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PCAC AND THE $\pi^0 \longrightarrow 2\gamma$ DECAY RATE*

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Abstract

The difference between applying PCAC to the amplitude for $\pi^0 \longrightarrow 2\gamma$ decay, which involves products of local currents, and to amplitudes involving composite hadrons where it has enjoyed its notable successes—viz., the Goldberger-Treiman relation, the Adler-Weisberger sum rule, and the Adler consistency condition—is analyzed. Using the Bell-Jackiw-Adler theory of the PCAC anomaly, we show that this difference provides a mechanism for removing the factor of 10 discrepancy that is usually claimed to exist between the observed decay rate and the one calculated on the basis of the original Gell-Mann/Zweig quark model with one triplet of fractionally charged quarks. An essential dynamical assumption is that pion pole dominance is valid only for those matrix elements of the divergence of the axial current taken with composite hadronic states; this is akin to features in the "weak PCAC" of Brandt and Preparata. A specific model of the hadron as a Bethe-Salpeter bound state of two point-like constituents is used to illustrate the underlying dynamical mechanism. It follows from this that there should be a sizable enhancement above the PCAC prediction by Adler for forward angle high energy very inelastic neutrino-hadron cross sections. Verification of this prediction will be a crucial test of our theory.

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I. INTRODUCTION AND DISCUSSION

The decay rate $\pi^0 \longrightarrow 2\gamma$ can be computed theoretically by assuming the validity of[1]

(A) The PCAC hypothesis[2]

(B) The theory of the PCAC anomaly discussed first by Bell, Jackiw,[3] and Adler[4,5]

In addition, it is necessary, in order to predict a definite number, to use a specific model for the currents. We shall, as our third assumption, adopt

(C) The quark model, i.e., a field theoretic model for the currents based on the original Gell-Mann and Zweig triplet of fractionally charged quarks.[6] This model with local currents provides a specific realization of Gell-Mann's current algebra.[7]

These three assumptions lead to a calculated decay rate that is smaller than the observed one[8] by a factor of $\approx \frac{1}{10}$; i.e.,

$$\left\{\tau^{-1}_{\text{calc}} = 0.8 \text{ eV}\right\} \approx \frac{1}{10} \left\{\tau^{-1}_{\text{obs}} = 7.7 \pm 0.9 \text{ eV}\right\} \tag{1}$$

Equation (1) presents a serious dilemma because each of the above three assumptions is of great value, having provided the basis of a substantial body of very important and successful relations between theory and experiment.

It is often stated that (1) provides evidence against the original Gell-Mann-Zweig quark model with one triplet of fractionally charged quarks.[9] In this paper, we study this question and argue against this conclusion by exhibiting the basic difference between applying PCAC to hadronic amplitudes where it has enjoyed its notable successes—viz., the Goldberger-Treiman relation,[10] the Adler-Weisberger sum rule,[11] and the Adler consistency condition[12]—and applying PCAC to amplitudes involving matrix elements of products of local currents, such as $\pi^0 \longrightarrow 2\gamma$.

The PCAC hypothesis (A) is basically an assumption of smoothness of the matrix elements of the divergence of the strangeness-conserving axial vector current[1]

$$\left\langle \beta(q_2) \middle| \partial_\lambda \left(\mathscr{F}_1^{5\lambda} + i\, \mathscr{F}_2^{5\lambda} \right) \middle| \alpha(q_1) \right\rangle . \tag{2}$$

It is assumed that (2) satisfies an unsubtracted dispersion relation in the momentum transfer, or mass, $q^2 = (q_2 - q_1)^2$, and that the contribution of the pion pole is dominant in the interval $0 < q^2 < \mu^2$. It follows, then, that matrix elements (2) can be replaced by corresponding matrix elements of the π-meson. The success of the Goldberger-Treiman relation[10] between the weak $\pi \longrightarrow \mu\nu$ decay rate and the strong pion-nucleon coupling provided the original justification for this smoothness assumption. The most notable subsequent success is the Adler consistency condition[12] relating the pion-nucleon coupling constant with the even (under crossing) pion-nucleon scattering amplitude near threshold. In conjunction with Gell-Mann's current algebra,[7] the PCAC hypothesis was also tested by the Adler-Weisberger relation[11] which is a sum rule relating the axial vector coupling constant in β-decay with the pion-nucleon S-wave scattering length in the odd channel under crossing. Subsequently the great power of PCAC, when jointed together with the current commutation relations, was shown in the derivation of theorems for soft pion processes,[1] typical of which are Callan-Treiman relations[13] between $K_{\ell 3}$ and $K_{\ell 2}$ decay, and between pion β-decay, $\pi^+ \longrightarrow \pi^o e^+ \nu$, and $\pi \longrightarrow \mu\nu$ decay.

The PCAC anomalies (B) appear in singular amplitudes involving current operators when gauge invariance and PCAC are naively applied.[3-5] One such example is the triangle graph of Fig. 1 for $\pi^o \longrightarrow 2\gamma$ decay via a spin 1/2 Dirac particle circulating in the loop. An order-by-order analysis of the Feynman graphs contributing to this process shows that these graphs obey an anomalous Ward identity for the axial vector current. Furthermore, in a renormalizable field theory, and in the usual PCAC limit of zero pion mass, all graphs computed to an arbitrary finite order in the strong and electromagnetic corrections to Fig. 1 do not contribute to the anomaly[4,5,14] which in this limit gives the decay rate.[15] Therefore, if we define the field theory by its order-by-order expansion, the anomaly in Fig. 1 can be computed exactly for $q^2 \to 0$. This definition in terms of its order-by-order expansion in the interaction is the only working hypothesis of quantum field theory.

The quark model (C) with local currents is a special and specific realization of Gell-Mann's current algebra and is the simplest and most useful field theoretic model for studying current commutators. Its simplicity and utility in organizing our observations of hadron spectroscopy are very well known as are the, to some, embarrassing questions of their statistics and non-observability.

Evidently the prospect of abandoning any one or more of the three assumptions (A) to (C) is not a priori attractive. Nevertheless the discrepancy in (1) remains and has led to various such proposals for its resolution.

Conceptually the simplest to abandon is (B). The notion of hadrons as bound states or composite systems has received much attention in various forms (bootstraps; parton models; Z = 0 self-consistency conditions) and the very fundamental question of whether or not the perturbation expansion, order by order to all finite orders, correctly represents essential bound state effects is unresolved.[16] However, a choice to abandon (B) while leading us out of the valley of difficulty does so at the price of closing all roads to progress on this problem; we merely deny that we can even address it at this time. Therefore, we shall retain (B) in this work.

Alternatively we can modify or abandon (C) by enlarging and elaborating the quark model with a larger family. Motivation for such attempts comes independently from the desire to resolve the question of why the quarks "seem" to obey symmetric statistics. This has led to alternate versions of quark models such as the Han-Nambu set of three integrally charged triplets[17] and the recent proposal of Gell-Mann[18] of three triplets of fractionally charged quarks. These proposals indeed resolve the dilemma of (1) as well as that of quark statistics but the price they pay is not insignificant. It might even be said that the essential simplicity of the model has been sacrificed. Such judgments are, of course, personal, but the motivation for this work comes directly from the desire to preserve the elementary quark model, and so we turn elsewhere for the resolution of (1).

There remains, then, only the possibility of inquiring into (A) and the smoothness assumption underlying the PCAC hypothesis. The rest of this paper will focus on this analysis. We shall give a well-defined operational interpretation of the smoothness or pion-pole-dominance assumption that preserves the successes of PCAC and at the same time resolves the dilemma posed by (1).

In particular, we assume that there are large corrections to the operator identification of the pion field with the divergence

$$D^{+}(x) \equiv \partial_{\lambda}\left(\mathscr{F}_{1}^{5\lambda}(x) + i\,\mathscr{F}_{2}^{5\lambda}(x)\right) \tag{3}$$

These contributions arise from massive chiral breaking terms in D—there being much more physics than the pion in the 0^- channel. The matrix elements of D connecting the vacuum to a state of 2γ's as calculated in the soft pion limit are thus modified from the $\pi^0 \longrightarrow 2\gamma$ decay rate. However, when we evaluate matrix elements involving extended composite hadrons (in contrast to the point-like currents circulating in the triangle of Fig. 1), these correction terms are suppressed by the hadron's form factors or overlap integrals, and the PCAC successes that follow from the identification of (3) with the pion field are restored.

In order to put content into these words, we work with a physical model of the hadrons as a bound state of point-like constituents, or quarks. For a specific, illustrative calculation, we use a physically artificial but mathematically well defined hadronic model: the hadron is treated as a composite of two point-like constituents described by the Bethe-Salpeter equation in the ladder model.[19] The general feature of this model that we use is the identification of constituent quarks as the current quarks. This model provides a basis for understanding both the asymptotic behavior of the elastic electromagnetic form factors and the scaling behavior of the inelastic structure functions in the Bjorken limit. We do not, however, in this paper <u>derive</u> PCAC from this model of the composite hadron. We simply adopt the PCAC hypothesis and show that its successes in application to the usual soft pion theorems for hadrons are compatible with failure for (1). We also make a new experimental prediction for forward inelastic neutrino scattering based on the analysis of the discrepancy (1). The problem of <u>deriving</u> PCAC in the composite hadron model, while currently under study, remains for the future.

The general physical assumption underlying this work is that <u>the operator world is non-chiral; matrix elements with low-mass hadrons, however, exhibit approximate chiral invariance. The success of PCAC is thus tied to a dynamical basis.</u>

In Section II, we develop this idea more fully using the Goldberger-Treiman relation to illustrate the essential differences as well as similarities between it and the calculation of $\pi^0 \longrightarrow 2\gamma$ decay. In Section III, we consider other applications of our PCAC model and show that the successes of PCAC (in particular, the Adler consistency condition and the Adler-Weisberger relation) are retained by our approach. In Section IV, we discuss a test of these ideas in high-energy neutrino scattering and derive a new result for the forward differential cross section that differs from Adler's original prediction. In the concluding Section V, we sum up.

The underlying physical ideas in this approach are similar to the discussion of "weak PCAC" given in 1970 by Brandt and Preparata.[20] They first developed in detail the general idea that pion-pole dominance is a good approximation as a result of dynamical considerations only for those matrix elements of D(x) that are taken between soft, composite hadronic structures. With respect to this pole dominance idea, the present work differs only in some details from Brandt and Preparata's. We sharpen some of their discussion of weak PCAC by joining it with a specific model of physical hadrons as bound states of point-like constituents.[19] Our motivation to adopt such a model is twofold:

1. Such a model, developed in detail for hadrons with two constituents and with the essential property that the relativistic wave function is finite for zero separation of the constituents, has been successful in describing the observed scaling property of the structure functions for inelastic electron scattering as well as the dipole shape ($\sim 1/t^2$) of the electromagnetic form factors.
2. It provides a framework for discussing and evaluating corrections to PCAC that is both specific and very simple. These corrections take a very different form from those discussed in Ref. 20.

We do not, in this paper, consider the question, which was also discussed by Brandt and Preparata,[20] of how the SU(3) ⊗ SU(3) symmetry is broken, and whether SU(2) ⊗ SU(2) or SU(3) is a better approximate symmetry, nor do we discuss processes dependent on such additional theoretical assumptions. We confine our attention here to the essential role played by composite hadronic structures in leading to a dynamical basis for understanding the successes of PCAC while at the same time correcting (1).

II. THE GOLDBERGER-TREIMAN RELATION AND $\pi^0 \longrightarrow 2\gamma$ DECAY

To derive the Goldberger-Treiman (G-T) relation,[10] we first construct the matrix element of $\mathscr{F}_1^{5\lambda} + i\,\mathscr{F}_2^{5\lambda}$ between single neutron and proton states, $|n(q_1)\rangle$ and $|p(q_2)\rangle$. This defines the axial vector coupling constant g_A:

$$\sqrt{\frac{q_1^0 q_2^0}{M^2}}\;\langle p(q_2)|\,\mathscr{F}_1^{5\lambda} + i\mathscr{F}_2^{5\lambda}|n(q_1)\rangle = \bar{u}_p(q_2)\left[-g_A(q^2)\,\gamma^\lambda\gamma^5 + q^\lambda h_A(q^2)\gamma^5\right]u_n(q_1) \tag{4}$$

For the divergence defined by (3), we have

$$\begin{aligned}\sqrt{\frac{q_1^0 q_2^0}{M^2}}\;\langle p(q_2)|D^+|n(q_1)\rangle &= \left(-2Mg_A(q^2) + q^2 h_A(q^2)\right)\bar{u}_p(q_2)\,i\gamma^5\,u_n(q_1) \\ &\equiv -\mathscr{D}(q^2)\,\bar{u}_p(q_2)\,i\gamma^5\,u_n(q_1)\end{aligned} \tag{5}$$

where $q^\lambda = (q_2 - q_1)^\lambda$; M is the (common) nucleon mass; $g_A \equiv g_A(0) = 1.2$; and $h_A(q^2)$ is the induced pseudoscalar term, with $h_A(0)$ finite; therefore

$$\mathscr{D}(0) = 2Mg_A\ . \tag{6}$$

Assuming $\mathscr{D}(q^2)$ to satisfy an unsubtracted dispersion relation in q^2 and separating out the pion pole term, we write

$$\mathscr{D}(q^2) = \frac{F_\pi \mu^2}{\mu^2 - q^2 - i\epsilon}\;\sqrt{2}\,g + \frac{1}{\pi}\int_{9\mu^2}^{\infty} \frac{\rho(\sigma^2)\,d\sigma^2}{\sigma^2 - q^2 - i\epsilon} \tag{7}$$

where $\sqrt{2}\,g$ is the nucleon coupling to the charged pion pole $\left(\frac{g^2}{4\pi} = 14.6\right)$ and F_π is the charged pion decay constant defined by the matrix element

$$\begin{aligned}\sqrt{2q_0}\;\langle\pi^+(q)|\,\mathscr{F}_1^{5\lambda} + i\,\mathscr{F}_2^{5\lambda}|0\rangle &= -iq^\lambda F_\pi \\ \sqrt{2q_0}\;\langle\pi^+(q)|D^+|0\rangle &= \mu^2 F_\pi\ .\end{aligned} \tag{8}$$

The first term in (7) corresponds to Fig. 2a, and the second term, to all else. This includes pion propagator and vertex corrections off-pole, as in Fig. 2b, and, in addition, and most importantly, all other physical processes in the same 0^- channel that cannot be so summarized. One such is illustrated in Fig. 2c by a 3π continuum with 0^- quantum members. The spectral weight function $\rho(\sigma^2)$ measures the strength of the continuum in this channel as determined by the coupling of D^+ to the vacuum and to the nucleon in Fig. 2.

For ease of writing and picturing, we summarize the second term of (7) by a pole, writing

$$\mathscr{D}(q^2) = \frac{F_\pi \mu^2}{\mu^2 - q^2} \sqrt{2}\, g + \frac{F' \mu'^2}{\mu'^2 - q^2} \sqrt{2}\, g'(\mu'^2) \tag{9}$$

i.e., we work in terms of a two-component theory of the $\mathscr{D}$-channel, one being the pion and the other a heavy π' of mass μ' ($> 3\,\mu$), decay constant F', and nucleon coupling $\sqrt{2}\ g'(\mu'^2)$. This is <u>not</u> an essential assumption and does not imply a peaking in the weight function $\rho(\sigma^2)$ at $\sigma^2 \sim \mu'^2$. Equation (9) is an approximation made on grounds of simplicity as we shall comment at appropriate points in our subsequent discussion.

According to (5) and (7), the pion at the pole $q^2 = \mu^2$ is <u>defined</u> by

$$\lim_{q^2 \to \mu^2} \left\{ \frac{\mu^2 - q^2}{F_\pi \mu^2} \sqrt{\frac{q_1^o q_2^o}{M^2}} \left\langle p(q_2) \middle| D^+ \middle| n(q_1) \right\rangle \right\} = -\sqrt{2}\, g\, \bar{u}_p(q_2)\, i\gamma^5 u_n(q_1) \, . \tag{10}$$

The G-T relation is obtained by taking the limit $q^2 \to 0$ appropriate to β-decay,

$$\lim_{q^2 \to 0} \left\{ \frac{\mu^2 - q^2}{F_\pi \mu^2} \sqrt{\frac{q_1^o q_2^o}{M^2}} \left\langle p(q_2) \middle| D^+ \middle| n(q_1) \right\rangle \right\} \tag{11}$$

which, by (5), (6), and (9), gives

$$\frac{2Mg_A}{F_\pi} = \sqrt{2}\, g \left\{ 1 + \frac{F'}{F_\pi} \frac{g'(\mu'^2)}{g} \right\} . \tag{12}$$

Pion pole dominance says that the correction term in the brackets on the right-hand side of (12) is small. Neglecting it, we have

$$F_\pi \approx \sqrt{2}\, M g_A/g \tag{13}$$

which is the G-T relation. Experiment, on the other hand, provides a calibration of the approximate nature of this smoothness assumption and we know that[21,22]

$$K \equiv \frac{F'}{F_\pi} \frac{g'(\mu'^2)}{g} \approx -0.08 \quad . \tag{14}$$

The "strong" or field theoretic version of PCAC which <u>defines</u> the divergence $D^+(x)$ in (3) as the pion interpolating field <u>operator</u>, up to a proportionality constant, interprets (14) as the off-shell correction to the pion-nucleon coupling $g(q^2)$ for $q^2 \to 0$ relative to its pole value $g \equiv g(\mu^2)$. It then becomes a problem to understand why the correction is as <u>large</u> as 8%. In the notation of (7), off-shell corrections to the pion would presumably carry the same proportionality factor μ^2 as the pole term, in which case $F'/F_\pi \sim \mu^2/\mu'^2 < \mu^2/(3\mu)^2 = 1/9$ and, as Pagels[23] has discussed, an 8% correction is not readily found. From our point of view, the "non-pion" continuum contributions from multi-particle states to (7) and (9) will be the most important. They are large because they do not vanish, nor is chiral symmetry restored, in the $\mu \to 0$ limit. Only the pion pole term itself vanishes as $\mu \to 0$ and its contribution is suppressed as in (7) and (9) by the factor μ^2. The equivalent operator statement of this approach, in the two-component approximation (9), can be summarized by

$$D^+(x) = \sqrt{2}\, F_\pi \mu^2 \phi_\pi^+(x) + \sqrt{2}\, F' \mu'^2 \phi_{\pi'}'^+(x) \tag{15}$$

where ϕ_π and $\phi_{\pi'}$ denote canonical fields for the π and π', respectively.

We turn next to a calculation of $\pi^0 \to 2\gamma$ based on the diagram of Fig. 1 and the result of Adler[4] that in the PCAC limit of $q^2 \to 0$, it is only this one triangle graph, with no insertions or radiative corrections that we need to evaluate.

At this point, we must specify exactly what it is that is circulating around the loop in Fig. 1, and what relation it has with a physical hadron such as the nucleon in the G-T relation. Following the success of recent studies of the electromagnetic form factors and of the scaling properties of the structure functions, we adopt a model of the physical hadron as a bound state of point-like constituents. In particular, for a working model, may we use the Bethe-Salpeter equation in the ladder approximation with scalar gluons.[24] It leads to nucleon wave functions that are finite at the origin, corresponding to strong interaction vertices that vanish for infinite momentum transfers. With these smoothness properties, a physical bound state circulating around the loop as in Fig. 3 will not contribute to the Adler anomaly and we need consider only the circulation of the elementary point-like constituents themselves.[4] Whether or not these are "observable" quarks, their currents are described by the quark model as per our original assumption (C).

Except for the fact that the divergence of the axial current is now being absorbed on a bare quark constituent—i.e., a bare quantum of the field theory—rather than the physical hadron as in the G-T relation, we can repeat the above steps for applying PCAC and extrapolating to zero pion mass. Formally, the $\pi^0 \to 2\gamma$ decay amplitude $\mathscr{F}_\pi\left((k_1+k_2)^2\right)$ is defined by

$$\lim_{\substack{(k_1+k_2)^2 \to \mu^2 \\ k_1^2 = k_2^2 = 0}} \left\{ \sqrt{2}\, \frac{\mu^2 - (k_1+k_2)^2}{F_\pi \mu^2} \left\langle \gamma(k_1,\epsilon_1)\gamma(k_2,\epsilon_2) \middle| D^3 \middle| 0 \right\rangle \right\} \tag{16}$$

$$= \frac{1}{\sqrt{4 k_{10} k_{20}}}\, \mathcal{E}_{\xi\tau\sigma\rho}\, k_1^\xi k_2^\tau \epsilon_1^\sigma \epsilon_2^\rho\, \mathscr{F}_\pi\left((k_1+k_2)^2\right).$$

According to the theory of the PCAC anomaly for triangle graphs, Fig. 1,

$$\lim_{k_1, k_2 \to 0} \left\{ \sqrt{2}\, \frac{\mu^2 - (k_1 + k_2)^2}{F_\pi \mu^2} \left\langle \gamma(k_1, \epsilon_1)\gamma(k_2, \epsilon_2) \middle| D^3 \middle| 0 \right\rangle \right\} \tag{17}$$

$$= \frac{1}{\sqrt{4 k_{10} k_{20}}}\, \mathcal{E}_{\xi \tau \sigma \rho} k_1^\xi k_2^\tau \epsilon_1^\sigma \epsilon_2^\rho \left[\frac{\sqrt{2}}{F_\pi} \left(- \frac{2\alpha}{\pi} \right) S \right]$$

or

$$\mathscr{F}_\pi(0) = - \frac{2\alpha}{\pi} \frac{\sqrt{2}}{F_\pi} \; S \tag{18}$$

The content of (17) is as follows: The naive divergence D which is the π, or in our case the $\pi + \pi'$ as in (15), has non-vanishing matrix elements in the soft pion limit, $k_1, k_2 \to 0$, only because of the singularity introduced by the electromagnetic currents. The appearance of this singularity in the presence of the electromagnetic current _is_ the anomaly and leads to the value on the right-hand side of (17). S is a constant parameter determined by the charges and axial couplings of the elementary Dirac quanta circulating around the loop[5] in Fig. 1. In quark triplet models, S is the average charge of the quarks participating in the charged β-decay currents:

$$\begin{aligned} S &= \tfrac{1}{2}\left(\tfrac{2}{3} - \tfrac{1}{3}\right) = \tfrac{1}{6} && \text{Gell-Mann-Zweig} \\ &= \tfrac{1}{2}(1 + 0) = \tfrac{1}{2} && \\ &= \tfrac{1}{2}(0 - 1) = -\tfrac{1}{2} && \text{Han-Nambu} \\ &= 3 \times \tfrac{1}{6} = \tfrac{1}{2} && \text{three Gell-Mann triplets} \end{aligned}$$

Equation (1) follows from the choice of $S = \frac{1}{6}$, together with the usual smoothness assumption of PCAC that

$$\mathscr{F}_\pi(\mu^2) \approx \mathscr{F}_\pi(0) \, . \tag{19}$$

<u>The question for us now is, how good is the extrapolation (19)?</u> In contrast to (11), we are here calculating a matrix element with high momentum, or light-cone singularities that give rise to the PCAC anomaly and the non-vanishing of the right-hand side

of (17). Aside from the anomaly itself, all the other contributions to $\pi^o \longrightarrow 2\gamma$ decay correspond to non-singular matrix elements. They are assumed to extrapolate smoothly from $q^2 = 0$, where they vanish, to the pion pole at $q^2 = \mu^2$, as they do in the G-T relation. The singular triangle graph of Fig. 1 with circulating point-like constituents is the new element in the calculation of $\pi^o \longrightarrow 2\gamma$ decay and we focus on its extrapolation in (19).

The correction to (19) can be calculated from (17) directly in terms of the coupling of D to the electromagnetic current that is the source of the two photons emerging in the decay. Since we want to illustrate the basic difference between applying PCAC to hadronic amplitudes and applying it to amplitudes with local currents or point-like constituents, and do not attempt a quantitative calculation of the 1/10 in (1), we continue to work with our simplified model. At the end of this section, we return to a more general and qualitative discussion that does not rely on a two-pole approximation in terms of the π and π' but retains the continuum in (7). Here we stay with a two-dimensional model of D as a π plus a heavy π'.

We see readily in the notation of (15), (16), and (17) that the anomaly can be written (schematically) as

$$\mathscr{F}_\pi(0) = -\frac{2\alpha}{\pi}\frac{\sqrt{2}}{F_\pi} S \propto \left\{\langle\pi|\gamma\gamma\rangle + \frac{F'}{F_\pi}\langle\pi'|\gamma\gamma\rangle\right\} \tag{20}$$

The identification of the anomaly with $\pi^o \longrightarrow 2\gamma$ decay depends on the smallness of the π' contribution on the right-hand side, i.e., on the ratio

$$\left(\frac{F'}{F_\pi}\right)\left(\frac{\langle\pi'|\gamma\gamma\rangle}{\langle\pi|\gamma\gamma\rangle}\right)$$

If $\frac{F'}{F_\pi} \lesssim 0\left(\mu^2/\mu'^2\right) < \frac{1}{10}$, as in the usual "strong" PCAC, then we can expect (19) to be accurate and the anomaly will give a good approximation to the observed

$\pi^o \longrightarrow 2\gamma$ decay rate. On the other hand, if we interpret the small (-8%) correction to the G-T relation in (14) as the result of a small coupling constant ratio g'/g, due to hadronic overlap integrals or form factors that suppress the coupling to the massive π', the ratio $\frac{F'}{F_\pi} \sim 0(1)$. In this case, (20) shows that the anomaly and $\pi^o \longrightarrow 2\gamma$ decay can be very different, there being no structure factor to suppress the π' relative to the π contribution to the triangle in Fig. 1. This latter is the approach we adopt here. It illustrates the general difference between applying pion pole dominance to the $\pi^o \longrightarrow 2\gamma$ decay amplitude and the G-T relation.

In the following, we make more explicit this observation by adopting a simple illustrative model. We assume that both the π and the π' couple to the circulating point-like currents (quarks) in Fig. 1 with γ^5 coupling. The result by straightforward calculation is that

$$\mathscr{F}_\pi(q^2) = C\left(\frac{\mu^2 - q^2}{\mu^2}\right)\left\{\frac{F_\pi \mu^2}{\mu^2 - q^2}\, g_Q + \frac{F' \mu'^2}{\mu'^2 - q^2}\, g'_Q\right\}$$

where C is a numerical constant, F_π and F' are as before the decay constants of the π and π', respectively, and g_Q and g'_Q are their coupling constants to the point-like quarks comprising the currents. In particular

$$\mathscr{F}_\pi(0) = \mathscr{F}_\pi(\mu^2)\left\{1 + \frac{F'}{F_\pi}\,\frac{g'_Q}{g_Q}\right\} \tag{21}$$

In writing (21), we have furthermore suppressed any possible dependence of the π' contribution on the masses of the point-like quarks circulating around the loop in Fig. 1. For massive quarks, this dependence is very weak; the correction to the coupling strength g'_Q is easily evaluated as $\left(1 - \frac{1}{12}\,\mu'^2/m_Q^2\right)$ to leading order for small mass ratios $\mu'^2/m_Q^2 < 1$. For the rest of this section, we shall ignore possible corrections of this type due to finite quark mass. In a renormalizable perturbation expansion, all such corrections to the ratio g'_Q/g_Q coming from this loop, or its higher order insertions including off-mass shell corrections to the assumed γ^5 coupling of the π', will vanish with the ratio $\left(\mu'^2/m_Q^2\right) \longrightarrow 0$.

Figure 4 shows the simplest example of a class of graphs contributing to the correction factor is (21).

We can remove the unknown ratio F'/F_π from (21) in terms of the $K \approx 8\%$ correction to the G-T relation in (14):

$$\begin{aligned} \mathscr{F}_\pi(0) &= \mathscr{F}_\pi(\mu^2)\left[1 + K(g/g_Q)\left(g'_Q/g'(\mu'^2)\right)\right] \\ &\approx \mathscr{F}_\pi(\mu^2)\left[1 - 0.08(g/g_Q)\left(g'_Q/g'(\mu'^2)\right)\right] \end{aligned} \tag{22}$$

To determine the size of the correction term in (22), and through it, the accuracy of the PCAC smoothness assumption for $\pi^0 \longrightarrow 2\gamma$, we must now discuss the hadronic couplings of the π and π' in the divergence D.

According to the theory of the form factors for composite particles developed in Ref. (24), as illustrated by Fig. 5, the ratio

$$g/g_Q = C_J\, \mathscr{I}(\mu^2) \tag{23}$$

where C_J is the appropriate Clebsch-Gordan coefficient and $\mathscr{I}(\mu^2)$ is an overlap integral between the initial and final wave functions when the hadron absorbs mass μ^2; i.e., it is just the form factor. The same overlap $\mathscr{I}(\mu^2)$ should apply for the axial as well as electromagnetic form factors in this model with point constituents since it is a property of the bound state wave functions and not of the elementary vertex. Likewise

$$\frac{g'(\mu'^2)}{g'_Q} = C_J \mathcal{I}(\mu'^2) \tag{24}$$

where C_J is the same coefficient, combining the same quantum numbers, and $\mathcal{I}(\mu'^2)$ is the same overlap integral but for an absorbed mass of μ'^2. Inserting (23) and (24) into (22) gives

$$\begin{aligned}\mathscr{F}_\pi(0) &= \mathscr{F}_\pi(\mu^2)\left[1 + K\,\mathcal{I}(\mu^2)/\mathcal{I}(\mu'^2)\right] \\ &\approx \mathscr{F}_\pi(\mu^2)\left[1 - 0.08\,\mathcal{I}(\mu^2)/\mathcal{I}(\mu'^2)\right]\end{aligned} \tag{25}$$

Equation (25) is the central result of this analysis. Qualitatively it tells us that the PCAC correction in $\pi^0 \to 2\gamma$ decay as computed here in terms of the PCAC anomaly differs from that in the G-T relation by a ratio of overlap integrals expressing the absence of hadronic structure functions.

According to the model of the hadron described above, the measured electromagnetic form factors should give a good clue to the dependence of $\mathcal{I}(q^2)$ on q^2. In particular, the recently reported measurement[25] on $e\bar{e} \to p\bar{p}$ near threshold at $q^2 = 4M^2$ yields $G_E(4M^2) \cong \frac{1}{4}$ so that

$$\mathcal{I}(4M^2) \cong \frac{1}{4\,[1 + 1.79]} \approx \frac{1}{11} \tag{26}$$

if the Dirac and Pauli form factors (as well as their isovector and isoscalar parts) are comparable, i.e., if $F_1(4M^2) \approx F_2(4M^2)$.

In contrast, if $F_2/F_1 \longrightarrow 0$ at $q^2 \approx 4M^2$,

$$\mathscr{J}(4M^2) = \frac{1}{4} \tag{27}$$

Since we expect $\mathscr{J}(\mu^2) \approx \mathscr{J}(0) = 1$, we conclude that the correction to PCAC in (25) for $\pi^0 \longrightarrow 2\gamma$ decay will be strongly enhanced if the effective π' mass is $\mu' \sim 1.5 - 2$ GeV. Comparing with (1), we see that a value of

$$\mathscr{J}(\mu'^2) \approx 2/17 \tag{28}$$

removes the discrepancy. If the dipole parameterization of the form factors, which fits the measurements for space-like momentum transfers, is applied here for time-like momenta near the $p\bar{p}$ production threshold region, it provides a rough fit to (26)

$$\left(\frac{.71\ \mathrm{GeV}^2}{q^2 - .71\ \mathrm{GeV}^2}\right)^2 \approx 1/16 \text{ at } q^2 = 4M^2 \tag{29}$$

With a mass $\mu' \approx 1.65$ GeV, the dipole form (29) gives the desired enhancement factor of 17/2.

Although we have no means of making an independent calculation of $\mathscr{F}_\pi(0)/\mathscr{F}_\pi(\mu^2)$ in terms of a known effective mass of the π', we do have a simple, specific, physical mechanism to show the quantitative failure of the smoothness assumption of PCAC for $\pi^0 \longrightarrow 2\gamma$ decay. The key point is that there is no small overlap factor in (21) to damp the high mass or π' contributions as there was in the G-T relation. Moreover (26), (28), and (29) show that an enhancement at only moderate masses in the π' channel is required to remove the discrepancy in (1). In fact, too large a mass leading to

$$\mathscr{J}(\mu'^2) < -K \sim 0.08$$

would reverse the sign of the amplitude (25) in conflict with independent analysis from other processes, in particular the Primakoff amplitude for π^0 photoproduction.[26]

Equation (21), together with (1) or (28), gives for the ratio of π' to π decay constants

$$F'/F_{\pi} = -0.7\ g_Q/g'_Q \tag{30}$$

If the π and π' have hadronic couplings of comparable magnitudes, then their decay constants will also be comparable. This is in sharp contrast to a "strong" PCAC theory that attributes the breaking of chiral symmetry to the finite pion mass,[27] so that $D^+ \propto \mu^2$, and in (9) and (20)

$$F'/F_{\pi} \sim 0(\mu^2/\mu'^2) \sim 1\% \tag{31}$$

In order to satisfy (30) with such a small ratio of F'/F_{π} would require a correspondingly huge ratio of g'_Q/g_Q which would itself be puzzling. Comparable decay constants lead to a relatively large $\pi^{o'}$ decay rate

$$\tau^{-1}_{\pi^{o'}\rightarrow 2\gamma} = \left|F_{\pi}/F'\right|^2 (\mu'/\mu)^3\ \tau^{-1}_{\pi^{o}\rightarrow 2\gamma} \approx 10\ \text{keV} \tag{32}$$

for $F'/F_{\pi} \sim 1$. This is roughly ten times larger than the $\eta(549) \longrightarrow 2\gamma$ decay rate and well below the level to cause an observable bump in the $\gamma\gamma$ cross section (as can be studied via the two-photon exchange contribution to $e\bar{e}$ scattering with colliding rings).[28]

There is no difficulty in our approach to finding the missing 8% in the G-T relation, (14), and the difficulty discussed by Pagels[23] disappears. This is because we have formally introduced a very large amount of chiral symmetry breaking into the operator D. The smoothness hypothesis of PCAC, and through it the G-T relation, is then recovered by appealing to dynamical arguments—to some, aesthetically less satisfying—based on the extended structure of physical hadrons.

The two-pole approximation of D as a light π plus a massive π' which has been used thus far, together with a specific Bethe-Salpeter bound-state hadron model, has allowed us to exhibit simply and concretely in (25) the difference between applying PCAC to the $\pi^o \to 2\gamma$ amplitude and to the G-T relation. We can, however, arrive at the same qualitative conclusion by applying the same physical ideas in more general terms. The spectral weight function in (7) is given by a sum of products of matrix elements for D to form real physical states, with the quantum numbers of the pion and with mass $\sqrt{\sigma^2} > 3\mu$, which then form a proton-antineutron pair, i.e.,

$$\rho(\sigma^2) = \sum_{\mathscr{A}} (2\pi)^4 \delta^4\left(P_{\mathscr{A}} - q_\sigma\right) \langle p\bar{n}^{(-)}|\mathscr{A}^{(+)}\rangle\langle\mathscr{A}^{(+)}|D^+|0\rangle \tag{33}$$

The σ^2 dependence of ρ will then be determined by the masses of those states $|\mathscr{A}\rangle$ that couple strongly both to D and to the $|p\bar{n}\rangle$ pair in the $^1S_0$ state; and even though the former coupling may be strong extending over a very broad mass range, we expect the latter to decrease rapidly with increasing $\sigma^2 > 1\ \mathrm{GeV}^2$ in analogy with the experimentally observed decrease of the electromagnetic form factors as described by (26) and (27). This suppression of high σ^2 contributions is illustrated schematically in Fig. 6, where it is contrasted with the corresponding contribution to the $\pi^o \to 2\gamma$ matrix element. For the $\pi^o \to 2\gamma$ decay, using PCAC and the theory of the anomaly at $q^2 = 0$, the states $|\mathscr{A}\rangle$ must couple to D and to a $q\bar{q}$ (or any other point-like constituent pair such as the bare quanta of a canonical field theory model) and no high σ suppression due to hadronic form factors is anticipated.[29] Qualitatively, the continuum corrections will then be much larger than in (33). Consequently, it will be a poor approximation to neglect them entirely in making the pion-pole dominance assumption when applying PCAC to (17), in contrast to (11). Equations (7) and (9), with the parameters discussed above as in (30), show that the continuum is, in general, for $|q^2| \gg \mu^2$, much more important than the pion-pole term with its μ^2 suppression factor.

Our analysis has assumed the correctness of the theory of the PCAC anomaly. This theory is valuable as the only working tool we have for computing the $\pi^o \longrightarrow 2\gamma$ decay rate from Fig. (1) which is exact, free of all higher order insertions and radiative corrections, in the $q^2 \longrightarrow 0$ limit. In this approach, all corrections to the calculated rate are concentrated in the PCAC extrapolation which we have calibrated against the known G-T correction and the observed behavior of hadronic structure functions. The more ambitious proposal of Brandt and Preparata[20] is to abandon the PCAC anomaly approach. They propose to take advantage of the smooth structure of the physical (composite) pion on its mass shell at $q^2 = \mu^2$ in calculating $\pi^o \longrightarrow 2\gamma$ decay, rather than introducing the divergence of the local axial current and, with it, the singularities leading to the PCAC anomaly. Although technically different, both approaches proceed from the same physical assumption that the structure of physical hadrons, and not the algebraic structure of the operators, is at the basis of the successes of PCAC.

Within our more restricted framework, we must still address the questions of what happens to the other successes of PCAC and to this we now turn.

III. APPLICATIONS OF PCAC

A. Soft Pion Results for the πN Amplitude

Two very important and impressive successes of PCAC are the Adler-Weisberger sum rule,[11] which also makes use of the Gell-Mann current commutation relations for axial charges, and the Adler consistency condition.[12] Both[30] can be derived as low energy theorems by soft pion techniques. A calculation of the contributions of the off-shell corrections to D—i.e., the π'—to these processes requires that we extend the study of Section II from vertex functions to scattering amplitudes. This extension, in turn, requires us to accept several assumptions presently included in the folklore of hadron scattering[31] amplitudes and apply them to the π' as well as the π interactions. In particular, the highly successful two-component Harari-Freund[32] duality model, according to which the absorptive part of the π-N amplitude can be written as the sum of two parts, viz., s-channel resonances plus a uniform background arising from t-channel Pomeron exchange, will be adopted and applied to the π' as well as the π interaction.

The invariant πN scattering amplitude for $\pi^a(q) + P_i \longrightarrow \pi^b(k) + P_f$ is written

$$\mathcal{M}(P_i, q, a; P_f, k, b) = -\sqrt{2k_0} \lim_{q^2 \to \mu^2} \left\{ \frac{\mu^2 - q^2}{\sqrt{2}\, F_\pi \mu^2} \langle P_f, \pi^b(k)^{(-)} | D^a(0) | P_i \rangle \right\} \tag{34a}$$

$$= -i \lim_{q^2, k^2 \to \mu^2} \left\{ \frac{(\mu^2 - k^2)(\mu^2 - q^2)}{\left(\sqrt{2}\, F_\pi \mu^2\right)^2} \int d^4x\, e^{ik\cdot x} \langle P_f | T(D^b(x), D^a(0)) | P_i \rangle \right\} \tag{34b}$$

$$= \bar{u}(P_f) \left\{ \left[A^{\pi N(-)}(\nu, t, \mu^{i2}, \mu^{f2}) + \not{q}\, B^{\pi N(-)} \right] \frac{1}{2} \left[\tau^a, \tau^b \right] + \delta^{ab} \left[A^{\pi N(+)} + \not{q}\, B^{\pi N(+)} \right] \right\} u(P_i) \tag{34c}$$

where the odd and even, under crossing, invariant amplitudes are defined in the standard notation and are considered as functions of the initial and final pion masses as well as of the energy and momentum transfer variables

$$\begin{aligned} \nu &\equiv (P_i + P_f) \cdot q/2M \\ t &= (k - q)^2 \end{aligned} \tag{35}$$

We must now investigate the corrections to pion pole dominance if we take the limit of one soft pion, $q_\mu \longrightarrow 0$ instead of the pole in (34a), or if we take the two-soft-pion limit, q_μ and $k_\mu \longrightarrow 0$ instead of the double pole in (34b).

The Adler consistency condition derived in the one-soft-pion limit for the even amplitude is

$$A^{\pi N(+)}(0, \mu^2, 0, \mu^2) = \left(g^2/M\right) K^{\pi NN}(0) \tag{36}$$

where $K^{\pi NN}(0)$ is the form factor at the π-N vertex at $q^2 = 0$: $g(0) \equiv g K^{\pi NN}(0)$. Dispersion theory accomplishes the extrapolation of the π-N amplitude to the unphysical energy below threshold in (36) for on-shell mesons and gives $A^{\pi N(+)}(0, 0, \mu^2, \mu^2)$. The problem for us here is how smooth is the required extrapolation to zero pion mass in (36). In Adler's original analysis,[12] the extrapolation

$$\Delta \equiv \frac{A^{\pi N(+)}(0, \mu^2, 0, \mu^2)}{K^{\pi NN}(0)} - A^{\pi N(+)}(0, 0, \mu^2, \mu^2) \tag{37}$$

was shown in a specific model to give corrections of only a few percent ($\approx 2\%$) and the agreement of (36) with experiment is to within 10%. The model used by Adler was based on the observation that the dominant contribution to the low-energy limit $A^{\pi N(+)}(0, 0, \mu^2, \mu^2)$ comes from the(3,3) resonance. In fact, in a narrow resonance approximation and in the static limit, the (3,3) resonance contributes 8/9ths of the right-hand side of (36) on the mass shell to $A^{\pi N(+)}(0, 0, \mu^2, \mu^2)$. Therefore, he reasoned that this same model could be used to a good approximation to evaluate the off-shell extrapolation (37):

$$\Delta \approx \frac{2}{\pi} \int_{\mu(1+\mu/2M)}^{\infty} \frac{d\nu'}{\nu'} \mathcal{I}m \left[\frac{A_{33}^{\pi N(+)}(\nu', \mu^2, 0, \mu^2)}{K^{\pi NN}(0)} - A_{33}^{\pi N(+)}(\nu', 0, \mu^2, \mu^2) \right] \quad (38)$$

We now ask what happens to this good agreement on the basis of our analysis. We expect, according to the discussion of the previous section, large corrections to PCAC in the absence of hadronic structure functions to damp the high q^2 contributions in the D-channel. Such damping occurs at the nucleon vertex in the G-T relation. However, the calculation of $A^{\pi N(+)}(0, \mu^2, 0, \mu^2)$ from (34a) involves the extrapolation in q^2 to be applied to a scattering amplitude that grows with increasing energy. We, therefore, must understand corrections to PCAC when we take the limit

$$\lim_{q_\mu \to 0} \left\{ \left(\frac{\mu^2 - q^2}{\sqrt{2}\, F_\pi \mu^2} \right) \langle P_f, \pi_{(k)}^{(b)(-)} | D^a | P_i \rangle \right\} \quad (39)$$

including the high-energy limiting behavior that gives rise to a subtraction term, as well as evaluating the dispersion integral in ν' including π-N states lying in the resonance region.

An important point here is Adler's observation that the dominant contribution to (36) comes from the low lying resonance region, and in particular $\approx 8/9$ths of the right-hand side of (36) comes from the (3,3) resonance alone. We denote this by Δ_R. The contribution of the ν channel resonances to the dispersion integral for Δ can be pictured as in Fig. 7.

The extrapolation of these contributions from the pion mass shell to $q_\mu \to 0$ introduces a vertex function for $N+(D) \to R$. This is similar to the G-T relation except that here the vertex function measures the overlap of the ground state nucleon with a composite resonance state. The measured form factors for exciting the low-lying resonances are, aside from threshold factors reflecting their spins, very similar[33] in their high q^2 behavior to the nucleon dipole form factor, (29). Therefore, a similar correction to the -8% found for the G-T relation should also apply to Δ_R as characteristic of the accuracy of the PCAC extrapolation at these vertices. To the extent that these resonances are the dominant contribution to the evaluation of the left-hand side of (36), the accuracy of the Adler consistency condition should be comparable to the G-T relation.

When we turn to the non-resonant contributions to Δ which are lodged in the high-energy behavior and the subtraction constant in the dispersion relation for (37), we cannot make as reliable an estimate. Let us first estimate an upper limit on these added contributions, and then refer to high-energy data for a more reasonable estimate. In either case, the added correction lies within the usual 10% tolerances for application of PCAC.

As a reasonable upper limit, we propose to perform the extrapolation to $q_\mu \to 0$ in (37) for $(\Delta - \Delta_R)$ by removing all suppression factors due to hadron structure and assuming the divergence D couples to point-like structures. There will be two classes of graphs as illustrated in Fig. 8, the one corresponding to high-lying s-channel resonances (whose average contributions are summarized in P', A_2, etc., trajectories with positive intercepts), and the other corresponding to pomeron ($\mathscr{P}$) exchange which incorporates the high-energy limiting behavior of $A^{\pi N(+)}$, as contained in the subtraction constant.

If the product of the coupling and decay constants of the π' is comparable to that of the π, as was the case in the coupling to "quarks" in (30), there could be anything from a doubling to a cancelling of the π contribution by the π' contribution in D, depending on their relative phases. However, we are talking here of less than 10% of the total contribution to (36) according to Adler's analysis, so that even this correction is of little significance at our level of accuracy in these considerations at present.

For a much stronger limit on the high-energy contribution, we turn directly to experimental data on inelastic diffraction scattering of high-energy pions by nuclear targets. This process measures the amplitude for

$$\pi(0^-) + (Z, A) \longrightarrow (0^-, 1^+, 2^-, \ldots.) + (Z, A) \tag{40}$$

via Pomeron exchange. In principle, the final state can be analyzed in terms of the individual quantum numbers in order to separate out the $\pi'(0^-)$ contribution. From the published analyses, the cross section for inelastic diffraction production from 60-GeV π's is substantially smaller than the elastic cross section[34]

$$\sigma_{\substack{\text{inel}\\ \text{diff}}} \sim 1 \text{ mb} < \frac{1}{5}\,\sigma_{\text{el}} \quad \text{per nucleon}$$

Furthermore, the observed cross section leads predominantly to 1^+ and/or 2^- final states in (40), with the A_1 resonance particularly prominent. The production of a π' (0^-) is considerably smaller and at most no more than a small fraction of the elastic cross section for π's.

We conclude that the π' contribution to the high-energy behavior of $A^{\pi N(+)}$ is small and was considerably overestimated by the earlier discussion. In any event, independent of theoretical conjectures as to the Pomeron's form factors or structure, or of the overlap of π and π' wave functions, the π' has little effect on the Adler consistency condition.

There is evidently room here for much more extensive and accurate numerical work, not only for the consistency condition for the πN amplitude, but also for those involving the strange particles. Our main qualitative result here has been to demonstrate that a large correction to the operator statement of pion pole dominance, which in our theoretical framework was required to remedy (1), does not disturb the Adler consistency condition along with the G-T relation.

Turning next to the Adler-Weisberger relation,[11] we consider the extrapolation away from the double pole in the matrix element on the right-hand side of (34b) to $q^\mu, k^\mu \longrightarrow 0$. Following the by-now standard procedure, one considers the S-wave amplitude, odd under-crossing, and keeps only first-order terms in the $k, q \longrightarrow 0$ limit.[30] Partially integrating the derivatives in D off of the axial currents and onto the exponential and time-ordering operator in taking this limit leaves only the equal time commutator of the axial charges, or by Gell-Mann, the isovector charge of the proton. All other terms vanish in this limit, there being no s-channel pole terms; and the equal-time commutator of the time component of the axial current with the divergence of the axial current, or the so-called σ-term, appears only in the δ_{ab} term even under crossing. The result is

$$\tilde{a}_{1/2} - \tilde{a}_{3/2} = \frac{3\mu}{4\pi F_\pi^2} \tag{41}$$

where the $\tilde{a}_I$ are the zero energy scattering lengths in the I = 1/2 and 3/2 isospin channels for a divergence

$$\left\{\frac{1}{\sqrt{2}\, F_\pi \mu^2}\, D\right\}$$

incident on a nucleon.

To give physical content to (41), we must relate the $\tilde{a}_I$ to the corresponding π-N scattering lengths a_I defined by (34) on the mass shell $q^2 = k^2 = \mu^2$. This we do by taking advantage of the good convergence property of the odd πN amplitude which satisfies an unsubtracted dispersion[35] relation in ν. The dominant contributions by $a_{1/2} - a_{3/2}$ come from the πN pole term and the first few s-channel resonances below a mass $m_{res} \sim 2$ GeV. The contributions of the latter to the dispersion integral can be represented as in Fig. 9. Since we are here once again computing vertex functions the extrapolation of these contributions to $q, k \longrightarrow 0$ from the pion mass shell can be made as we have already described for Δ_R in the Adler consistency condition. The overlap integrals for both the nucleon pole and low-lying resonance contributions will have the same or comparable numerical values as in the correction to the G-T relation. Therefore, we conclude that the success of the Adler-Weisberger relation to the same 8% accuracy characteristic of the G-T relation is assured by the composite "soft" structure of the nucleon and its low-lying resonances.

In this section, we have arrived at the same conclusions as Brandt and Preparata[20] in their applications of weak PCAC to these processes, and for basically the same reason. We have computed processes in which π' is suppressed because of the structure of the hadron and the low-lying resonances at the important vertices where the π' is interacting. Therefore, for these processes, the world looks approximately chiral for dynamical reasons.

B. Additional Soft Pion Results

A full discussion of various other applications of soft pion techniques and the smoothness assumption of PCAC can be found in the works of Brandt and Preparata.[20] Here we are concerned only with those points specifically relevant to the composite model which is the basis of this paper.

The role of the π' will be suppressed, as in the G-T relation, in those amplitudes for low-energy photo- and electroproduction, and for the π-π scattering lengths that are dominated by low-lying resonance contributions; and this covers most of the predictions.

The relation of the axial form factors for $K_{\ell 4}$ decay to $K_{\ell 3}$ decay so beautifully predicted by PCAC[13,36] is not altered since the π' must enter a vertex as illustrated in Fig. 10, and the large mass it brings will lead to a small overlap of the initial K and final π wave functions. This is analogous to the nucleon overlap in the G-T relation; the axial current of the $(\ell \nu)$ pair emerges with relatively low momenta only and therefore has little effect on this overlap. However, the anomaly contribution to the vector form factors can be affected by the π' as in the $\pi^0 \longrightarrow 2\gamma$ decay[37] and this is now under study.

Similarly, the relation of $K_{\ell 3}$ decay itself to $K_{\ell 2}$ decay, like that of $\pi_{\ell 3}$ to $\pi_{\ell 2}$ decay, should be in accord[38] with PCAC and little altered by the presence of the π' because the presence of the initial K or π wave functions in these two cases will suppress the π' contribution. There is no anomaly contributing to these amplitudes.

The σ-term in πN scattering, which has received much recent attention and some controversy as to how big it is, is defined by the $q^\mu, k^\mu \longrightarrow 0$ soft pion limit of the even amplitude in (34b). Since the value of σ depends on the value of the subtraction constant in the forward dispersion relation in the energy variable for this amplitude, and we have no firm theoretical basis for performing an off-mass shell extrapolation to the $q^2 = k^2 = 0$ point for this constant,[39] there is nothing we can say here as to how the π' contributes to σ. This circumstance is in sharp contrast to our discussion of the Adler consistency condition (36) which, as we saw, was determined very largely by low-lying resonance contributions.

IV. HIGH ENERGY NEUTRINO CROSS SECTIONS IN THE FORWARD DIRECTION

In this section, we discuss a test of our hypothesis that D is strongly coupled to hadrons through the π', i.e., off-the-mass shell, as well as through the π on the mass shell. This test is provided by the Adler proposal[40] for measuring forward inelastic scattering of high-energy neutrinos (ν or $\bar{\nu}$) from nucleons (N):

$$\nu + N \longrightarrow \ell + \text{anything} \tag{42}$$

When the lepton mass is neglected, the transition current $\left(\bar{\nu}\gamma^{\lambda}(1-\gamma^5)\ell\right)$ is proportional to the momentum transfer $q^{\lambda} = \left(k(\nu) - k(\ell)\right)^{\lambda}$ for forward inelastic scattering. Therefore, if we invoke CVC to remove the vector part of the weak hadronic interaction, we find

$$q_{\lambda} \left\langle \alpha \left| J_V^{i\lambda}(x) + J_A^{i\lambda}(x) \right| N \right\rangle = i \left\langle \alpha \left| D^i \right| N \right\rangle \tag{43}$$

PCAC identifies the right-hand side of (43) with the amplitude

$$\pi^i + N \longrightarrow \text{anything} \tag{44}$$

for an off-shell pion of mass q^2. The underlying assumption in making this identification is that there is a smooth extrapolation of the πN cross section from the pion mass shell in (44) to a small space like mass q^2 in (43). In our theory, we must evaluate this extrapolation in terms of the π' contribution and must therefore add to (44)

$$\pi^{i'} + N \longrightarrow \text{anything} \tag{45}$$

The importance of presenting such a test of our π' hypothesis is clear. So far, we have shown that none of the important successes of PCAC is disturbed by the π'. However, at this point, the π' is playing a somewhat academic role — providing an excuse for the factor $\frac{1}{10}$ in (1) but hiding its tracks elsewhere. Clear and independent evidence of the π' would be important.

In search of a measurable process to provide such a test, we are led to turn to high-energy reactions like (42) in order to escape the suppression effects on the vertices with π' 's when they interact only with extended structures like the nucleon ground state and low-lying resonances. In the high-energy regime, we are once again faced, as we were in the resonance region, with the need to make specific assumptions, or models, in discussing strong interaction dynamics for the π'. Therefore, a decisive test that is completely independent of any additional dynamical assumptions cannot be constructed. In particular, an extension of our ideas on the π' interaction with low-lying hadronic states to apply to the π' interaction with the pomeron, $\mathscr{P}$, is required in order to construct a relation between (42), and (44) and (45). We have already introduced this topic in discussing the Adler consistency condition in Section III, but there we needed only to show that an upper limit could be given that did not disturb the good agreement of (36). Here we must develop a more complete picture on which to base a new and hopefully observable prediction of a direct effect of the π'.

It is known from measured total πN cross sections that the $\pi\pi\mathscr{P}$ vertex is large for forward scattering. In contrast, as we argued from experiment, below (40), the off-diagonal amplitude $\pi\pi'\mathscr{P}$ is small, reflecting, in our theory, the poor overlap of the wave functions of the light (140 MeV) π with the massive (~ 1.6 GeV) π' states when their vertex is weakly disturbed by the $\mathscr{P}$—i.e., for high s, low t forward scattering. It is natural to assume that the diagonal $\pi'\pi'\mathscr{P}$ vertex is also large, since the overlap between initial and final wave functions is again good. Furthermore, there is no apparent diminution of the cross section for massive external particles and PP and KP total cross sections are comparable to the πP. Therefore, as our first assumption, for high energy scattering of D by a nucleon we propose that the cross sections are given by the sum of the diagonal π and π' cross sections and that the $\pi\pi'$ interference can be neglected, as illustrated in Fig. 11, In contrast to the reduction in $\pi^0 \longrightarrow 2\gamma$ decay by a factor of $\frac{1}{10}$, due to interference between the π and π' vertices with the point-like quarks, we predict that the π' will increase the neutrino cross section above the Adler prediction relating (42) and (44) via pion pole dominance. The numerical value of this increase depends on the relative coupling strengths of the π and π' to hadrons, as we discuss shortly. Adler's result can be written[40]

$$\left(\frac{d\sigma}{d\Omega_\ell}\right)_{0^o} = \frac{2}{\pi^2} \int_{M+\mu}^{\sqrt{2M\ E_\nu + M^2}} \frac{dW}{M}\left(\frac{W}{M}\right)^2 \left[\frac{2ME_\nu - W^2 + M^2}{W^2 - M^2 + \mu^2}\right]^2 \left\{G M^2 (0.9\, F_\pi)/\mu\right\}^2$$

$$\cdot \left(\frac{\mu}{M}\right)^2 \frac{|C_\pi|^2}{M} \mathcal{I}m\, f_{\pi N}\, (W, 0^o) \tag{46}$$

where $\left(\frac{d\sigma}{d\Omega_\ell}\right)_{0^o}$ is the forward differential cross section (42) for leptons on nucleons N, E_ν is the incident neutrino laboratory energy, W is the total invariant mass of the final hadron system produced in (44) by an on-shell pion, $G = 10^{-5}/M^2$ is the Fermi constant, the factor 0.9 introduces the correction to the G-T relation for $F_\pi = 0.96\mu$, and $\mathcal{I}m\, f_{\pi N}\, (W, 0^o)$ denotes the imaginary part of the forward πN elastic scattering amplitude at total energy W. It is expressible in terms of the total πN cross section through the optical theorem

$$\mathcal{I}m\, f_{\pi N}\, (W, 0^o) = \frac{q_{lab}}{4\pi}\, \sigma^{\pi N}(W) \tag{47}$$

The off-mass shell extrapolation of the pion mass from $q^2 = \mu^2$ to the small space-like mass

$$q^2 = \left(k(\nu) - k(\ell)\right)^2 = -m_\ell^2 \left(\frac{W^2 - M^2 + \mu^2}{2ME_\nu - W^2 + M^2}\right) \tag{48}$$

is contained in the kinematical factor

$$C_\pi(\mu^2, q^2) = \frac{\mu^2}{\mu^2 - q^2} \quad . \tag{49}$$

For large W resulting from very inelastic collisions, it is clear that we are here studying PCAC for high-energy π' 's. The soft pion results discussed earlier were sensitive to high-energy π' 's only through the dispersion integrals or subtraction constants. Here we can adjust the experimental conditions, in principle, to measure directly the amplitude for large W and thereby study directly corrections to (46) due to high-energy π' 's above the low-energy and resonance region where their effects are suppressed by form factors.

We incorporate the contribution of the π' into (46) using (47) as follows:

$$(0.9\, C_\pi F_\pi)^2 \mathcal{I}m\, f_{\pi N}(W,0^o) = (0.9\, F_\pi)^2 \frac{q_{lab}}{4\pi} \sum_n{}' \left| \frac{\mu^2}{\mu^2 - q^2} A(\pi N \to \mathcal{N}_n) \right|^2$$

$$\Rightarrow \frac{q_{lab}}{4\pi} \sum_n{}' \left| F_\pi \frac{\mu^2}{\mu^2 - q^2} A(\pi N \to \mathcal{N}_n) + F' \frac{\mu'^2}{\mu'^2 - q^2} A(\pi' N \to \mathcal{N}_n) \right|^2 \tag{50}$$

where A denotes the inelastic scattering amplitude to all energy-conserving states $|\mathcal{N}\rangle$, as summed over by $\sum_n{}'$, and the other notation is as before. Equation (50) gives the explicit form of the extrapolation of the pion off the mass shell to the mass q^2 in (48) according to our theory. Neglecting the very small correction to $\frac{\mu'^2}{\mu'^2 - q^2} \approx 1$, we now replace (46) by

$$\left(\frac{d\sigma}{d\Omega_\ell}\right)_{0^0} = \frac{1}{2\pi^3} \int_{M+\mu}^{\sqrt{2ME_\nu + M^2}} \frac{dW}{M} \left(\frac{W}{M}\right)^2 \left[\frac{2ME_\nu - W^2 + M^2}{W^2 - M^2 + \mu^2}\right]^2 \left\{GM^2\right\}^2 \left(\frac{\mu}{M}\right)^2 \frac{q_{lab}}{M}$$

$$\cdot \frac{1}{\mu^2} \sum_n{}' \left| F_\pi C_\pi A(\pi N \longrightarrow \mathscr{N}_n) + F' A(\pi' N \longrightarrow \mathscr{N}_n) \right|^2 \quad . \tag{51}$$

For W in the resonance region, the predominant contribution to A comes from S-channel resonances and the bracket in (51) will reduce to the usual PCAC prediction

$$\text{i.e.,}\ F' A(\pi' N \longrightarrow \mathscr{N}_{Res}) \simeq -0.1\, F_\pi A(\pi N \longrightarrow \mathscr{N}_{Res}) \quad . \tag{52}$$

In this way, the structure of the hadron reestablishes the 0.9 factor for the G-T correction.

For high energies W above the resonance, we are concerned with the uniform background in the πN cross sections which, according to the Harari-Freund two-component duality theory, comes from Pomeron exchange and must be added to the resonances in writing the imaginary part of the amplitude. This contribution of the t-channel Pomeron exchange dominates the πp interaction at large W, i.e., for the very inelastic processes. We also expect this to be true for the π' amplitude and adopt this as an added assumption. With the neglect of the $\pi\pi'\mathscr{P}$ interference, for reasons already discussed, we have in (51)

$$\left\{ \sum_n{}' \left| F_\pi C_\pi A(\pi N \longrightarrow \mathscr{N}_N) + F' A(\pi' N \longrightarrow \mathscr{N}_n) \right|^2 \right\} \tag{53}$$

$$\underset{\text{large W}}{\Rightarrow} \left\{ \left| F_\pi C_\pi \right|^2 \sigma_{\pi N}(\infty) + |F'|^2 \sigma_{\pi' N}(\infty) \right\}$$

To give a specific value to the correction in (51), we need to know the ratio

$$R_1 = |F'|^2 \sigma_{\pi' N}(\infty) / F_\pi^{\ 2} \sigma_{\pi N}(\infty) \tag{54}$$

whereas what we have deduced already from the $\pi^0 \longrightarrow 2\gamma$ decay rate in (30) is the ratio

$$R_2 = \left(F' g'_Q / F_\pi g_Q\right)^2 \approx 1/2 \ . \tag{55}$$

To the extent that the elementary couplings are comparable, and therefore, as for example in a multiperipheral model, so are the $W \rightarrow \infty$ cross section ratios, we <u>conjecture</u>

$$R_1 \approx R_2 \approx 1/2 \ . \tag{56}$$

In principle, colliding beam experiments on two-photon exchange processes $e\bar{e} \rightarrow e\bar{e}(n\pi)$ can give information in the ratio F'/F_π via (32) and measured contributions to the $\gamma\gamma$ cross sections. In terms of (56), we can summarize our results for (51) as a correction to the Adler prediction:

$$\left(\frac{d\sigma}{d\Omega_\ell}\right)_{0^0} = \left(\frac{d\sigma}{d\Omega_\ell}\right)_{0^0}^{\text{Adler}} + \frac{1}{2\pi^3} \int_{\approx 2.5\,M}^{\sqrt{2ME_\nu + M^2}} \frac{dW}{M} \left(\frac{W}{M}\right)^2 \left[\frac{2ME_\nu - W^2 + M^2}{W^2 - M^2 + \mu^2}\right]^2 \left\{GM^2 F_\pi/\mu\right\}^2$$

$$\cdot \left(\frac{\mu}{M}\right)^2 \frac{q_{\text{lab}}}{M} \left\{\left(1 - (0.9)^2\right) |C_\pi|^2 + R_1\right\} \sigma_{\pi N}(\infty) \ . \tag{57}$$

The lower limit of the integral in (57) represents a rough dividing line between the region dominated by s-channel resonances and the region dominated by the Pomeron exchange.

Equation (57) is more likely a lower limit on the correction to Adler since we have arrived at it by dividing the πN amplitude into two contributions: the low-lying resonances plus the background or Pomeron exchange. There are also the higher resonances whose average contribution to the absorptive amplitude manifests itself as Regge exchanges with positive intercepts—viz., the $\mathscr{P}'$, A_2, etc. It is possible that for these higher excitations, the contribution of the π' will behave more like (53) than (52) in which case they, too, will enhance (57).

We have made an approximate evaluation of the size of this correction to the Adler prediction in (57). For 50-GeV neutrinos, roughly one-half of the cross section comes from the region $W > 2.5M$ and this translates into a 25% additive correction to the PCAC prediction according to the ratio in (56). Such an added contribution may best be identified by looking for energy, angle, charge, or perhaps polarization differences between pion-induced and the forward-neutrino-induced cross sections when W is varied and increases to values $W > 2.5$ M. Further detailed analysis into the practical possibility of exploiting the unique feature of this cross section as a probe of PCAC for very energetic, almost real pions, as opposed to soft ones in the $q^{\mu} \longrightarrow 0$ limit, is in progress.[41]

V. CONCLUSIONS

In this paper, we have demonstrated and exploited the difference between applying PCAC to $\pi^0 \rightarrow 2\gamma$ decay and to hadronic amplitudes in which it has enjoyed its notable successes. Our essential dynamical assumption is that pion pole dominance is valid only for those matrix elements of the divergence of the axial current taken with composite hadronic states. As illustrated by our specific composite model of the hadron with point-like constituents, it follows from this underlying physical assumption that the factor of 10 discrepancy in (1) between calculated and observed $\pi^0 \rightarrow 2\gamma$ rates cannot, per se, be used as evidence to discard the original quark model with one triplet of fractionally charged quarks.

In contrast to our dynamical model, field theoretic models of PCAC generally attribute the breaking of chiral symmetry to the small but non-zero pion mass.[42] Such theories identify the divergence of the axial current with the canonical pion field and contain no large corrections to the PCAC soft pion extrapolation in calculating $\pi^0 \rightarrow 2\gamma$ decay. However, we have argued on dynamical grounds that the O^-D channel contains much more than the pion pole. Indeed, as seen in (7), the pion pole term is strongly suppressed by μ^2. The much larger continuum contribution in (7), or the π' in (9), which is not suppressed by μ^2, provides the -8% correction to the G-T relation and removes the dilemma of (1). If our model is right, Section IV shows that a substantial correction to the PCAC prediction for forward angle, high-energy, very inelastic, neutrino-hadron cross sections should be observed. This will be a crucial test of our theory.

Two principal criticisms can be made of the specific model we have employed in our calculations:

(1) On the basis of the Bethe-Salpeter ladder model, we have identified the constituent quarks with the current quarks. Although mathematically

well defined, this model provides a limited and highly artificial physical picture of a hadron.

(2) The PCAC hypothesis has been simply adjoined to this composite hadronic model and has not been <u>derived</u> as a consistent consequence of it.

However independent of our particular motivation of preserving a naive 3-quark picture, or of our specific composite dynamical hadron model, the general point remains to be emphasized, as in the discussion below (20), that there is a qualitative difference between applying PCAC to smooth hadronic amplitudes, such as in the G-T relation, and applying it to a singular one involving local currents as in $\pi^0 \rightarrow 2\gamma$ decay. In this essential point, we are agreeing with the idea of "weak PCAC" discussed by Brandt and Preparata.[20]

Crewther[43] has recently derived a relation, based on the short-distance expansion of Wilson,[44] between the size of the high-energy asymptotic $e\bar{e}$ total annihilation cross section to hadrons and the $\pi^0 \rightarrow 2\gamma$ decay rate. The ratio is dependent on the charges and the number of different kinds of quarks appearing in the currents. An input of his in deriving this relation is PCAC and the extrapolation to the soft pion limit for $\pi^0 \rightarrow 2\gamma$ decay. It follows, however, from our work that, while formally correct, this relation cannot be used for physical comparisons of data since PCAC fails quantitatively when applied to $\pi^0 \rightarrow 2\gamma$ decay.

The implications of all this are not yet clear for the quark model. The data on the high energy $e\bar{e}$ annihilation, while showing large values for this cross section, do not yet determine its functional behavior with energy.[45] We do not yet know whether the predicted high energy limiting behavior of $\sigma \sim \frac{1}{E^2}$ has been reached and as of now, we do not know the Crewther ratio. On the theoretical end, there are still fundamental problems in constructing dynamical models for bound quarks that do not escape from one another and cannot be seen.[46] What is clear is that the most successful calculation of $\pi^0 \longrightarrow 2\gamma$ is still the first and simplest one in lowest order perturbation theory by Steinberger,[47] who calculated Fig. 1 with a circulating "bare proton." He also first revealed the anomaly though it was not so appreciated for a long time.[48]

Acknowledgments

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$$\frac{a_{1/2} - a_{3/2}}{3\mu} = \frac{2 f_r^2}{\mu^2} - \frac{1}{4\pi^2} P \int_{\mu}^{\infty} \frac{d\omega'}{k'} \left\{ \sigma_{tot}^{\pi^+ p}(\omega') - \sigma_{tot}^{\pi^- p}(\omega') \right\}$$

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Figure Captions

1. Triangle diagram for $\pi^o \longrightarrow 2\gamma$ decay.

2. Contributions to the dispersion relation (7) for $\mathscr{D}(q^2)$.

3. Contribution of a bound state to the triangle graph for $\pi^o \longrightarrow 2\gamma$ decay.

4. Example of correction to $\pi^o \longrightarrow 2\gamma$ decay in Eq. (21).

5. Diagram for the form factor of a composite particle in the Bethe-Salpeter ladder model.

6. Schematic picture of the mass dependence of the weight functions for the G-T relation as contrasted with $\pi^o \longrightarrow 2\gamma$ decay.

7. Resonance contribution to the absorptive part of Eq. (37).

8. Contributions to (37) from high-lying resonances, as summarized by P', A_2, etc., trajectories with positive intercepts, and from the Pomeron ($\mathscr{P}$) exchange.

9. Resonance contribution to the absorptive part of the dispersion integral for the absorptive part of (41) with both pions at zero mass.

10. Graph showing application of PCAC to $K_{\ell 4}$ to $K_{\ell 3}$ ratio.

11. Contributions to high-energy forward scattering of a divergence D by a nucleon.

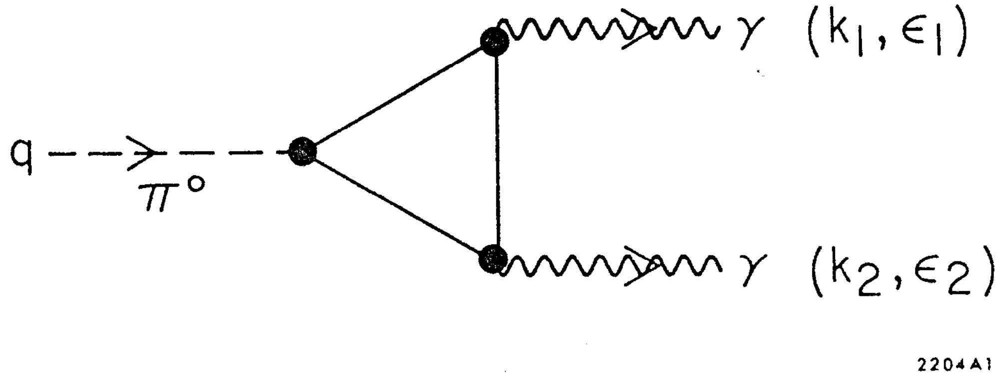


Fig. 1

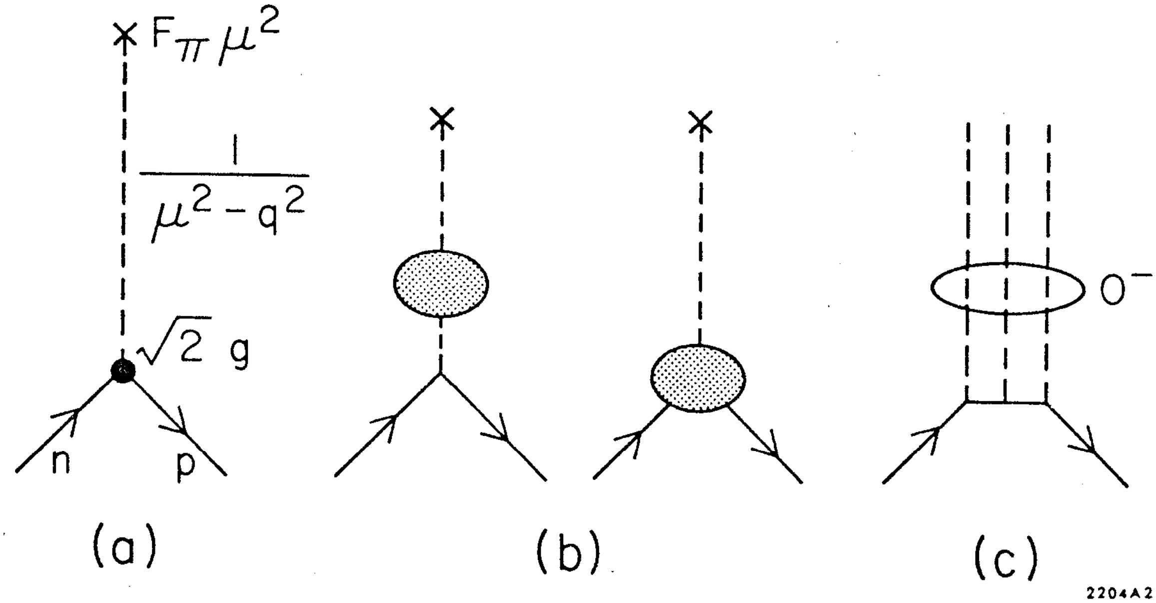


Fig. 2

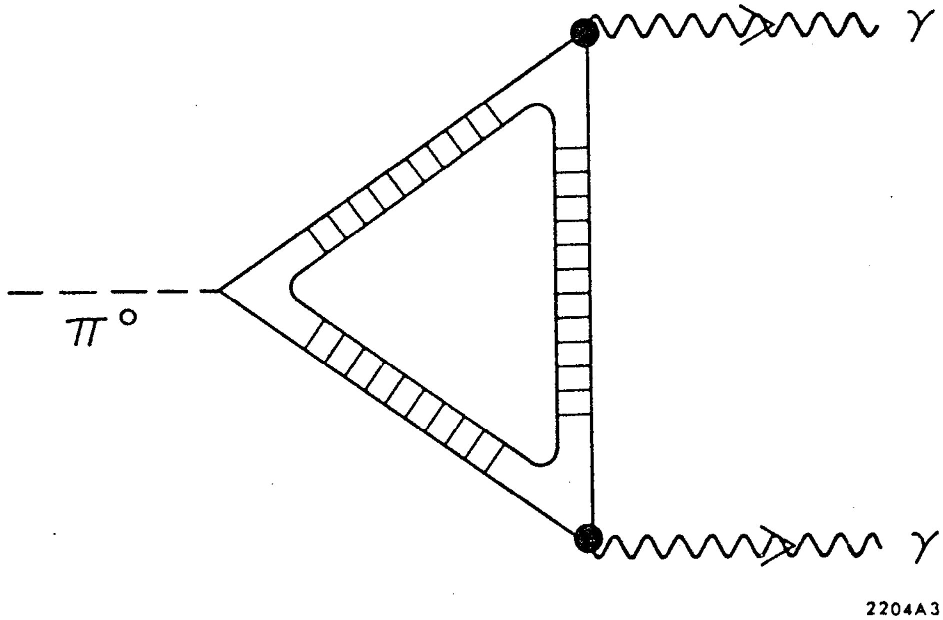


Fig. 3

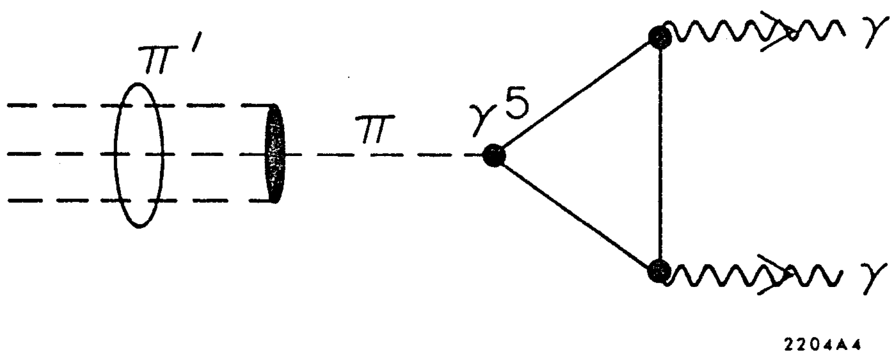


Fig. 4

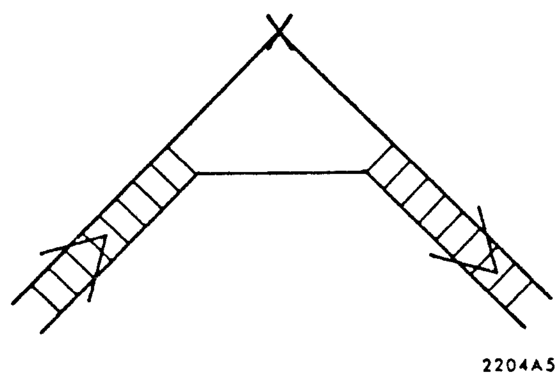
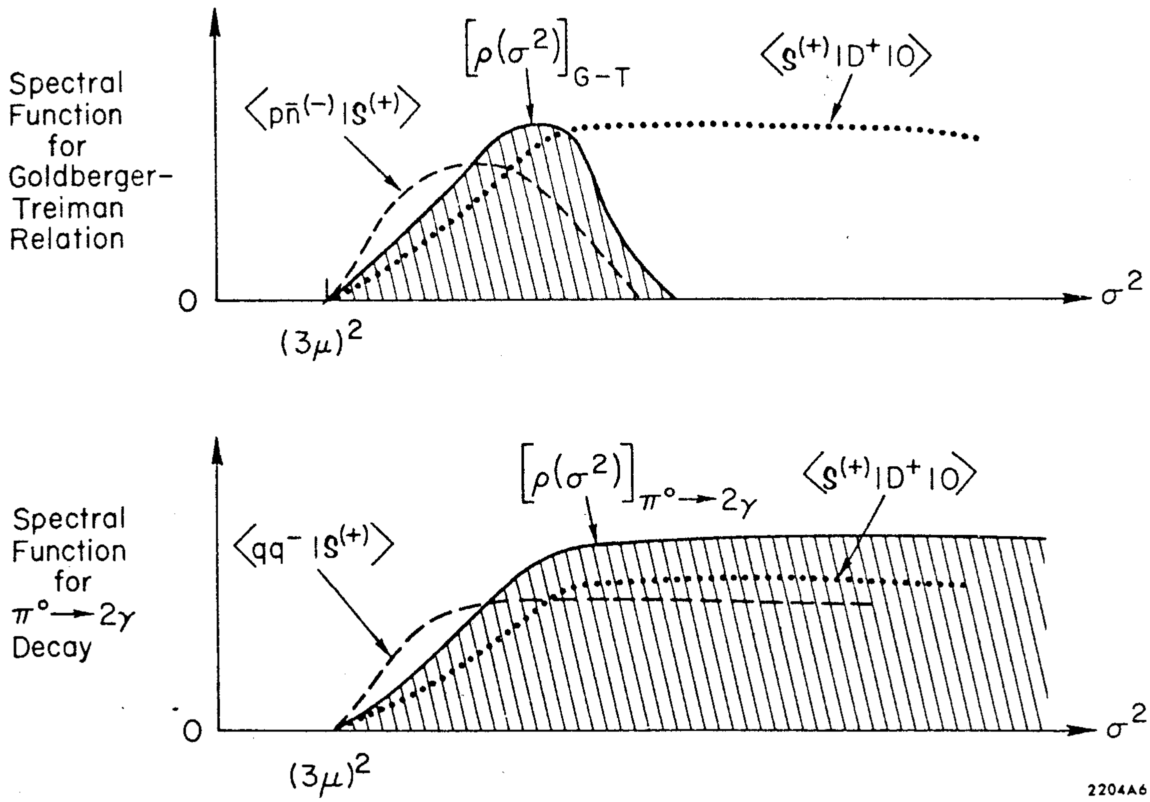


Fig. 5



Spectral Function for Goldberger-Treiman Relation
$\langle p\bar{n}^{(-)}|S^{(+)}\rangle$
$[\rho(\sigma^2)]_{G-T}$
$\langle S^{(+)}|D^+|0\rangle$
0
$(3\mu)^2$
σ^2
Spectral Function for $\pi^\circ \rightarrow 2\gamma$ Decay
$\langle qq^-|S^{(+)}\rangle$
$[\rho(\sigma^2)]_{\pi^\circ\rightarrow 2\gamma}$
$\langle S^{(+)}|D^+|0\rangle$
0
$(3\mu)^2$
σ^2
2204A6

Fig. 6

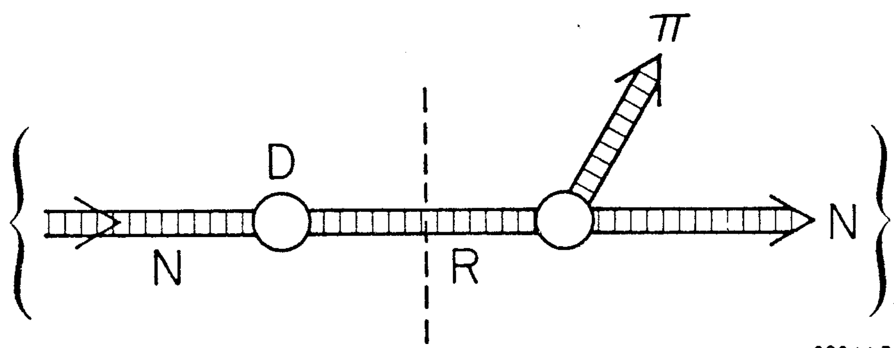
$\mathrm{Abs}\left\{\Delta_R\right\} =$ 

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Fig. 7

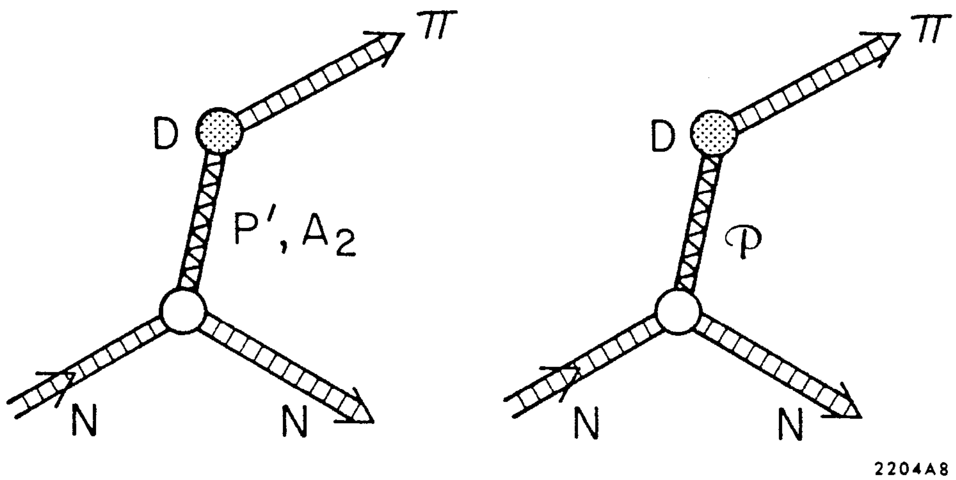

Fig. 8

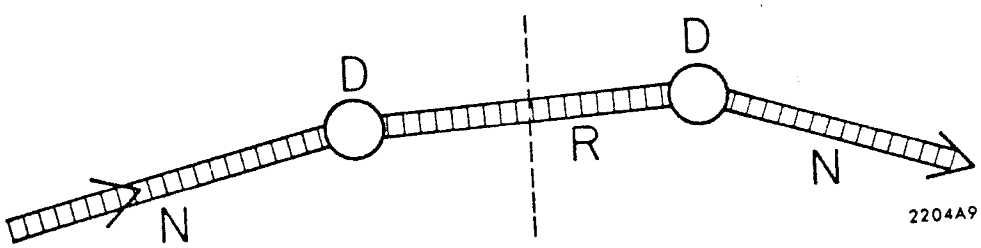

Fig. 9

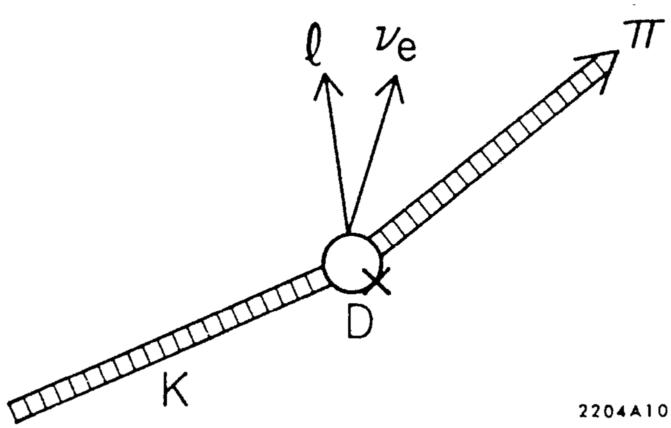

Fig. 10

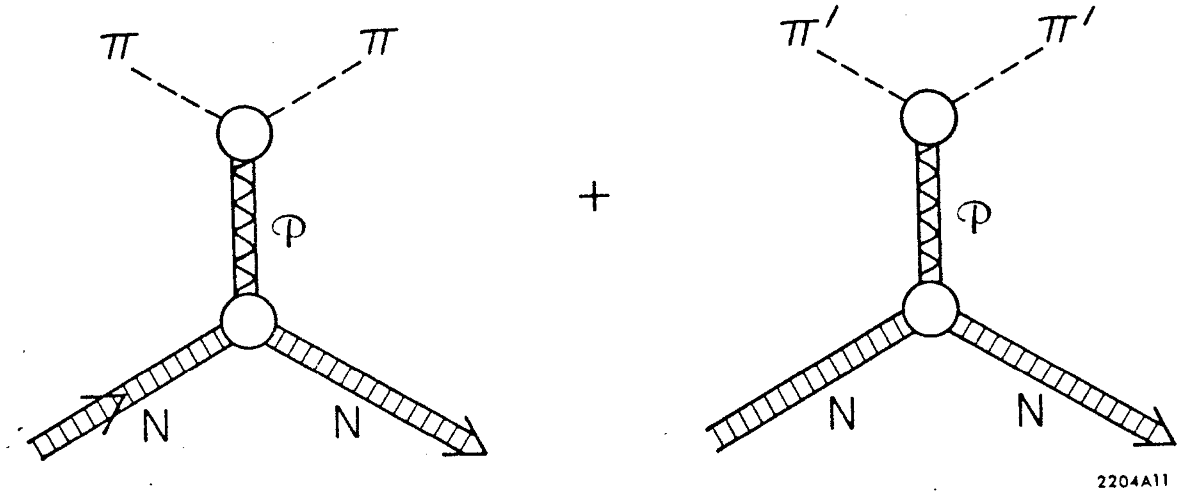

Fig. 11