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LIGHT CONE AND SHORT DISTANCE SINGULARITIES*

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I. Introduction

The subject of singularity structure of operator products at almost light-like distances has received much attention in the last several years. It is a generalization of earlier studies of short distance structure of operator products. There has been much activity recently in the latter subject too. The idea of "scale invariance" at short and almost light-like distances, or generalizations of this idea, are central to these approaches.

Short distance expansions of operator products were introduced by Wilson[1] as an abstraction of his studies in model field theories. These have the form

$$A(x)\,B(y) \underset{(x_\mu \to y_\mu)}{\approx} \sum_{[\alpha]} C^{[\alpha]}(x-y)\; F^{[\alpha]}(y) \tag{1}$$

Where A, B and $F^{[\alpha]}$ are local operators and $C^{[\alpha]}(x-y)$ singular c-number functions. The index $[\alpha]$ characterizes Lorentz as well as internal quantum numbers. To any degree of accuracy in (x-y) it is assumed that only a finite number of terms appear in the expansion Eq. (1). This expansion is a generalization and an explicitly covariant form of the Bjorken-Johnson-Low expansion.[2] Its applications are in studies of high momentum limits. The degree of singularity of the functions $C^{[\alpha]}$ is given by the "asymptotic dimensions" of the operators involved, namely the dynamical dimension governing short distance behavior.[1] Wilson argues, that such dimensionality will in general be different from the canonical value as appearing in a formal Lagrangian consideration, unless there are special reasons against that. Thus local current algebra yields dimension 3 for the $SU(3) \otimes SU(3)$ currents. The energy-momentum tensor has dimension 4.

The light cone expansion is an expansion of products of operators when their space-time distance approaches light-like separations. It was suggested as a generalization of the short distance expansion, to study high virtual mass limits in deep inelastic lepton hadron scattering. It has the form[3,4,5]

$$A(x)\,B(y) \underset{(x-y)^2 \to 0}{\approx} \sum_{[\alpha]} C^{[\alpha]}(x-y)\; F^{[\alpha]}(x,y) \tag{2}$$

where $F^{[\alpha]}(x,y)$ are <u>bilocal operators</u>, depending on the two points x and y and regular at $(x-y)^2=0$. In fact, it turns out that they are analytic in (y-x), as follows from the spectral conditions in deep inelastic scattering. The expansions of the form Eq. (2) and the existence of bilocal operators were <u>postulated</u> to hold in nature, namely for the fully interacting theories.[3,4,5] It was an abstraction from Wick's expansion for free fields and from the existence of such a light cone expansion in the Thirring model.[6] In the latter case the light cone singularities are not canonical, but rather depend on the coupling constant. Matrix elements of the bilocal operators are directly measurable in deep inelastic scattering experiments, which exhibit simple scaling phenomena and therefore imply the appearance of canonical light cone singularities.[7,8,9]

Expanding the bilocal operators in a Taylor series,

$$F^{[\alpha]}(x,y) = \sum_{n=0}^{\infty} (x-y)^{\alpha_1} \ldots (x-y)^{\alpha_n}\, F^{[\alpha]}_{\alpha_1 \ldots \alpha_n}(y) \tag{3a}$$

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$$F^{[\alpha]}_{\alpha_1 \ldots \alpha_n}(y) = \frac{1}{n!}\left[\frac{\partial}{\partial x^{\alpha_1}} \cdots \frac{\partial}{\partial x^{\alpha_n}} F^{[\alpha]}(x,y)\right]_{x=y} \tag{3b}$$

We get, for each light cone singularity, an infinite number of local terms in Wilson's expansion Eq. (1).[10] Inversely, if we have a Wilson expansion with an infinite number of terms of local operators with increasing spin and the same singularity function $C^{[\alpha]}$, we may sum them up to one bilocal operator and obtain generalized scaling phenomena.[5,7]

Wilson's expansions with sums like in (3a) have been demonstrated to hold in renormalizable quantum field theories[11] to any order in the coupling constant. To any finite order, scaling is violated by logarithmic terms. Summation of infinite sets of diagrams considered so far show no possibility of obtaining bilocal operators and scaling.[12] Instead, even when one considers sets in which only power singularities appear (where one considers neither self-energy nor vertex corrections), the singularities near the light cone depend both on the spin n of the local operators and the coupling constant.[12] So far there is no nontrivial model of quantum field theory in which canonical light cone singularities are exhibited.[13]

We should emphasize that the study of light cone singularity structure emerged from the scaling observed at SLAC,[8] a phenomena remarkably predicted by Bjorken.[14] An earlier approach that emerged from scaling is the parton model,[15] followed by cutoff field theory calculations.[16] Later, "soft field theory"[16a] calculations were developed, and duality ideas were also incorporated.[17]

A very important step in the development of the light cone approach is the quark algebra structure suggested by Fritzsch and Gell-Mann,[18] which is to assume free quark field algebra for the SU(3) × SU(3) structure on the light cone. This made clear the connection with the parton approach in deep inelastic scattering, and shed light on which results of the parton model are of a general nature and which dependent on specific assumptions peculiar to that model.

This paper is organized as follows. In Section II we discuss deep inelastic electron-nucleon scattering. In II.A we review the light cone dominance analysis, and in II.B discuss the Regge behavior in the deep inelastic limit and the relation with equal-time commutator sum rules. In Section III light cone expansions of operator products are considered. We review the general structure in III.A. In III.B we discuss the Thirring model, where anomalous dimensions appear. It also exhibits the phenomena of "softening" for composite operators, namely that their dimensionality may be less than the sum of the dimensionalities of the constituent fields, and it may also be canonical (as is the case for the currents, but not the scalar and pseudoscalar densities). We then review the deep inelastic scattering (III.C) and the Cornwall-Norton sum rules (III.D), the latter in relation with results from summations in field theory. In Section III.E we discuss the subject of fixed poles and the polynomial residue in the mass variables of the "photons". In Section IV we discuss the quark algebra on the light cone as suggested by Fritzsch and Gell-Mann. It is the light cone singularities that are exhibited in IV.A as for currents constructed out of free fields. The resulting bilocal operators have matrix elements which include all the complexity of strong interactions, and have no resemblance to a scale invariant theory (mass parameters and Regge trajectories play there an important role). In IV.B we review the results for deep inelastic scattering and discuss the asymptotic sum rules derived. In IV.C we review other tests of the light cone algebra as applied to nonforward matrix elements and also tests for the algebra of the bilocals. In IV.D we show what extra nonleading terms are needed to ensure current conservation. These extra terms are interaction dependent, in contrast to the leading singularity, the structure of which is interaction independent (it is model dependent in the sense of the kind of constituents used). In Section V we discuss total e^+e^- annihilation into hadrons and $\pi^0 \to 2\gamma$. The relation between the two following from consistency considerations of operator product expansions and quark schemes are reviewed. In Section VI we discuss single particle inclusive annihilation, namely e^+e^- annihilation with the detection of the momentum of a given hadron in the final state. The scaling properties are reviewed. Special attention is given to the question of asymptotic multiplicity. It is shown that canonical light cone singularities, scaling and logarithmic multiplicities are consistent, as follows from the singularity structure of products of two electromagnetic currents and two hadronic sources at short distances[91] (for the difference between the space-time points of the electromagnetic currents). In Section VII we mention other problems and approaches. In particular, one photon amplitude processes (like form factors, exclusive electroproduction and $pp \to \mu^+\mu^- X$), summations of perturbation graphs, conformal symmetry approach (very interesting approximate bootstrap schemes were recently studied in that limit[112]), null plane quantization and sum rules, and finite QED.

Finally, we should emphasize that the most important issues ahead are:

(1) Checking scaling and relations among structure functions for higher virtual mass and energy carried by the currents. Also checking of the various sum rules.

(2) More studies, experimentally and theoretically, of details of final states: distributions, charge ratios, multiplicities, etc. These in both the scattering and annihilation regions.

II. Deep Inelastic Electron-Nucleon Scattering

A. Light Cone Dominance

Consider deep inelastic electron nucleon scattering. For an unpolarized target and in the one photon exchange approximation, the differential cross section is given by

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2 \cos^2 \frac{\theta}{2}}{4E^2 M \sin^4 \frac{\theta}{2}} \left[W_2(q^2\nu) + 2tg^2 \frac{\theta}{2} W_1(q^2\nu)\right] \tag{4}$$

where E and E' are initial and final electron energies and $\Omega(\theta)$ the scattering angle in the laboratory frame, p and M the four-momentum and mass of the target, q the virtual photon four momentum, and $M\nu = q\cdot p$. W_1 and W_2 are given by

$$W_{\mu\nu}(q,p) = \frac{1}{2}\sum_s \int d^4x\, e^{iqx} <ps|\left[J_\mu(x),\ J_\nu(0)\right]|ps> =$$

$$= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1(q^2,\nu) + \frac{1}{M^2}\left(p_\mu - \frac{p\cdot q}{q^2} q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2} q_\nu\right) W_2(q^2,\nu) \tag{5}$$

where j_μ is the electromagnetic current. Bjorken's scaling is[14]

$$\begin{aligned} W_1(q^2,\nu) &\xrightarrow[B]{} F_1(\omega) \\ \nu W_2(q^2,\nu) &\xrightarrow[B]{} F_2(\omega) \end{aligned} \tag{6}$$

where B is the limit of $q^2 \to -\infty$ (space like) and $\nu \to \infty$, with $\omega = 2M\nu/(-q^2)$ fixed. In this limit most contributions come from the singularities near the light cone of the current commutator in Eq. (5).[19, 19a] Light-cone analysis of $W_{\mu\nu}(q,p)$ then proceeds through the introduction of the causal functions $V_L(x^2, p\cdot x)$ and $V_2(x^2, p\cdot x)$, defined as

$$<p|\left[J_\mu(x),\ J_\nu(0)\right]|p> = \left(g_{\mu\nu}\Box - \partial_\mu\partial_\nu\right) V_L(x^2, p\cdot x)$$

$$+ \left[p_\mu p_\nu \Box - \left(p_\mu\partial_\nu + p_\nu\partial_\mu\right)(p\cdot\partial) + g_{\mu\nu}(p\cdot\partial)^2\right] V_2(x^2, p\cdot x) \tag{7}$$

Bjorken scaling is obtained by[7]

$$\begin{aligned} V_L(x^2, p\cdot x) &= -(2\pi i)\,\epsilon(x_0)\,\delta(x^2)\, f_L(p\cdot x)\,. \\ V_2(x^2, p\cdot x) &= (2\pi i)\,\epsilon(x_0)\,\theta(x^2)\, f_2(p\cdot x) \end{aligned} \tag{8}$$

as leading light-cone singularities. For a vanishing longitudinal cross section,[8] (as may be indicated by experiment) we get that $f_L = 0$, and thus the leading singularity for V_L is also a $\theta(x^2)$. In general, for f_L non-vanishing, we obtain

$$\begin{aligned} F_1(\omega) &= (2\pi)^3\left[g_2'\left(\frac{1}{\omega}\right) - \frac{1}{\omega} g_L\left(\frac{1}{\omega}\right)\right] \\ F_2(\omega) &= (2\pi)^3\left(\frac{2M}{\omega}\right) g_2'\left(\frac{1}{\omega}\right) \end{aligned} \tag{9}$$

where

$$\begin{aligned} f_L(p\cdot x) &= \int_{-1}^{1} d\lambda\, g_L(\lambda)\, e^{i\lambda p\cdot x} \\ f_2(p\cdot x) &= \int_{-1}^{1} d\lambda\, g_2(\lambda)\, e^{i\lambda p\cdot x} \end{aligned} \tag{10}$$

The λ integration is limited to $|\lambda| \leq 1$ by the spectral conditions, which also implies that f_1 and f_2 are analytic in $(p\cdot x)$. In these considerations we assume that there are no strongly varying parts to the commutator inside the light cone, which may contribute in the scaling limit. This is certainly a reasonable physical assumption. Moreover, Jaffe[20] calculated the contribution of a $\delta(x^2-a^2)$ singularity, and found that it has, relative to a $\delta(x^2)$ contribution, an extra factor of $\nu^{-3/4}$ times an oscillatory factor $\left(1-\frac{1}{\omega}\right)^{1/4} \exp\left[ia\sqrt{2M\nu}\left(1-\frac{1}{\omega}\right)\right]$. Here $g(1)=0$ and $g'(1)\neq 0$ was assumed. In another calculation an extra factor of $1/\nu$, with no extra oscillations, was found.[21] Here $g(1)\neq 0$ was assumed, and the method of evaluation was equivalent to averaging over oscillations. [One can understand the connection as follows. From Jaffe's calculation, Eq. (8) in Ref. 20, taking $g(1)\neq 0$, one gets a $\nu^{-1/4}$ suppression as compared with $\delta(x^2)$, times a factor $\left(1-\frac{1}{\omega}\right)^{-1/4} \exp\left[ia\sqrt{2M\nu}\left(1-\frac{1}{\omega}\right)\right]$ for $\omega\neq 1$ and $\nu\to\infty$. Now, since $\int_0^1 d\xi\, g(\xi)\,(1-\xi)^{-1/4}\exp\left[ia\sqrt{2M\nu\,(1-\xi)}\right] \xrightarrow[\nu\to\infty]{} \nu^{-3/4} g(1)$, the oscillatory factor is equivalent to an extra $\nu^{-3/4}$. The result can be obtained from Eq. (7) in Ref. 20 directly integrating as above. Thus even a $\delta(x^2-a^2)$ singularity, which is certainly too singular for any realistic situation, is less important than a $\delta(x^2)$ in the Bjorken limit.]

B. Regge Behaviour and Sum Rules

It had been suggested[22] that the scaling functions exhibit Regge behaviour for $\omega\to\infty$. This is certainly an extra assumption, since Regge behaviour, which is the limit $\nu\to\infty$ for fixed q^2, may be given by non-leading light cone singularities.[5,7] Adopting this unification of Regge and scaling limits, we get for the contribution of a Regge pole with intercept $\alpha(0)$,

$$\begin{aligned} F_1(\omega) &\xrightarrow[\omega\to\infty]{} \omega^{\alpha(0)} \\ F_2(\omega) &\xrightarrow[\omega\to\infty]{} \omega^{\alpha(0)-1} \end{aligned} \tag{11}$$

This implies

$$\begin{aligned} g_L(\lambda) &\underset{\lambda\to 0}{\sim} |\lambda|^{-[\alpha(0)+1]}\, C_L[\alpha(0)] \\ g_2(\lambda) &\underset{\lambda\to 0}{\sim} |\lambda|^{[1-\alpha(0)]}\, C_2[\alpha(0)] \end{aligned} \tag{12}$$

For any $\alpha(0)\geq 0$ Eq. (12) implies that the first Fourier transform does not exist as a usual integral. It has to be understood, of course, as a generalized function.[5,7,23] We replace, for $\lambda>0$ and $1>\alpha(0)>0$,

$$|\lambda|^{-[\alpha(0)+1]} \longrightarrow \frac{1}{2i\sin\pi\alpha(0)}\left\{(-\lambda+i\epsilon)^{-[\alpha(0)+1]} - (-\lambda-i\epsilon)^{-[\alpha(0)+1]}\right\}$$

in the integrals. Thus

$$\int_{-1}^{1} d\lambda\, g_L(\lambda)\, e^{i\lambda(p\cdot x)} = 2\int_0^\infty d\lambda\, g_L(\lambda)\cos\lambda(p\cdot x)$$

and the contribution of a given Regge pole with $1\geq\alpha>0$ may be evaluated as

$$\frac{1}{i\sin\pi\alpha}\int_0^\infty d\lambda\left[(-\lambda-i\epsilon)^{-(\alpha+1)} - (-\lambda-i\epsilon)^{-(\alpha+1)}\right]\cos\lambda(p\cdot x) =$$

$$= \frac{1}{i\sin\pi\alpha}\int_{-\infty}^{\infty} d\lambda\left[(-\lambda+i\epsilon)^{-(\alpha+1)} - (-\lambda-i\epsilon)^{-(\alpha+1)}\right]\cos\lambda(p\cdot x) =$$

$$= \frac{(p\cdot x)}{i\alpha\sin\pi\alpha}\int_{-\infty}^{\infty} d\lambda\left[(-\lambda+i\epsilon)^{-\alpha} - (-\lambda-i\epsilon)^{-\alpha}\right]\sin\lambda(p\cdot x)$$

$$= -\frac{2p\cdot x}{\alpha}\int_0^\infty d\lambda\,\lambda^{-\alpha}\sin\lambda(p\cdot x) = -\frac{2}{\alpha(0)}\,|(p\cdot x)|^{\alpha(0)}\int_0^\infty d\lambda\,\lambda^{-\alpha}\sin\lambda =$$

$$= -\frac{2}{\alpha(0)}\,|(p\cdot x)|^{\alpha(0)}\,\Gamma[1-\alpha(0)]\sin\tfrac{1}{2}\pi[1-\alpha(0)] = 2|(p\cdot x)|^{\alpha(0)}\,\Gamma[-\alpha(0)]\cos\tfrac{1}{2}\pi\alpha(0)$$

Thus,

$$f_L(p\cdot x) = \int_{-1}^{1} d\lambda\, g_L(\lambda)\, e^{i\lambda(p\cdot x)} = \int_{-\infty}^{\infty} d\lambda\, g_L(\lambda)\, e^{i\lambda(p\cdot x)} =$$

$$= \int_{-\infty}^{\infty} d\lambda \left\{ g_L(\lambda) - \sum_{\alpha>0} C_L(\alpha)\, |\lambda|^{-(\alpha+1)} \right\} e^{i\lambda\, p\cdot x} + \sum_{\alpha>0} D_L(\alpha)\, |(p\cdot x)|^{\alpha} \tag{13}$$

For α=1 we can take a limit $\alpha \to 1$, or take $\frac{1}{2}\left[\frac{1}{(\lambda+i\epsilon)^2} + \frac{1}{(\lambda-i\epsilon)^2}\right]$, from the start in $g_L(\lambda)$.[5] As for α=0, making $C_L(\alpha) \propto \alpha$ we see that in the limit $\alpha \to 0$ we get a constant contribution to $f_L(0)$ and a part $\propto \delta(\lambda)$ in $g_L(\lambda)$. Such a part does not contribute to $F_1(\omega)$, but contributes a subtraction term to T_1. Since $f_L(0) = S$ gives the matrix element of the operator Schwinger term, we have the sum rule[24, 25]

$$S = t_L + \int_{-\infty}^{\infty} d\lambda \left\{ g_L(\lambda) - \sum_{\alpha>0} C_L(\alpha)\, |\lambda|^{-(\alpha+1)} \right\} \tag{14}$$

where we exclude a J=0 singularity in g_L. t_L is a kronecker delta singularity at J=0 in the real part. If present, it will show up also in photoproduction processes (see discussion in III. E). Whenever spin 0 or field algebra spin-1 couplings are present, we have a longitudinal cross section.[26] In the first case we have a non-vanishing S, while in the latter S=0.[27]

Note that Regge contributions influence the high $(p\cdot x)$ behaviour of matrix elements of bilocal operators. As is clear from Eq. (13), the contribution of a Regge pole is a $|(p\cdot x)|^{\alpha}$ to $|(p\cdot x)| \to \infty$ (the integral in Eq. (13) gives a vanishing contribution in that limit). Similarly, $f_2(p\cdot x) \to |(p\cdot x)|^{\alpha(0)-2}$.

Another way to take into account the Regge singularities, including α=0 in g_L, is the following. Define

$$\tilde{f}_L = f_L - f_L^R\,, \qquad \tilde{g}_L = g_L - g_L^R$$

where g_L^R, f_L^R are the Regge contributions. Take

$$g_L^R = \sum_{\alpha>0} \left[C_L(\alpha)\, |\lambda|^{-(1+\alpha)} + C_L(0)\, \theta(1-|\lambda|)\, \frac{1}{|\lambda|} \right]$$

Then,

$$\tilde{f}_L(p\cdot x) = \int_{-\infty}^{\infty} d\lambda\, \tilde{g}_L(\lambda)\, e^{i\lambda(p\cdot x)} = \int_{-\infty}^{\infty} d\lambda\, \tilde{g}_L(\lambda) + \int_{-\infty}^{\infty} d\lambda\, \tilde{g}_L(\lambda)\, [\cos\lambda(p\cdot x) - 1]$$

Now we take,

$$f_L^R(p\cdot x) = \int_{-\infty}^{\infty} d\lambda\, g_L^R(\lambda)\, [\cos\lambda(p\cdot x) - 1]$$

It is easy to show that

$$f_L^R(p\cdot x) = \left\{ \sum_{\alpha>0} (-2)\frac{C_L(\alpha)}{\alpha}\, |(p\cdot x)|^{\alpha}\, \Gamma(1-\alpha)\, \sin\frac{1}{2}\pi(1-\alpha) - 2\, C_L(0) \int_0^{|p\cdot x|} \frac{d\xi}{\xi}\, (1-\cos\xi) \right\}.$$

Thus the J=0 singularity has a $\ln|(p\cdot x)|$ as $|(p\cdot x)| \to \infty$, while its contribution for $p\cdot x$=0 is zero as for the α>0 poles. We finally have

$$f_L(p\cdot x) = \int_{-\infty}^{\infty} \tilde{g}_L(\lambda) + \int_{-1}^{1} d\lambda\, g_L(\lambda) \left(e^{i\lambda\, p\cdot x} - 1\right) \tag{15}$$

This gives, for $p\cdot x$=0, the same result as (14) when we separate a $t_L\,\delta(\lambda)$ term from g_L. Comparing Eq. (15) with Eq. (10) we see that we have effectively set $\int_{-\infty}^{\infty} d\lambda\, g_L^R(\lambda) = 0$.[28]

III. Light Cone Expansions of Operator Products

A. General Structure and Examples

In the previous section we were concerned with the singularity structure for one matrix element. In order to get relations among various experiments we need an operator statement. This is provided, for deep inelastic processes, by the light cone expansion[4,5] as in Eq. (2),

$$A(x)\ B(y) = \sum_{[\alpha]} C^{[\alpha]}(x-y)\ F^{[\alpha]}(x,y) \tag{16}$$

To be specific about the Lorentz structure, we write

$$A(x)\,B(y) = \sum_{[\alpha]} S_+^{[\alpha]}(x-y) \sum_n (x-y)^{\alpha_1} \ldots (x-y)^{\alpha_n}\ F^{[\alpha]}_{\alpha_1 \ldots \alpha_n}(x,y) \tag{17}$$

where $S_f^{[\alpha]}(x-y)$ is a scalar singular function,

$$S_+^{[\alpha]}(x) = \left(-x^2 + i\epsilon\, x_0\right)^{d^{[\alpha]}} \Gamma\left(-d^{[\alpha]}\right) \tag{18}$$

The value of $d^{[\alpha]}$ is given by the dimensions of the operators as

$$-2d^{[\alpha]} = d(A) + d(B) - \left[d\left(F^{[\alpha]}_{\alpha_1 \ldots \alpha_n}\right) - n\right] \tag{19}$$

Thus the degree of singularity is determined by the difference

$$d_n^{[\alpha]} = d\left(F^{[\alpha]}_{\alpha_1 \ldots \alpha_n}\right) - n \tag{20}$$

between dimension and spin, of the operator.[29] The smaller $d_n^{[\alpha]}$ the more stronger the singularity near the light cone.

When the operator product $B(y)\ A(x)$ is considered, we have

$$B(y)\ A(x) = \sum_{[\alpha]} S_-^{[\alpha]}(x-y) \sum_n (x-y)^{\alpha_1} \ldots (x-y)^{\alpha_n}\ F^{[\alpha]}_{\alpha_1 \ldots \alpha_n}(x,y) \tag{21}$$

where

$$S_-^{[\alpha]}(x) = \left(-x^2 - i\epsilon\, x_0\right)^{d^{[\alpha]}} \Gamma\left(-d^{[\alpha]}\right) \tag{22}$$

Note that apart from the sign change of ϵ in Eq. (21) as compared with Eq. (17), everything is the same. This follows from locality and analyticity of the bilocal operators. Thus, in order for $[A(x),\ B(y)]$ to vanish at space-like separations, the bilocal operators in Eq. (17) and Eq. (21) have to be the same for (x-y) space-like, and by the assumed analyticity have to be the same operator everywhere.

For the commutator the singularity is

$$S^{[\alpha]}(x) = \Gamma\left(-d^{[\alpha]}\right)\left[\left(-x^2 + i\epsilon\, x_0\right)^{d^{[\alpha]}} - \left(-x^2 - i\epsilon\, x_0\right)^{d^{[\alpha]}}\right] \tag{23}$$

which for $d^{[\alpha]} \to n$, $n = 0, 1, 2, \ldots$ is

$$S^{[\alpha]}(x) \xrightarrow[d^{[\alpha]} \to n]{} \frac{2i\pi}{n!}\ \epsilon(x_0)\ \theta(x^2)\ (x^2)^n \tag{24}$$

For the time ordered product[7] the singularity is

$$S_F^{[\alpha]}(x) = (-x^2 + i\epsilon)^{d^{[\alpha]}}\, \Gamma\left(-d^{[\alpha]}\right) \tag{25}$$

The simplest example of light cone expansions is provided within the framework of free field theory, and so far the only example with canonical singularities. Thus taking $J_\mu(x) = i:\phi^+ \overleftrightarrow{\partial}_\mu \phi:$, where ϕ is a free scalar field, we have

$$i\left[J_\mu(x), J_\nu(y)\right] = (-)\,[\Delta(x-y)]\,\overleftrightarrow{\partial}_\mu^{(x)}\overleftrightarrow{\partial}_\nu^{(y)}\left[\Delta_1(x-y)\right] - [\Delta(x-y)]\,\overleftrightarrow{\partial}_\mu^{(x)}\overleftrightarrow{\partial}_\nu^{(y)}\left[:\phi^+(x)\phi(y):+:\phi^+(y)\phi(x):\right] \qquad (26)$$

where

$$\Delta(x) = -\frac{1}{(2\pi)^3}\int d^4p\, e^{-ipx}\,\epsilon(p_0)\,\delta(p^2-M^2) \sim -\frac{1}{2\pi}\,\epsilon(x_0)\,\delta(x^2) \qquad \text{near } x^2=0$$

and

$$\Delta_1(x) = \frac{1}{(2\pi)^3}\int d^4p\, e^{-ipx}\,\delta(p^2-M^2) \sim \frac{i}{2\pi^2}\,P\frac{1}{x^2} \quad .$$

The leading c-number singularities in Eq. (26) are proportional to

$$2x_\mu x_\nu\ \delta'''(x^2) + 3\,g_{\mu\nu}\ \delta''(x^2) \qquad (27)$$

Note that the leading term, namely the first one, is not separately conserved. The sum of the leading δ''' and next to leading δ'' are conserved. As for the operator term, the leading singularity is a $x_\mu x_\nu\ \delta''(x^2)$ with the scalar bilocal $\left[:\phi^+(x)\,\phi(y):+:\phi^+(y)\,\phi(x):\right]$. The divergence for the leading singularity has no $\delta'''(x^2)$ term, and the $\delta''(x^2)$ term is cancelled by a corresponding term of a next to leading singularity. Terms of the order of $\delta'(x^2)$ in the divergence are cancelled only after use of the equations of motion for ϕ. Since we are going to generalize to the case of the full currents, where we have no equations of motion, current conservation will be ensured only after adding more terms in Eq. (26). An explicitly current conserving form is given in Eq. (36).

It is interesting to comment, that anomalous dimensions appear in the study of solutions of the Dirac equation in a Coulomb potential Ze^2/r. Thus the behaviour near the origin, $r\to 0$, of the wave function with a given angular momentum is

$$\psi_j \sim r^{-1+\left[\left(j+\frac{1}{2}\right)^2 - (Z\alpha)^2\right]^{1/2}} \quad .$$

which depends on the coupling constant. A similar situation appears in the Schrödinger equation with a $1/r^2$ potential. In both cases, anomalous dimensions appear when the potential has the same dimension as the kinetic energy term.

B. The Thirring Model

Investigations in the Thirring model showed that anomalous dimensions appear there.[1,6,30] Recently, an operator solution of the Thirring model was exhibited in terms of the full light cone expansion for products of Fermion fields.[6] Recall that the Thirring model has a massless spinor field in one space dimension interacting through $L_I = -g :j_\mu j^\mu:$. Define

$$u = t + x$$
$$v = t - x$$

Since both the axial current j^5_μ and the vector current j_μ are conserved, and since $j^5_\mu = \epsilon_{\mu\nu} j^\nu$, it turns out that $j_+ = j_0 + j_1$ depends on u only and $j_- = j_0 - j_1$ depends on v only. Since j_μ has no divergence and no curl, we may be tempted to write $j_\mu = \partial_\mu \phi$, where ϕ is a massless scalar field. However, in one space dimension a massless scalar field does not exist, since the Fourier transform of the propagator $1/p^2$ does not exist. One can introduce regularization procedures.[31] However, it is possible to avoid all these problems and also others related to singular products in equations of motion and in defining currents.[32] Our way of obtaining the solution is essentially algebraic, through commutation rules and consistency requirements.[6] The commutation rules are

$$\begin{aligned}\left[j_+(u),\ j_+(u')\right] &= 2i\,c\,\delta'(u-u')\\ \left[j_-(v),\ j_-(v')\right] &= 2i\,c\,\delta'(v-v')\end{aligned} \qquad (28)$$

c is a number, which serves to normalize the current. Also

$$\left[j_+(u),\ \psi(u'v')\right] = -\ (a+\bar{a}\gamma_5)\ \psi(u'v')\ \delta(u-u')$$
$$\left[j_-(v),\ \psi(u'v')\right] = -\ (a-\bar{a}\gamma_5)\ \psi(u'v')\ \delta(v-v') \tag{29}$$

Equations (28) and (29) result from equal-time commutators and conservation of j_μ. However, "a" and "$\bar{a}$" cannot be equal to their canonical value 1 unless $g=0$.[32] We take these commutation rules as a starting point. The resulting expansion is

$$\psi_1(uv)\ \psi_1^+(u'v') = f_0\left[i(u-u') + \epsilon\right]^{-\frac{(a+\bar{a})^2}{4\pi c}} \left[i\,(v-v') + \epsilon\right]^{-\frac{(a-\bar{a})^2}{4\pi c}} :\exp\left\{\frac{i}{2c}\ (a+\bar{a})\int_u^{u'} j_+(\xi)\ d\xi + (a-\bar{a})\int_v^{v'} j_-(\eta)\ d\eta\right\}: \tag{30}$$

For $\psi_2\ \psi_2^+$ replace $\bar{a} \to -\bar{a}$.

The normal ordering in Eq. (30) is with respect to the frequencies in the decomposition of the current. Comparing with the equations of motion we get

$$a - \bar{a} = gc \tag{31a}$$

while from locality

$$a\,\bar{a} = n\pi c \tag{31b}$$

where 1/2 n is the spin of the field, implying n=1 for spin 1/2. We thus obtain

$$d[\psi] = \frac{1}{2} + \frac{g^2 c}{4\pi} \tag{32}$$

Also, for the scalar and pseudoscalar densities, properly defined[6]

$$d(\bar{\psi}\psi) = d\ (\bar{\psi}\gamma_5\psi) = \left(\frac{\bar{a}}{a}\right) \tag{33}$$

The generator of scale transformations is

$$D = \frac{1}{4c}\left[\int u:j_+^2(u):\ du + \int v:j_-^2(v):\ dv\right] \tag{34}$$

Its commutation rules with ψ, as calculated by use of Eqs. (29), give us directly $d[\psi]$ through[33]

$$i\left[D, \psi(x)\right] = \left[x^\mu\ \partial_\mu + \left(\frac{1}{2}+\frac{g^2 c}{4\pi}\right)\right]\psi(x) \tag{35}$$

The main conclusions we can draw from this model are:

(a) Currents are more regular than the respective products of fields and obey simple commutation rules.

(b) Scalar and pseudoscalar densities, which have no algebraic reason to have canonical dimensions, indeed have anomalous dimensions. Their dimensions are canonical only for $g=0$.[34]

C. Products of Electromagnetic Currents

In the case of electromagnetic currents, we write the light cone expansion as[5]

$$\left[J_\mu(x),\ J_\nu(y)\right] = \left[\partial_\mu^{(y)}\partial_\nu^{(x)} - g_{\mu\nu}\,\partial_\alpha^{(x)}\partial^{\alpha(y)}\right]\left[C_L(x-y)\ F_L(x,y)\right] + \left[g_{\mu\alpha}\,\partial_\nu^{(x)}\partial_\beta^{(y)} + g_{\nu\beta}\,\partial_\alpha^{(x)}\partial_\mu^{(y)} - g_{\mu\alpha}g_{\nu\beta}\,\partial_\lambda^{(x)}\partial^{\lambda(y)} - g_{\mu\nu}\,\partial_\alpha^{(x)}\partial_\beta^{(y)}\right]\left[C_2(x-y)\ F_2^{\alpha\beta}(x,y)\right] + \dots \tag{36}$$

The other terms do not contribute to forward spin averaged matrix elements. Forward matrix elements are analyzed as in the previous section, with

$$\left[<p|F_2^{\alpha\beta}(x,0)|p>\right]_{x^2=0} = p^\alpha p^\beta f_2(p\cdot x) + \ldots \tag{37}$$

The other contributions are less leading in the scaling limit. Note that when the longitudinal cross section has no scaling contribution, namely C_L has no $\delta(x^2)$ and a leading $\theta(x^2)$ singularity only, then the extra terms in (37) of the form $p^\alpha x^\beta + p^\beta x^\alpha$ are as important in their contribution to V_L.[7, 35] A $x^\alpha x^\beta$ term in Eq. (37) has an extra $1/\nu$ suppression as compared with $(p^\alpha x^\beta + p^\beta x^\alpha)$ (we exclude $g^{\alpha\beta}$ terms, since those are identical in Eq. (36) to the V_L term).

D. Generalized Cornwall-Norton Sum Rules

The Cornwall-Norton sum rules[36] express integrals over moments of the scaling functions in terms of commutators of corresponding numbers of time derivatives of a space component of the current with a space component, at infinite momentum. They are,

$$\lim_{q^2\to-\infty} \frac{(-q^2)\, 2^{2n}}{M^2\pi}\int_1^\infty \frac{W_2(q^2\omega')d\omega'^2}{\omega'^{2n+2}} = \lim_{|\vec{p}|\to\infty}\int \frac{d^3\vec{x}}{|\vec{p}|^{2n+2}}\, e^{i\vec{q}\cdot\vec{x}}\Big\{<p|\left[\partial_0^{2n+1} J_z(\vec{x}),\, J_z(\vec{0})\right]|p> - <p|\left[\partial_0^{2n+1} J_x(\vec{x}),\, J_x(\vec{0})\right]|p>\Big\} \tag{38a}$$

$$\lim_{q^2\to-\infty} (-)\,\frac{2^{2n+2}}{\pi}\int_1^\infty \frac{W_1(q^2\omega')d\omega'^2}{\omega'^{2n+4}} = \lim_{|\vec{p}|\to\infty}\int \frac{d^3\vec{x}}{|\vec{p}|^{2n+2}}\, e^{i\vec{q}\cdot\vec{x}} <p|\left[\partial_0^{2n+1} J_x(\vec{x}), J_x(\vec{0})\right]|p> \tag{38b}$$

where we choose $\vec{p}$ in the z direction and $\vec{q}$ in the y direction. Thus the existence of the scaling limit implies the existence of the spin -(2n+2) operator coefficient of $\delta^{(3)}(\vec{x})$ in $<p|\left[\partial_0^{2n+1} J_i(\vec{x}),\, J_j(\vec{0})\right]|p>$. However, if we have a non-leading singularity which is not an integer power of (x^2), we may get infinite contributions from lower spin operators.[25] (This does not happen for half-odd integer powers. For details see Ref. 25.)

The general results for the moments of W_1 and W_2 for $q^2\to(-\infty)$ can be obtained from Wilson's short distance expansion. We do not assume the existence of a light cone expansion and bilocal operators, and therefore write, instead of $C_2(x-y)\, F_2^{\alpha\beta}(x,y)$ in Eq. (36), the form

$$V_2^{\alpha\beta}(x,0) \sim \sum_{n=0}^\infty C_n(x)\, x_{\alpha_1}\ldots x_{\alpha_{2n}}\, F_2^{\alpha\beta\alpha_1\ldots\alpha_{2n}}(0);\quad C_n(x) = \left[(-x^2+i\epsilon x_0)^{d_n} - (-x^2-i\epsilon x_0)^{d_n}\right]\Gamma(-d_n) \tag{39}$$

The terms we omitted have less singular C_n. Taking the singularity structure of the time ordered product, and going to the limit of $q_\mu\to\infty$ for all components, with $q^2\to-\infty$ (namely $q_\mu = \lambda\tilde{q}_\mu$ with $\lambda\to\infty$ and $\tilde{q}$ any fixed space-like vector), and then taking $p_0\to\infty$, we obtain

$$T_2 \to \sum_{n=0}^\infty A_n (p\cdot q)^{2n}\, (-q^2)^{-d_n-2n-1}$$

The first limit of $\lambda\to\infty$ implies that for each n it is the smallest d_n that is leading, and the second limit of $p_0\to\infty$ justifies in keeping only the spin (n+2) part for the $F^{\alpha\beta\,\alpha_1\ldots\alpha_{2n}}(0)$. We could have taken simultaneously $\lambda\to\infty$ and $p_0\to\infty$ such that $p_0/\lambda\to 0$, to obtain the same result.

Writing an unsubtracted dispersion relation for T_2 and taking the same limits we obtain,

$$T_2 \to \sum_{n=0}^\infty \frac{\omega^{2n}}{\pi}\int_1^\infty \frac{W_2(q^2\omega')d\omega'^2}{\omega'^{2n+2}}$$

Comparing the two expressions we get, for $q^2 \to -\infty$,

$$(-q^2)\int_1^\infty \frac{W_2(q^2\omega')\, d\omega'^2}{\omega'^{2n+2}} = B_n\,(-q^2)^{-d_n} \tag{40a}$$

Scaling means that $d_n = 0$ for all n. In that case the infinite sum of local operators in Eq. (39) defines a bilocal operator.

Studies of infinite sums of ladder graphs in perturbation theory show that in general the right hand side of Eq. (40) is not a power of q^2.[37] Thus the notion of dimensionality of operators in Eq. (39) is non-existant. However, for simple ladder sums the right hand side is a power, but then d_n does depend on n. For example, for the infinite sum of simple ladders in a theory of a charged spinor interacting with a pseudoscalar through a γ_5 coupling, we have[12]

$$d_n = \frac{g^2}{16\pi^2}\,\frac{1}{(2n+2)(2n+3)} \tag{40b}$$

This also serves to show, that Bjorken scaling cannot be derived from Wilson's expansion and the assumption of unsubtracted dispersion relations for T_2.[38]

One can argue that $d_0 = 0$ in Eq. (40a), as observed at SLAC. From the theoretical point of view, since $d\left[F_2^{\alpha\beta\,\alpha_1\ldots\alpha_{2n}}\right] = 4 + 2n + 2d_n$, and since $F_2^{\alpha\beta}$ presumably has a part which is the energy-momentum tensor[39] and therefore dimension 4, we expect $d_0 = 0$. If d_1 is anomalous, $d_1 > 0$, we have a situation in which νW_2 goes to zero for $q^2 \to -\infty$ at each fixed ω. We thus see that for n=0 there is non-uniformity of the left hand side, in the sense that the limit $(-q^2) \to \infty$ cannot be put inside the integral. In fact, this non-uniformity is there for any n. The reason is the following. We must have an infinite series of increasing d_n in Eq. (40a), since if there is only a finite number of different d_n there would not be an unsubtracted dispersion relation for T_2 unless all d_n are equal (since any single term in the expansion Eq. (39) contributes to T_2 but not W_2 for $q^2 < 0$ [5]). Thus, for any given d_n there is a $d_{n'}$ with $n' > n$ and $d_{n'} > d_n$. If we can interchange the limits $q^2 \to \infty$ with the integration in Eq. (40a) for a certain n, this would imply that $(-q^2)^{d_n+1}\, W_2(q^2, \omega')$ scales, which would then violate the sum rule Eq. (40a) for n'. The conclusion is that either all d_n are equal, in which case we get scaling, or that there is an infinite number of different d_n. This also shows that one cannot prove Bjorken scaling from Wilson's expansion and unsubtracted dispersion relation for T_2.[38] All one can show is that it is impossible to have all but a certain finite number of d_n equal.

E. Fixed Poles

It was demonstrated in various works that a J=0 fixed pole exists in the amplitude T_2, either using a light cone approach[40] or a parton type "soft" field theory.[41,42] It has been argued that in $1/q^2\ T_2$, the residue of the fixed pole is independent of q^2.[43]

Let us return to V_2 of Eq. (7). For the leading light cone contribution we have

$$T_2 \sim 16\pi^2 M^2(-q^2)\int_{-1}^{1} \frac{d\lambda\, g_2(\lambda)}{(-q^2 - 2\lambda M\nu + i\epsilon)^2} \tag{41}$$

Subtracting all Regge contributions with $\alpha > 0$ (assume no contributions to g_2 at $\alpha = 0$), then

$$\tilde{T}_2(q^2\nu) \approx 16\pi^2 M^2(-q^2)\int_{-1}^{1} \frac{d\lambda\, \tilde{g}_2(\lambda)}{(-q^2 - 2\lambda M\nu + i\epsilon)^2} \tag{42}$$

Thus, for $\nu \to \infty$,

$$\tilde{T}_2(q^2\nu) \underset{\nu\to\infty}{\longrightarrow} 4\pi^2\,\frac{(-q^2)}{\nu^2}\int_{-1}^{1} \frac{d\lambda}{\lambda^2}\,\tilde{g}_2(\lambda) \tag{43}$$

and the integral is convergent since $g_2(\lambda)$ vanishes faster than $|\lambda|$ at $\lambda \to 0$ (see Eq. (12) for $\alpha(0) < 0$). But are we justified in taking only the leading singularity near the light cone, if we deal with fixed q^2? It turns out that this is alright for $\tilde{T}_2$. For suppose we take a less leading singularity. Then its contribution to T_2 is of the form

$$T_2^{[d]} \sim (-q^2) \int_{-1}^{1} \frac{d\lambda\, g_2^{[d]}(\lambda)}{(-q^2 - 2\lambda M\nu + \mu^2 + i\epsilon)^{2+d}} \tag{44}$$

where $d > 0$ and where we also introduce an "effective" μ^2, which represents less leading contributions. In the scaling limit

$$T_2^{[d]} \to (-q^2)\, \nu^{-2-d} \int_{-1}^{1} \frac{d\lambda\, g_2^{[d]}(\lambda)}{\left(\frac{1}{\omega} - \lambda + i\epsilon\right)^{2+d}}$$

A Regge pole with intercept α will be generated by a term $|\lambda|^{1-\alpha}$ in $g_2^{d}(\lambda)$ for $\lambda \to 0$, as in Eq. (12). This is so since Regge behaviour is obtained from the small λ or high $(p\cdot x)$ behaviour of the matrix elements of the bilocal operators, and is therefore independent of the type of singularity near the light cone.[5,7] Thus, for $\nu \to \infty$ and $(d+\bar{\alpha}) > 0$

$$T_2^{[d]} \to (-q^2)\, \nu^{-2-d} \left(\frac{\nu}{-q^2 + \mu^2}\right)^{d+\bar{\alpha}}$$

where $\bar{\alpha}$ is the leading singularity with negative intercept. Thus considering $(\nu^2/(-q^2))\, \tilde{T}_2 = R_2(t)$, we see that the sum of contributions of the leading light cone singularity and a representative of the non-leading singularities is,

$$R_2(t) \sim 4\pi^2 \int_{-\infty}^{\infty} \frac{d\lambda}{\lambda^2}\, g_2(\lambda, t) + C\left(\frac{\mu^2}{\mu^2 - q^2}\right)^{d+\bar{\alpha}(t)} \left(\frac{\nu}{M}\right)^{\bar{\alpha}(t)} \quad . \tag{45}$$

where we now look in a non-forward direction and allow for momentum transfer t dependence (the first term can also be rewritten as

$$4\pi^2 \int_{-\infty}^{\infty} \frac{d\lambda}{\lambda}\, g_2'(\lambda t)$$

thus making contact with Eq. (9)). For $\bar{\alpha}(t) < 0$ only the first term survives, which shows the appearance of a fixed pole in $1/(-q^2)\, T_2$ at J=0 with a residue that is independent of q^2. However, when $\bar{\alpha}(0)$ is very close to zero, it may not be able to separate its contribution by present data through finite energy sum rules. Since we expect d=1 for the next to leading singularity (as mass term corrections, for example), we see that the second term is especially important near $q^2=0$, while for $q^2 \to -\infty$ only the first term obviously survives. An effective change in the value of the residue of the fixed pole around $(-q^2) \sim \mu^2$ in a phenomenological analysis may therefore not be surprising. The value of μ^2 should be around that value where scaling begins in $(-q^2)$.

One can of course separate the first term in Eq. (45) by looking at Compton scattering for $t \neq 0$, since for $R_L=0$, a J=0 fixed pole in T_2 implies a kronecker delta singularity at J=0 in T_1, which is q^2 independent and survives at $q^2=0$. One may also detect the t-dependence in amplitudes with one real photon and one off-shell, like bremsstrahlung in electron-nucleon scattering,[44] since the residue of the fixed singularity does not depend on the photon masses (see below). Our assumption is that $\bar{\alpha}(t)$ changes with t, since it comes from the matrix element of the bilocal, and there is no reason for that to be fixed. (Anything that can move — moves!)

The fact, that the fixed pole term is dominated by the light cone singularity even at low (q^2) follows from the standard phase variation argument in Eq. (5) defining $W_{\mu\nu}$. For choose the proton at rest and

$$q = (\nu, 0, 0, \sqrt{\nu^2 - q^2}) \approx \left(\nu, 0, 0, \nu + \frac{M}{\omega}\right) \quad .$$

Thus most contributions come from $|x_0-x_3| \lesssim 1/\nu$ and $|x_3| \lesssim \omega/M$. It is thus $x^2 \lesssim 2\omega/M = 4/(-q^2)$ that are important in deep inelastic. However, if one subtracts all Regge contributions first, namely the behaviour in $|x_0| \approx |x_3|$ is damped for high values, then it is sufficient to have $\nu \to \infty$ to get to the light cone. In the analysis for non-forward direction and different photon "masses" in Compton scattering one has matrix elements of the bilocal operator between different momentum states,

$$\left[\langle p|F_2^{\alpha\beta}(x,0)|p'\rangle\right]_{x^2=0} = P^\alpha P^\beta \times \int d\alpha\, d\beta\, e^{i(\alpha p\cdot x+\beta p'\cdot x)}\, g_2(\alpha,\beta,t) \tag{46}$$

where $P = p+p'$. The spectral conditions restrict the integration variables to a finite region which, in the variables $\alpha_1 = \alpha+\beta$ and $\alpha_2 = \alpha-\beta$, turns out to be the area within the lines connecting the four points ($\alpha_1 = \pm 1$, $\alpha_2 = 0$) and ($\alpha_1 = 0$, $\alpha_2 = 1 \pm \sqrt{1-4M^2/t}$).[45] One can thus rewrite Eq. (46) as

$$\left[\langle p|F_2^{\alpha\beta}\left(\tfrac{1}{2}x, -\tfrac{1}{2}x\right)|p'\rangle\right]_{x^2=0} = P^\alpha P^\beta \times \int d\lambda_1\, d\lambda_2\, h_2(\lambda_1\lambda_2 t)\, e^{i\lambda_1 \frac{p+p'}{2}\cdot x}\, e^{i\lambda_2\sqrt{1-\frac{4M^2}{t}}\,\frac{p-p'}{2}\cdot x} \tag{46a}$$

with λ_1, λ_2 bounded by four lines connecting the points $(\pm 1, 0)$ and $(0, \pm 1)$. Thus in Eq. (42) we now have a double integration and a denominator of the form $(-q^2 - 2\alpha M\nu - 2\beta M\nu' + i\epsilon)^2$, where $M\nu' = q\cdot p'$. To arrive at Eq. (45) is now straightforward ($\mathfrak{G}_2$ there is an integral over λ_2 of $\tilde{h}_2$). One also see that there is no dependence of the residue on any of the "photon" masses. The full analysis in the non-forward direction shows that there is also an additional fixed pole in a spin-flip amplitude.[44]

Finally, we would like to comment that there may be a fixed pole at J=0 in the T_2 amplitude coming from non-leading singularities, of the form

$$\frac{\mu^2}{\mu^2-q^2}\;\frac{q^2}{\nu^2}$$

which is non-polynomial in q^2. However, such a term will show up also as a fixed pole in electroproduction of the hadronic states with the mass μ^2, that give rise to the discontinuity at $q^2 = \mu^2$. One may of course replace $1/(q^2-\mu^2)$ by $\int dm^2 \rho(m^2)/(q^2-m^2)$, with $\int \rho(m^2)\, dm^2 = 1$, to get the same effect as before for the J=0 singularity, but now with a continuum contribution for the discontinuity in q^2. The quantum numbers of these hadronic states are those of the electromagnetic current. One can of course also have a fixed singularity (kronecker delta) at J=0 in the T_L amplitude, which from the leading light cone singularity implies a t_L term in the Schwinger term sum rule Eq. (14). Such a singularity also implies J=0 fixed singularities in electroproduction of hadronic states, and also changes the relation between T_1 and T_2 fixed singularities. It also ruins the polynomial dependence, since it may be of the form $q^2/(q^2-\mu^2)$. It may be argued in this case that such terms are absent, since they are not produced by dispersion integrals over the imaginary part, but by real subtractions only. (The dispersion integral has no J=0 fixed behaviour once the $\alpha > 0$ Regge contributions are subtracted.) However, for $R_L = 0$ such terms appear in T_1.

Note, that from a general light cone singularity d we get a fixed pole at $\alpha = -d$ in T_2, with a residue $(-q^2)\, f(t)$ for all q^2. This follows from Eq. (44) by arguments similar to the above. Note that now it will appear also in W_2[46] (for non-integer d).

IV. The Fritzsch-Gell-Mann Algebra

A. Quark Algebra on the Light Cone

A very important step forward in the study of deep inelastic processes was the hypothesis of Fritzsch and Gell-Mann,[18] that not only is the leading light cone singularity given by a free field of spin-1/2 constituents, but that so is also the whole SU(3) ⊗ SU(3) structure of the bilocals of the leading singularity.[47] This implied many relations, and it thus became clear which results of the parton model are a consequence of the SU(3) ⊗ SU(3) structure on the light cone and which depend on specific assumptions of that model.

To obtain the commutation relations, one writes the electromagnetic and weak currents in terms of quark fields,

$$J_\mu^{E.M.} = \frac{2}{3}\bar{p}\gamma_\mu p - \frac{1}{3}\bar{n}\gamma_\mu n - \frac{1}{3}\bar{\lambda}\gamma_\mu\lambda =$$

$$= \bar{\psi}\gamma_\mu \frac{1}{2}\left(\lambda_3 + \frac{1}{\sqrt{3}}\lambda_8\right)\psi \tag{47a}$$

$$J_\mu^W = \left[\bar{p}\gamma_\mu(1-\gamma_5)n\right]\cos\theta_c + \left[\bar{p}\gamma_\mu(1-\gamma_5)\lambda\right]\sin\theta_c =$$

$$= \bar{\psi}\gamma_\mu(1-\gamma_5)\frac{1}{2}\left[(\lambda_1+i\lambda_2)\cos\theta_c + (\lambda_4+i\lambda_5)\sin\theta_c\right]\psi \tag{47b}$$

and then computes the commutators as for free fields. One then postulates that the type of singularities and the SU(3) ⊗ SU(3) structure are the same for nature. The space dependence of the bilocal operators is unknown – it is measured in deep-inelastic electron and neutrino scattering experiments. One should emphasize, that only the light cone singularities are of a free field nature. The matrix elements include all the complications of strong interactions and may not have any resemblance with a scale invariant limit of setting all mass parameters to zero. Defining

$$J_\mu^{a\pm} = :\bar{\psi}(x)\gamma_\mu(1\pm\gamma_5)\frac{1}{2}\lambda^a\psi(x): \tag{48}$$

we get

$$\left[J_\mu^{a\pm}(x),\ J_\nu^{b\pm}(y)\right] \underset{(x-y)^2\approx 0}{\sim} i f^{abc} \times \Big\{S_\nu^{c\pm}(x,y)g_{\mu\alpha} + S_\mu^{c\pm}(x,y)g_{\nu\alpha} - g_{\mu\nu}S_\alpha^{c\pm}(x,y)$$

$$\mp \epsilon_{\mu\nu\rho\alpha} A^{c\pm\rho}(x,y)\Big\}\partial^\alpha D(x-y) + d^{abc}\{S \leftrightarrow A\} \tag{49}$$

where

$$D(z) = -\frac{1}{2\pi}\epsilon(z_0)\delta(z^2)$$

and

$$\begin{aligned} S_\mu^{a\pm}(x,y) &= :\bar{\psi}(x)\gamma_\mu(1\pm\gamma_5)\left(\frac{1}{2}\lambda^a\right)\psi(y): + (x\leftrightarrow y) \\ A_\mu^{a\pm}(x,y) &= :\bar{\psi}(x)\gamma_\mu(1\pm\gamma_5)\left(\frac{1}{2}\lambda^a\right)\psi(y): - (x\leftrightarrow y) \end{aligned} \tag{50}$$

The commutators $\left[J_\mu^{a+}(x),\ J_\nu^{b-}(y)\right]$ are less singular near the light cone by one power of x^2, namely have a leading $\delta((x-y)^2)$ singularity rather than the $\delta'((x-y)^2)$ as in Eq. (49). They are also proportional to mass terms.

We now adopt the structure of Eq. (49) to <u>hold in nature</u> for the leading light cone singularity.

One can try and argue that the leading light cone singularity is not going to be modified in renormalizable field theories, proceeding as if canonical considerations are valid and "subtleties" of renormalization of infinities can be ignored.[29,48] One then discovers, that the leading bilocal is not changed for interactions with scalars or pseudoscalars, while for neutral vector mesons, "gluons", it gets multiplied by a line integral

$$:\bar{\psi}(x)\,\Gamma\psi(y): \rightarrow\ :\bar{\psi}(x)\, e^{ig\int_y^x v_\mu(\xi)\, d\xi^\mu}\,\Gamma\psi(y):$$

where $v_\mu(x)$ is the gluon field and g is the gluon-quark coupling constant. One does not have to worry about ordering problems in the definition of the exponential since (x-y) is almost light-like and the Gupta-Bleuler commutation rules are taken for the gluon field, and thus any two parts of the line integral commute.

One can go further and postulate closed commutation rules among bilocal operators,[18] which yield light cone singularities multiplying the same set of bilocal operators. For two bilocals $F_1(x_1y_1)$ and $F_2(x_2y_2)$, this is assumed to hold when all four points are near to one light ray (all six distances are almost light like), as indicated from canonical considerations of quarks with gluon interactions.[29]

B. Results for Deep Inelastic Scattering

The analysis of deep inelastic processes proceeds as in our discussion in section II. One takes

$$\left[\langle p|S^c_\mu(x)|p\rangle\right]_{x^2=0} = p_\mu\, f^c_S(p\cdot x) + x_\mu\, g^c_A(p\cdot x) \tag{51a}$$

$$\left[\langle p|A^c_\mu(x,0)|p\rangle\right]_{x^2=0} = p_\mu\, f^c_A(p\cdot x) + x_\mu\, g^c_S(p\cdot x) \tag{51b}$$

where the subscripts S and A mean symmetric or antisymmetric in $p\cdot x$, respectively. The terms that go with x_μ do not contribute to the leading terms which give scaling. However, since in an approach with underlying fermion fields the longitudinal cross sections are zero[26] in the scaling limit, these terms contribute to $W_L = \left(1-\frac{\nu^2}{q^2}\right)W_2 - W_1$ terms proportional to $\frac{1}{\nu}$ [35] (the p_μ terms contribute a $W_L = W_2 = \frac{1}{\nu}F_2$ part, and the contribution from next to leading $\delta(z^2)$ terms starts as $1/\nu^2$).

For neutrino-nucleon scattering,[49,50] the matrix elements that enter are

$$\begin{aligned} W^{(\nu)}_{\mu\nu}(q,p) &= \frac{1}{2}\sum_s \int d^4x\, e^{iqx} \langle p\, s|\left[J^W_\mu(x),\ J^{W+}_\nu(0)\right]|p\, s\rangle = \\ &= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)W^{(\nu)}_1(q^2,\nu) + \frac{1}{M^2}\left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right)W^{(\nu)}_2(q^2,\nu) \\ &\quad - \frac{i}{2M^2}\epsilon_{\mu\nu\alpha\beta}\, p^\alpha q^\beta\, W^{(\nu)}_3(q^2,\nu) \\ &\quad + \frac{1}{M^2} q_\mu q_\nu\, W^{(\nu)}_4(q^2,\nu) + \frac{1}{2M^2}\left(q_\mu p_\nu + q_\nu p_\mu\right)W^{(\nu)}_5(q^2,\nu) + \frac{i}{2M^2}\left(q_\mu p_\nu - q_\nu p_\mu\right)W^{(\nu)}_6(q^2,\nu) \end{aligned} \tag{52}$$

and

$$W^{(\bar{\nu})}_{\mu\nu}(q,p) = -\,W^{(\nu)}_{\nu\mu}(-q,p) \quad .$$

T invariance sets $W^{(\nu)}_6 = 0$. Since we have $SU(3)\otimes SU(3)$ symmetry on the light cone, with all currents conserved, the scaling limit for νW_4 and νW_5 is zero. If the next to leading singularities are given by a structure like mass term corrections, then it is rather $\nu^2 W_4$ and $\nu^2 W_5$ that scale. The latter also do not contribute to the scattering cross sections in the limit of zero lepton masses. As for W_3,

$$\nu W_3(q^2\nu) \rightarrow F_3(\omega) \tag{53}$$

in the scaling limit. The positivity conditions here are

$$\left(1-\frac{\nu^2}{q^2}\right)W_2 \geq W_1 \geq \frac{\sqrt{\nu^2 - q^2}}{M}\,|W_3|$$

The algebra Eq. (49) implies

$$\omega F_2(\omega) = 2M\, F_1(\omega) \tag{54}$$

for all processes. Setting $\theta_c=0$, we get[51]

$$6\,\omega\left[F_2^{ep}(\omega) - F_2^{en}(\omega)\right] = \left[F_3^{\nu p}(\omega) - F_3^{\nu n}(\omega)\right] \tag{55}$$

One obviously also has the Adler sum rule[52] in the scaling limit,

$$\int_1^\infty \frac{d\omega}{\omega}\left[F_2^{\nu p}(\omega) - F_2^{\nu n}(\omega)\right] = -2 \tag{56}$$

and the Gross-Llewellyn Smith sum rule[53]

$$\int_1^\infty \frac{d\omega}{\omega^2}\left[F_3^{\nu p}(\omega) + F_3^{\nu n}(\omega)\right] = -6 \tag{57}$$

The first follows from the equal time commutation relations between time components, while the second follows from the d coupling part of the commutator between space components of vector and axial-vector currents. Note that the Adler sum rule can be derived by the $p \to \infty$ method, and holds for any $q^2 < 0$ fixed in the form[52,54]

$$\int_0^\infty d\nu\left[W_2^{\nu p}(\nu,q^2) - W_2^{\nu n}(\nu,q^2)\right] = -2$$

Equation (56) is the $q^2 \to -\infty$ limit of it. The Gross-Llewellyn Smith sum rule cannot be derived by the $p \to \infty$ method, since it involves a commutator between space components where z-diagram contributions are important.[54] However in some cases, one can include z-diagram contributions in fixed q^2 sum rules using null-plane commutators.[55] This is not the case for the sum rule Eq. (57), since in null plane commutators one deals with $\int dq^-\left[W_{\mu\nu}(q,p)\right]_{q^+=0}$, where $q^\mp = q^0 \mp q^3$, and therefore the only combination that can come out is

$$\int_0^\infty \frac{d\nu}{p^+}(\nu W_3) \sim \frac{q_\perp^2}{p^+}\int_1^\infty d\omega\; F_3(\omega q^2) \quad .$$

One would then get that the integral on the right hand side is q^2 independent (it is actually infinite). We thus see that light cone expansions put all current components on the same footing as far as sum rules at $q^2 \to -\infty$ are concerned. Bjorken's sum rule for W_1 [56] coincides here with the Adler sum rule, since the longitudinal cross section vanishes.

So far the scaling phenomena and all other relations following from the algebra Eq. (49) are consistent with experiments.[8,9] For spin dependent amplitudes and sum rules see Ref. 57.

Note that other sum rules, derived within the parton model with extra specific assumptions regarding the "sea" of pairs,[58]

$$\int_1^\infty \frac{d\omega}{\omega}\left[F_2^{ep}(\omega) - F_2^{en}(\omega)\right] = \frac{1}{3} \tag{58a}$$

$$\int_1^\infty \frac{d\omega}{\omega^2}\, F_2^{en} = \frac{2}{9} \tag{58b}$$

cannot be derived here. The first one is not related to any local commutator; rather, the left hand side is proportional to $\int_0^\infty \frac{d(p\cdot x)}{(p\cdot x)}\, f_A^3(p\cdot x)$, namely an integration over a line on the light cone running to infinity.[58] The left hand side of Eq. (58b) is related to a commutator of a time derivative of a space component with a space component (n=0 in Eq. (38a)), the value of which between neutron states does not follow from any algebraic structure.

When considering the combination

$$\sum_a \int d^3\vec{x} \langle p|\left[\partial_0 J_i^a(\vec{x}),\; J_j^a(\vec{0})\right]|p\rangle \,,$$

one gets that the leading light cone singularity contributes a term which is the part of the kinetic energy carried by the quark fields, namely $\propto \langle p|\bar{\psi}\left(\gamma_i \overleftrightarrow{\partial}_j + \gamma_j \overleftrightarrow{\partial}_i\right)\psi|p\rangle$. Since only the kinetic energy part of $\theta_{\mu\nu}$ contributes to the $p_\mu p_\nu$ part of the matrix element, we get a sum rule,[18]

$$6\int_1^\infty \left[F_2^{ep}(\omega) + F_2^{en}(\omega)\right]\frac{d\omega}{\omega^2} - \int_1^\infty \left[F_2^{\nu p}(\omega) + F_2^{\nu n}(\omega)\right]\frac{d\omega}{\omega^2} = \frac{4}{3}(1-\epsilon) \tag{59}$$

where ϵ is the fraction of energy carried by constituents that do not couple to the currents, like neutral gluons. Recent experiments indicate that $\epsilon = 0.46 \pm 0.21$,[9] namely about half of the energy is carried by neutrals.

All the results of the light cone algebra depend on the hypothesis that the current constituents have free field leading singularities. The relation between current quarks and constituent quarks, the latter appearing in quark model spectroscopy considerations, is a subject of recent activity.[59,60]

Other implications result from the internal group structure and positivity, and are in the form of inequalities which hold for all ω. These were first discovered within the parton model,[61] and then shown to hold from general light cone considerations.[62,63] We mention here the bounds on the ratio between en and ep structure functions

$$\begin{matrix}\text{(SU(2) symmetry) } 4 \\ \text{(SU(3) symmetry) } 3\end{matrix}\Bigg\} \geq \frac{F_2^{en}(\omega)}{F_2^{ep}(\omega)} \geq \frac{1}{4} \tag{60}$$

and the bound

$$\frac{F_2^{(\nu p)}(\omega)}{4\,F_2^{(en)}(\omega) - F_2^{(ep)}(\omega)} \leq \begin{cases} \frac{12}{5} & \text{(SU(2))} \\ \frac{48}{23} & \text{(SU(3))} \end{cases} \tag{61}$$

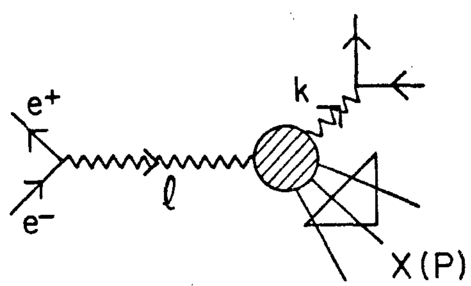
The latter being severe for those ω where the ratio in Eq. (60) is close to the lower limit.

There have been recently discussions regarding the rate of convergence of the Adler sum rule. It is argued that[64] either the convergence is very slow ($\omega \sim 100$?) or that $F_2^{\nu n}/F_2^{\nu p}$ is large for $\omega < 5$. However, it can be shown[65] that the ratio is going to be large whenever F_2^{en}/F_2^{ep} is near to a 1/4. It may be possible to saturate the sum rule up to $\omega \sim 30-40$ with a ratio $F_2^{\nu n}/F_2^{\nu p}$ of 3-4 for $\omega < 5$.[65]

C. Further Implications - Non-forward Matrix Elements

Since the singularity structure near the light cone is a c-number, the scaling laws will be the same for all processes in which the bilocals of the leading singularity have non-vanishing matrix elements. In particular, varying the momenta of the states in the matrix elements of bilocal operators constitute a severe test of the idea of c-number singularities. Such matrix elements occur in amplitudes with two currents, and to get to the light cone we need both "masses" of the two currents to become large, in either space like or time like directions.

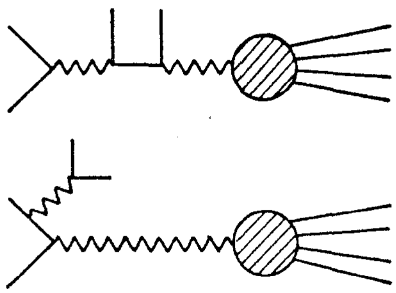
One can consider e^+e^- annihilation into a $\mu^+\mu^-$ pair and a given hadronic state.[66] We are interested in the part of the amplitude which is the diagram



Here the hadronic state is with charge conjugation C=1. With $P=\ell-k$ and $Q=\frac{1}{2}(\ell+k)$, the matrix element is

$$M_{\mu\nu} = \int d^4x\, e^{iQx} \langle X | T^* \left[J_\mu\left(\frac{1}{2}x\right), J_\nu\left(-\frac{1}{2}x\right)\right] | 0 \rangle \tag{62}$$

One can consider here the BJL limit of $Q_0 \to \infty$ with $\vec{Q}$ fixed, which is in the physical region. The scaling limit here is $\nu = Q\cdot P \to \infty$ with $\omega = 2Q\cdot P/Q^2$ fixed ($\omega \le 1$). In this limit one can use the light cone expansion for the time ordered product in Eq. (62). Moreover, by squaring the matrix element and summing over X, and then letting $M_X \to \infty$ (first $\nu \to \infty$ with P and ω fixed), one can check the assumption that the bilocals obey a closed algebra when all distances are light like.[18,29] If correct, one obtains an explicit expression for the cross section as a function of ω in the above limit. One has to separate the contribution of the diagrams where the hadrons are in C=-1 states,



These can be calculated in terms of $e^+e^- \to$ all. For more details see Ref. 66.

Other processes which involve two high off shell currents are inclusive electroproduction of $\mu^+\mu^-$ pairs[67] and $e^\pm e^- \to e^\pm e^- X$.[68] In the former one can relate the inclusive cross section, again assuming the algebra of bilocals, to total electroproduction. In the latter process of colliding beams one has the advantage that in certain regions the diagrams where both exchanged photons are space like dominate, thus simplifying the analysis of connection with experiments.[68]

All these processes have cross sections smaller by 2 - 4 orders of magnitude than present day experimental techniques. The colliding beam processes have the largest cross sections for near future study (SLAC, DESY).

D. Current Conservation

The form Eq. (49) for the commutation relations near the light cone for vector currents is consistent with current conservation to leading order, namely when applying $\partial^{\mu(x)}$ we do not get $\delta''(x^2)$ terms because $\Box D(x)=0$. However, we do get terms with $\delta'(x^2)$ singularities in general. These should be cancelled by the corresponding contributions of the next to leading singularities near the light cone, which in the current commutators involve $\delta(x^2)$ singularities, and therefore $\delta'(x^2)$ terms when a divergence is taken.

Note that if we try and write the general terms contributing to W_1^{ab} and W_2^{ab} in an explicitly conserved way like for the case of the electromagnetic currents Eq. (36), we get that this gives $\left[j_0^a(\vec{x}), j_0^b(\vec{y})\right]=0$ for local functions V_L^{ab} and V_2^{ab}, a result known long ago.[69]

We can write the next term in Eq. (49) as $C_{\mu\nu}^{ab}(x,y)\, D(x-y)$. Also, we do not want $\langle p| C_{\mu\nu}^{ab}(x,y) |p\rangle$ to have a $p_\mu p_\nu$ term, since such a term contributes to the leading scaling behaviour. Let us demonstrate our results for the f^{abc} part of the commutator.[70] Introducing Z=x+y and Δ=x-y, we get from the conservation conditions

$$\left(\partial^{\alpha X} S_\alpha\right)\Delta_\nu + \left(\partial_\alpha^Z S_\nu - \partial_\nu^Z S_\alpha + \epsilon_{\mu\nu\rho\alpha}\, \partial^{\mu\Delta} A_5^\rho\right)\Delta^\alpha + \left(C_{\alpha\nu} - C_{\nu\alpha}\right)\Delta^\alpha = \Delta^2 g_\nu \tag{63a}$$

$$\left(\partial^{\alpha\Delta} S_\alpha\right)\Delta_\nu + \left(\partial_\alpha^\Delta S_\nu - \partial_\nu^\Delta S_\alpha + \epsilon_{\mu\nu\rho\alpha}\, \partial^{\mu Z} A_5^\rho\right)\Delta^\alpha + \left(C_{\alpha\nu} + C_{\nu\alpha}\right)\Delta^\alpha = \Delta^2 h_\nu \tag{63b}$$

where g_ν and h_ν are new bilocals, which do not contribute to the next to leading light cone singularity. Since $\left[S_\alpha(x,y)\right]_{x=y} = 2J_\alpha(x)$ which is conserved, it follows that

$$\partial^{\alpha Z} S_\alpha(x,y) = \Delta^\alpha \tilde{S}_\alpha(x,y) \qquad (64)$$

Therefore,

$$\left(\partial^{\alpha Z} S_\alpha\right) \Delta_\nu = \Delta^\alpha \Delta_\nu \tilde{S}_\alpha = \Delta^\alpha\left(\Delta_\nu \tilde{S}_\alpha - \Delta_\alpha \tilde{S}_\nu\right) + \Delta^2 \tilde{S}_\nu$$

and Eq. (63a) implies

$$C_{\alpha\nu} - C_{\nu\alpha} = \Delta_\alpha \tilde{S}_\nu - \Delta_\nu \tilde{S}_\alpha - \left(\partial^Z_\alpha S_\nu - \partial^Z_\nu S_\alpha + \epsilon_{\mu\nu\rho\alpha}\, \partial^{\mu\Delta} A^\rho_5\right) + \left(\tilde{C}_{\alpha\nu} - \tilde{C}_{\nu\alpha}\right) \qquad (65a)$$

where $\Delta^\alpha(\tilde{C}_{\alpha\nu} - \tilde{C}_{\nu\alpha}) = \Delta^2(g_\nu - \tilde{S}_\nu)$. Since we do not include in C or $\tilde{C}$ terms proportional to Δ^2, it follows that $g_\nu = \tilde{S}_\nu$ and

$$\tilde{C}_{\alpha\nu} - \tilde{C}_{\nu\alpha} = \epsilon_{\alpha\nu\lambda\sigma} \Delta^\lambda D^\sigma(x,y) \qquad (65b)$$

As for solving (63b), we observe that

$$\partial^\Delta_\alpha S_\nu - \partial^\Delta_\nu S_\alpha + \epsilon_{\mu\nu\rho\alpha}\, \partial^{\mu Z} A^\rho_5 = F_{[\alpha\nu]\lambda} \Delta^\lambda \qquad (66)$$

since each term on the left hand side vanishes for $\Delta \to 0$. Thus, observing that $F_{[\alpha_1\alpha_2]\alpha_3}$ is antisymmetric in the first two indices,

$$\Delta^\alpha F_{[\alpha\nu]\lambda} \Delta^\lambda = \Delta^\alpha \Delta^\lambda F_{[\lambda\nu]\alpha} = \Delta^\alpha \Delta^\lambda \left[F_{[\lambda\nu]\alpha} + F_{[\lambda\alpha]\nu}\right]$$

and hence, from (63b),

$$C_{\alpha\nu} + C_{\nu\alpha} = \Delta^\lambda \left[F_{[\lambda\alpha]\nu} + F_{[\lambda\nu]\alpha}\right] - \left(\partial^{\beta\Delta} S_\beta\right) g_{\alpha\nu} + \left(\tilde{C}_{\alpha\nu} + \tilde{C}_{\nu\alpha}\right) \qquad (67a)$$

where again, excluding Δ^2 terms in $\tilde{C}$, we have

$$\tilde{C}_{\alpha\nu} + \tilde{C}_{\nu\alpha} = \Delta_\alpha \Delta_\nu\, \tilde{C}(x,y) \qquad (67b)$$

Note that only in the free field case, neglecting masses, can we have $C_{\alpha\nu} = 0$. This is so since in any interacting theory $\tilde{S}_\nu \neq 0$, since $\tilde{S}_\nu = 0$ means an infinite number of local conserved quantities through $\partial^{\alpha Z} S_\alpha(x,y) = 0$.[71] The same is true for the left hand side of Eq. (66), which vanishes only for free fields.

V. Total Annihilation $e^+e^- \to$ all, and $\pi^0 \to 2\gamma$

The total electron-positron annihilation cross section into hadrons is given by

$$\sigma(s) = \frac{4\pi\alpha^2}{3s}\,\rho(s) \tag{68}$$

where q is the total momentum and $s = q^2$, and the function $\rho(s)$ is related to the commutator of electromagnetic currents by

$$\langle 0|\left[J_\mu(x), J_\nu(0)\right]|0\rangle = \frac{1}{3(2\pi)^6}\int d^4q\, e^{-iqx}\left(q_\mu q_\nu - q_{\mu\nu} q^2\right)\rho\left(q^2\right) \tag{69}$$

Here we are dealing with a vacuum expectation value of a commutator, and therefore the short distance and light-cone structure coincide. The asymptotic behavior of $\sigma(s)$ is therefore given by the short distance structure of the left-hand side of Eq. (69). Using free-field singularities near the light cone, we obtain that $\sigma(s) \propto \frac{1}{s}$ for $s \to \infty$.[72,73] However, the coefficient cannot be determined unless we also assume that the unrenormalized fields from which the current is constructed satisfy canonical commutation relations. In such a case, we obtain

$$\rho(s) = \frac{\sigma_{e^+e^- \to \text{all}}(s)}{\sigma_{e^+e^- \to \mu^+\mu^-}(s)} \longrightarrow \left[\sum_{s=\frac{1}{2}} Q_i^2 + \frac{1}{4}\sum_{s=0} Q_i^2\right] \tag{70}$$

Note that when one calculates the short distance structure for free fields using $j_\mu = :\bar{\psi}\gamma_\mu\psi:$, one obtains for the vacuum expectation value near $x^2 = 0$,

$$\begin{aligned}\langle 0|\left[j_\mu(x), j_\nu(0)\right]|0\rangle &\sim \left\{\mathrm{Tr}\left[\gamma_\mu\gamma^\alpha\gamma_\nu\gamma^\beta\right]\right\}\cdot \\ &\left\{\left[\partial_\alpha\Delta^-(x)\right]\left[\partial_\beta\Delta^-(x)\right] - \left[\partial_\alpha\Delta^+(x)\right]\left[\partial_\beta\Delta^+(x)\right]\right\} \sim \frac{i}{3\pi^3}\,\epsilon(x_0)\,\delta'''(x^2)\left(g_{\mu\nu}x^2 - 2x_\mu x_\nu\right)\end{aligned} \tag{71}$$

and thus one obtains for the time-space commutators, for $x_0 \to 0$,

$$\langle 0|\left[j_0(x), j_k(0)\right]|0\rangle \xrightarrow[x_0\to 0]{} \frac{i}{6\pi^2|x_0|^2}\,\partial_k\delta^{(3)}(\vec{x}) + \frac{i}{12\pi^2}\,\partial_k\Delta\,\delta^{(3)}(\vec{x}) \tag{72}$$

Note that the infinite Schwinger term[74] is here obtained without any point-splitting in defining the current. This shows the advantage of short-distance expansions over direct use of equal-time commutators. Mass corrections introduce terms $\propto m^2\partial_k\delta^{(3)}(\vec{x})$.

The prediction for the ratio in Eq. (70) depends now on the constituent scheme used. From $\sigma_L \approx 0$ in deep inelastic electron scattering, we assume no spin-zero constituents. For the Gell-Mann-Zweig (GMZ) fractional charge quarks, one obtains $\rho(s) \to R = \frac{2}{3}$. When an extra SU(3) quantum number is introduced,[60,75] "color," then R = 2. "Color" quarks ("red," "white," and "blue") obey ordinary Fermi-Dirac statistics. All physical states are singlets under the "color" group. Thus mesons are constructed as $\delta_{ab}q_a^i\bar{q}_b^j$ and baryons as $\epsilon_{abc}q_a^iq_b^jq_c^k$, where a b c are "color" indices and i j k usual SU(3) ones.

Recent experiments at CEA[76] indicate that $R = \frac{2}{3}$ is excluded, and are consistent with R = 2, as for the "color" quarks. (One should remember, however, that the $\frac{1}{s}$ law has not yet been verified.) The latter scheme also is in agreement with the observed $\pi^0 \to 2\gamma$ decay rate, as given by the Adler-Bell-Jackiw anomaly,[75] while the GMZ quarks give a value smaller by a factor of 9. For a discussion of the various quark schemes, see ref. 75. Note that the Hahn-Nambu quarks[77] of integral charge yield the same value for the $\pi^0 \to 2\gamma$ decay as "color" quarks while they predict R = 4 for annihilation. When "charm" states do not contribute, one gets R = 2 for Hahn-Nambu quarks, too. The "charm" quarks do not contribute to $\pi^0 \to 2\gamma$ because of their quantum numbers. In the Hahn-Nambu scheme,[78,79] the current can be written as

$$J_\mu = \bar{\psi}^i_a Q^i \psi^i_a + \bar{\psi}^i_a \bar{Q}_a \psi^i_a \tag{73}$$

where $Q^i = \left(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right)$ as in GMZ and $\bar{Q}_a = \left(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}\right)$. The first term, $J^{(1)}_\mu$, is a singlet under $[SU(3)]_{\text{"charm"}}$ and an octet in the usual SU(3), and the second, $J^{(2)}_\mu$ is an octet under $[SU(3)]_{\text{"charm"}}$ and a singlet under SU(3). Thus, if one asserts that "charm" hadron states have very high mass, then when considering matrix elements between usual hadrons, the relations are as for GMZ quarks.[78] However, for total annihilation, we have an extra factor 3, as compared with GMZ, since each index "a" in $J^{(1)}_\mu$ contributes equally to the vacuum expectation value of the commutator, thus yielding R = 2. For $\pi^0 \rightarrow 2\gamma$, all quarks contribute to the triangle anomaly, since they are virtual. However, since the matrix element involved is $\langle \pi^0 | T J_\mu J_\nu | 0 \rangle$, the combination $J^{(1)}_\mu J^{(2)}_\nu$ does not contribute since it is an octet in "charm" and $J^{(2)}_\mu J^{(2)}_\nu$ does not contribute since it is a singlet in SU(3). Thus only $J^{(1)}_\mu J^{(1)}_\nu$ contributes. Here again the amplitude is 3 times the GMZ value. Note that when "charm" states contribute, the Hahn-Nambu scheme predicts $\frac{\sigma_{en}}{\sigma_{ep}} \geq \frac{1}{2}$, which is excluded by present experiments.[8]

Crewther[80] showed that a relation exists between total annihilation, π^0 decay, and a space-space commutator $[V_i(\vec{x}), V_j(\vec{0})] = i\,\epsilon_{ijk} A^{(Q^2)}_k \delta^{(3)}(\vec{x})$ which appears in Bjorken's polarization sum rule.[81] Here $A^{(Q^2)}_k = \bar{\psi}\gamma_5\gamma_k Q^2 \psi$, and only the isovector part enters into the relation. Also, only the isovector contribution of total annihilation enters. Denoting the coefficient of the isovector part of A_k by K (which can be measured by the difference between proton and neutron for polarized electron on polarized target scattering), and R_1 the isovector contribution to total annihilation, the relation is

$$S = KR_1 , \tag{74}$$

where S appears in the $\pi^0 \rightarrow 2\gamma$ amplitude as[82,83]

$$S = -\frac{1}{12}\pi^2 \epsilon^{\mu\nu\alpha\beta} \iint d^4x\, d^4y\, x_\mu y_\nu \, \langle 0 | T^* J_\alpha(x) J_\beta(0) \partial^\gamma J^5_\gamma(y) | 0 \rangle \tag{75}$$

For GMZ quarks, $S = \frac{1}{6}$, $R_1 = \frac{1}{2}$, and $K = \frac{1}{3}$. For the "color" quarks, S and R_1 are multiplied by 3. From experiment, $S \approx \frac{1}{2}$.[84]

Crewther[80] derives this relation by a consistency consideration, first using a short-distance expansion in $x \rightarrow 0$ and then in $y \rightarrow 0$ in Eq. (75), and the free field form for the three-point function at short distances.[85] This relation is of great importance, since it connects π^0 decay to other processes so that we can get the decay amplitude without any need for renormalized perturbation theory methods. This is relevant since our light-cone expansions do not hold in the latter approaches. In fact, the form for the three-point function for all points near one light ray was demonstrated by Bardeen, Fritzsch and Gell-Mann[75] to follow from consistency considerations in comparing the different ways of reducing that function by light-cone expansions of pairs of currents.

An interesting problem is that of constraints imposed on operator product expansions from the free field form for the three-point function, namely, from the existence of an anomaly. It turns out that one gets constraints on Wilson's short-distance expansion, namely, that line integrals of local operators appear in an expansion of a product of two currents.[86] Light-cone expansions with bilocal operators are not implied.

VI. Single Particle Inclusive e^+e^- Annihilation

Light-cone expansions were generalized to include products of more than two operators[87,88] to discuss single-particle inclusive experiments in e^+e^- annihilation and eN scattering. Recently, it was pointed out that certain regularity assumptions of the terms multiplying the light-cone singularity lead to finite multiplicities in e^+e^- annihilation.[89] It was also pointed out that introducing singularities to get logarithmic increase in multiplicity ruins scaling.[90] Moreover, such a behavior is inconsistent with the spectral conditions.

A careful examination of the singularity structure reveals that one can get a consistent formulation which yields both scaling and logarithmic multiplicities from the leading light-cone singularity.[91] The logarithmic multiplicity is obtained by a certain singularity structure at short distances of the term multiplying the light-cone singularity. It does not affect the scaling because the relation between this singularity and the light-cone singularity has to be changed (as compared with that in ref. 89).

We consider $e^+e^- \rightarrow H + x$, where the four-momentum p of H is observed, and $p^2 = M^2$. Define[92]

$$\bar{W}_{\mu\nu}(q,p) = \frac{1}{2\pi}\int d^4x\, e^{iq\cdot x} \sum_X \langle 0|J_\mu(x)|H(p)X\rangle$$

$$\langle H(p)X|J_\nu(0)|0\rangle = \bar{W}_1(\nu, q^2)\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)$$

$$+ \frac{1}{M^2}\bar{W}_2(\nu,q^2)\left(p_\mu - \frac{M\nu}{q^2}q_\mu\right)\left(p_\nu - \frac{M\nu}{q^2}q_\nu\right) \tag{76}$$

where we also sum over the spin of particle H and $M\nu = q \cdot p$. Assuming a one-photon exchange amplitude, we get

$$\frac{d^2\sigma}{d\xi\, d(\cos\theta)} = \frac{\pi\alpha^2}{q^2\xi^3}\left[\sigma_T(1+\cos^2\theta) + \sigma_L(1-\cos^2\theta)\right] \tag{77}$$

where $\xi = \frac{q^2}{2M\nu}$, θ is the scattering angle of H in the e^+e^- center-of-mass frame, and

$$\left.\begin{aligned} \sigma_T &= \bar{W}_1 \\ \sigma_L &= \bar{W}_1 + \left(\frac{\nu}{2M\xi} - 1\right)\bar{W}_2 \end{aligned}\right\} \tag{78}$$

where in Eq. (77) terms of order $\frac{m_e^2}{q^2}$ or $\frac{M\xi}{q^2}$ were ignored. (Note that here $\bar{W}_2$ need not be positive. It is, in fact, negative for $\sigma_L = 0$.)

$$\frac{d\sigma}{d\xi} = \frac{4\pi\alpha^2}{3q^2\xi^3}\left[2\sigma_T(\xi,q^2) + \sigma_L(\xi,q^2)\right] \equiv \frac{4\pi\alpha^2}{3q^2\xi^3} f(\xi,q^2) \tag{79}$$

We have

$$\int_1^{q/2M} d\xi \frac{d\sigma}{d\xi} = \bar{n}(q^2)\,\sigma_{tot}(q^2) \tag{80a}$$

$$\int_1^{q/2M} \frac{d\xi}{2\xi}\frac{d\sigma}{d\xi} = \sigma_{tot}(q^2) \tag{80b}$$

where $\bar{n}(q^2)$ is the average multiplicity of hadrons H and $\sigma_{tot}(q^2)$ is the total e^+e^- annihilation cross section (in (80b) we assume for simplicity one type of hadrons present). Assuming $\sigma_{tot}(q^2) \sim \frac{1}{q^2}$, we get

$$\int_1^{q/2M} \frac{d\xi}{\xi^3} f(\xi, q^2) = A\,\bar{n}(q^2) \tag{81a}$$

$$\int_1^{q/2M} \frac{d\xi}{\xi^4} f(\xi, q^2) = 2A \tag{81b}$$

A is related to the rate of decrease of $\sigma_{tot}(q^2)$. In coordinate space

$$\bar{W}_{\mu\nu}(q,p) \sim \int d^4x\, d^4y\, d^4z\, e^{iq(x-y)}\, e^{ipz}$$
$$\langle 0 | \left[T^*\left(J_\mu(x) J_H^+(0)\right)\right] \left[\bar{T}^*\left(J_\nu(y) J_H(z)\right)\right] | 0 \rangle \tag{82}$$

where J_H is the source of H (suppressing spin indices) and $\bar{T}^*$ denotes a covariant anti-time-ordered product (operators with earlier times are to the left). Defining

$$\bar{f}_{\mu\nu}(u^2, p\cdot u) = \int d^4y\, d^4z\, e^{ipz} \langle 0 | \left[T^*\ J_\mu(u+y) J_H^+(0)\right] \left[\bar{T}^*\ J_\nu(y) J_H(z)\ \right] | 0 \rangle \tag{83}$$

we have

$$\bar{W}_{\mu\nu}(q,p) \sim \int d^4u\, e^{iqu}\, \bar{f}_{\mu\nu}(u^2, p\cdot u) \tag{84}$$

and standard arguments imply light-cone dominance $|u^2| \lesssim \frac{1}{q^2}$ for $q^2 \to \infty$ and fixed ξ. These arguments hold also for $\xi \to \infty$ as long as $\frac{M\xi}{q} \to 0$. Thus we may get light-cone dominance terms also for large ξ, where one may get an increase of $\bar{n}$ with q^2. For $f(\xi, q^2) \to F(\xi)$ as $q^2 \to \infty$, with $F(\xi) \sim \xi^2$ for large ξ, we get a logarithmic multiplicity. Since from $\frac{q}{2\mu} \le \xi$ (with $\mu > M$) the contribution to $\bar{n}$ is finite, most of the contribution comes from $\frac{M\xi}{q} \to 0$, in which case light-cone dominance applies. We should remember, however, that a logarithmic increase in $\bar{n}$ may come from a non-scaling term altogether, like, for example,

$$f(\xi, q^2) \sim \frac{\ell g\, q}{q} f_1(\xi) + f_2(\xi) \tag{85}$$

with $f_1(\xi) \sim \xi^3$ as $\xi \to \infty$ and

$$\int_1^\infty \frac{d\xi}{\xi^3} f_2(\xi) \text{ finite.}$$

One then gets that only f_2 contributes to the energy sum rule, Eq. (80b), and only f_1 to the logarithmic increase in $\bar{n}$, as $q^2 \to \infty$.

We define, in analogy with Eq. (7),

$$\bar{W}_{\mu\nu} = \left(-g_{\mu\nu}\, q^2 + q_\mu q_\nu\right) \bar{V}_L(q^2\nu)$$
$$+ \left[(q\cdot p)(q_\mu p_\nu + q_\nu p_\mu) - p_\mu p_\nu q^2 - g_{\mu\nu}(q\cdot p)^2\right] \bar{V}_2(q^2\nu) \tag{86}$$

and take $\bar{V}_L = 0$, as follows from the fact that the tensor structure of $\bar{f}_{\mu\nu}(u^2, p\cdot u)$ is taken to be the same as that for deep inelastic electron scattering. Namely, the light-cone expansion for the product of two currents and two hadronic sources, as in Eq. (83), is taken to have the same tensor structure as the product of two currents, when the space-time distance between the coordinates of the two currents in the above two cases approaches the light cone. A discussion similar to what follows can be applied to $\overline{V}_L \neq 0$.

We expect

$$\overline{V}_2\,(q^2\nu) \sim \int d^4u\, e^{iqu}\left[\ell n\, \mu^2\left(-u^2 + i\,\epsilon\, u_0\right)\right]\overline{f}(p\cdot u) \tag{87}$$

where μ is some mass parameter. The singularity structure is that of the next to leading part in a two-point function,

$$\frac{1}{(2\pi)^3}\int d^4q\, e^{-iqu}\,\theta\,(q_0)\delta(q^2-\mu^2) = \frac{1}{4\pi^2}\,\frac{1}{-u^2+i\epsilon u_0} + \frac{1}{16\pi^2}\,\mu^2\ell n\,\mu^2\left(-u^2+i\epsilon u_0\right) + \ldots . \tag{88}$$

This is dictated by the fact that in Eq. (83), $J_\mu(x)$ is always to the left of $J_\nu\,(y)$, and for scaling the singularity in $\overline{V}_2$ in u^2 is of zero order. Writing

$$\overline{f}(p\cdot u) = \int d\alpha\, \overline{g}(\alpha)\, e^{-i\alpha p\cdot u} \tag{89}$$

we obtain

$$\begin{aligned}\overline{V}_2(q^2\nu) &\sim \int d\alpha\,\overline{g}(\alpha)\int d^4u\, e^{i(q-\alpha p)}\,\ell n\,\mu^2\left(-u^2+i\,\epsilon\, u_0\right)\\ &\propto \int d\alpha\,\overline{g}(\alpha)\,\theta\left(q_0-\alpha p_0\right)\left[\delta\left((q-\alpha\, p)^2-\mu^2\right)-\delta\left((q-\alpha\, p)^2\right)\right]\\ &\sim \int d\alpha\,\overline{g}(\alpha)\;\theta\,(\nu-\alpha M)\;\delta'\left((q-\alpha\, p)^2\right)\end{aligned} \tag{90}$$

Note that $\theta\,(k_0)\,\delta'\,(k^2)$ has no Fourier transform due to an infrared divergence, as is obvious from Eq. (88). However, for calculating $\overline{V}_2$ for $q^2 \to \infty$, the last two expressions in Eq. (90) are equivalent. We obtain

$$\overline{V}_2(q^2\nu) \sim \nu^{-2}\,\overline{g}'\,(\xi) \tag{91}$$

The other root $\alpha_+ \sim \frac{2\nu}{M}$ of $(q-\alpha p)^2 = 0$ does not contribute due to the $\theta(\nu-\alpha M)$ factor. The spectral conditions also imply $\overline{g}(\xi) = 0$ for $\xi \leq 1$. To get logarithmic increase in multiplicity, we need $\overline{g}'\,(\xi) \sim \xi^2$ for large ξ, which means $\overline{f}(p\cdot u) \sim \frac{1}{(p\cdot u)^4}$ for small $p\cdot u$.

If one starts with the Fourier transform of a commutator, one gets an $\epsilon\,(x_0)\,\theta\,(x^2)$ light-cone singularity for $\overline{V}_2$. The procedure in ref. 89 is to take over this singularity but modify the bilocal such as to pick up the part relevant to the annihilation process. However, as pointed out in ref. 90, in such a case the root $\alpha_+ \sim \frac{2\nu}{M}$ also contributes, and in case $g(\xi)$ grows as $\xi \to \infty$, we get a violation of scaling. Moreover, the spectral conditions are not maintained. One can argue that these $g\left(\frac{2\nu}{M}\right)$ terms are cancelled by less leading singularities with more singular $p\cdot u$ behavior. However, the ξ^2 term in $f(\xi,\,q^2)$ does not survive in the leading light-cone singularity, and the non-leading singularities do not produce such a term. Thus one cannot just modify the function multiplying the light-cone singularity when starting from a commutator; one also has to modify the structure of the light-cone singularity.

If we subtract, in Eq. (83), the expression with $u \to -u$, we obtain that the integrand for $\overline{V}_2$ of Eq. (87) is

$$\left[\ell n\,\mu^2\left(-u^2+i\,\epsilon\, u_0\right)\right]\overline{f}(p\cdot u) - \left[\ell n\,\mu^2\left(-u^2-i\,\epsilon\, u_0\right)\right]\overline{f}(-p\cdot u) \tag{92}$$

However, unlike the case of deep inelastic scattering, here $f(p\cdot u)$ is definitely not purely symmetric, and therefore we do not have a

$$\left[\ln\mu^2\left(-u^2+i\epsilon u_0\right)\right]-\left[\ln\mu^2\left(-u^2-i\epsilon u_0\right)\right] = \epsilon\left(u_0\right)2i\pi\,\theta\,(u^2)$$

light-cone singularity only.

It is instructive to examine the above structure in lowest order perturbation theory for a ϕ^3 type interaction, which yields Bjorken scaling for all ladder graphs.[13] Taking a scalar current

$$J_\mu(x) = :\phi^+(x)\overleftrightarrow{\partial}_\mu\,\phi(x):$$

we have a light-cone expansion for the connected part (denoted by subscript c),

$$\left[J_\mu(x)\,J_\nu(y)\right]_c \approx \frac{1}{-u^2+i\epsilon u_0}\,\overleftrightarrow{\partial}_\mu^{(x)}\overleftrightarrow{\partial}_\nu^{(y)}\left\{\left[\phi^+(x)\,\phi(y)\right]_c+\left[\phi(x)\,\phi^+(y)\right]_c\right\} \tag{93}$$

Consider now

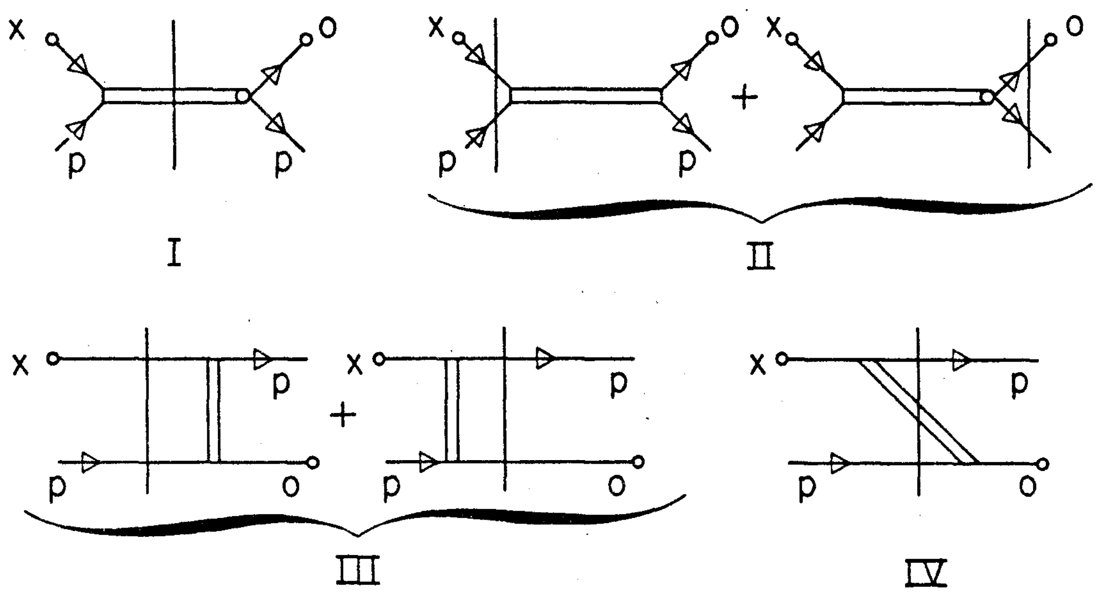
$$\int d^4x\, e^{iqx}\left[\langle p|J_\mu(x)\,J_\nu(0)|p\rangle\right]_c$$

in the Bjorken limit, with $|p\rangle$ being a state created as $\phi^+|0\rangle$. Take a $\phi^+\phi B$ interaction, where B is a neutral scalar field. Define

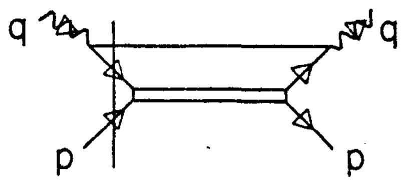
$$g_1(\alpha) = \int d(p\cdot x)\, e^{i\alpha p\cdot x}\left[\langle p|\phi^+(x)\,\phi(0)|p\rangle\right]_{c\atop x^2=0} \tag{94a}$$

$$g_2(\alpha) = \int d(p\cdot x)\, e^{i\alpha p\cdot x}\left[\langle p|\phi(x)\,\phi^+(0)|p\rangle\right]_{c\atop x^2=0} \tag{94b}$$

Evaluating g_1 and g_2 by introducing intermediate states, we get



I + II give $g_1(\alpha)$, and III + IV give $g_2(\alpha)$. Note that II, III, and IV contribute to $q^2 > 0$ only, since for example II means, for the Compton amplitude, the contribution



Defining

$$G(\alpha) = \frac{1+\alpha}{\alpha^2 + \alpha\left(2 - \frac{\mu^2}{M^2}\right) + 1} \tag{95}$$

where μ is the mass of the quantum of B, we get for the various graphs

$$\left.\begin{array}{ll} \mathrm{I} : & \theta(1+\alpha)\,G(\alpha) \\ \mathrm{II} : & -\theta(\alpha)\,G(\alpha) \\ \mathrm{III} : & -\theta(\alpha)\,G(-\alpha) \\ \mathrm{IV} : & \theta(\alpha-1)\,G(-\alpha) \end{array}\right\} \tag{96}$$

where an overall proportionality constant is omitted. Thus[93]

$$\left.\begin{array}{l} g_1(\alpha) = \theta(-\alpha)\,\theta(1+\alpha)\,G(\alpha) \\ g_2(\alpha) = \theta(\alpha)\,\theta(1-\alpha)\,G(-\alpha) \end{array}\right\} \tag{97}$$

and $g_1(\alpha) + g_2(\alpha)$ appears in deep inelastic scattering. However, for our process of single-particle inclusive annihilation, only IV contributes. Here $g(\alpha) \sim \frac{1}{\alpha}$ for large α, which means a $\ln(p\cdot u)$ singularity for small $(p\cdot u)$.[94] To get a logarithmic multiplicity in this case of scalar constituents, one needs here a $\frac{1}{(p\cdot u)^2}$ singularity, as follows from the structure in Eq. (93). "Soft" field theories yield at most a $\frac{1}{(p\cdot u)}$ behavior[95] and thus a finite multiplicity.

VII. Other Problems and Approaches

Let us mention here other problems and other approaches that we have not discussed.

A. The Parton Model

It has been discussed in many papers.[15,96] It has also been applied to inclusive single particle electroproduction in the parton fragmentation region,[97] and to large angle hadron-hadron scattering.[98] In both cases the light cone dominance does not directly apply.

B. One Photon Amplitudes

These include exclusive electroproduction,[95] and considerations regarding form factors.[100] For a review and criticism see Ref. 101. One may argue that in the case of electroproduction of pions the contribution of the light cone singularity of $\left[J_\mu(x),\ J_H(0)\right]$, where $J_H(0)$ is the source of the pion field, is that of a fixed pole and therefore important also when the mass associated with J_H is finite.[102]

C. $pp \to \mu^+\mu^- X$

The theoretical analysis of this process is still controversial.[103] The prediction of Drell and Yan,[104] from arguments of parton-antiparton annihilation dominance, is a simple scaling law

$$\frac{d\sigma}{dq^2} = \frac{\alpha^2}{(q^2)^2}\, f\left(\frac{q^2}{S}\right) \tag{98}$$

where q^2 is the mass of the $\mu^+\mu^-$ pair and $s = (p_1+p_2)^2$. Here the leading light cone singularity, the $1/x^2$ term, does not appear, but only the next one, which is regular on the light cone. Writing

$$\frac{d\sigma}{dq^2} = -\frac{\alpha^2}{q^2}\,\frac{1}{6\pi^3}\,\frac{1}{\sqrt{S(S-4M^2)}}\int \frac{d^3q}{q_0}\int d^4x\, e^{-iqx}\ \langle p_1p_2|J_\mu(x)J^\mu(0)|p_1p_2\rangle \tag{99}$$

We need $\langle p_1p_2|J_\mu(x)J^\mu(0)|p_1p_2\rangle$ near the light cone. Using a form as in Eq. (36), with $C_L=0$ and $C_2 = \ln(-x^2+i\epsilon x_0)$, as indicated from the scaling at SLAC, we obtain

$$\begin{aligned}
\frac{d\sigma}{dq^2} &\sim \int \frac{d^3\vec{q}}{q_0}\int d^4x\, e^{-iqx}\,\ln(-x^2+i\epsilon x_0)\; f(p_1\cdot x, p_2\cdot x, S)\\
&= \int \frac{d^3\vec{q}}{q_0}\int d\alpha\, d\beta\, g(\alpha,\beta,S)\int d^4x\, e^{i(\alpha p_1+\beta p_2-q)\cdot x} \times \ln(-x^2+i\epsilon x_0)\\
&\sim \int \frac{d^3\vec{q}}{q_0}\int d\alpha\, d\beta\, g(\alpha,\beta,S)\ \theta(\alpha E_1+\beta E_2-q_0)\,\delta'\left((\alpha p_1+\beta p_2-q)^2\right)\\
&= \int \frac{d^3\vec{q}}{q_0}\int d\alpha\, d\beta\, g(\alpha,\beta,S)\ \theta(\alpha E_1+\beta E_2-q_0)\\
&\quad \times \delta'\left(q^2 - 2\alpha p_1\cdot q - 2\beta p_2\cdot q - \alpha\beta S\right)
\end{aligned} \tag{100}$$

The spectral conditions imply that

$$\begin{aligned} 0 \le \alpha \le 1\\ 0 \le \beta \le 1 \end{aligned} \tag{101}$$

and then the contribution comes from either

$$\begin{aligned} 1 \ge \alpha \ge \xi\\ \xi \ge \beta \ge \frac{\lambda_2-\omega}{1-\lambda_1} \end{aligned} \tag{102a}$$

or

$$1 \geq \beta \geq \xi$$
$$\xi \geq \alpha \geq \frac{\lambda_1 - \omega}{1-\lambda_2} \tag{102b}$$

where

$$\omega = \frac{q^2}{S}$$
$$\lambda_1 = \frac{2q \cdot p_1}{S} \tag{103}$$
$$\lambda_2 = \frac{2q \cdot p_2}{S}$$

$$\xi = \frac{q \cdot p}{S} + \sqrt{\frac{(q \cdot p)^2}{S^2} - \frac{q^2}{S}} = \frac{1}{2}(\lambda_1 + \lambda_2) + \sqrt{\frac{1}{4}(\lambda_1 + \lambda_2)^2 - \omega}$$

We thus see that a choice like

$$g(\alpha, \beta, S) = \delta(\alpha)\, \tilde{g}(\beta, S) + \delta(\beta)\, \tilde{g}(\alpha, S)$$

does not contribute at all (this was the form taken in Ref. 105). The procedure of Drell and Yan amounts to setting $g(\alpha, \beta, S) = 0$ for the leading singularity and considering only the contribution of the next to leading as

$$\langle p_1 p_2 | J^{\mu}(x)\, J_{\mu}(0) | p_1 p_2 \rangle \sim \langle p_1 | \left[\bar{\psi}(x)\gamma_{\mu}\right]_{\alpha} [\psi(0)]_{\beta}\, | p_1 \rangle \, \times$$
$$\times \langle p_2 | \left[\bar{\psi}(0)\gamma_{\mu}\right]_{\beta} [\psi(x)]_{\alpha} | p_2 \rangle \sim p_1^{\alpha} p_2^{\beta}\, \mathrm{Tr}\left[\gamma_{\alpha}\gamma_{\mu}\gamma_{\beta}\gamma^{\mu}\right] \times$$
$$\times f(p_1 \cdot x)\, \bar{f}(p_2 \cdot x) \sim S f(p_1 \cdot x)\, \bar{f}(p_2 \cdot x)$$

So that

$$\frac{d\sigma}{dq^2} \sim \frac{\alpha^2}{(q^2)^2} \int d\lambda_1\, d\lambda_2\, g(\lambda_1)\, \bar{g}(\lambda_2)\, \delta(1 - \lambda_1 \lambda_2 \omega)$$

$$\omega = \frac{S}{q^2}$$

Brandt and Preparata[106] take the form Eq. (100), with $g(\alpha, \beta, S)$ exponentially damped in S or in power thereof. Thus $d\sigma/dq^2$ decreases faster than any power in q^2 for fixed ω.

D. Can Quarks Escape?

In deep inelastic scattering, the bilocal operators that appear have quantum numbers of a quark at one point and of an antiquark at the other. The distance between the two points is light like, with the space distance of order ω/M. Thus it appears that, although light like, the space distance may be arbitrary large. The fact that one does not have asymptotic quark states is no doubt due to a complicated dynamic. Possible infinite potential wells may provide the answer, as argued by Johnson.[107] For a discussion of this problem in the various quark schemes see Ref. 75.

E. Studies in Perturbation Theory

The studies of Drell, Levy and Yan[16] demonstrated the emerging of scaling by introducing transverse momentum cutoff. If one does not introduce such a cutoff, perturbation diagrams do not scale, but lead to violations by powers of $\ell g\ q^2/m^2$.[12] Summation of infinite sets of ladders without self-energy corrections leads to power singularities in Wilson's expansions, but no bilocal structure (as we discussed in Section III. D). The Callan-Symanzyk equation[108] shows that Green's functions may have power type behaviour at most for certain values of the coupling constant. This in general does not lead to Bjorken scaling unless one has canonical dimensionality.

F. Conformal Symmetry

Wilson's "skeleton" limit,[1,109] is the limit when strong interactions become conformally invariant. This limit has been studied recently by many authors.[110,111] In particular, an interesting bootstrap scheme for the two and three point functions was studied. Such an approach originates from the fact that the two and three point functions are determined by conformal invariance (up to proportionality constants) from the dimension of the field, and the higher point functions can be computed by skeleton graph expansions. The integral Schwinger-Dyson equations then turn, by use of conformal invariance, into algebraic equations for the dimension of the field and the coupling constant.[112] We should mention again, as in Section IV, that one should take results of scale invariance only for the singularities, since matrix elements of bilocal operators involve mass parameters (like Regge trajectory slopes, etc).

G. Null Plane Quantization and Sum Rules

Quantization on a null plane instead of an equal time surface was investigated by several authors.[113,47] When considering sum rules,[55] one has here an advantage over the infinite momentum approach[54] in that z-diagrams are taken into account (as verified for the cases of free fields, where the $p \to \infty$ approach already leads to problems for space-space commutators). See also our discussion in Section IV.B. How to take class II diagrams into account is discussed in Ref. 114.

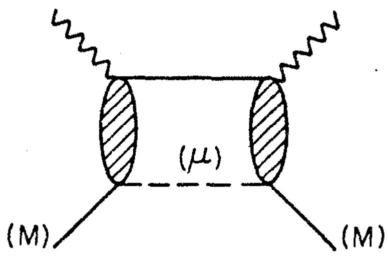
H. Relation Between Scattering and Annihilation Scaling Functions

It has been argued[92] that the $\bar{F}_i(\omega)$ defined for single particle inclusive annihilation be analytic continuations in ω of the deep inelastic functions according to

$$\bar{F}_1(\omega) = \mp F_1(\omega) \qquad \bar{F}_2(\omega) = \pm F_2(\omega) \tag{104}$$

where the upper sign is for fermions and lower for bosons.

Such a relation is in general not expected, since the scaling functions are cross sections which have no simple analyticity properties. Even when $F_i(\omega)$ can be continued analytically, Eq. (104) need not hold. Equation (104) was shown to hold in ladder models with stable particle exchanges. When propagators are modified to include self-energy cuts, Eq. (104) does not hold any more.[115] It is instructive to consider the relation implied by a box diagram (with scalar currents and particles),



where the exchange  represents some spectral function. Here $F(\omega)$ is continuable, but the relation now is

$$\bar{F}(\omega) = -\mathrm{Re}\ F(\omega) + \frac{1}{8}\pi g^2 \int_{L(\omega)}^{\infty} \rho^2(m^2)\, dm^2 \tag{105}$$

where

$$L(\omega) = M^2 \left(\omega + \frac{\omega}{\omega - 1}\ \frac{\mu^2}{M^2}\right) .$$

Note that for ω near 1 the continuation is expected to hold, since $L(\omega) \to \infty$ and there is essentially no contribution from the spectral function term. ($\mathrm{Im}\ F(\omega)$ is also vanishingly small there.) For $\omega = 1 + \mu/M$ we have the minimal value for $L(\omega)$, which is $(M+\mu)^2$. Note that parametrizing $F_2 = C\,(\omega-1/\omega)^3$ with $F_1=(\omega/2M)\,F_2$ the analytic continuation relations to annihilation lead to structure functions which yield a logarithmic multiplicity (see our Section VI). However, the continuation should not be believed for $\omega \to 0$.

The Gribov-Lipatov relations[37] between the scattering and annihilation, as derived in perturbation theory for $g^2 \ll 1$ and $g^2\, \ell n\; q^2/m^2 \sim 1$, are

$$\bar{F}_2\left(\frac{1}{\omega}, \eta\right) = -\omega^3\, F_2(\omega, \eta)$$
$$\eta = \frac{3}{4}\, \ell g \left[1 - \frac{g^2}{12\pi^2}\, \ell g\, \frac{Q^2}{m^2}\right]^{-1} \tag{106}$$

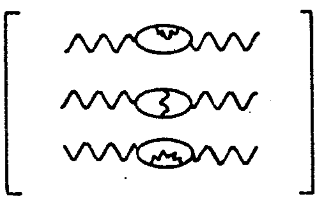
This also yields a logarithmic growth in multiplicity for $F_2 \xrightarrow[\omega\to\infty]{} \text{(const)}$.

I. Early Scaling

Light cone dominance does not account for the early scaling observed at SLAC,[8] namely for $(-q^2) \gtrsim 1\ \mathrm{BeV}^2$. Any attempt to explain this result must involve dynamics, since non-leading light cone singularities are involved.[116,117] We should mention that the Bloom-Gilman variable[118] $\omega' = \omega + M^2/(-q^2)$ extends the scaling to lower q^2. It was also suggested[119] that the variable $(2M\nu + M^2)/\left[(-q^2) + a^2\right]$, with $a^2 \sim 0.4$, is the only variable for all q^2.

J. Finite QED

It was recently argued by Adler[120] that the Baker-Johnson-Willey condition[121] for a finite photon propagator, which is an eigenvalue equation for the bare coupling constant α_0, should be changed to be the same condition but for the physical coupling constant α.[122] A finite photon propagator yields a finite QED for an appropriate choice of gauge, when the bare electron mass vanishes.[121,123] Their condition is $F^{[1]}(\alpha_0) = 0$, where $F^{[1]}(\alpha_0)$ is defined as the coefficient of $\ell n\,(-q^2/m^2)$ in the single fermion loop part $\pi_c^{[1]}(-q^2/m^2,\ \alpha_0)$ of the renormalized photon proper self energy,

$$\pi_c^{[1]}\left(-\frac{q^2}{m^2}, \alpha_0\right) =$$  $$+\ \alpha_0$$  $$+ \ldots \tag{107}$$

$$= G^{[1]}(\alpha_0) + F^{[1]}(\alpha_0)\, \ell n \left(-\frac{q^2}{M^2}\right) + (\text{vanishing terms as } q^2 \to \infty)\ .$$

Their condition is a consequence of the Gell-Mann-Low condition[24] $\psi(\alpha_0) = 0$, which however involves all self-energy diagrams (coefficient of logarithmic divergences). Adler shows[120,122] that $\psi(\alpha_0) = 0$ implies that $F^{[1]}(\alpha_0) = 0$ is an infinite order zero. He furthermore argues, that in such a case one has an extra solution for finite QED, which is $F^{[1]}(\alpha) = 0$, coming from a different order of summation. The condition $F^{[1]}(\alpha_0) = 0$ comes from summing first all photon self-energy parts, thus obtaining an asymptotic photon propagator α_0/q^2, and then inserting those into the vacuum polarization graphs Eq. (107). One can, however, first sum all single fermion loop vacuum polarizations, then all two loops, etc. One then gets the condition $F^{[1]}(\alpha) = 0$, since the single loop sum now involves α/q^2 for the photon propagator. Adler argues, that this is the condition chosen in nature. The fact that the zero is of infinite order has implications on attempts of evaluating it.[122] For speculations on experimental implications see Ref. 125.

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