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MAGNETIC DIPOLE AND ELECTRIC QUADRUPOLE MOMENTS OF $W^{\pm}$ MESON*

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ABSTRACT

It is shown that the magnetic dipole and the electric quadrupole moments of $W^{\pm}$ meson must be equal to e/m and $-e/m^2$ respectively if we demand either that the Drell-Hearn sum rule is satisfied up to α^2 or that the helicity of $W^{\pm}$ is conserved in the scattering from an arbitrary electromagnetic field at high energies and at small but finite scattering angles.

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I. INTRODUCTION

$W^{\pm}$ vector bosons, which are supposed to mediate the weak interactions,[1] have (in addition to the charge) a magnetic dipole moment and an electric quadrupole moment.[2,3] We assume that the electromagnetic interaction of the $W^{\pm}$ bosons is invariant under the time reversal and the parity operation, hence the electric dipole moment[4] is zero. The values of these moments greatly affect the total production cross sections, the energy-angle distributions and the decay correlations in the processes such as $e^+e^- \rightarrow W^+W^-$,[5] $\gamma Z \rightarrow W^+W^-$ + anything,[6] $\nu_\mu Z \rightarrow \mu W$ + anything,[7] etc. Therefore if $W^{\pm}$ bosons are discovered it is relatively easy to find these moments. It is interesting to speculate what these moments should be. $W^{\pm}$ bosons are assumed to have no strong interactions,[8] hence the observable moments are expected to be not greatly affected by the radiative corrections, in analogy to the magnetic moment of an electron which[9] is given by

$$\mu_e = \frac{e}{2m_e}\left(1 + \frac{\alpha}{2\pi} - \frac{\alpha^2}{\pi^2}\,0.328479 + \ldots\right) \equiv \frac{e}{4m_e}\, g_e \,.$$

As is well known, this is the consequence of the quantum electrodynamics of a spin 1/2 particle assuming no anomalous magnetic moment (Pauli term) in the Lagrangian. The absence of the Pauli term in the Lagrangian is commonly believed to be due to the fact that its presence would render the theory unrenormalizable. Also the concept[10] of "principle of minimum interaction" was invented to describe the absence of the Pauli term in the leptons and the concept was widely applied to the electromagnetic interaction of spin 0 and spin 1/2 hadrons. For charged spin 1 particles,[11] the principle of minimum interaction does not yield a unique magnetic moment, but once the magnetic moment is given the electric quadrupole is determined, i.e., if $\mu = e(1+\kappa)/(2m)$ then $Q = -e\kappa/m^2$.

Weinberg,[12] and many others after him, proposed a theory to unify the electromagnetic and weak interactions using the Higg's phenomena. In this theory, the photon-$W^\pm$ boson coupling is of the Yang-Mill type, which implies that to the lowest order in α, the magnetic moment and the electric quadrupole moment of $W^\pm$ bosons are given respectively by

$$\mu = e/m \qquad \text{and} \qquad Q = -e/m^2 \quad , \tag{1.1}$$

where e and m are the charge and the mass respectively of $W^\pm$.

In this paper we show two additional arguments which can be regarded as supporting the values of μ and Q given by Eq. (1.1). The first is to consider the Drell-Hearn sum rule,[13,14]

$$\pi s \left(\frac{\mu}{s} - \frac{e}{m}\right)^2 = \int_0^\infty \left[\sigma_P(\omega) - \sigma_A(\omega)\right] \frac{d\omega}{\omega} \quad . \tag{1.2}$$

where s is the spin of the particle (s=1 for $W^\pm$), $\sigma_P(\omega)$ (or $\sigma_A(\omega)$) is the total cross section for $\gamma + W^\pm$ with the spins of γ and $W^\pm$ parallel (or antiparallel) to each other in the laboratory system, and ω is the incident photon energy. Now if the sum rule is true, it must also be true for each order in α. The right hand side starts with terms of order α^2 because the W is assumed to have no strong interaction. Thus as pointed out by Weinberg,[14] the term linear in α in the left hand side must vanish. This implies that to the lowest order in α, all non-strongly interacting particles have magnetic moments given by

$$\mu_0 = se/m \quad . \tag{1.3}$$

Since the magnetic moments of a particle and an antiparticle must have the same magnitude and be opposite in sign, we must have

$$\mu = \frac{se}{m}\left(1 + a_1\alpha + a_2\alpha^2 + \ldots\right) \quad . \tag{1.4}$$

Substituting (1.4) into the left hand side of (1.2), we obtain

$$\frac{4\pi^2\alpha^3 s}{m^2} (a_1 + \alpha a_2 + \ldots)^2 ,$$

which does not have terms proportional to α^2. Hence terms proportional to α^2 in the right hand side must vanish.[15] α^2 terms in the rhs of (1.2) are just the spin dependent part of the lowest order γ+W Compton scattering cross section, which in general depends upon μ and Q. In Section II, we show that the integration in the rhs of (1.2) diverges if either $\mu \neq e/m$ or $Q \neq -e/m^2$, but when $\mu = e/m$ and $Q = -e/m^2$, the integration in the right hand side of (1.2) yields zero.

Another argument, which we shall present in Section III, is the helicity conservation.[16] The argument is not very convincing but interesting. Nature seems to like helicity conservation at high energies. We show that the helicity of $W^{\pm}$ is conserved in the scattering of $W^{\pm}$ from an arbitrary electromagnetic field at high energies and at small but finite angles, if and only if $\mu = e/m$ and $Q = -e/m^2$. This argument gives also $\mu = e/2m$ for an electron (or muon). However for a spin 1/2 particle, the helicity is conserved at high energies and <u>at all angles</u> if and only if $\mu = e/2m$, whereas for a spin 1 particle even if $\mu = e/m$ and $Q = -e/m^2$, the helicity will <u>not</u> be conserved unless $\theta \ll 1$. This fact is due to the conservation of angular momentum. Therefore we can not demand the helicity conservation in the electromagnetic scattering of a spin 1 particle unless $\theta \ll 1$.

In Appendix A, we discuss how to identify various form factors in the manifestly covariant vertex functions with the charge radius, magnetic dipole and electric quadrupole moments commonly used in the nonrelativistic nuclear physics.

II. DRELL-HEARN SUM RULE

In this section, we study the Drell-Hearn sum rule (1.2) for $W^{\pm}$ bosons with arbitrary magnetic dipole and electric quadrupole moments. We consider the sum rule in the order α^2, in which case the total cross sections σ_P and σ_A in (1.2) come from the lowest order Compton scattering. We shall show that the integral (1.2) converges if and only if μ and Q are given by (1.1) and when it converges the integral vanishes.

The Feynman rules for the quantum electrodynamics of $W^{\pm}$ bosons with arbitrary μ and Q have been given by H. Aronson,[7] and are shown in Fig. 1 and Table I. There also exists a four-W direct coupling term but this is not relevant in our calculation. When the W bosons represented by p and p' in Fig. 1 are on the mass shell the vertex function V can be written in a simpler form as follows:

$$V_{\mu\alpha\beta} \xrightarrow[\substack{p^2=m^2\\ p'^2=m^2}]{} ie\left[(p+p')_\mu\left\{g_{\alpha\beta}\left(1+\tfrac{1}{2}\lambda q^2/m^2\right)-\lambda m^{-2}q_\alpha q_\beta\right\}\right.$$
$$\left. + (1+\kappa+\lambda)\left(g_{\alpha\mu}\, q_\beta - g_{\beta\mu}q_\alpha\right)\right] \quad . \qquad (2.1)$$

The matrix element of a current operator $J_\mu(0)$ can be written as

$$<\vec{p}'h'|J_\mu(0)|\vec{p}h> = i\left(\epsilon'_{h'}\right)^*_\beta \epsilon_{h\alpha}\, V_{\mu\alpha\beta} \equiv \Gamma^\mu_{h'h} \qquad (2.2)$$

where $\epsilon'_{h'}$ (or ϵ_h) is the polarization vector of $\vec{p}'$ (or $\vec{p}$) with a helicity h' (or h). In the appendix, we show that the parameters κ and λ are related to the magnetic dipole moment by

$$\mu = e\,(1+\kappa+\lambda)/(2m) \qquad (2.3)$$

and the electric quadrupole moment by

$$Q = -e\,(\kappa-\lambda)/m^2 \quad . \qquad (2.4)$$

We note that the expression for Q given by Aronson[18] contains an error. In the appendix we also show that the mean square charge radius of a charged vector particle having a vertex function (2.1) is given by

$$R^2 = (\kappa + \lambda)/m^2 \quad . \tag{2.5}$$

It can be shown from $q_\mu \Gamma^\mu_{h'h} = 0$, invariance under parity and time reversal and $p \cdot \epsilon_h = p' \cdot \epsilon'_{h'} = 0$ that $\Gamma^\mu_{h'h}$ has a tensor structure

$$\Gamma^\mu_{h'h} = -e\Big[G_1(q^2)\,(p+p')_\mu\, \epsilon^*_{h'} \cdot \epsilon_h + G_2(q^2)\,\big(\epsilon_h \cdot q\, \epsilon^*_{h'\mu} - \epsilon'^*_{h'} \cdot q\, \epsilon_{h\mu}\big)$$

$$+ G_3(q^2)\,(p+p')_\mu \big(\epsilon'^*_{h'} \cdot q\big)(\epsilon_h \cdot q)\, m^{-2}\Big] \quad , \tag{2.6}$$

where $G_1(q^2)$, $G_2(q^2)$ and $G_3(q^2)$ are real functions of q^2. Thus Eq. (2.1) can be regarded a particular form of (2.6) due to a specific assumption about the form of the Lagrangian. Since W's are assumed to have no strong interaction, the only quantity which can have the dimension of q^2 is m^2, hence G's are in general functions of q^2/m^2 and the dimensionless quantities such as λ and κ. For example (2.1) gives

$$G_1(q^2) = 1 + \frac{1}{2}\lambda q^2/m^2 \,, \qquad G_2 = 1 + \kappa + \lambda, \quad \text{and} \quad G_3 = -\lambda \quad . \tag{2.7}$$

If one assumes other forms of Lagrangian, G_2 and G_3 may also be functions of q^2/m^2 instead of being constants. These considerations are important when we discuss the helicity conservation in the next section.

The terms proportional to λ in Table I can be regarded as anomalous because they can not be derived from the principle of the minimal interaction. As emphasized by T. D. Lee,[11] the terms proportional to κ should not be regarded as anomalous because the free Lagrangian, L_{free}, is not uniquely defined for a spin 1 particle and one gets different values of κ from different expressions by L_{free} by replacing in L_{free}, $\partial/\partial x_\mu \rightarrow \partial/\partial x_\mu - ie A_\mu$. Since the purpose of this paper is to show that $\kappa=1$ is more normal than other values, it is convenient to define a new parameter

$$\eta \equiv \kappa - 1 \quad . \tag{2.8}$$

In terms of η and λ, the magnetic moment μ and the quadrupole moment Q can be written respectively as

$$\mu = (2 + \eta + \lambda)e/(2m) \quad ,$$

and

$$Q = -(1 + \eta - \lambda)e/m^2 \quad .$$

The Feynman diagrams for the Compton scattering are shown in Fig. 2, and the matrix elements can be written as $\epsilon^{\alpha}\epsilon'^{*\beta}e^{\mu}e'^{*\nu}\ M_{\mu\alpha\nu\beta}$, where

$$M_{\mu\alpha\nu\beta} = M^{a}_{\mu\alpha\nu\beta} + M^{b}_{\mu\alpha\nu\beta} + M^{c}_{\mu\alpha\nu\beta} \quad . \tag{2.9}$$

M^a, M^b, and M^c correspond to the diagrams a, b, and c respectively of Fig. 2 and they can be written as follows:

$$\begin{aligned} M^{a}_{\mu\alpha\nu\beta} = \frac{1}{2x}\Big[&(k-p)_{\rho}\, g_{\mu\alpha} + \eta k_{\rho}\, g_{\mu\alpha} - \frac{\lambda}{m^2}\, g_{\mu\alpha}\left\{x(p-k)_{\rho} - m^2 k_{\rho}\right\}\Big] \\ &\left(g_{\rho\sigma} - \frac{(p+k)_{\rho}\,(p'+k')_{\sigma}}{m^2}\right)\Big[2p'_{\nu}\, g_{\sigma\beta} - (p'-k')_{\sigma}\, g_{\nu\beta} - 2k'_{\beta}\, g_{\nu\sigma} \\ &+\eta\left(g_{\beta\nu}\, k'_{\sigma} - g_{\sigma\nu}\, k'_{\beta}\right) - \frac{\lambda}{m^2}\Big\{g_{\beta\nu}\left(x(p'-k')_{\sigma} - m^2 k'_{\sigma}\right) \\ &+ m^2 g_{\nu\sigma}\, k'_{\beta} - p'_{\nu}p'_{\sigma}k'_{\beta} + p'_{\nu}k'_{\sigma}k'_{\beta}\Big\}\Big] \quad . \end{aligned} \tag{2.10}$$

$$M^b_{\mu\alpha\nu\beta} = \frac{-1}{2y}\Bigg[2p_\nu g_{\alpha\rho} + 2k'_\alpha g_{\nu\rho} - (k'+p)_\rho\, g_{\nu\alpha} + \eta\left(g_{\rho\nu}k'_\alpha - g_{\alpha\nu}k'_\rho\right)$$

$$- \frac{\lambda}{m^2}\left\{-m^2 g_{\rho\nu}k'_\alpha + g_{\nu\alpha}\left(m^2 k'_\rho - y(p+k')_\rho\right) + p_\nu k'_\alpha (p+k')_\rho\right\}\Bigg]$$

$$\left(g_{\rho\sigma} - \frac{(p-k')_\rho (p'-k)_\sigma}{m^2}\right)\Big[2p'_\mu g_{\sigma\beta} - (p'+k)_\sigma\, g_{\mu\beta} + 2k_\beta g_{\mu\sigma} + \eta(g_{\sigma\mu}k_\beta - g_{\beta\mu}k_\sigma)$$

$$- \frac{\lambda}{m^2}\left\{g_{\beta\mu}\left(m^2 k_\sigma - y(p'+k)_\sigma\right) - m^2 g_{\mu\sigma}k_\beta + p'_\mu k_\beta (p'+k)_\sigma\right\}\Big] \ . \quad (2.11)$$

M^c is the seagull term U divided by ie^2 in Table I. In the above equations, we have introduced the notations

$$x = k\cdot p = k'\cdot p' \qquad \text{and} \qquad y = k'\cdot p = k\cdot p' \ . \qquad (2.12)$$

We have ignored the terms proportional to k_μ, p_α, k'_ν and p'_β because they yield zero after contractions with the polarization vectors. We have also ignored the terms proportional to k_α and p_μ because in the Drell-Hearn sum rule we are interested only in the polarization vectors of the initial photon and the target W boson which are orthogonal to k and p. We choose the coordinate system in which the incoming photon direction is the z axis and the scattering takes place in the xz plane:

$$k = (\omega,\ 0,\ 0,\ \omega) \ ,$$

$$k' = (\omega',\ \omega' \sin\theta,\ 0,\ \omega' \cos\theta) \ ,$$

$$p = (m,\ 0,\ 0,\ 0) \ ,$$

$$x = m\omega$$

and

$$y = m\omega' = m\omega/\left[1 + \omega m^{-1}(1 - \cos\theta)\right] \ .$$

The helicity of the incident photon is chosen to be +1,

$$e = -\frac{1}{\sqrt{2}}(0,\ 1,\ i,\ 0) \ ,$$

and the spin of the target W boson is either parallel or antiparallel to the incident photon direction

$$\epsilon^{\pm} = \mp \frac{1}{\sqrt{2}} (0,\ 1,\ \pm i,\ 0) \quad .$$

The relative phases between e, ϵ^{+} and ϵ^{-} do not enter into our problem, hence we may let $\epsilon^{+} = e$ and $\epsilon^{-} = e^{*}$. The difference of the two differential cross sections is then (we have ignored the difference between upper and lower indices for simplicity)

$$\frac{d\sigma_p}{d\Omega} - \frac{d\sigma_A}{d\Omega} = e_{\mu} e^{*}_{\mu'} (e_{\alpha} e^{*}_{\alpha'} - e^{*}_{\alpha} e_{\alpha'}) \frac{\alpha^2}{4m^2} \left(\frac{y}{x}\right)^2 M_{\mu\alpha\nu\beta} M^{*}_{\mu'\alpha'\nu\beta'} \left(g_{\beta\beta'} - \frac{p'_{\beta} p'_{\beta'}}{m^2} \right) \quad . \tag{2.13}$$

We have done this calculation using the algebraic computer program written by A. C. Hearn.[18] The results are too long to be reproduced here. However for the case $\eta = \lambda = 0$, both the calculations and the results are fairly simple and we shall treat this case separately. When either $\lambda \neq 0$ or $\lambda = 0$ and $\eta \neq 0$, we need only to pick up the terms which are the most divergent and this can be done without using a computer. For this purpose, let us define

$$\chi_{\mu\mu'\alpha\alpha'} = \left(\frac{y}{x}\right)^2 M_{\mu\alpha\nu\beta} M^{*}_{\mu'\alpha'\nu\beta'} \left(g_{\beta\beta'} - \frac{p'_{\beta} p'_{\beta'}}{m^2} \right) \tag{2.14}$$

and

$$\chi = e_{\mu} e^{*}_{\mu'} (e_{\alpha} e^{*}_{\alpha'} - e^{*}_{\alpha} e_{\alpha'}) \chi_{\mu\mu'\alpha\alpha'} \quad . \tag{2.15}$$

χ is dimensionless and can be written as

$$\chi = \sum_{\substack{\ell+c+s=n \\ \ell \geq -4 \\ c \geq 0 \\ s=0,1}} C_{\ell cs} \frac{x^{\ell} y^{c} z^{s}}{m^{2n}} \quad , \tag{2.16}$$

where

$$z = |e\cdot k'|^2 = |e^*\cdot k'|^2 = |e\cdot p'|^2 = |e^*\cdot p'|^2 = \frac{1}{2}\,\omega'^2 \sin^2\theta \quad .$$

$C_{\ell cs}$ is a constant. The condition $n=\ell+c+s$ is obvious from the dimensional consideration. The conditions $\ell \geq -4$ and $c \geq 0$ can be seen easily from the matrix elements. $s=0,1$ can be understood in the following way: The fourth rank tensor $\chi_{\mu\mu'\alpha\alpha'}$ can be constructed from the metric tensor g and available vectors p, p', k and k' of which only three are independent. Since p and k are orthogonal to e and e*, we need to consider only the matric tensor g and the vector k'. The tensor $k'_\mu k'_{\mu'} k'_\alpha k'_{\alpha'}$ yields zero after contraction with the polarization vectors in (2.15). Hence only s=0 and s=1 contribute to χ in (2.16).

In order to see energy dependence of $\sigma_p - \sigma_A$, we integrate $x^\ell y^c z^s/m^{2n}$ with respect to the solid angle. The results are:

<u>s=0</u>

$$\int x^\ell y^c/m^{2n}\ d\cos\theta \xrightarrow[\omega\to\infty]{}$$

$$(\omega/m)^{\ell+c-1}/(c-1) = (\omega/m)^{n-1}/(c-1) \qquad \text{if } c > 1 \ ,$$

$$(\omega/m)^\ell\ \ln(2\omega/m) = (\omega/m)^{n-1}\ \ln(2\omega/m) \qquad \text{if } c = 1 \ ,$$

and

$$2(\omega/m)^\ell = 2(\omega/m)^n \qquad \text{if } c = 0 \ . \qquad (2.17)$$

<u>s=1</u>

$$\int x^\ell y^c z/m^{2n}\ d\cos\theta \xrightarrow[\omega\to\infty]{}$$

$$(\omega/m)^{\ell+c}/(c^2+c) = (\omega/m)^{n-1}/(c^2+c) \qquad \text{if } c > 0 \ ,$$

and

$$(\omega/m)^\ell\ \ln(2\omega/m) = (\omega/m)^{n-1}\ \ln(2\omega/m) \qquad \text{if } c = 0 \ .$$

In general the most divergent terms are those with the maximum n and the minimum c. However in the following we shall show that when s=0 and c=0, n is necessarily small, $n \leq 0$, whereas the most divergent terms are $n \geq 2$ unless $\lambda = 0$ and $\eta = 0$. Therefore the case represented by (2.17) can be dropped from the consideration. Now in all other cases the energy dependence is either ω^{n-1} or $\omega^{n-1} \ln \frac{2\omega}{m}$, and these two energy dependences can not have mutual cancellations. Therefore we need only to consider the terms with the maximum n without having to worry about the possible cancellations by terms which have a smaller n but with a different c.

We first establish the above mentioned fact that only terms with $n \leq 0$ can have s=0 and c=0. Only the matrix element M_b can give c=0. Since we are interested only in the terms with the largest n and the smallest s we can ignore all terms containing m^2, y, k'_μ, p'_μ, k'_α and p'_α in the numerators of M_b. With this simplification, the part of M_b which we are interested in is

$$M^b_{\mu\alpha\nu\beta} \longrightarrow - \frac{1}{2y} (2+\eta+\lambda) \left[2p_\nu k_\beta g_{\alpha\mu} + x g_{\nu\alpha} g_{\beta\mu} (2+\eta+\lambda)\right] . \qquad (2.18)$$

Substituting this expression into Eq. (2.14), we note that terms contracted with $g_{\beta\beta'}$ has $n \leq 0$. In order to consider the terms contracted with $p'_\beta p'_{\beta'}/m^2$, we note that

$$p'_\beta M^b_{\mu\alpha\nu\beta} \longrightarrow - \frac{1}{2y} (2+\eta+\lambda) \left[2p_\nu y g_{\alpha\mu} + x g_{\nu\alpha} p'_\mu (2+\eta+\lambda)\right] \qquad (2.19)$$

which does not contribute to the s=0, c=0 terms in x. This proves that only terms with $n \leq 0$ can have s=0 and c=0. This fact assures us that the most divergent terms in x are those with the maximum n if $n_{max} \geq 2$.

Our next task is then to pick up terms with the largest n. Before doing this, we note the following properties of the vertex function V given in Table I.

Let

$$V_{\mu\alpha\beta} = A_{\mu\alpha\beta} + \eta B_{\mu\alpha\beta} + \lambda m^{-2} C_{\mu\alpha\beta} ,$$

where $A_{\mu\alpha\beta}$ is independent of η and λ. The following can be easily verified:

(a) $p_\alpha C_{\mu\alpha\beta} = 0$ and $p'_\beta C_{\mu\alpha\beta} = 0$.

These relations are true even when p' and p are off the mass shell. Hence the tensor $p'_{\beta'} p'_\beta/m^2$ in the spin sum in (2.14) as well as the similar terms in the W boson propagators, $(p+k)_\rho (p'+k')_\sigma/m^2$ in (2.10) and $(p-k')_\rho (p'-k)_\sigma/m^2$ in (2.11), do not contribute to the λ^4 terms in χ.

(b) $p_\alpha B_{\mu\alpha\beta} \neq 0$, $p'_\beta B_{\mu\alpha\beta} \neq 0$ but $p_\alpha p'_\beta B_{\mu\alpha\beta} = 0$ if $q^2 = 0$. This means that in considering the most divergent η^4 terms in χ we may ignore $p'_\beta p'_{\beta'}/m^2$ in (2.14) but have to retain $(p+k)_\rho (p'+k')_\sigma/m^2$ in (2.10) and $(p-k')_\rho (p'-k)_\sigma/m^2$ in (2.11).

(c) $p_\alpha A_{\mu\alpha\beta} \neq 0$, $p'_\beta A_{\mu\alpha\beta} \neq 0$ but $p'_\alpha p_\beta A_{\mu\alpha\beta} = 0$ if $q^2 = 0$.

<u>Case $\lambda \neq 0$</u>

Using (a), we see immediately that the most divergent terms, when $\lambda \neq 0$, are proportional to λ^4/m^8 and the seagull diagram does not contribute terms of this order to χ. After some simple calculation we find terms which are proportional to λ^4/m^8 in χ and the result can be written as

$$\frac{d\sigma_p}{d\Omega} - \frac{d\sigma_A}{d\Omega} = \frac{-\alpha^2\lambda^4}{2m^2} \frac{y^2}{x^2} \left(\frac{x^4 - 6x^3y + 9x^2y^2 - 2xy^3}{m^8}\right) + \ldots \quad . \qquad (2.20)$$

After integrating with respect to the solid angle, we obtain

$$\sigma_p(\omega) - \sigma_A(\omega) \xrightarrow[\omega \to \infty]{} \frac{-\alpha^2\pi}{2m^2} \lambda^4 \left(\frac{\omega}{m}\right)^3 + 0 \left(\frac{\omega^2}{m^2}\right) \qquad (2.21)$$

which shows that the integration (1.2) diverges when $\lambda \neq 0$.

Case $\lambda = 0, \ \eta \neq 0$

Using (b), we see immediately that the most divergent terms, when $\lambda = 0$ and $\eta \neq 0$, are proportional to η^4/m^4 and the seagull diagram does not contribute terms of this order to χ. After some simple calculation we find the most divergent part of χ and the result can be written as

$$\frac{d\sigma_p}{d\Omega} - \frac{d\sigma_A}{d\Omega} = \frac{-\alpha^2 \eta^4}{8m^2} \frac{y^2}{x^2} \frac{(x^2 - xy)}{m^4} + \ldots \quad . \tag{2.22}$$

After integrating with respect to the solid angle, we obtain

$$\sigma_p(\omega) - \sigma_A(\omega) \xrightarrow[\omega \to \infty]{} \frac{-\alpha^2}{8m^2} \eta^4 \left(\frac{\omega}{m}\right) \quad , \tag{2.23}$$

which shows that the integration (1.2) diverges when $\eta \neq 0$.

Case $\lambda = 0, \ \eta = 0$

In this case, the matrix elements are very simple and the cross section can be expressed as follows

$$\frac{d\sigma_p}{d\Omega} = \frac{\alpha^2}{m^2} \frac{y^2}{x^2} \left[\frac{2z^2}{y^2} - z\left(\frac{m^2}{y^2} + 4\frac{x}{y^2}\right) + \frac{x^2}{y^2}\right] \tag{2.24}$$

$$\frac{d\sigma_A}{d\Omega} = \frac{d\sigma_p}{d\Omega} + \frac{\alpha^2}{m^2} \frac{y^2}{x^2} \left[z\left(\frac{4}{x} - \frac{4}{y} + \frac{8x}{y^2}\right) + 6 - 4\left(\frac{x}{y} + \frac{y}{x}\right) + 2\frac{y^2}{x^2}\right] \tag{2.25}$$

From these two equations one can show easily by an explicit calculation that

$$\int_0^\infty \frac{d\omega}{\omega} \int \left(\frac{d\sigma_p}{d\Omega} - \frac{d\sigma_p}{d\Omega}\right) d\Omega = 0 \quad . \tag{2.26}$$

We conclude that both η and λ have to be zero in order that the Drell-Hearn sum rule is satisfied.

III. HELICITY CONSERVATION AT HIGH ENERGIES

In this section we show that in the electromagnetic scattering of a charged spin 1 particle the necessary and sufficient conditions for the helicity conservation at high energies and at small but finite scattering angles are $\eta = 0$ and $\lambda = 0$. This can be demonstrated by an explicit calculation for each helicity amplitude. In this section we shall use the coordinate system shown in Fig. 3, where p, p' and q have the following components:

$$q = \left(0,\ 0,\ 0,\ 2p \sin\frac{\theta}{2}\right)$$

$$p = \left(E,\ p\cos\frac{\theta}{2},\ 0,\ -p\sin\frac{\theta}{2}\right)$$

$$p' = \left(E,\ p\cos\frac{\theta}{2},\ 0,\ p\sin\frac{\theta}{2}\right) \quad .$$

The helicity states of the incident and outgoing W's can be represented by the vectors:

$$|\vec{p}+\rangle = \epsilon_+ = \frac{1}{\sqrt{2}}\left(0,\ \sin\frac{\theta}{2},\ -i,\ \cos\frac{\theta}{2}\right) \quad ,$$

$$|\vec{p}-\rangle = \epsilon_- = \frac{-1}{\sqrt{2}}\left(0,\ \sin\frac{\theta}{2},\ i,\ \cos\frac{\theta}{2}\right) \quad ,$$

$$|\vec{p}0\rangle = \epsilon_0 = \left(\frac{p}{m},\ \frac{E}{m}\cos\frac{\theta}{2},\ 0,\ -\frac{E}{m}\sin\frac{\theta}{2}\right) \quad ,$$

$$|\vec{p}'+\rangle = \epsilon'_+ = \frac{-1}{\sqrt{2}}\left(0,\ \sin\frac{\theta}{2},\ i,\ -\cos\frac{\theta}{2}\right) \quad ,$$

$$|\vec{p}'-\rangle = \epsilon'_- = \frac{1}{\sqrt{2}}\left(0,\ \sin\frac{\theta}{2},\ -i,\ -\cos\frac{\theta}{2}\right) \quad ,$$

$$|\vec{p}'0\rangle = \epsilon'_0 = \left(\frac{p}{m},\ \frac{E}{m}\cos\frac{\theta}{2},\ 0,\ \frac{E}{m}\sin\frac{\theta}{2}\right) \quad .$$

Let us define the helicity amplitudes by

$$\Gamma^{(\mu)}_{h'h} \equiv e^{-1} <\vec{p}'h'|J_\mu|\vec{p}h> = \epsilon'^{*}_{h'\beta}\,\epsilon_{h\alpha}\,V_{\mu\alpha\beta}/(ie)$$

$$= -(p+p')_\mu\left[G_1\epsilon'^{*}_{h'}\cdot\epsilon_h - \lambda m^{-2}(q\cdot\epsilon'^{*}_{h'})(q\cdot\epsilon_h)\right] - g\,(\epsilon_{h\mu}\,q\cdot\epsilon'^{*}_{h'} - \epsilon'^{*}_{h'\mu}\,q\cdot\epsilon_h) \tag{3.1}$$

where

$$g = 1+\kappa+\lambda\,, \qquad G_1 = 1-2\lambda\,p^2 m^{-2}\sin^2\frac{\theta}{2}$$

and $V_{\mu\alpha\beta}$ is the vertex function defined by Eq. (2.1). In our frame $q_0=0$, hence from the current conservation, $q_0J_0=q_3J_3$, we have $J_3=0$. Thus we need to consider only the matrix elements of J_0 and $J_\pm = \mp(J_x \pm iJ_y)/(2)^{1/2}$. Because of the symmetries, not all 27 helicity amplitudes are independent. From the invariance under the reflection, $y \leftrightarrow -y$, we have

$$\Gamma^{o}_{h'h} = (-1)^{h-h'}\,\Gamma^{o}_{-h'-h} \tag{3.2}$$

and

$$\Gamma^{\pm}_{h'h} = (-1)^{h-h'+1}\,\Gamma^{\mp}_{-h'-h}\;. \tag{3.3}$$

This can be shown in the following way: Let us use $|h>$ and $|h'>$ to represent the spin states of the particles at rest. Then the helicity states can be written as:

$$|\vec{p}h>\equiv e^{-i\left(\frac{\pi}{2}+\frac{\theta}{2}\right)J_2}\,e^{-i\xi K_3}|h> \tag{3.4}$$

$$|\vec{p}'h'>\equiv e^{-i\left(\frac{\pi}{2}-\frac{\theta}{2}\right)J_2}\,e^{-i\xi K_3}|h'> \tag{3.5}$$

where $\xi = \sinh^{-1}(p/m)$, $e^{-i\xi K_3}$ is the boost operator in the z direction which is the axis of quantization of the state and $e^{-i(\pi/2+\theta/2)J_2}$ is the operator to rotate the z axis to the direction of $\vec{p}$ for the initial state and $\exp\left[-i\left(\frac{\pi}{2}-\frac{\theta}{2}\right)J_3\right]$ is the similar rotation operator for the final state. Under the operation $Y = \mathscr{P} e^{-i\pi J_2}$, where $\mathscr{P}$ is parity operator, we have

$$Y J_0 Y^{-1} = J_0 \quad , \tag{3.6}$$

$$Y J_{\pm} Y^{-1} = -J_{\pm} \quad , \tag{3.7}$$

$$Y|\vec{p}h> \ = \eta_p (-1)^{s-h} |\vec{p}-h> \quad , \tag{3.8}$$

and

$$Y|\vec{p}'h'> = \eta_p (-1)^{s-h'} |\vec{p}'-h'> \quad , \tag{3.9}$$

where $\eta_p = -1$ is the parity and s=1 is the spin of the particle. The last two equations come from the facts that Y commutes with $\exp\left[-i\left(\frac{\pi}{2}+\frac{\theta}{2}\right)J_2\right]$, $e^{-i\xi K_3}$ and $\exp\left[-i\left(\frac{\pi}{2}-\frac{\theta}{2}\right) J_2\right]$, and

$$Y|h> = \eta_p (-1)^{s-h} |-h> \quad . \tag{3.10}$$

We note that (3.8) and (3.9) are satisfied by our polarization vectors ϵ_h and $\epsilon'_{h'}$. For example changing the sign of the y component of ϵ_+ yields $-\epsilon_-$. The desired relations (3.2) and (3.3) follow immediately from (3.6) through (3.9).

From the hermiticity of the current operators J_0, J_x and J_y and the invariance under time reversal we have

$$\Gamma^o_{h'h} = (-1)^{h-h'} \Gamma^o_{hh'} \tag{3.11}$$

and

$$\Gamma^{\pm}_{h'h} = (-1)^{h-h'} \Gamma^{\pm}_{hh'} \quad . \tag{3.12}$$

Let us derive these two relations in the following. Under the time reversal operation, we have

$$T(J_0, J_x, J_y)\, T^{-1} = (J_0, -J_x, -J_y) \tag{3.13}$$

and

$$ {}_T\langle h'\vec{p}'|T J_\mu T^{-1}|h\vec{p}\rangle_T = \langle h'\vec{p}'|J_\mu|h\vec{p}\rangle^* \equiv \Gamma^{\mu *}_{h'h} \quad , \tag{3.14} $$

where

$$ |h\vec{p}\rangle_T \equiv T|h\vec{p}\rangle = e^{-i\pi J_2}|h\vec{p}\rangle \quad . \tag{3.15} $$

Substituting (3.15) and (3.13) into the left hand side of (3.14) and using

$$ e^{i\pi J_2}(J_0, -J_x, -J_y)\, e^{-i\pi J_2} = (J_0, J_x, -J_y) \quad , \tag{3.16} $$

we obtain

$$ \Gamma^{o*}_{h'h} = \Gamma^{o}_{h'h} \,, \qquad \Gamma^{x*}_{h'h} = \Gamma^{x}_{h'h} \qquad \text{and} \qquad \Gamma^{y*}_{h'h} = -\Gamma^{y*}_{h'h} \,. \tag{3.17} $$

On the other hand the hermiticity, $J^+_\mu = J_\mu$ gives the following results: If

$$ \Gamma^\mu_{h'h} \equiv \langle h'|e^{i\xi K_3}\, e^{i\left(\frac{\pi}{2}-\frac{\theta}{2}\right)J_2}\, J_\mu\, e^{-i\left(\frac{\pi}{2}+\frac{\theta}{2}\right)J_2}\, e^{-i\xi K_3}|h\rangle \quad , \tag{3.18} $$

then

$$ \Gamma^{\mu *}_{h'h} = \langle h|e^{i\xi K_3}\, e^{i\left(\frac{\pi}{2}+\frac{\theta}{2}\right)J_2}\, J_\mu\, e^{-i\left(\frac{\pi}{2}-\frac{\theta}{2}\right)J_2}\, e^{-i\xi K_3}|h'\rangle \quad . \tag{3.19} $$

In (3.19) the particle with the helicity h (or h') is moving in the direction of $\vec{p}$ (or $\vec{p}'$). In the right hand sides of the desired relations (3.11) and (3.12), the particle with the helicity h (or h') is moving in the direction of $\vec{p}'$ (or $\vec{p}$). If we rotate the coordinate system around the z axis by 180°, $\vec{p}'$ becomes $\vec{p}$ and vice versa. Inserting $e^{iJ_3\pi}\,e^{-iJ_3\pi}$ between all adjacent factors in (3.19) and using

$$ e^{-iJ_3\pi}\left(e^{i\xi K_3},\ e^{i\left(\frac{\pi}{2}+\frac{\theta}{2}\right)J_2},\ e^{-i\left(\frac{\pi}{2}-\frac{\theta}{2}\right)J_2},\ J_0,\ J_x,\ J_y\right)e^{iJ_3\pi} $$

$$ = \left(e^{i\xi K_3},\ e^{-i\left(\frac{\pi}{2}+\frac{\theta}{2}\right)J_2},\ e^{i\left(\frac{\pi}{2}-\frac{\theta}{2}\right)J_2},\ J_0,\ -J_x,\ -J_y\right) , $$

$$ e^{-iJ_2\pi}(J_0, -J_x, -J_y)\, e^{iJ_2\pi} = (J_0, J_x, -J_y) $$

and

$$e^{-iJ_3\pi}\,|h'\rangle = (-1)^{h'}\,|h'\rangle$$

we obtain

$$\Gamma^{o,x*}_{h'h} = (-1)^{h-h'}\,\Gamma^{o,x}_{hh'} \quad \text{and} \quad \Gamma^{y*}_{h'h} = (-1)^{h-h'+1}\,\Gamma^{y}_{hh'} \qquad (3.20)$$

Combining (3.17) with (3.20) we obtain the desired relations (3.11) and (3.12). The consequence of the four symmetry relations given by (3.2), (3.3), (3.11) and (3.12) is that we need to consider only 10 amplitudes instead of 27; the relations between various amplitudes and the expression for all amplitudes are given below:

Helicity conserving amplitudes (h=h')

$$\Gamma^{o}_{++} = \Gamma^{o}_{--} = 2E\left[G_1\cos^2\frac{\theta}{2} + \frac{\lambda}{2m^2}\,p^2\sin^2\theta\right]$$

$$\Gamma^{o}_{00} = 2E\left[G_1\left(1-2E^2m^{-2}\sin^2\frac{\theta}{2}\right) - 4\lambda E^2p^2m^{-4}\sin^4\frac{\theta}{2}\right]$$

$$\Gamma^{+}_{++} = -\Gamma^{-}_{--} = -2^{\frac{1}{2}}p\cos\frac{\theta}{2}\left[G_1\cos^2\frac{\theta}{2} + \frac{\lambda p^2}{2m^2}\sin^2\theta\right] - 2^{-\frac{1}{2}}gp\sin\theta\left(\sin\frac{\theta}{2}+1\right)$$

$$\Gamma^{+}_{--} = -\Gamma^{-}_{++} = -2^{\frac{1}{2}}p\cos\frac{\theta}{2}\left[G_1\cos^2\frac{\theta}{2} + \frac{\lambda p^2}{2m^2}\sin^2\theta\right] - 2^{-\frac{1}{2}}gp\sin\theta\left(\sin\frac{\theta}{2}-1\right)$$

$$\Gamma^{+}_{00} = -\Gamma^{-}_{00} = -2^{\frac{1}{2}}p\cos\frac{\theta}{2}\left[G_1\left(1-\frac{2E^2}{m^2}\sin^2\frac{\theta}{2}\right) - \frac{4\lambda E^2p^2}{m^4}\sin^4\frac{\theta}{2}\right] - \frac{2^{\frac{1}{2}}gE^2p}{m^2}\sin\frac{\theta}{2}\sin\theta$$

Helicity nonconserving amplitudes (h-h'≠0)

Double helicity flip |h-h'| = 2

$$\Gamma^{o}_{+-} = \Gamma^{o}_{-+} = 2E\left[G_1\sin^2\frac{\theta}{2} - \frac{1}{2}\lambda p^2m^{-2}\sin^2\theta\right]$$

$$\Gamma^{+}_{+-} = \Gamma^{+}_{-+} = -\Gamma^{-}_{+-} = -\Gamma^{-}_{-+} = -2^{\frac{1}{2}}p\cos\frac{\theta}{2}\left[G_1\sin^2\frac{\theta}{2} - \frac{1}{2}\lambda p^2m^{-2}\sin^2\theta\right] + 2^{-\frac{1}{2}}gp\sin\theta\sin\frac{\theta}{2}$$

Single helicity flip $|h-h'| = 1$

$$\Gamma^{0}_{0+} = -\Gamma^{0}_{0-} = -\Gamma^{0}_{+0} = \Gamma^{0}_{-0}$$

$$= 2E\left[G_1 2^{-\frac{1}{2}} E\, m^{-1} \sin\theta + \lambda 2^{\frac{1}{2}} Ep^2 m^{-3} \sin\theta \sin^2\frac{\theta}{2}\right] - 2^{-\frac{1}{2}} gp^2 m^{-1} \sin\theta$$

$$\Gamma^{+}_{+0} = -\Gamma^{+}_{0+} = -\Gamma^{-}_{0-} = \Gamma^{-}_{-0}$$

$$= 2^{\frac{1}{2}} p \cos\frac{\theta}{2}\left[G_1 2^{-\frac{1}{2}} E\, m^{-1} \sin\theta + 2^{\frac{1}{2}}\lambda\, Ep^2 m^{-3} \sin\theta \sin^2\frac{\theta}{2}\right]$$

$$- gEp\, m^{-1} \sin\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2} - 2\sin^2\frac{\theta}{2}\right)$$

$$\Gamma^{+}_{-0} = -\Gamma^{+}_{0-} = \Gamma^{-}_{+0} = -\Gamma^{-}_{0+}$$

$$= -2^{\frac{1}{2}} p \cos\frac{\theta}{2}\left[G_1 2^{-\frac{1}{2}} E\, m^{-1} \sin\theta + 2^{\frac{1}{2}}\lambda\, E p^2 m^{-3} \sin\theta \sin^2\frac{\theta}{2}\right]$$

$$+ g\, Epm^{-1} \sin\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2} - 2\sin^2\frac{\theta}{2}\right)$$

From these amplitudes we observe the following:

1. When $\theta \ll m/E$, all the helicity nonconserving amplitudes become negligible compared with the helicity conserving ones independent of values of g and λ. Hence no conditions on g and λ can be obtained under this condition.

2. If we demand that the helicity is conserved even when $\theta \approx m/E \ll 1$, then we obtain g=2 and λ=0. We note that the amplitudes with double helicity flip yield only the condition λ=0, because the terms proportional to G_1 and g are small as long as $\theta \ll 1$.

3. When g=2 and λ=0, the helicity is conserved as long as $\theta \ll 1$ and $m/E \ll 1$.

This concludes the demonstration of the fact mentioned at the beginning of this section. We have considered the scattering of $W^{\pm}$ by an electromagnetic field in the lowest order in α. In the actual scattering, an infinite number of photons are exchanged. However in the electromagnetic scattering of a charged particle at a finite angle, it is most probable (because of $1/q_n^2$ for each photon propagator and $q_1 + q_2 + \ldots + q_n + \ldots = q$) that practically all the momentum transfer is carried by a single photon and the rest of the photons (an infinite number of them) are soft (i.e., small angle scattering). Now the small angle scattering does not flip helicity as observed in 1 above. Thus we expect our result to be true even if an infinite number of photons are exchanged. The restriction $\theta \ll 1$ comes from the angular momentum conservation. In order to see this, let us consider the extreme case $\theta = \pi$, which corresponds to the brick wall system discussed in the appendix. In the brick wall system the conservation of angular momentum in the helicity amplitude $\Gamma^i_{h'h}$ $(i = 0, \pm 1)$ gives $i + h + h' = 0$. Hence in order for the helicity to be conserved in the 180^o scattering we must have $i = -2h$, which is impossible if $h = \pm 1$. In the electromagnetic scattering of a charged spin 1/2 particle, the helicity is conserved at high energies even at $\theta = 180^o$ if $\mu = e/2m$.

IV. CONCLUDING REMARKS

We have shown that the special values of the magnetic dipole moment and the electric quadrupole moment given by (1.1) for the charged W boson have two desirable features: (1) satisfaction of the Drell-Hearn sum rule in the orders α and α^2 and (2) the helicity conservation at high energies in the electromagnetic scattering. These two features are also shared by the only known charged nonstrongly-interacting particles: the electron and the muon. However it is quite possible that nature is more complicated than what we think it might be. For example $W^{\pm}$ bosons may have an electric dipole moment which violates both P and T invariances, or they may interact strongly among themselves. Indeed Salzman and Salzmann[4] suggested that the small CP violation in the decay of K_2 may be due to the existence of the electric dipole moment of $W^{\pm}$ and many people[8] have considered the possibility of strong interactions among W's in order to overcome the divergence difficulties of the weak interaction. If P and T invariances are violated, the Drell-Hearn sum rule has to be rederived. If W has strong interactions among themselves, then the right hand side of Eq. (1.1) will be dominated by the terms of order α, and in this case the magnetic dipole moment is no longer $\mu = e/m$. A large deviation from this value will indicate the existence of the strong interaction of W. The values of μ and Q given by (1.1) are of course what the unified theory of weak and electromagnetic interaction of Weinberg et al.[12] gives. The arguments given in this paper can therefore be regarded as rendering some extra supports for such a theory.

APPENDIX A

MAGNETIC DIPOLE AND ELECTRIC QUADRUPOLE MOMENTS AND MEAN SQUARE CHARGE RADIUS

In this appendix we discuss the problem of identifying various form factors in a relativistically covariant vertex function with the electric and magnetic multipole moments defined in the nonrelativistic nuclear physics. Fully relativistic multipole expansion of an electromagnetic vertex function has been treated by Durand, DeCelles and Marr[3] (hereafter referred to as DDM) in the helicity formalism. However it is not immediately obvious how the multipole moments defined by DDM are related to the multipole moments commonly used in the nonrelativistic nuclear physics. Of course there is a one to one correspondence between the two, because in both cases the multipole moments are defined by the rotational properties of various irreducible tensor operators. Therefore in principle we need to know only the proportionality constants between the two conventions. In the nonrelativistic nuclear physics the electric quadrupole moment Q is defined as

$$Q = \int \langle ss|(3z^2-r^2)\,\rho(\vec{x})|ss\rangle\, d^3x = 2\int \langle ss|r^2\rho(\vec{r})\,p_2(\cos\theta)|ss\rangle\, d^3x \tag{A.1}$$

where $|ss\rangle$ represents the state with spin s and s_z=s, and $\rho(\vec{x}) \equiv J_0(0,\vec{x})$ is the charge density operators normalized such that

$$\int \langle ss_z|\rho(\vec{x})|ss_z\rangle\, d^3x = e \quad . \tag{A.2}$$

The mean square charge radius is given by

$$R^2 \equiv \int \langle ss_z|r^2\rho(\vec{x})|ss_z\rangle\, d^3x/e \quad . \tag{A.3}$$

The magnetic dipole moment $\vec{\mu}$ is defined classically by its energy in the magnetic field $\vec{B}$,

$$\text{Energy} = -\vec{\mu}\cdot\vec{B} \quad . \tag{A.4}$$

In quantum mechanics, the interaction energy between a current $J_\mu(0,\vec{x})$ and an electromagnetic field $A_\mu(0,\vec{x})$ is given by

$$\int J_\mu(0,\vec{x})\, A_\mu(0,\vec{x})\, d^3x \quad . \tag{A.5}$$

Let the electromagnetic potential $A_\mu(0,\vec{x})$ in (A.5) be

$$A_0 = 0 \quad \text{and} \quad \vec{A} = (\hat{e}_x - i\hat{e}_y)\, e^{-i\vec{q}\cdot\vec{r}}/2^{\frac{1}{2}} \tag{A.6}$$

where $\vec{q} = \hat{e}_z q$. Then the magnetic field is

$$\vec{B} = \vec{\nabla}\times\vec{A} = -i\vec{q}\times\vec{A} \quad . \tag{A.7}$$

Substituting (A.7) into (A.4) and (A.6) into (A.5) and equating the two expressions, we obtain

$$\mu_- = \lim_{q\to 0} \frac{1}{q}\int J_-(0,\vec{x})\, e^{-i\vec{q}\cdot\vec{r}}\, d^3x \quad .$$

Applying the Wigner-Eckart theorem, we have

$$\mu \equiv \langle ss|\mu_z|ss\rangle = \frac{\langle ss|10\ ss\rangle}{\langle ss_z'|1-1\ ss_z\rangle} \lim_{q\to 0}\frac{1}{q}\int \langle ss_z'|J_-(0,\vec{x})|ss_z\rangle\, e^{-i\vec{q}\cdot\vec{r}}\, d^3x \; . \tag{A.8}$$

Equations (A.1, A.2, A.3, A.8) define the quantities e, QK^2 and μ in terms of matrix elements of nonrelativistic quantum mechanics in which the particle is assumed to be infinitely heavy.

Our next task is to find out the relationship between these nonrelativistic matrix elements and the relativistic vertex functions. Let us choose the helicity amplitudes in the brick wall system for this comparison. The desired relation is then

$$e\Gamma^{(\mu)}_{h'h}/(2E) \equiv \langle -\vec{p}h' | J_\mu(0) | \vec{p}h\rangle/(2E)$$

$$\xrightarrow[q/m\to 0]{} (-1)^{s+s_z'} \int \langle ss_z' | J_\mu(0,\vec{x}) | ss_z\rangle\, e^{-i\vec{q}\cdot\vec{x}}\, d^3x \tag{A.9}$$

where

$$s = \text{spin}, \qquad s_z = h, \qquad s_z' = -h', \qquad \vec{q} = 2\vec{p},$$

and

$$E = (p^2 + m^2)^{\frac{1}{2}} \;.$$

The factor $(-1)^{s+s_z'}$ comes from the fact that in the nonrelativistic quantum mechanics, we have quantized the spin states of both the initial and final states along the direction of $\vec{q}$, whereas in the helicity representation the final state is quantized along the direction opposite to $\vec{q}$, thus

$$e^{-iJ_2\pi}|h'\rangle = (-1)^{s-h'}|-h'\rangle = (-1)^{s+s_z'}|s_z'\rangle \quad . \tag{A.10}$$

In this appendix we have chosen $q_\mu = (p - p')_\mu$ which is opposite to the convention used in Fig. 1, because we want to use the convention of DDM in the definitions of the helicity amplitudes. Throughout this appendix we shall use q to represent $|\vec{q}|$. The over all normalization and the sign of the left hand side of Eq. (A.9) can be checked by using (A.2). The factor 2E is put there so that when the form factor for a spin 0 particle is unity we obtain $R^2=0$. The brick wall system was chosen because (1) in this frame $q_0=0$ for the elastic scattering, hence from the gauge invariance $(q_0J_0=qJ_z)$ we have $J_z=0$, (2) the selection rule due to the conservation of the angular momentum is very simple, namely, if we write $\Gamma^{\Lambda}_{h'h}$ where $\Lambda=\pm1,0$, we have $\Lambda+h'+h=0$.

The magnetic moment μ can be calculated readily from (A.9) and (A.8). In order to calculate Q and R^2 we perform a multipole analysis of the left hand side a la DDM, and compare the results with the multipole analysis of the right hand side using the nonrelativistic quantum mechanics. The helicity amplitude $\Gamma^{(\mu)}_{h'h}$ can be obtained from the vertex function given by Eq. (3.1) except that the sign of q is changed and the helicity states now have the following representations:

$$
\begin{aligned}
\epsilon_+ &= -2^{-\frac{1}{2}}(0,\ 1,\ i,\ 0) \\
\epsilon_- &= 2^{-\frac{1}{2}}(0,\ 1,\ -i,\ 0) \\
\epsilon_0 &= (p/m,\ 0,\ 0,\ E/m) \\
\epsilon'_+ &= \epsilon_- \\
\epsilon'_- &= \epsilon_+ \\
\epsilon'_0 &= (p/m,\ 0,\ 0,\ -E/m)
\end{aligned}
\tag{A.11}
$$

After straight forward calculations we obtain

$$\Gamma^{-}_{0+} = -2pEg/m \ , \tag{A.12}$$

$$\Gamma^{o}_{00} = -2E\left[G_1 + 2p^2m^{-2}\left(G_1 - g + 2\lambda E^2/m^2\right)\right] \ , \tag{A.13}$$

and

$$\Gamma^{o}_{-+} = 2EG_1 \quad . \tag{A.14}$$

From (A.8), (A.9) and (A.12), we obtain

$$
\begin{aligned}
\mu &= \frac{<11|1011>}{<10|1\text{-}111>}\,\frac{1}{q}\,(-1)\,\frac{e}{2E}\,\Gamma^{-}_{0+} \\
&= eg/(2m) = e(1+\kappa+\lambda)/(2m) \quad .
\end{aligned}
\tag{A.15}
$$

To obtain R^2 and Q we first decompose $\Gamma^{o}_{h'h}$ into multipole moments using Eq. (109) of DDM:

$$\Gamma^{o}_{h'h} = \begin{pmatrix} 1 & 0 & 1 \\ h' & 0 & h \end{pmatrix} Q_0 + \begin{pmatrix} 1 & 2 & 1 \\ h' & 0 & h \end{pmatrix} Q_2 \ , \tag{A.16}$$

Using (A.13), (A.14) and (A.16), we may write Q_0 and Q_2 in terms of Γ^o_{00} and Γ^o_{1-1}:

$$Q_0 = \left(-\Gamma^o_{00} + 2\Gamma^o_{-+}\right) /(3)^{1/2}$$

$$= 2(3)^{1/2}\, E\left[G_1 - 2p^2/(3m^2)\left(G_1 - g + 2\lambda E^2/m^2\right)\right] \quad . \tag{A.17}$$

$$Q_2 = \left(\Gamma^o_{00} + \Gamma^o_{-+}\right) (3/10)^{-1/2}$$

$$= -4Ep^2 m^{-2}\left(G_1 - g + 2\lambda E^2/m^2\right) (3/10)^{-1/2} \quad . \tag{A.18}$$

The right hand side of Eq. (A.9) can also be expanded in terms of multipole moments in the following way: We first expand the exponential factor $\exp(-i\vec{q}\cdot\vec{r})$ by

$$e^{-i\vec{q}\cdot\vec{r}} = e^{-iqr\cos\theta} = \sum_{J=0}^{\infty} (-i)^J (2J+1)\, j_J(qr)\, P_J(\cos\theta) \tag{A.19}$$

From (A.19) we may write the right hand side of (A.9) as

$$(-1)^{1+s_z'} \int \langle 1s_z' | J_0(0,\vec{x}) | 1s_z\rangle\, e^{-i\vec{q}\cdot\vec{x}}\, d^3x$$

$$= \begin{pmatrix} 1 & 0 & 1 \\ h & 0 & h \end{pmatrix} Q_0^{NR} + \begin{pmatrix} 1 & 2 & 1 \\ h & 0 & h \end{pmatrix} Q_2^{NR} \tag{A.20}$$

where

$$Q_J^{NR}(q^2) = \frac{(2J+1)\,(-i)^J}{\begin{pmatrix} s & J & s \\ -s & 0 & s \end{pmatrix}} \int \langle ss | \rho(\vec{x})\, P_J(\cos\theta) | ss\rangle \quad j_J(qr)\, d^3x \quad . \tag{A.21}$$

Expanding the spherical Bessel functions up to q^2, we obtain

$$j_0(qr) = 1 - \vec{q}^{\,2} r^2/6 \tag{A.22}$$

and

$$j_2(qr) = \vec{q}^{\,2} r^2/15 \quad . \tag{A.23}$$

From (A. 9, A. 16, A. 20, A. 21 and A. 22), we obtain

$$eQ_0/(2E) = Q_0^{NR} = 3^{\frac{1}{2}}(1 - \vec{q}^{\,2}R^2/6)\ e \tag{A.24}$$

From (A. 9, A. 16, A. 20, A. 21 and A. 23), we obtain

$$eQ_2/(2E) = Q_2^{NR} = -(5/6)^{1/2}\, Q\vec{q}^{\,2} \quad . \tag{A.25}$$

From (A. 24 and A. 17), we obtain

$$R^2 = (\kappa + \lambda)/m^2 \quad . \tag{A.26}$$

From (A. 25 and A. 18), we obtain

$$Q = -e(\kappa - \lambda)/m^2 \quad . \tag{A.27}$$

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19. Note that q defined here is the negative of q used in Fig. 1. We have used this convention in the appendix in order to conform to the convention used in Durand _et al._'s paper (Ref. 3).

TABLE I

The Values of V and U ($\lambda' = \lambda/m^2$, $\kappa = \eta + 1$)

$$V_{\mu\alpha\beta} = ie\Big[g_{\alpha\beta}(p+p' + \lambda' p'\cdot q\, p - \lambda'\, p\cdot q p')_\mu$$

$$- g_{\alpha\mu}(p - \kappa\, q + \lambda' p'\cdot q\, p - \lambda' p'\cdot pq)_\beta$$

$$- g_{\beta\mu}(p' + \kappa q - \lambda' p\cdot q\, p' + \lambda' p'\cdot pq)_\alpha$$

$$- \lambda' p_\mu p'_\alpha q_\beta + \lambda' p'_\mu q_\alpha p_\beta\Big]$$

$$= ie\Big[g_{\alpha\beta}(p+p')_\mu - g_{\alpha\mu}(p-q)_\beta - g_{\beta\mu}(p'+q)_\alpha$$

$$+ \eta\,(g_{\alpha\mu}q_\beta - g_{\beta\mu}q_\alpha)\Big]$$

$$+ \lambda m^{-2}\Big[g_{\alpha\beta}(p'\cdot q\, p_\mu - p\cdot q p'_\mu) - g_{\alpha\mu}(p'\cdot q\, p_\beta - p'\cdot pq_\beta)$$

$$+ g_{\beta\mu}(p\cdot q\, p'_\alpha - p'\cdot pq_\alpha) - p_\mu p'_\alpha q_\beta + p'_\mu q_\alpha p_\beta\Big]$$

$$U_{\mu\nu\alpha\beta} = -ie^2(2g_{\mu\nu}\, g_{\alpha\beta} - g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu})$$

$$-ie^2\lambda'\Big\{g_{\mu\nu}\, g_{\alpha\beta}(q-q')\cdot(p'-p) - g_{\alpha\mu}g_{\beta\nu}\,(q\cdot p' + q'\cdot p)$$

$$+ g_{\alpha\nu}\, g_{\beta\mu}(q\cdot p + q'\cdot p') - g_{\alpha\beta}\Big[q_\nu\,(p'-p)_\mu - q'_\mu(p'-p)_\nu\Big]$$

$$+ g_{\mu\nu}\Big[p_\beta(q-q')_\alpha - p'_\alpha(q-q')_\beta\Big] + g_{\beta\nu}\,(q_\alpha p'_\mu + q'_\alpha p_\mu + q'_\alpha p'_\mu - q'_\mu p'_\alpha)$$

$$- g_{\alpha\nu}\,(q'_\beta p_\mu - q'_\mu p_\beta + q_\beta p_\mu + q'_\beta p'_\mu) - g_{\mu\beta}(p'_\nu\, q_\alpha - p'_\alpha q_\nu + p_\nu\, q_\alpha + p'_\nu\, q'_\alpha)$$

$$+ g_{\mu\alpha}(p_\nu\, q_\beta - p_\beta q_\nu + q_\beta p'_\nu + q'_\beta p_\nu)\Big\}$$

LIST OF FIGURES

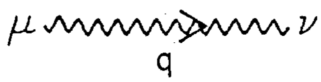
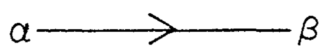
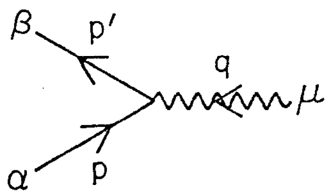
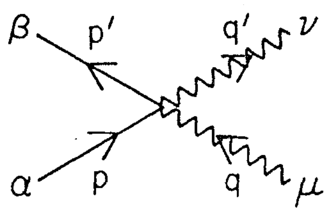
Element	Graph	Value
Internal photon line		$-i\, g_{\mu\nu}/q^2$
Internal W meson line		$-i\, \dfrac{g_{\alpha\beta} - p_\alpha p_\beta m^{-2}}{p^2 - m^2}$
γWW vertex		$V_{\mu\alpha\beta}$ = See Table I
Seagull		$U_{\mu\nu\alpha\beta}$ = See Table I

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Fig. 1

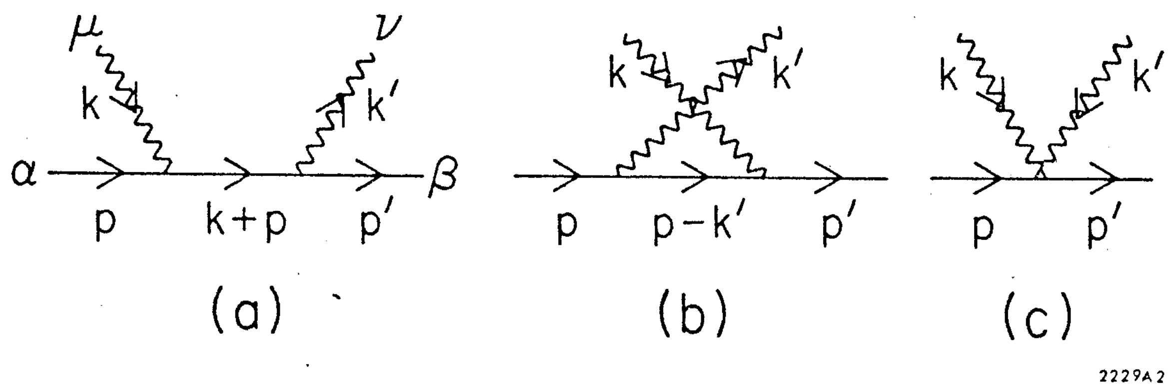

Fig. 2

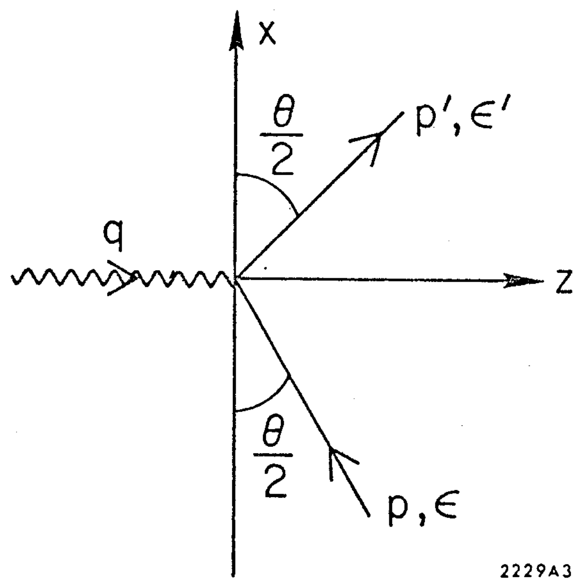

Fig. 3