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MULTIPLICITIES IN LEPTON-HADRON PROCESSES*

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Abstract

The average multiplicity in deep inelastic electro- and neutrinoproduction at large ω ($\omega \sim s/Q^2 + 1$) is related in Feynman's version of the parton model to the average multiplicities in high-energy electron-positron annihilation and hadron-hadron scattering. The relation is:

$$\langle n(s,Q^2)\rangle_{\substack{eP\\ \nu P}} \sim C_{e^+e^-}\, \ell n\left(Q^2/M_{\perp}^2\right) + C_h\, \ell n(\omega - 1)$$

where $C_{e^+e^-}$ and C_h are, respectively, the coefficients of $\ell n\left(s/M_{\perp}^2\right)$ in the multiplicities from $e^+ - e^-$ and P - P into hadrons, and $M_{\perp}$ is an average transverse mass.

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One of the most interesting aspects of Feynman's version of the parton model[1,2] is the intimate relation between all high-energy processes producing hadronic final states. If such a connection exists, the first evidence should come from the multiplicities observed in $e^+ - e^-$, P - P, and e - P (or ν - P) collisions. In particular, the average hadronic multiplicity in lepton-hadron scattering, which depends on two variables (s and Q^2), might be related to the multiplicities in leptonic and/or hadronic colliding beams, which both depend on only one variable (Q^2 or s). For that reason, we have investigated the multiplicity in e - P and ν - P scattering expected in Feynman's model. Before deriving the prediction for the multiplicity and comparing with other models, we first review the basic ideas of the parton approach.

The parton model,[1,3] which is based on the field theory concept that there exists an amplitude to find various numbers and momenta of the basic fields in the hadron, can predict many features of inclusive deep-inelastic current scattering (assuming a point-like coupling of the parton) even though the mechanism by which partons become observable hadrons is obscure. A more ambitious approach such as Feynman's encompasses purely hadronic processes[1] and the observation of hadrons in the final state.[1,2,4-8] In this model, the result of all high-energy collisions can be understood from the parton wave function of the hadron. In a frame where the hadron moves with a large momentum P, the wave function has the property that the probability for finding a parton depends on the transverse momentum $p_\perp$ with a rapid fall-off and for finite z on the fraction z of longitudinal momentum ($z = p_L/P$). Moreover, for partons with finite p_L, the probability is finite and independent of the rapidity y, of the distribution of finite z partons, and of the type of hadron (which guarantees factorization). Finally, two assumptions concern short-range correlation: (i) partons interact only if their rapidity difference is of order one or smaller, and (ii) the distribution of final state hadrons at y depends on the nature of the parton distribution within a finite range of y. Also, of course, the final hadron distribution in a collision must be independent of the frame chosen for the wave functions.

The above picture can be applied to a variety of high-energy processes. In hadron-hadron scattering, the two wave functions join together, smearing near $y = 0$, to produce the standard rapidity plot for the inclusive distribution $\frac{1}{\sigma_{tot}} \frac{d\sigma}{dy}$: two fragmentation regions and one central region (Fig. 1a). The dominant contribution to the multiplicity at high energies comes from the central region:

$$\langle n(s) \rangle_{PP} \sim C_h \, \ell n \left(s/M_{1\perp}^2\right) \tag{1}$$

where C_h is the height of the universal hadronic plateau and $M_{1\perp}$ is an average transverse mass. In $e^+ - e^-$ annihilation, the virtual photon of mass $\sqrt{Q^2}$ produces a parton and anti-parton each of which subsequently fragments into hadrons. Actually, a description more in keeping with the model pictures the unstable rapidity gap between the two initial fast partons filling in with partons, creating a final distribution, which then produces hadrons. The result is a rapidity plot consisting of a central region of length $\ell n \left(Q^2/M_{1\perp}^2\right)$ and two finite fragmentation regions—one each for the parton and anti-parton (Fig. 1b). In the parton fragmentation region,[2,4-7] the distribution scales as a function of $x = \frac{2p_{1L}}{Q}$ (p_{1L} in C.M.) and $p_{1\perp}$, and the quantum numbers on the average are conjectured to be those of the parton.[2] The multiplicity again comes from the neutral central region (flat in rapidity):

$$\langle n(Q^2) \rangle_{e^+e^-} \sim C_{e^+e^-} \, \ell n \left(Q^2/M_{1\perp}^2\right) \tag{2}$$

where $C_{e^+e^-}$ is the height of the universal parton plateau (i.e., the plateau resulting from the fragmentation of a parton).

In deep inelastic e - P (or ν-P) scattering, the picture is especially simple in the Breit frame of the virtual photon and parton[2,6,7] (Fig. 2) where the momenta are:

$$q_\mu = (0,\ 0,\ 0,\ -2\,x\,P) \qquad P_\mu = (E,\ 0,\ 0,\ P)$$

$$Q^2 = -q^2 = 4\,x^2\,P^2 \qquad \omega = \frac{2\,m\,\nu}{Q^2} = \frac{1}{x}$$

$$M\nu = P_\mu\, q^\mu = 2\,x\,P^2 \qquad s = (\omega - 1)\,Q^2 + M^2$$

In this frame, a parton of momentum xP has its momentum reversed after being struck by the space-like photon. This struck parton at $y \sim -\ell n\ 2\,x\,P$ (assuming a mass scale $\sim$ 1 GeV) produces its characteristic finite fragmentation region, scaling in $z = \frac{2\,p_{1L}}{Q}$ (p_{1L} in Breit frame) and $p_{1\perp}$, and central region of height $C_{e^+e^-}$ stretching to $y_{Breit} = 0$. The remaining partons in the proton extend from $y_{Breit} \sim 0$ to $y \sim \ell n\ 2\,P(1-x)$ with a hole for the missing parton at $y \approx \ell n\ 2\,x\,P$. (The relative positions in rapidity of the hole, the parton, and the proton as a function of x for fixed Q^2 are shown in Fig. 3.) The effect on the rapidity plot caused by the absent parton, or hole, has been analyzed by Bjorken[9] for large ω. We give an equivalent analysis in Feynman's language. The partons with rapidity greater than y_{hole} are already in equilibrium and, consequently, produce a plateau of length $\ell n\,(\omega - 1)$ and height C_h extending to the proton fragmentation region. The disturbance or instability caused by the hole produces a finite fragmentation region at y_{hole} as well as a readjustment of the height of the final parton distribution between zero rapidity and y_{hole}. The quantum numbers of the current minus the parton are found on the average in the hole fragmentation region where the final hadron rapidity distribution scales as a function of $z = \frac{2\,p_{1L}}{Q}$ and $p_{1\perp}$. (Because of the equal densities of partons and anti-partons at small x, the average charge in the parton and hole fragmentation regions approaches zero in electroproduction.) In the plateau region, a kink at $y_{Breit} = 0$, i.e., unequal heights to the right and left, is not possible because the final hadron distribution must be independent of the frame chosen for the wave functions. Otherwise, the position of the kink would be frame-dependent. Consequently, the central region for the current is flat in rapidity between the parton and hole fragmentation regions with a height $C_{e^+e^-}$. The resulting invariant rapidity plot is shown in Fig. 4 (corresponding to a slice in Fig. 3 at small x).

The dominant contribution to the multiplicity for large Q^2 and ω comes from the combination of the two central regions:

$$\left\langle n\left(s, Q^2\right)\right\rangle_{\substack{eP \\ \nu P}} \sim C_{e^+e^-} \ln\left(Q^2/M^2_{1\perp}\right) + C_h \ln(\omega - 1) \tag{3}$$

The factorizability of the parton model is the essential ingredient in deriving this result. If only one of ω or Q^2 is large, the multiplicity is effectively given by the larger of the two terms in Eq. (3). If $C_{e^+e^-} = C_h$, as speculated by Feynman[2] and Bjorken,[10] the multiplicity would go as $\ln s$ in all high-energy processes. Within the context of Feynman's model, it seems unlikely that $C_{e^+e^-} \ll C_h$. Otherwise, the effect of the hole on the partons in the proton between $y_{Breit} = 0$ and y_{hole} would, most likely, not be strong enough to reduce the plateau height from C_h to $C_{e^+e^-}$.

If hadronic reactions provide a guide to the length of all fragmentation regions, a kinematical configuration with $\ln \omega \geq 4$ and $\ln Q^2 \geq 4$ would be necessary to check the prediction experimentally, which is not possible with existing machines but would be with PEP (proposed positron-electron-proton colliding beam facility).[11] It is feasible, however, to demonstrate separately the presence of each plateau by taking first large ω and small $Q^2 \sim 1$ GeV^2 to see the hadronic plateau and second small $\omega \sim 3$ and large Q^2 to see the current plateau. Also, an interesting experimental question is the extent to which $\langle n(s, Q^2) \rangle_{\substack{eP \\ \nu P}}$ equals the sum of $\langle n(Q^2) \rangle_{e^+e^-}$ and $\langle n(s \sim \omega M^2_{1\perp}) \rangle_{PP}$ at non-asymptotic values of ω and Q^2. Of course, experiments with $e^+ - e^-$ colliding beams will provide the first test of a current plateau.

The parton model result for the multiplicity may be contrasted with the predictions of other models. Chou, Yang, and others[12-15] argue from "limiting fragmentation" that a substantial component of the multiplicity at small ω comes from "pulverization" of the photon:

$$\langle n(s, Q^2) \rangle_{eP} \propto \nu^{C/\omega}$$

where C is a constant (smaller than 0.5). In the region of large ω, the various authors disagree on the amount of pulverization products in the final state. Reference (15) claims that for large Q^2, the multiplicity is maximal, i.e.,

$$\langle n(s, Q^2) \rangle_{eP} \underset{\text{large } Q^2}{\propto} Q(\omega - 1)^{\frac{1}{2}}$$

while for the Regge region, $s \gg Q^2$, the multiplicity is

$$\langle n(s, Q^2) \rangle_{eP} \underset{s \gg Q^2}{\propto} Q^2 \ell n\, s$$

If present experimental results are any indication, the last prediction is in serious trouble.[16] Multiperipheral models[17,18] predict that the multiplicity scales and is proportional to $\ell n\, \omega$, i.e., the contribution from the current fragmentation region is finite as in some models for e^+e^- annihilation.[19] A general Mueller analysis leaves undetermined the multiplicity in the current fragmentation region since variations in Q^2 are involved.

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Figure Captions

1. (a) Inclusive distribution $\frac{1}{\sigma_{tot}} \frac{d\sigma}{dy}$ versus y for high energy hadron-hadron scattering. $M_{1\perp}$ is an average transverse mass.

   (b) Same for electron-positron annihilation into hadrons.

2. Parton distributions before (a) and after (b) interaction in the Breit frame of the virtual photon and parton.

3. A graph of x versus y_{Breit} for fixed $\ell n\, Q/M_{1\perp} \sim 2$. The various finite fragmentation regions are located as follows: that of the parton borders the left kinematic boundary, that of the hole is centered at the dotted line, and that of the proton borders the right kinematic boundary. Only for $x < .5$ does the hole lie inside the kinematic boundaries.

4. Inclusive distribution $\frac{1}{\sigma_{tot}} \frac{d\sigma}{dy}$ versus y for deep inelastic electro- or neutrinoproduction at large ω. $M_{1\perp}$ is an average transverse mass.

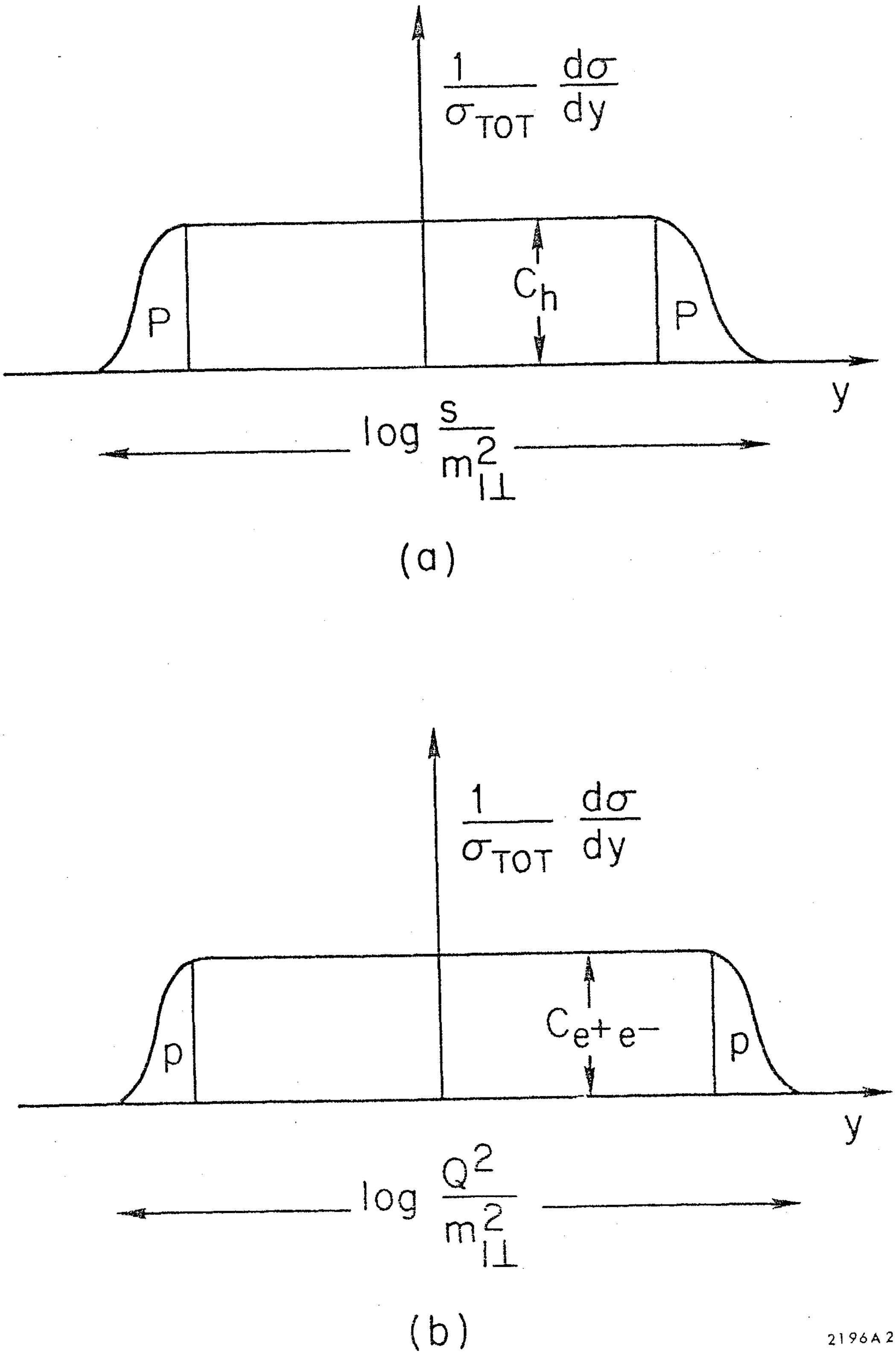

$\frac{1}{\sigma_{TOT}} \frac{d\sigma}{dy}$
C_h
P
P
y
$\log \frac{s}{m_{1\perp}^2}$
(a)
$\frac{1}{\sigma_{TOT}} \frac{d\sigma}{dy}$
$C_{e^+e^-}$
p
p
y
$\log \frac{Q^2}{m_{1\perp}^2}$
(b)
2196A2

Fig. 1

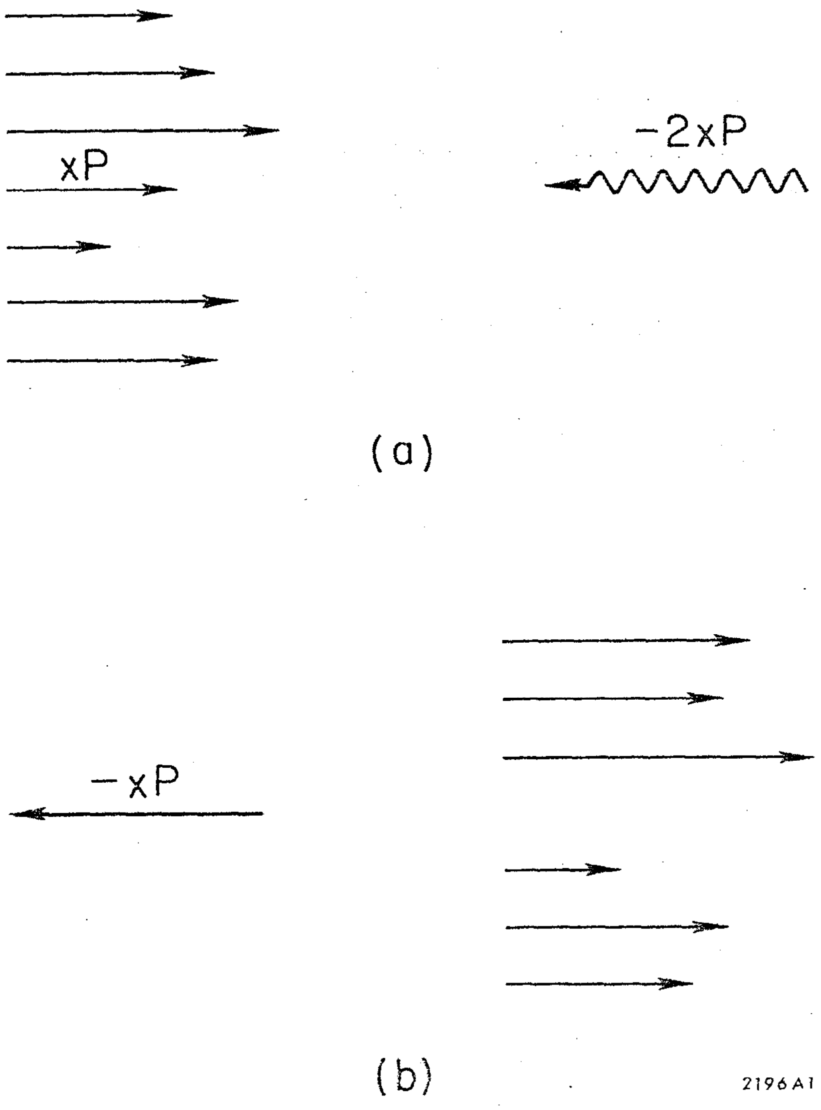
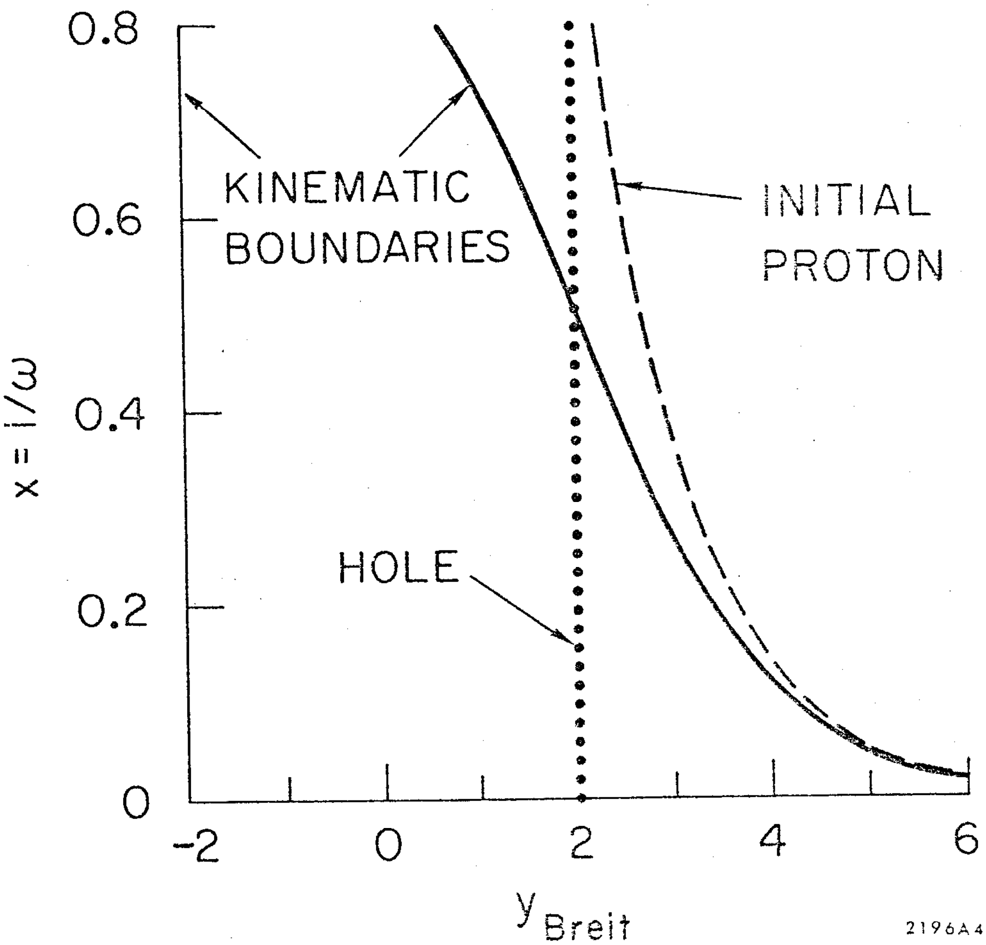
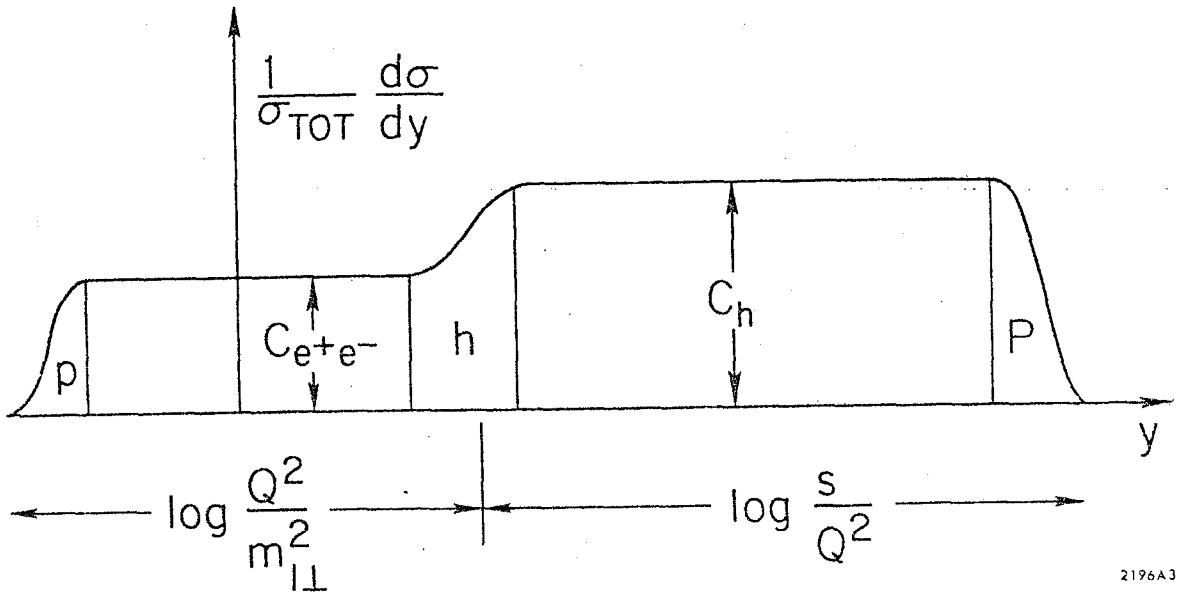


Fig. 2



0.8
0.6
0.4
0.2
0
x = 1/ω
KINEMATIC
BOUNDARIES
INITIAL
PROTON
HOLE
-2
0
2
4
6
y Breit
2196A4

Fig. 3



$\frac{1}{\sigma_{TOT}}\frac{d\sigma}{dy}$
p
$C_{e^+e^-}$
h
C_h
P
y
$\log \frac{Q^2}{m_{1\perp}^2}$
$\log \frac{s}{Q^2}$
2196A3

Fig. 4