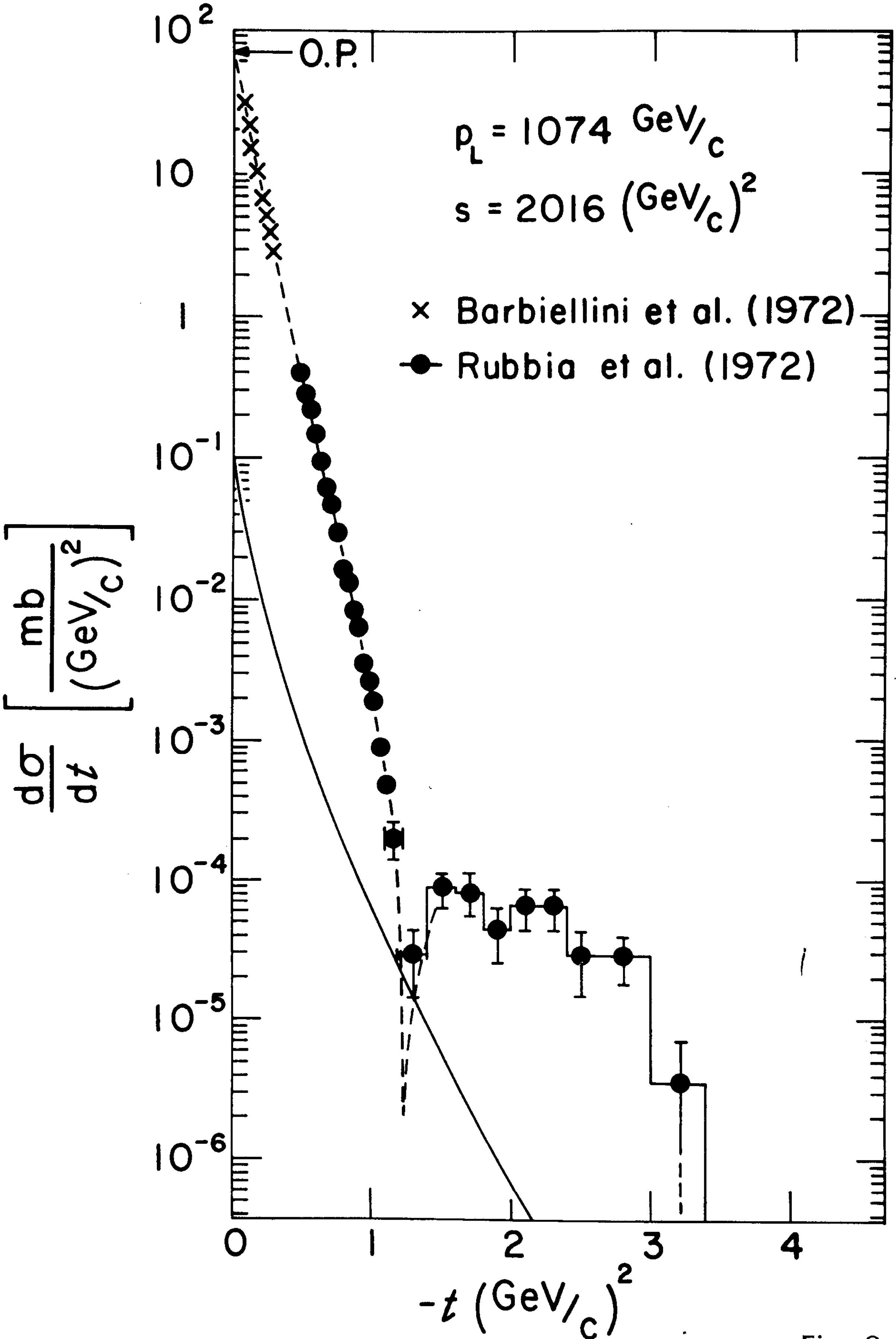

O.P.
p_L = 1074 GeV/c
s = 2016 (GeV/c)^2
× Barbiellini et al. (1972)
Rubbia et al. (1972)
dσ/dt [mb/(GeV/c)^2]
-t (GeV/c)^2

Fig. 3

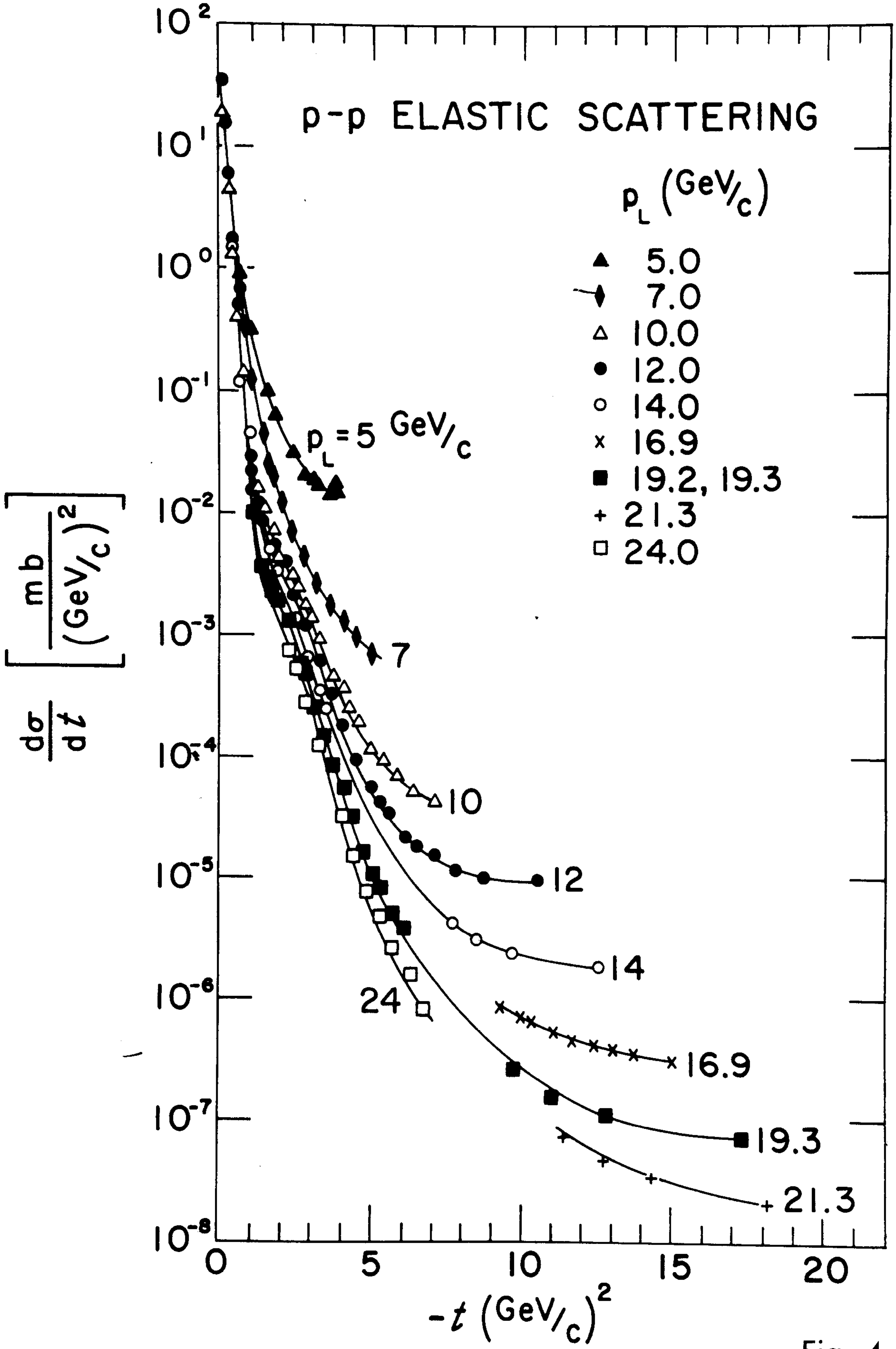

p-p ELASTIC SCATTERING
p_L (GeV/c)
5.0
7.0
10.0
12.0
14.0
16.9
19.2, 19.3
21.3
24.0
p_L = 5 GeV/c
7
10
12
14
16.9
19.3
21.3
24
$\frac{d\sigma}{dt}$ $\left[\frac{mb}{(GeV/c)^2}\right]$
$-t$ $(GeV/c)^2$

Fig. 4