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RECENT THEORETICAL WORK ON e^+e^- ANNIHILATION AND CONTINUATION FROM INELASTIC ELECTRON SCATTERING *

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Two lines of theoretical developments have emerged as a result of the observed scaling behavior in DIES (deep inelastic electron scattering). One is the parton model and the other is the light-cone algebra; and we ask what are their implications for the e^+e^- colliding beam cross sections that we've been learning about.

The parton model, with its point-like constituents of the proton which scatter incoherently in very inelastic electromagnetic or weak interactions at high energies, leads to Bjorken scaling for energy and momentum transfers exceeding ≈ 1 GeV. This result defines the mass scale for partons as well as a dimension of roughly 10^{-14} cm for point-like behavior. It remains for the future—hopefully, the very near future—to tell whether a further refinement of scale to dimensions $\approx 10^{-15}$ cm will reveal deviations from scaling, perhaps due to a parton structure resulting from its gluon cloud—i.e., the radiation and self-reaction effects associated with the exchange of the quanta or gluons binding the partons in the proton. This is the way all other scaling laws have broken when probed on higher resolution scales, and such deviations should be kept in mind no matter what formal approach is adopted. If we ignore this possibility, the

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parton model predicts

$$\sigma_{e\bar{e} \to \text{hadrons}} = N\left(\frac{4\pi\alpha^2}{3s}\right) \text{ for } s > M_p^2 \tag{1}$$

with

$$N = \sum Q^2_{J=1/2} + 1/4 \sum Q^2_{J=0}$$

We must always face, in this approach, the embarrassing question of where are these constituents if they really exist and if they are light enough to be produced so that we have precocious scaling as in DIES.

To avoid this embarrassment or paradox and construct a more general basis for understanding scaling, it is desirable to cast off the strict literal interpretation of a parton model and abstract just those general features being probed. This is the idea of the light-cone approach.

What we learned from DIES and the observation of scaling is that the singularities of a current commutator near the light cone are canonical—i.e., the same as in free field theory, since what is measured is the Fourier transform of this singularity.

Turning to $e\bar{e}$ annihilation, the object under study is

$$\sigma_{e\bar{e} \to \text{hadrons}} = \frac{4\pi\alpha^2}{3s}\,\rho(s)$$

with

$$\rho(s) = \frac{12\pi^2}{s} \int d^4x\, e^{i\sqrt{s}\,t} \langle 0|[J_\perp(x), J_\perp(0)]|0\rangle \tag{2}$$

and

$$0\left(\frac{1}{\sqrt{s}}\right) \sim t > |\vec{x}|\text{, i.e., the tip of the light cone.}$$

Again one is near the light cone at high energies and is measuring a Fourier transform of a singularity in the commutator which, if it is canonical, $\longrightarrow$ scaling—i.e.,

$$\rho(s) \sim \text{const}$$

and

$$\sigma_{e^+e^- \longrightarrow h} = \left(\frac{4\pi\alpha^2}{3s}\right) N$$

This result was first given by Bjorken[1] in 1966 from a sum rule in terms of equal time commutators and by Gribov, Ioffe, and Pomeranchuk[2] in terms of canonical dimensions of the Schwinger term for hadrons. Presumably, this behavior sets in when no large masses are around to impede the approach to the light cone, and it is important to keep in mind that it is not yet clear when that will be. In fact, it could be that the large numbers observed for (1) reflect individual ρ', ρ'', resonances, and that scaling is yet to be found at higher energies.

To go further and get the <u>number</u> N in (1), we must postulate the algebraic structure of the current operators in detail. It is necessary to make use not only of the symmetry properties of the algebra, but to treat the current operator as actually factorizable into products of quark operators. Technically, this is necessary because we only know how to get a value for N by using the propagator sum rule for a quark propagator—i.e., the result that asymptotically for small distances or high momenta, the quark propagator approaches a free propagator. It is clear that with these literal manipulations, we also open the possibility of creating single quarks as indicated schematically when we use closure to insert a complete set of states, i.e.,

$$\langle 0|q^\dagger(x)\,q(0)|0\rangle = \sum_{\substack{\text{quark}\\ \text{states } S}} \langle 0|q^\dagger(x)|S\rangle\langle S|q(0)|0\rangle \qquad (3)$$

Their propagator vanishes and so does the annihilation cross section below the quark production threshold. This is a murky business of how to avoid creating the quarks. If the model nucleon consists of 3 Fermi-Dirac quarks, $N = 2/3$. But then one needs some very high potential wall to confine the individual quarks and avoid single quark production. This requires an inquiry into possible dynamical origins of such a wall, as have been discussed by K. Johnson[3] recently. Another way to prevent quarks from emerging is to describe them by parastatistics of rank 3, with a physical restriction that all physical particles are bosons or fermions so individual quarks never appear; or equivalently to assume, as Gell-Mann has recently proposed,[4] that there are three colors of quarks, and to restrict all physical states to be singlets in the SU3 of color. Then a current operator with the structure

$$J \sim \bar{q}_R q_R + \bar{q}_W q_W + \bar{q}_B q_B \,, \quad \text{i.e., a color singlet,} \tag{4}$$

forms only singlet states from the vacuum. In this case, $N = 2$, and the problem of incorporating into a quark-gluon field theory model the restriction that all physical states are color singlets has not been solved. The full current operator[5] contributes in (2), and so $N = 2$, even though we rule out the possibility of creating higher states of color away from the light cone, as illustrated in Fig. 1.

Although it is without any internal inconsistencies, the light cone approach is murky with regard to quark production, and we would like to avoid literal treatment of currents as a factored operator when abstracting algebraic relations from field theory models. It is still a challenge to fix N without introducing quark states. Recently Crewther[6] has shown how to construct a relation of N to the decay rate for $\pi^0 \longrightarrow 2\gamma$ without explicitly introducing the quark states. Using PCAC in the soft pion limit $\mu_\pi \longrightarrow 0$, one needs for calculating $\pi^0 \longrightarrow 2\gamma$

$$\langle 0 | J_\alpha (x) J_\beta (y) \partial_\lambda J^5_\lambda (z) | 0 \rangle \,. \tag{5}$$

In the zero frequency limit of PCAC, it is the light-cone behavior of the current products in (5) that determines the decay rate as shown by K. Wilson.[7] Since we can go to the light cone in several independent ways among the three space-time variables, but must get consistent results no matter which directions we choose, there are constraints on the form of singularities. These constraints force a relation of the connected parts of operator products in (5) with the disconnected one needed in (2) for $\sigma_{e\bar{e} \quad hadrons}$; and their constant of proportionality is given by the symmetry properties of the current algebra. This is as shown by Bardeen, Fritzsch, and Gell-Mann.[5] In his work, Crewther[6] first derived this connection on the basis of the short-distance algebra, plus an assumption of the conformal symmetry of the world.

In concluding this discussion, I have two comments:

1. Accepting the measured decay rate for $\pi^o \rightarrow 2\gamma$, we obtain from the Crewther relation N = 2 if the currents satisfy an underlying SU3 quark algebra. If we enlarge the symmetry group to SU4 to accommodate "charm," $N = 3\frac{1}{3}$. The added "charmed" quark has 2/3 charge, and 0 isotopic spin and strangeness. It is desired, for example, in the Weinberg theory with neutral currents[8] to suppress $K_\ell^o \rightarrow \mu\bar{\mu}$ to observed levels.

2. The application of PCAC to singular products of local currents as in (5) for $\pi^o \rightarrow 2\gamma$ may not be as accurate as in its demonstrated successes for soft pion theorems in which the divergence of the axial current interacts with an extended composite hadron. The PCAC extrapolation for $\pi^o \rightarrow 2\gamma$ decay has been questioned[9] in a version of weak PCAC that preserves all other successes. In this case, one cannot compute N from Crewther's relation but must return to the factorized form in (3).

We turn next to inclusive cross sections. The question discussed in contributions by Gatto, Menotti, and Vendramin,[10] and Gatto and Preparata[11] is what, if any, is the general connection between the structure functions in DIES

$$e^- p \longrightarrow e^- + \text{anything:} \qquad W_1,\ \nu W_2 \qquad \omega = \frac{2\,P\cdot q}{Q^2} > 1$$

$$e^- e^+ \longrightarrow p + \text{anything:} \qquad \overline{W}_1,\ \nu \overline{W}_2 \qquad \omega < 1$$

In several models, a cut-off Yukawa field theory[12] and a multiperipheral ladder model in which stable particles propagate,[13] it has been shown that the $\overline{W}$ scale and can be determined by a simple analytic continuation

$$\begin{aligned} \overline{W}_1(\omega) &= -W_1(\omega) \\ \nu\,\overline{W}_2(\omega) &= +\nu\, W_2(\omega) \end{aligned} \qquad (6)$$

This has also been shown to be true for the leading term as $(\omega - 1) \longrightarrow 0$ in a Bethe-Salpeter ladder model for the bound state.[14] But it is not a general result.[15] The problem is that no longer are we dealing with a simple commutator of currents. Added terms appear in the commutator, which contains all four contributions shown in Fig. 2, and their sum actually vanishes for $\omega < 1$ when we have canonical scaling.[16]

With canonical scaling (6) follows from the formal substitution rule interchanging ingoing and outgoing fermions, but the usefulness of these equations lies in the possibility of giving them the character of an analytic continuation—i.e., continuing W from $\omega > 1$ to $\omega < 1$ to determine $\overline{W}$. For practical purposes, the continuation is important near $\omega \sim 1$.

It is not in general easy to accomplish this continuation because the W are squares of moduli of amplitudes and, as such, have not in general the good analytic properties for making such a continuation. So one has to prove under what assumptions the $W(\omega)$ are continuable to $\omega < 1$ and that the $\overline{W}$ are actually the continuations of W.

By studies of single and double box diagrams with stable and unstable particle exchanges, Menotti[15] has reviewed various theories and observed that in general the continuation breaks down for a class of box graphs with unstable internal particle lines as illustrated in Fig. 3.

These produce cuts for $\text{Re}\,\omega \leq 1$. The question of immediate practical importance is what effect these cuts have on the behavior of the structure functions near $\omega = 1$ so that one can make predictions for the annihilation cross sections near $\omega = 1^-$ by continuing the experimentally observed behavior of DIES near $\omega = 1^+$. Menotti showed that, in the Bethe-Salpeter ladder model, there is a cut along the real axis for $\omega \leq 1$ which, in general, interferes with the continuation. However, its contribution is proportional to $(\omega - 1)^5$ near $\omega = 1^-$ and, therefore, is a higher order correction to the leading threshold behavior $(\omega - 1)^3$. He also analyzed the dependence of the cut near $\omega = 1$ on the mass spectrum of the unstable particle propagator in Fig. 3, showing the conditions which allow the continuation to be made for the leading threshold behavior.

A general light-cone analysis (reported in another session in some detail) by Gatto and Preparata[11] has shown that, with canonical dimensions, the scaling of the total cross section (1) implies scaling for the inclusive cross section $e\bar{e} \rightarrow p$ + anything, but does not, in general, lead to scaling as in (6).

Finally, Gribov and Lipatov[17] have completed a study of the behavior of electroproduction structure functions in two field-theory models, summed to all orders of $g^2 \log(Q^2/m^2)$; $g^2 \ll 1$. The theories are neutral pseudoscalar (bare p, π^0) and neutral vector (bare p, ω^0). The important diagrams turn out to be t-channel ladder graphs, in which propagators and vertices are "exact," i.e., computed to all orders of $g^2 \log(Q^2/m^2)$.

The very same results were confirmed recently by Christ, Hasslacher, and Mueller[18] in a less laborious way by studying the Callen-Symanzik equations. For the neutral vector model, they are also contained in studies by Fishbane and Sullivan.[19] Among the conclusions are:

1. $\sigma_S/\sigma_T \approx 0.$

2. $W_1 = W_1(\omega, \xi)$ where

$$\xi = \frac{3}{4} \log\left[1 - \frac{g^2}{12\pi^2} \log \frac{Q^2}{m^2}\right]^{-1}$$

   in the neutral vector model, i.e., it does not scale but grows slowly with Q^2 at fixed ω.

3. A crossing relation

$$\nu\overline{W}_2(\omega, \xi) = -\left(\frac{1}{\omega^3}\right) \nu W_2\left(\frac{1}{\omega}, \xi\right)$$

This last has as its consequence that the multiplicity of protons grows as $\ell n\, Q^2$ if $\nu W_2(\omega) \longrightarrow$ constant for large ω, i.e.,

$$\overline{n}_p = \frac{1}{\sigma} \int_{m/\sqrt{Q^2}}^{1} \frac{d\sigma}{d\omega} \sim \ell n\, Q^2$$

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Figure Captions

Fig. 1. Forbidden production of final states that are not color singlets.

Fig. 2. Amplitudes contributing to the absorptive part of forward Compton scattering for time-like virtual photons.

Fig. 3. Example of contribution to $\overline{W}_{\mu\nu}$ that violates crossing relation (6).

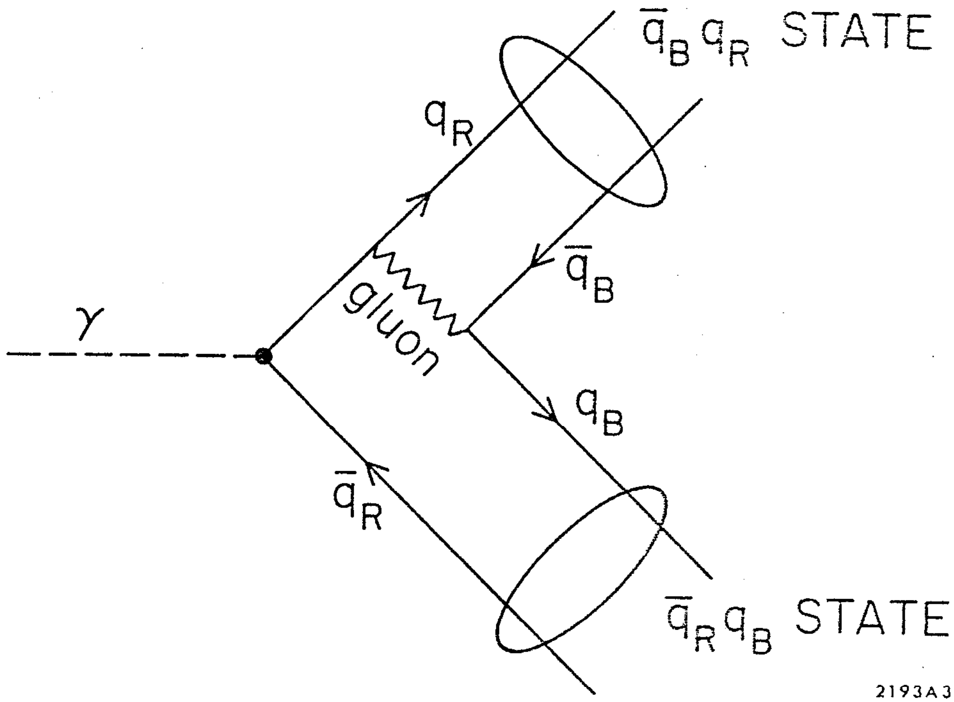
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Fig. 1

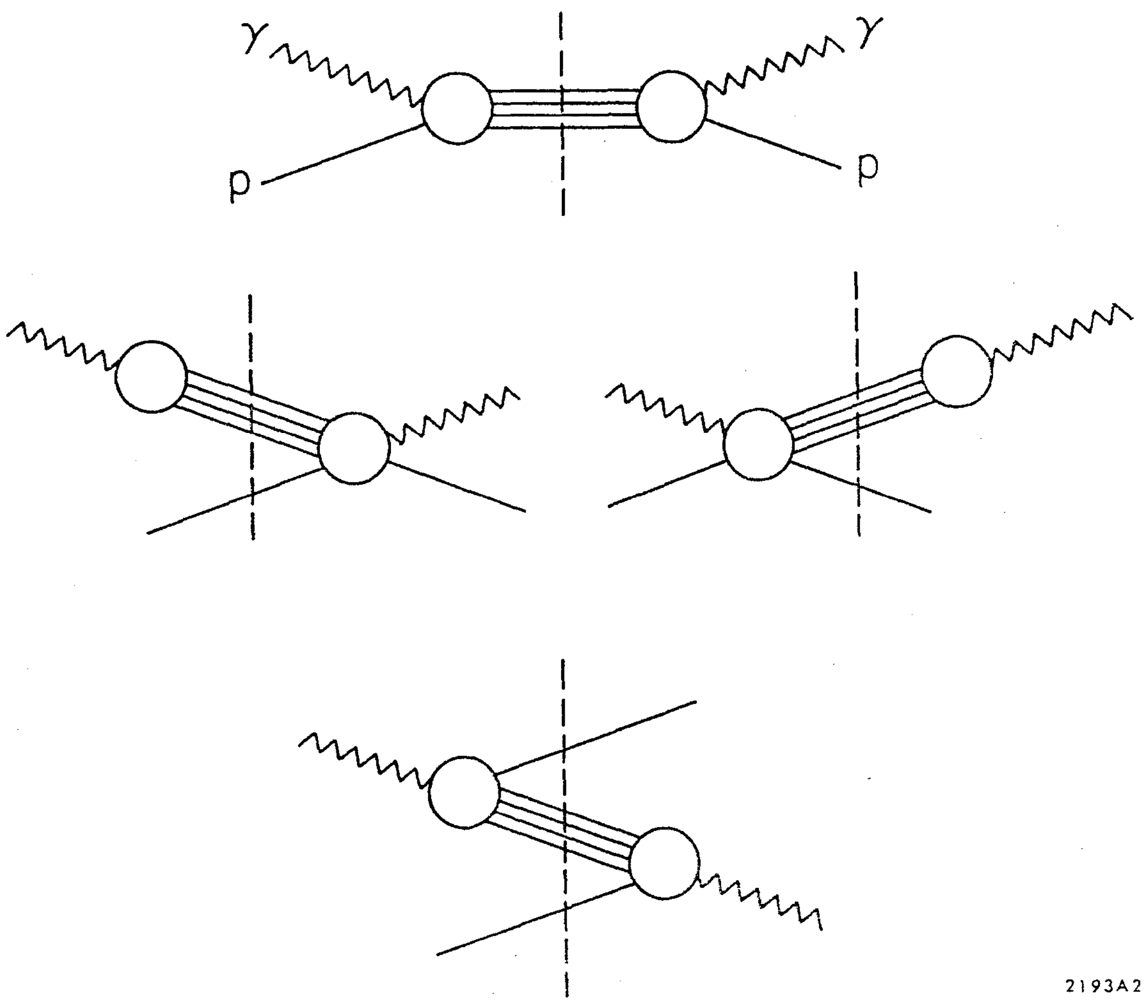

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Fig. 2

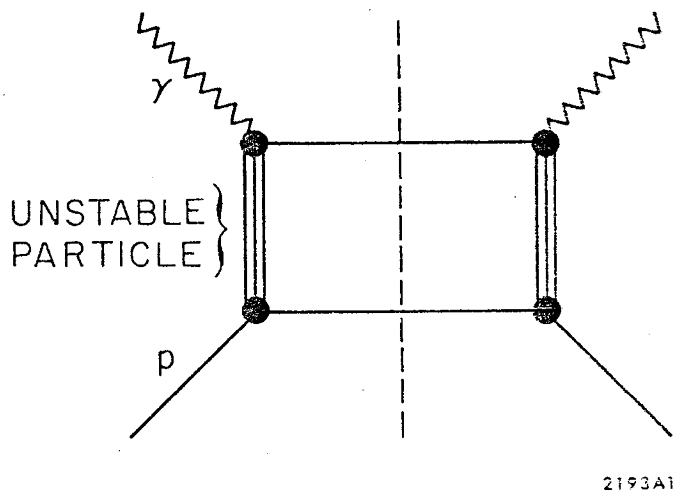

Fig. 3