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THE EIKONAL APPROXIMATION FOR THE BILOCAL VERTEX FUNCTION AND νW_2 IN A FERMION-NEUTRAL VECTOR GLUON MODEL*

E. Eichten

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

Abstract

The eikonal approximation is used to investigate the forward one-Fermion matrix element of the bilocal operator that appears in the most singular term of the canonical light cone current commutators for the Fermion-neutral vector gluon model. The relationship exhibited between this matrix element of the bilocal operator and the leading behavior of νW_2 allows results to be obtained for νW_2 in this model. A simple set of graphs contributing to the matrix element of the bilocal operator is calculated. This gives a result for νW_2 in agreement with explicit calculations in perturbation theory for the corresponding set of graphs of νW_2.

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1. Introduction

The determination of the structure functions for deep inelastic electron-proton scattering is of great theoretical interest. Insight may be gained by examining simple field theoretic models. In particular, the inelastic structure functions have been studied for a Fermion-neutral vector gluon model in perturbation theory.[1-3]

In this paper, I would like to suggest that the eikonal techniques may be used to investigate the bilocal operator that appears in the most singular term of the light-cone current commutators in the Fermion-neutral vector gluon field theory. Since the forward one-Fermion matrix element of this bilocal operator can be related to the leading behavior of νW_2 (as $-q^2 \longrightarrow \infty$, ω fixed), the behavior of νW_2 can be investigated by the method. In particular, the contribution to this matrix element of the bilocal operator corresponding to structureless gluon graphs can be easily calculated in the eikonal approximation. The results obtained in these approximations for νW_2 agree with the explicit calculations in perturbation theory made for the relevant graphs of νW_2 by P. M. Fishbane and J. D. Sullivan.[3]

In Section 2, the form of the bilocal operator will be obtained and the relationship between the forward one-Fermion matrix element of this bilocal operator and νW_2 will be derived. In Section 3, the eikonal approximation to the bilocal operator matrix element will be obtained and various approximations made to obtain numerical results. The relationship between the matrix element of the bilocal operator and νW_2 is used to obtain a result for νW_2 within these approximations. In Section 4, the approximations are discussed and further calculations suggested.

2. The Bilocal Vertex Function and Its Relation to νW_2

The electromagnetic current commutators at equal x^+ have been computed canonically for the Fermion-neutral vector gluon theory.[4] The results are

$$-i\left[J^+(x), J^\nu(0)\right]_{x^+=0} = \frac{i}{4}\partial^x_\alpha\left[\left(\bar{\psi}(x)\gamma^+\gamma^\alpha\gamma^\nu \exp\left\{-ig_0\int_0^x dz_\beta A^\beta(z)\right\}\psi(0) - h.c.\right)\epsilon(x^-)\delta^2(\vec{x}_\perp)\right]$$

Now the leading term in the operator expansions as $x^2 \to 0$ of the unordered product of currents can be obtained from these light-cone commutators, and can be written

$$J^\mu(x)J^\nu(0) \xrightarrow[x^2\to 0]{} 0^{\mu\nu\sigma}(x|0)\,\partial^x_\sigma\left[\Delta^{(-)}(x)\right] + \text{less singular terms}$$

where $\Delta^{(-)}(x)$ is the negative frequency part of the Pauli-Jordan commutator function. The most singular part of $\Delta^{(-)}$ is

$$\frac{-i}{4\pi^2}\,\frac{1}{x^2 - i\epsilon x^-}\ .$$

$0^{\mu\nu\sigma}(x|0)$ is determined by the light-cone commutators to be

$$0^{\mu\nu\sigma}(x|0) = -1\left(\bar{\psi}(x)\gamma_\alpha \exp\left\{-ig_0\int_0^x dz_\beta A^\beta(z)\right\}\psi(0) - h.c.\right)\left(g^{\mu\sigma}g^{\nu\alpha} + g^{\nu\sigma}g^{\mu\alpha}\right.$$

$$\left. - g^{\mu\nu}g^{\sigma\alpha}\right) + i\epsilon^{\mu\sigma\nu\delta}\left(\bar{\psi}(x)\gamma_5\gamma_\delta \exp\left\{-ig_0\int_0^x dz_\beta A^\beta(z)\right\}\psi(0) - h.c.\right)$$

The connected spin-averaged one-Fermion forward matrix element of the unordered product of the electromagnetic currents is therefore

$$\langle p|J^{\mu}(x)\,J^{\nu}(0)|p\rangle_{CSA} \xrightarrow[x^2=0]{} (-1)\left(g^{\mu\sigma}g^{\nu\alpha} + g^{\mu\alpha}g^{\nu\sigma} - g^{\mu\nu}g^{\sigma\alpha}\right)$$

$$\langle p|\bar{\psi}(x)\gamma_{\alpha}\exp\left\{-i\,g_0\int_0^x dz_{\beta}A^{\beta}(z)\right\}\psi(0) - \text{h.c.}|p\rangle\, \partial_{\sigma}^{x}\Delta^{(-)}(x)$$

R. Jackiw and R. E. Waltz have shown[5] that for the one-Fermion forward matrix element, the T product and the unordered product coincide as $x^2 \longrightarrow 0$; thus we may write

$$\langle p|J^{\mu}(x)\,J^{\nu}(0)|p\rangle_{CSA} \xrightarrow[x^2\to 0]{} (-1)\left(g^{\mu\sigma}g^{\nu\alpha} + g^{\nu\sigma}g^{\mu\alpha} - g^{\mu\nu}g^{\sigma\alpha}\right) \tag{2.1}$$

$$\langle p|T\left(\bar{\psi}(x)\gamma_{\alpha}\exp\left\{-i\,g_0\int_0^x dz_{\beta}A^{\beta}(z)\right\}\psi(0)\right) - \text{h.c.}|p\rangle_{CSA}\, \partial_{\sigma}^{x}\Delta^{(-)}(x)$$

The bilocal vertex function is defined by

$$\Gamma^{\mu}(x^2\cdot p) \equiv \langle p|T\left(\bar{\psi}(x)\gamma^{\mu}\exp\left\{i\,g_0\int_0^x dz_{\beta}A^{\beta}(z)\right\}\psi(0)\right)|p\rangle_{CSA}, \tag{2.2}$$

and defining $D(x^2,\ x\cdot p)$ and $C(x^2,\ x\cdot p)$ by

$$\mathrm{Im}\,\Gamma^{\mu}(x^2,\ x\cdot p) = \frac{p^{\mu}}{2M}D(x^2,\ x\cdot p) + \frac{x^{\mu}M}{2}C(x^2,\ x\cdot p)\quad , \tag{2.3}$$

the leading behavior of $\langle p|J^{\mu}(x)\,J^{\nu}(0)|p\rangle_{CSA}$ is given by

$$\langle p|J^{\mu}(x)\,J^{\nu}(0)|p\rangle_{CSA} \xrightarrow[x^2\to 0]{} -\frac{i}{M}D(x^2,\ x\cdot p)\,(p^{\mu}\partial^{\nu} + p^{\nu}\partial^{\mu} - g^{\mu\nu}\,p\cdot\partial)\,\Delta^{(-)}_{(x)} \tag{2.4}$$

+ less singular terms

Since this matrix element of the unordered product of currents is just the Fourier transform of $W^{\mu\nu}$ for $q^2 < 0$, we have

$$(2\pi)^2 \frac{E_p}{M} \int d^4x\, e^{iq\cdot x} \langle p|J^\mu(x)\, J^\nu(0)|p\rangle_{CSA} = -\left(g^{\mu\nu} - q^\mu q^\nu/q^2\right) W_L\left(q^2, q\cdot p\right)$$

$$+ \frac{1}{M^2}\left[p^\mu p^\nu - \frac{q\cdot p}{q^2}\left(p^\mu q^\nu + p^\nu q^\mu\right) + g^{\mu\nu}(q\cdot p)^2/q^2\right] W_2\left(q^2, q\cdot p\right) \tag{2.5}$$

Comparison of Eq. 2.4 and 2.5 for $\langle p|J^\mu(x)\, J^\mu(0)|p\rangle_{CSA}$ gives the relation to leading order as $x^2 \longrightarrow 0$

$$2\pi^2 \frac{E_p}{M} \Delta^{(-)}(x)\, D(x^2, x\cdot p) \xleftrightarrow[\text{singularity}]{\text{Leading}} \frac{1}{M}\int \frac{d^4q}{(2\pi)^4}\, e^{-iq\cdot x}\left(\frac{-q\cdot p}{q^2}\right) W_2\, q^2, q\cdot p) \tag{2.6a}$$

Or in momentum space, the leading behavior of $\nu W_2(q^2, q\cdot p)$ as $-q^2 \longrightarrow \infty$, ω fixed, is $(\omega = -q^2/2q\cdot p)(\nu = q\cdot p/M)$

$$\nu W_2\left(q^2, q\cdot p\right) \xrightarrow[\omega\ \text{fixed}]{-q^2\to\infty} -q^2 \int d^4x\, e^{-iq\cdot x}\, \Delta^{(-)}(x)\, D\left(x^2, x\cdot p\right)\left(\frac{M}{E_p}\right)^{-1} 2\pi^2 \tag{2/6b}$$

This relation allows the determination of the leading behavior of $\frac{1}{M} q\cdot p\, W_2(q^2, q\cdot p)$ from the behavior of the bilocal vertex function Γ^μ as $x^2 \longrightarrow 0$.

3. The Eikonal Approximation for the Bilocal Vertex Function

It is convenient to use functional techniques to rewrite $\Gamma^\mu(x^2, x\cdot p)$ in a manner amenable to the eikonal approximation. For this purpose, additional external potential interactions are introduced.

$$\mathscr{L}(x) = \bar\psi(x)(i\not\partial - m)\psi(x) - \frac{1}{4} F^{\mu\nu}(x)\, F_{\mu\nu}(x) - g_0\, \bar\psi(x)\gamma^\mu\psi(x)\, A_\mu(x)$$

$$+ \delta m\, \bar\psi(x)\psi(x) + \bar\eta(x)\psi(x) + \bar\psi(x)\eta(x) + A^\mu(x)\, B_\mu(x)$$

Then the vacuum-vacuum transition amplitude is

$$Z(\eta,\bar{\eta},B) \equiv \langle 0_{out}|0_{in}\rangle_{\eta\bar{\eta}B} = \langle 0|T\left(\exp\left\{i\int d^4x\left[-g_0\,\bar{\psi}^{in}(x)\gamma^{\mu}\,\psi^{in}(x)\right.\right.\right.$$

$$\left.\left.\left.A^{in}_{\mu}(x) + \delta_M\,\bar{\psi}^{in}(x)\psi^{in}(x) + \bar{\eta}(x)\psi^{in}(x) + \eta(x)\bar{\psi}^{in}(x) + A^{\mu\,in}(x)\,B_{\mu}(x)\right]\right\}\right)|0\rangle$$

The L.S.Z. reduction formula and functional techniques can now be used to rewrite Γ^{μ} as

$$\Gamma^{\mu}(x^2, x\cdot p) = -\frac{1}{Z_2}\,\frac{M}{(2\pi)^3 E_p}\int d^4z\int d^4z'\,e^{ip(z-z')}\left[\bar{u}(p,\lambda)\left(i\overrightarrow{\partial\!\!\!/}_{z'} - m\right)\right]_{\sigma}$$

$$\left\{S_{F\sigma\alpha}\left(x,z';ig_0\,\delta/\delta B^{\mu}\right)\gamma^{\mu}_{\alpha\beta}\,S_{F\beta\rho}\left(z,0;ig_0\,\delta/\delta B^{\mu}\right) + \gamma^{\mu}_{\alpha\beta}\,S_{F\beta\alpha}\right.$$

$$\left.\left(x,0;ig_0\,\delta/\delta B^{\mu}\right)\left[S_{F\sigma\rho}\left(z',z;ig_0\,\delta/\delta B^{\mu}\right)\right]\right\}\left[\left(-i\overleftarrow{\partial\!\!\!/}_z - m\right)\right.$$

$$\left.u(p,\lambda)\right]_{\rho}\exp\left[-g_0\int_0^x d\omega_{\beta}\,\frac{\delta}{\delta B_{\beta}(\omega)}\right]\frac{Z(0,0,B)}{Z(0,0,0)}\Bigg|^{C.S.A.}_{B=0}$$

where $S_F\left(x,0;ig_0\,\delta/\delta B^{\mu}\right)$ is the Feynman propagator for a Fermion interacting with an external potential $ig_0\,\delta/\delta B^{\mu}$, i.e.,

$$+\,i\,S_F\left(x,0;i\,g_0\,\delta/\delta B^{\mu}\right) \equiv \langle 0_{out}|T\,\psi(0)\,\bar{\psi}(x)|0_{in}\rangle_{\delta/\delta B^{\mu}}$$

$$= \langle 0|T\left(\psi^{in}_{(0)}\,\bar{\psi}^{in}_{(x)}\exp\left\{i\int d^4z\,\bar{\psi}^{in}_{(z)}\gamma^{\mu}\,\psi^{in}_{(z)}\,i\,g_0\,\frac{\delta}{\delta B^{\mu}_{(z)}}\right\}\right)|0\rangle.$$

Finally the L.S.Z. reduction formula for a Fermion in an external potential $i\,g_0\,\delta/\delta B^{\mu}(x)$ can be used to rewrite Γ^{μ} as

$$\Gamma^{\mu}\left(x^2, x \cdot p\right) \equiv \Gamma^{\mu}_{a}\left(x^2, x \cdot p\right) + \Gamma^{\mu}_{b}\left(x^2, x \cdot p\right)$$

$$\Gamma^{\mu}_{a}\left(x^2, x \cdot p\right) = \frac{1}{Z_2} \bar{\psi}\left(x; p, \lambda; i g_0 \delta/\delta B^{\mu}\right) \gamma^{\mu} \psi\left(0; p, \lambda; i g_0 \delta/\delta B^{\mu}\right)$$

$$\exp\left\{-g_0 \int_0^x d\omega_\nu \, \delta/\delta B_\nu(\omega)\right\} \frac{Z(0, 0, B^{\mu})}{Z(0, 0, 0)} \Bigg|_{B^{\mu} = 0}^{C.S.A.} \tag{3.1}$$

$$\Gamma^{\mu}_{b}\left(x^2, x \cdot p\right) = \frac{1}{Z_2} \operatorname{tr}\left[\gamma^{\mu} S_F\left(x, 0; i g_0 \delta/\delta B_{\mu}\right)\right] \frac{\langle p, \lambda \text{ out} | p, \lambda \text{ in}\rangle_{\delta/\delta B}}{\langle 0 \text{ out} | 0 \text{ in}\rangle_{\delta/\delta B}}$$

$$\exp\left\{-g_0 \int_0^x d\omega_\nu \, \delta/\delta B_\nu(\omega)\right\} \frac{Z(0, 0, B^{\mu})}{Z(0, 0, 0)} \Bigg|_{B^{\mu} = 0}^{C.S.A.}$$

where

$$\psi\left(0; p, \lambda; i g_0 \delta/\delta B^{\mu}\right) = \langle 0 \text{ out} | \psi(0) | p, \lambda \text{ in}\rangle_{\delta/\delta B} / \langle 0 \text{ out} | 0 \text{ in}\rangle_{\delta/\delta B}$$

and

$$\bar{\psi}\left(x; p, \lambda; i g_0 \delta/\delta B^{\mu}\right) \equiv \langle p, \lambda \text{ out} | \bar{\psi}(x) | 0 \text{ in}\rangle_{\delta/\delta B} / \langle 0 \text{ out} | 0 \text{ in}\rangle_{\delta/\delta B}$$

are the wave functions (for a Fermion interacting with an external potential $i g_0 \delta/\delta B_{\mu}(x)$) that represent a free Fermion of momentum p and spin λ at time $t \to -\infty$ and $t \to +\infty$, respectively. Also

$$\frac{\langle p, \lambda \text{ out} | p, \lambda \text{ in}\rangle_{\delta/\delta B}}{\langle 0 \text{ out} | 0 \text{ in}\rangle_{\delta/\delta B}}$$

is the forward one Fermion to one Fermion transition amplitude in the external potential $i g_0 \, \delta/\delta B^{\mu}(x)$.

Equation 3.1 is an exact formal expression for $\Gamma^{\mu}(x^2, x \cdot p)$. The contributions Γ^{μ}_{a} and Γ^{μ}_{b} to Γ^{μ} are separately gauge-invariant so it is meaningful to discuss these terms separately. The Feynman graphs that contribute to Γ^μ_a and Γ^μ_b in lowest order are shown in Fig. 1.

In order to apply the eikonal approximation to Γ^μ, the frame $p^+ \longrightarrow \infty$, $\vec{p}_\perp = 0$ ($p^2 = m^2$) is chosen and Γ^μ (x^2, $x \cdot p$) is considered in the region $x^+ = 0$ (so $x^2 \leq 0$). The eikonal approximation is now made by simply replacing $\bar{\psi}(x; p, \lambda; g_0 \delta/\delta B^\mu)$, $\psi(0; p, \lambda; g_0\delta/\delta B^\mu)$ and $\langle p, \lambda\, \text{out} | p, \lambda\, \text{in}\rangle_{\delta/\delta B} / \langle 0\, \text{out} | 0\, \text{in}\rangle_{\delta/\delta B}$ by their eikonal approximations.[6]

$$\bar{\psi}\left(x; p, \lambda; g_0 \delta/\delta B^\mu\right) \xrightarrow{\text{E.A.}} -i \sqrt{\frac{m}{E_p}}\, (2\pi)^{-3/2}\, \bar{u}(p,\lambda)\, e^{ip\cdot x} \exp\left\{(-1)\, g_0 \int_0^\infty d\tau\, \eta^\mu \frac{\delta}{\delta B^\mu\left(x+\eta_\xi \tau\right)}\right\} \tag{3.2a}$$

$$\psi\left(0; p, \lambda; g_0 \delta/\delta B^\mu\right) \xrightarrow{\text{E.A.}} -i \sqrt{\frac{m}{E_p}}\, (2\pi)^{-\frac{3}{2}} \exp\left\{(-1)\, g_0 \int_{-\infty}^0 d\sigma \eta_\xi^\mu \frac{\delta}{\delta B^\mu(\sigma\eta_\xi)}\right\} u(p,\lambda) \tag{3.2b}$$

$$\frac{\langle p', \lambda'\, \text{out}\; p, \lambda\, \text{in}\rangle_{\delta/\delta B}}{\langle 0\, \text{out}\; 0\, \text{in}\rangle_{\delta/\delta B}} \xrightarrow{\text{E.A.}} \delta_{\lambda\lambda'} \delta^3(\vec{p} - \vec{p}') - \frac{m}{\sqrt{E_p E_{p'}}} (2\pi)^{-3} \int d^4x\, e^{i(p'-p)\cdot x}$$

$$g_0\, \bar{u}(p',\lambda')\, \gamma^\mu\, u(p,\lambda)\, \delta/\delta B^\mu(x) \exp\left\{(-1)\, g_0 \int_{-\infty}^{\infty} d\tau\, \eta_\xi^\mu \frac{\delta}{\delta B^\mu(x+\tau\,\eta_\xi)}\right\} \tag{3.2c}$$

where $\eta_\xi = (0,0,1,1)$.

Now Γ^μ_a is more easily calculated than Γ^μ_b in the eikonal approximation. For $\Gamma^\mu_a(x^2, x\cdot p)\Big|_{\substack{x^+=0\\ p^+\to\infty\\ \vec{p}_\perp=0}} = \Gamma^\mu_a\left(-\vec{x}_\perp^2,\, x^- p^+\right)$ the eikonal approximation gives

$$\Gamma^\mu_a\left(-\vec{x}_\perp^2,\, x^- p^+\right) \xrightarrow{\text{E.A.}} -\frac{e^{ip\cdot x}}{Z_2 (2\pi)^3} \frac{p^\mu}{m} \frac{m}{E_p} \exp\left\{(-1)\, g_0 \int_0^\infty d\tau\, \eta_\xi^\mu \frac{\delta}{\delta B^\mu\left(x+\tau\,\eta_\xi\right)}\right\} \exp\left\{(-1)\, g_0 \right.$$

$$\left. \int_{-\infty}^0 d\sigma\, \eta_\xi^\mu \frac{\delta}{\delta B^\mu(\sigma\eta_\xi)}\right\} \exp\left\{-g_0 \int_0^x d\omega_\nu \frac{\delta}{\delta B_\nu(\omega)}\right\} \frac{Z(0,0,B^\mu)}{Z(0,0,0)}\Bigg|_{B^\mu=0} \tag{3.3}$$

Using the definition of Z,

$$Z(0,0,B^\mu) = \langle 0 \left| T\left(\exp\left\{ i \int d^4x \left[-g_0 \bar{\psi}^{in}(x)\gamma^\mu \psi^{in}(x) A^{in}_\mu (x) + \delta m \bar{\psi}^{in}(x)\psi^{in}(x) \right.\right.\right.\right.$$

$$\left.\left.\left.\left. + A^{in}_\mu(x) B^\mu(x) \right] \right\} \right) \right| 0 \rangle .$$

Rewriting $Z(0,0,B^\mu)$ in terms of one-particle connected amplitudes and assuming that under charge conjugation, C, A(x) transforms as $CA^\mu(x)C^{-1} = -A^\mu(x)$, we have

$$Z(0,0,B^\mu) = \exp\left\{ \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} g_0^{2n} \left[\prod_{i=1}^{2n} d^4x_i\, B^{\mu_i}(x_i) \right] \langle 0_{out} \left| T\left(A_{\mu_1}(x_1) \ldots \right.\right.\right.$$

$$\left.\left.\left. A_{\mu_{2n}}(x_{2n}) \right) \right| 0_{in} \rangle^{o.p.c.}_{\eta=\bar{\eta}=B^\mu=0} \right\}$$

Therefore, using this form for $Z(0,0,B^\mu)$ and evaluating the shift operators and then setting $B^\mu = 0$, Eq. (3.3) becomes

$$\Gamma^\mu_a\left(-\vec{x}_\perp^2, x^- p^+\right) \xrightarrow{E.A.} -\frac{e^{ip\cdot x}}{(2\pi)^3 Z_2} \frac{p^\mu}{m} \frac{m}{E_p} \exp\left\{ \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (g_0)^{2n} \prod_{i=1}^{2n} \left[\int d^4x_i \left(-\int_0^\infty d\tau_i \right.\right.\right.$$

$$\eta_\xi^{\mu_i} \delta^4(x_i - x - \eta_\xi \tau_i) - \int_{-\infty}^0 d\sigma_i \eta_\xi^{\mu_i} \delta^4\left(x_i - \sigma_i \eta_\xi\right) - \int_0^1 d\rho_i x_i^{\mu_i} \delta^4(x_i - \qquad (3.4)$$

$$\left.\left.\left. x\rho_i) \right) \right] \langle 0_{out} \left| T\left(A_{\mu_1}(x_1) \ldots A_{\mu_{2n}}(x_{2n}) \right) \right| 0_{in} \rangle^{o.p.c.} \right\}$$

In order to obtain a numerical result, the further simplification is made that gluon structure is neglected so

$$\langle 0_{out} \left| T\left(A_{\mu 1}(x_1) \ldots A_{\mu_{2n}}(x_{2n}) \right) \right| 0_{in} \rangle \longrightarrow \delta_{n1}\left(-i g_{\mu_1 \mu_2} \right) D_F\left(x_1 - x_2 \right)$$

where the representation

$$D_F(x) = \frac{-1}{16\pi^2} \int_0^\infty \frac{d\lambda}{\lambda^2} \exp\left\{-i\left[x^2 \frac{1}{4\lambda} + \mu^2\lambda - i\epsilon\right]\right\}$$

is used for the propagator and μ^2 is a gluon mass introduced to eliminate infrared divergences. The contribution to Γ_a^μ obtained by neglecting gluon structure is denoted by $\tilde{\Gamma}_a^\mu$.

The remaining integrations can be done[7] to obtain the eikonal approximation for the leading behavior[8] of $\tilde{\Gamma}_a^\mu$ as $\vec{x}_\perp^2 \to 0$.

$$\tilde{\Gamma}_a^\mu\left(-\vec{x}_\perp^2, x^-p^+\right) \xrightarrow[\vec{x}_\perp^2 \to 0]{\text{E.A.}} \frac{1}{Z_2} \frac{p^\mu}{E_p} \frac{e^{ip\cdot x}}{(2\pi)^3} \exp\left\{-g_0^2/2\pi^2 \int_0^1 \frac{d\alpha}{\alpha}\left[e^{-ix\cdot p\alpha} - 1\right] \ln\left(|\vec{x}_\perp|\mu\alpha\right)\right\}$$

or for the renormalized vertex function

$$\tilde{\Gamma}_{Ra}^\mu\left(-\vec{x}_\perp^2, x^-p^+\right) \xrightarrow[\vec{x}_\perp^2 \to 0]{\text{E.A.}} \frac{p^\mu}{E_p} \frac{e^{ip\cdot x}}{(2\pi)^3} \exp\left\{-g^2/2\pi^2 \int_0^1 \frac{d\alpha}{\alpha}\left[e^{-i\alpha x\cdot p} - 1\right] \ln\left(|\vec{x}_\perp|\alpha\mu\right)\right\}. \tag{3.5}$$

Now the contribution to $D(x^2, x\cdot p)$ from $\tilde{\Gamma}_{Ra}^{\mu\,\text{Eikonal}}$ is given by Eq. (2.3),

$$(2\pi)^3 \frac{E_p}{M} D(x^2, x\cdot p) \xrightarrow[x^2 \to 0]{} 2\,\text{Im}\; e^{ip\cdot x} \exp\left\{-g^2/2\pi^2 \int_0^1 \frac{d\alpha}{\alpha}\left(e^{-ix\cdot p\alpha} - 1\right) \ln\left((-x^2)^{\frac{1}{2}}\mu\alpha\right)\right\}$$

Using the relation (2.6b) between $D(x^2, x\cdot p)$ and $W_2(q^2, q\cdot p)$, the contribution to νW_2 from $\tilde{\Gamma}_{aR}^\mu$ can be calculated

$$\nu W_2(q^2,\omega) \xrightarrow[\substack{-q^2 \to \infty \\ \omega\ \text{fixed}}]{} -\frac{\pi}{2} q^2 \int d^4x\, e^{iq\cdot x} \frac{\left(\frac{-i}{4\pi^2}\right)}{x^2 - i\epsilon x^-} \;\text{Im}\left\{e^{ix\cdot p} \exp\left[-g^2/2\pi^2 \int_0^1 \frac{d\alpha}{\alpha}\left(e^{-ix\cdot p\alpha} - 1\right) \ln\left((-x^2)^{\frac{1}{2}}\mu\alpha\right)\right]\right\}$$

Within the approximation of keeping only leading order terms in each order of perturbation theory, the Fourier transform can be reduced to

$$\nu W_2(q^2, \omega) \xrightarrow[\substack{-q^2 \to \infty \\ \omega \text{ fixed}}]{} \frac{\omega}{\pi} \theta(q \cdot p) \int_{-\infty}^{\infty} d(x \cdot p) \left[e^{i(x \cdot p)(1-\omega)} - e^{-i(x \cdot p)(1+\omega)} \right]$$

$$\exp\left\{ g^2/4\pi^2 \int_0^1 \frac{d\xi}{\xi} \left[e^{-i\xi x \cdot p} - 1 \right] \ell n\left(\frac{2q \cdot p}{\mu^2} \right) \right\} \tag{3.6}$$

The remaining integration does not give any elementary function. However, as $1 - \omega \longrightarrow 0_+$, the leading behavior in $1 - \omega$ may be calculated. The result then becomes

$$\nu W_2(q^2, \omega) \xrightarrow[-q^2 \to \infty]{(1-\omega) \to 0_+} g^2/4\pi^2 \frac{1}{1-\omega} \ell n(-q^2/\mu^2) \exp\left\{ \frac{g^2}{4\pi^2} \ell n(-q^2/\mu^2) \ell n(1-\omega) \right\} \tag{3.7}$$

The structure of νW_2 has been investigated in perturbation theory by making use of the optical theorem to relate $W^{\mu\nu}$ to the imaginary part of the forward spin-averaged Compton amplitude for virtual photon-Fermion scattering. P. M. Fishbane and J. D. Sullivan[3] have calculated, by this method, the graphs corresponding to the contribution from Γ^{μ}_{aR} due to the graphs with no gluon "structure" (i.e., $\tilde{\Gamma}^{\mu}_{aR}$). Their results are exactly the same as Eq. (3.7) near $\omega = 1$.

4. Discussion of the Assumptions and Results

The eikonal approximation for Γ^{μ}_{a} was made by eikonally approximating the wave functions $\bar{\psi}$ and ψ in a frame-dependent way, since the eikonal approximation keeps only the leading terms in p^+ in the frame $p^+ \longrightarrow \infty$, $\vec{p}_{\perp} = 0$, $(p^2 = m^2)$. Since Γ^{μ}_{a} depends only on x^2 and $x \cdot p$, dropping terms non-leading in p^+ may not be justified as p^+ comes into Γ^{μ} only through $x^- p^+$. For $x \cdot p$ large, however, this approximation is meaningful. Since the region $x \cdot p$ large can be seen by Eq. (3.6) to be related to the leading behavior in $1-\omega$ of νW_2 as $1-\omega$ approaches zero, the results of the eikonal approximation should agree with explicit calculations of νW_2 in the region $-q^2 \to \infty, 1-\omega \to 0_+$. This is seen to be the case in Section 3.

If all the graphs of νW_2 are considered, three types may be distinguished: graphs in which the two currents act on different Fermion lines; graphs in which the two currents both act on the initial Fermion line (this is, of course, also the final Fermion line); and graphs in which the two currents act on the same line of Fermion propagators but this line does not connect to the initial or final Fermion line. Graphs of νW_2 in which the currents $J^\mu(x)$ and $J^\nu(0)$ act on different Fermion lines, as illustrated in Fig. 2, have no corresponding contributions in $\Gamma^\mu(x^2, x \cdot p)$. When graphs of this type contribute to the leading behavior of νW_2 in the Bjorken limit, the relation 2.6 between the leading behavior of νW_2 and the behavior of Γ^μ near $x^2 = 0$ will fail in perturbation theory since Γ^μ has no contributions corresponding to this type of graph in νW_2. Graphs of νW_2 in which the two currents act on the line of propagators connecting the initial and final Fermion correspond to the graphs of Γ^μ contained in Γ^μ_a. The relation is illustrated in Fig. 3. It is found in perturbation theory[1,3] that of these graphs, the leading behavior comes from the graphs in which the gluons have no structure. These are just the graphs calculated in $\tilde{\Gamma}^\mu_a$.

Finally there are the graphs of νW_2 in which the two currents act on the same line of Fermion propagators, but this line does not connect to the initial or final Fermion. These "pair production" graphs correspond to the graphs of Γ^μ contained in Γ^μ_b. The relation is illustrated in Fig. 4. The explicit calculations of V. N. Gribov and L. N. Lipatov[1] for νW_2 show that these pair production graphs do contribute to the leading behavior of νW_2 in the Bjorken limit, so Γ^μ_b must also be calculated. A simple method to apply the eikonal approximation to calculate the leading behavior of Γ^{μ}_{b} (as $x^2 \to 0$) has not yet been found.

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References and Footnotes

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5. R. Jackiw and R. E. Waltz, Phys. Rev. D6, 702 (1972).
6. For the calculations of the wave functions in an external potential, see E. Eichten, Phys. Rev. D4, 1225 (1971).
7. A limit on the divergent σ (or τ) integration is imposed by the eikonal approximation and the fact that $x^- p^+$ is held fixed at a finite value in this calculation. A comparison of the eikonal result in order g_0^2 with the exact expressions for these graphs checks the validity of this limit. The limit is $|\sigma| < 2p+\lambda$ (or $|\tau| < 2p+\lambda$).
8. By leading behavior of $\tilde{\Gamma}^\mu_a$ is meant that the leading term of the exponent is kept (as $x^2 \to 0$). This is equivalent to keeping only the leading term in x^2 as $x^2 \to 0$ in each order of the perturbative expansion of $\tilde{\Gamma}^\mu_a$.

Figure Captions

Fig. 1. Figure 1a represents the graphs of Γ_a^μ in order g_0^2. Figure 1b represents the graphs of Γ_b^μ in order g_0^4.

Fig. 2. Examples of unitarity graphs contributing to $W^{\mu\nu}$ in a Fermion-neutral vector gluon model where the two currents, J^μ and J^ν, act on different Fermion lines.

Fig. 3. The correspondence between graphs of Γ_a^α and graphs of $W^{\mu\nu}$. Figure 3a represents two graphs contributing to Γ_a^α and Fig. 3b represents the corresponding unitarity graphs of $W^{\mu\nu}$.

Fig. 4. The correspondence between graphs of Γ_b^α and graphs of $W^{\mu\nu}$. Figure 4a represents a graph contributing to Γ_b^α and Fig. 4b represents the corresponding unitarity graphs of $W^{\mu\nu}$.

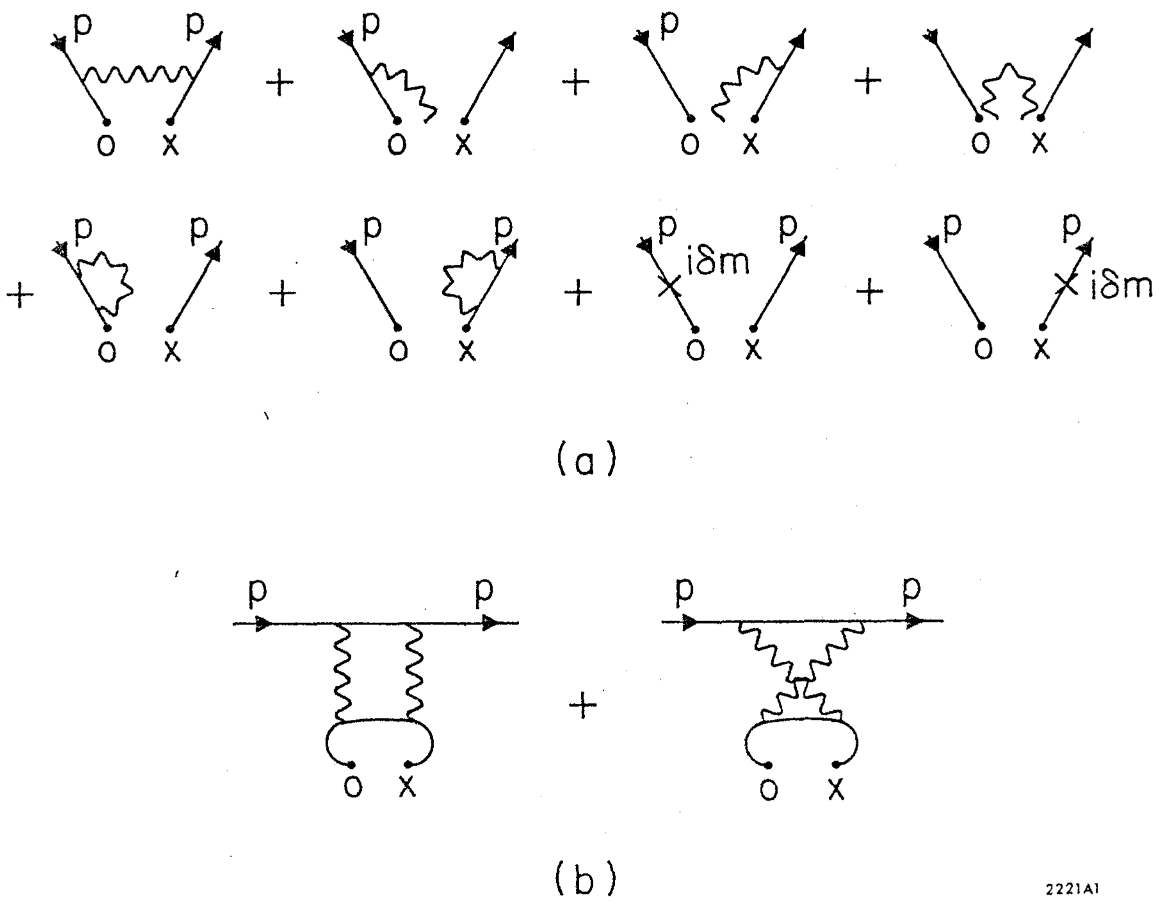


Fig. 1

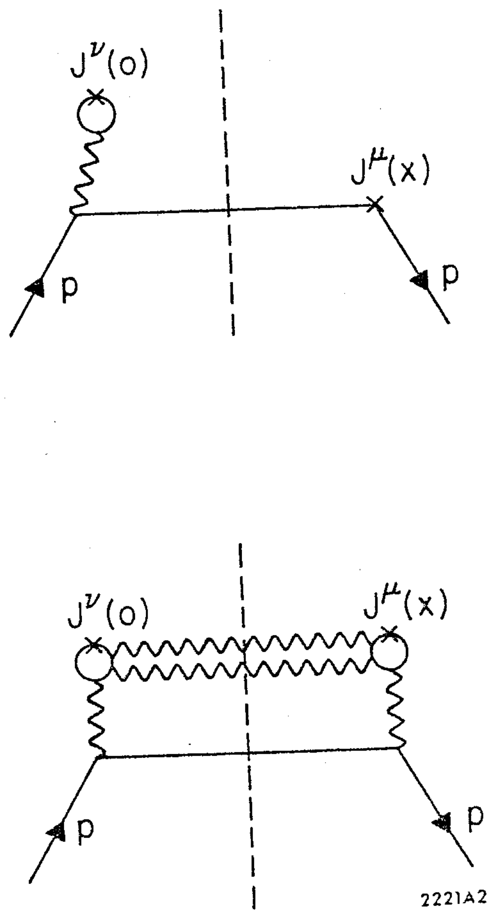
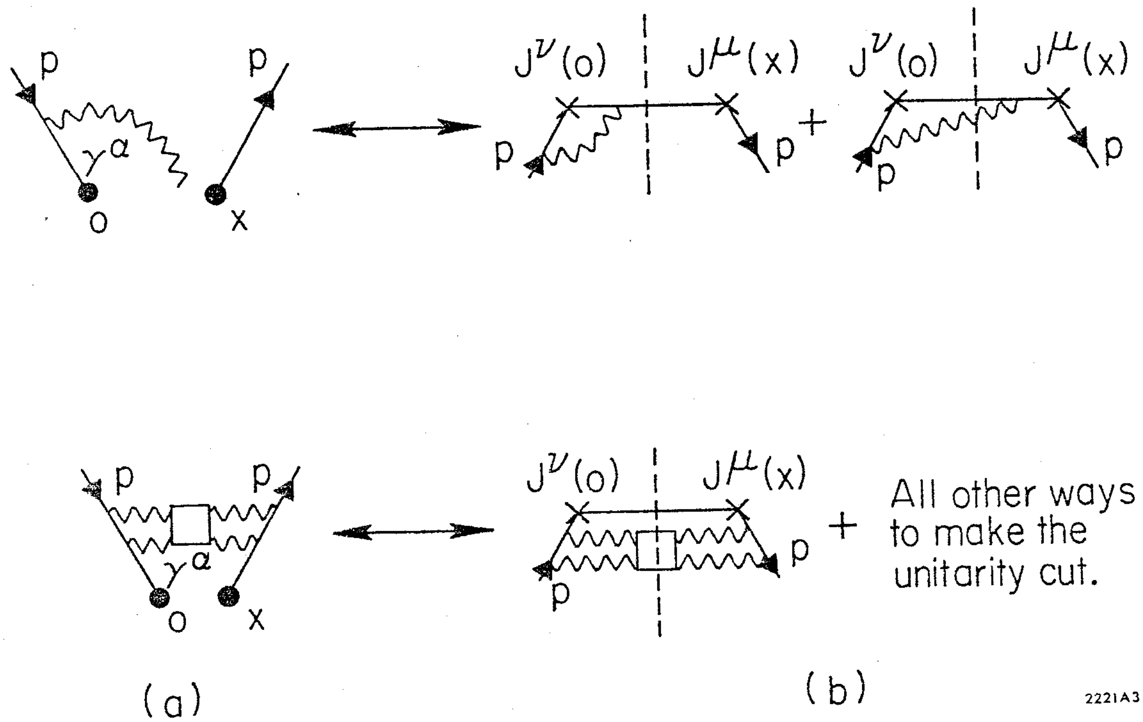


Fig. 2



p
p
γ^α
o
x
$J^\nu(o)$
$J^\mu(x)$
p
p
+
$J^\nu(o)$
$J^\mu(x)$
p
p
p
p
γ^α
o
x
$J^\nu(o)$
$J^\mu(x)$
p
p
+
All other ways
to make the
unitarity cut.
(a)
(b)
2221A3

Fig. 3

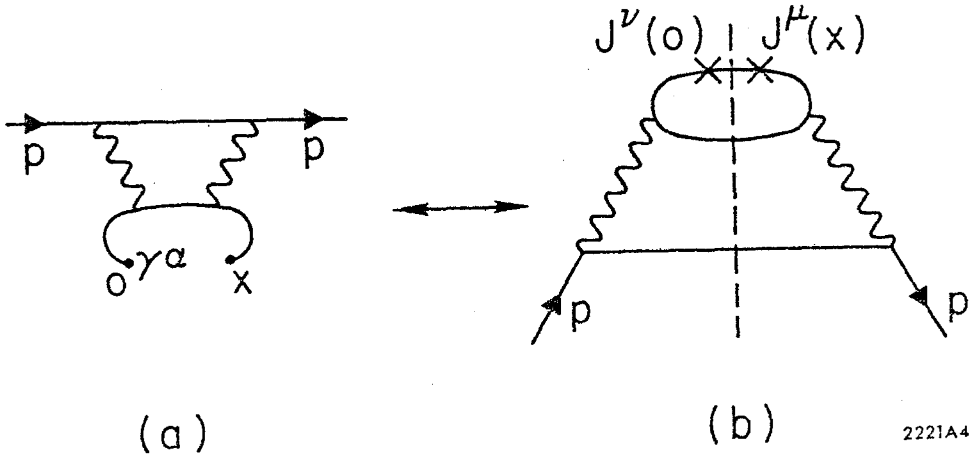


Fig. 4