SLAC-PUB-1131
(TH) and (EXP)
October 1972

DRELL-YAN SCALING IN SEMI-INCLUSIVE ELECTROPRODUCTION*

E. W. Colglazier

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

and

F. Ravndal

California Institute of Technology
Pasadena, California 91109

Abstract

The scaling behavior of deep inelastic single-particle inclusive electroproduction (or neutrino production) expected from the parton model has been given by Drell and Yan as $\nu^3 \mathscr{W}_2$ when the observed hadron has a finite fraction of the virtual photon's laboratory momentum. Although Drell and Yan give the correct scaling behavior with their choice of variables, we show that with variables more appropriate experimentally, the proper scaling is $\nu^2 \mathscr{W}_2$. Recent data previously thought inconsistent with the parton model are now found to be in agreement, which is especially important since scaling in this kinematic region cannot be predicted from a light-cone analysis.

(Submitted to Comments and Addenda, Phys. Rev.)

*Work supported by the U. S. Atomic Energy Commission.

Single particle inclusive electroproduction (or neutrino production) in the deep inelastic region provides several interesting features: (i) an extra variable (Q^2) not found in hadronic reactions, (ii) a combination of the Bjorken scaling limit with Feynman scaling or limiting fragmentation, and (iii) a kinematic region accessible to parton and not light-cone analysis. In the parton model, the scaling behavior has been derived by Drell and Yan[1] using field theory with a transverse momentum cut-off.

The kinematic variables and structure functions for spin-averaged inelastic electron scattering with the detection of the final electron plus one hadron (P_1) are given by Drell and Yan as follows:

$$Q^2 = -q^2 > 0 \qquad \nu = P \cdot q/M \qquad \nu_1 = P_1 \cdot q/M_1 \qquad \kappa_1 = P \cdot P_1/M$$

$$\frac{d^4\sigma}{dQ^2\, d\nu\, d\kappa_1\, d\nu_1} = \frac{4\pi\alpha^2}{\left(Q^2\right)^2}\left(\frac{\epsilon'}{\epsilon}\right)\left[\mathcal{W}_2 \cos^2(\theta/2) + 2\, \mathcal{W}_1 \sin^2(\theta/2)\right] \tag{1}$$

where ϵ is the incident energy and ϵ', θ are the energy and angle of the scattered electron in the laboratory system. (The azimuthal angle of the hadron is integrated over.) The kinematic region of deep inelastic scattering where the observed hadron in the laboratory system has a finite fraction (denoted as z) of the virtual photon's momentum (equal to laboratory energy in the Bj limit) is best described as the parton fragmentation region,[1-5] i.e.,

$$\nu, Q^2 \longrightarrow \infty, \ \omega = \frac{2M\nu}{Q^2} \text{ finite}, \ z \equiv \frac{1}{\omega_1} \equiv \frac{\kappa_1}{\nu} \text{ finite}, \ u_1 \equiv \frac{M_1\nu_1}{M\nu} = -\frac{1}{\omega\omega_1} + 0\left(\frac{1}{\nu}\right)$$

(The Feynman scaling variable z is also the ratio of momentum of the hadron to that of the parton struck by the photon in the parton-photon Breit frame[2] or of the hadron to its maximum momentum in the center-of-mass frame.)

The scaling behavior in the parton fragmentation region is derived by Drell and Yan to be:

$$\lim_{Bj} M\nu^2 \mathcal{W}_1 = \mathcal{F}_1(\omega, \omega_1, u_1) = \delta\left(u_1 + \frac{1}{\omega\omega_1}\right) \sum_{\lambda} F_{1\lambda}(\omega) f_\lambda(\omega_1) \tag{2}$$

$$\lim_{Bj} \nu^3 \mathcal{W}_2 = \mathcal{F}_2(\omega, \omega_1, u_1) = \delta\left(u_1 + \frac{1}{\omega\omega_1}\right) \sum_{\lambda} F_{2\lambda}(\omega) f_\lambda(\omega_1) \tag{3}$$

with spin-1/2 partons implying:

$$\mathcal{F}_1(\omega, \omega_1, u_1) = \frac{1}{2}\,\omega\,\mathcal{F}_2(\omega, \omega_1, u_1) .$$

The delta function in Eq. (2) and (3) which equates u_1 and $-\frac{1}{\omega\omega_1}$ results from the finite bound imposed on the transverse momenta. Actually, the finite quantities u_1 and $-\frac{1}{\omega\omega_1}$ are only approximately equal in the parton fragmentation region; their difference is of order $1/\nu$, i.e.,

$$u_1 + \frac{1}{\omega\omega_1} = -\frac{1}{\nu}\left(\frac{P_\perp^2 + M_1^2}{2M\,\omega u_1} + \frac{Mu_1}{2\omega}\right) + 0\left(\frac{1}{\nu^2}\right) \tag{4}$$

Consequently, u_1 and $-\frac{1}{\omega\omega_1}$ are not appropriate variables experimentally, since their difference, which expresses one of the kinematic degrees of freedom, vanishes as $\nu \longrightarrow \infty$. A better choice of variables would be $P_\perp^2$ and $z \equiv \frac{1}{\omega_1}$, in terms of which the scaling behavior is easily seen to be (removing the ν from the delta function)[6,7]:

$$\lim_{Bj} M^2 \nu\, \mathcal{W}_1 = F_1\left(\omega, z, P_\perp^2\right) \tag{5}$$

$$\lim_{Bj} M\nu^2 \mathcal{W}_2 = F_2\left(\omega, z, P_\perp^2\right) \tag{6}$$

This scaling behavior is equivalent to the scaling of $\frac{1}{\sigma} E \frac{d^3\sigma}{d^3 p}$, since:

$$\frac{1}{\sigma\left(\nu,\, Q^2\right)} E_1 \frac{d^2\sigma\left(\nu, Q^2\right)}{P_\perp\, d\, P_\perp\, dP_{1\,z}} = \frac{\nu^2 \mathscr{W}_2}{M_1\, \nu\, W_2} \tag{7}$$

where $\sigma(\nu,\, Q^2)$ and $\nu\, W_2$ are the total cross section and scaling structure function, respectively, for virtual photon-hadron scattering. (The relation from spin-1/2 partons has been used to eliminate $\mathscr{W}_1$ and W_1.) Thus, the above choice of variables also demonstrates the equivalence of Feynman scaling or limiting fragmentation and Drell-Yan scaling in the parton fragmentation region. The light-cone formalism, which, like the other two, predicts $\lim_{Bj} M\nu^2 \mathscr{W}_2 = \tilde{F}_2\left(\omega,\, u_1,\, P_\perp^2\right)$ in the proton fragmentation region, cannot predict scaling in the parton fragmentation region.

Recent data[8] taken in the parton fragmentation region for a limited range of $P_\perp^2$ and fixed ω were thought to be inconsistent with Drell-Yan scaling because a plot of $\nu^3 \mathscr{W}_2$ versus $1/\omega_1$ did not exhibit scaling (Fig. 1a). However, as shown above, the correct experimental test of Drell-Yan scaling for a range of $P_\perp^2$ is a plot of $\nu^2 \mathscr{W}_2$ versus $1/\omega_1$. Graphed in this way, the data are consistent with the scaling predictions of the parton model (Fig. 1b).

One of us (EWC) would like to thank his colleagues at SLAC, especially Professor S. Drell and Dr. J. Cleymans, for numerous helpful discussions.

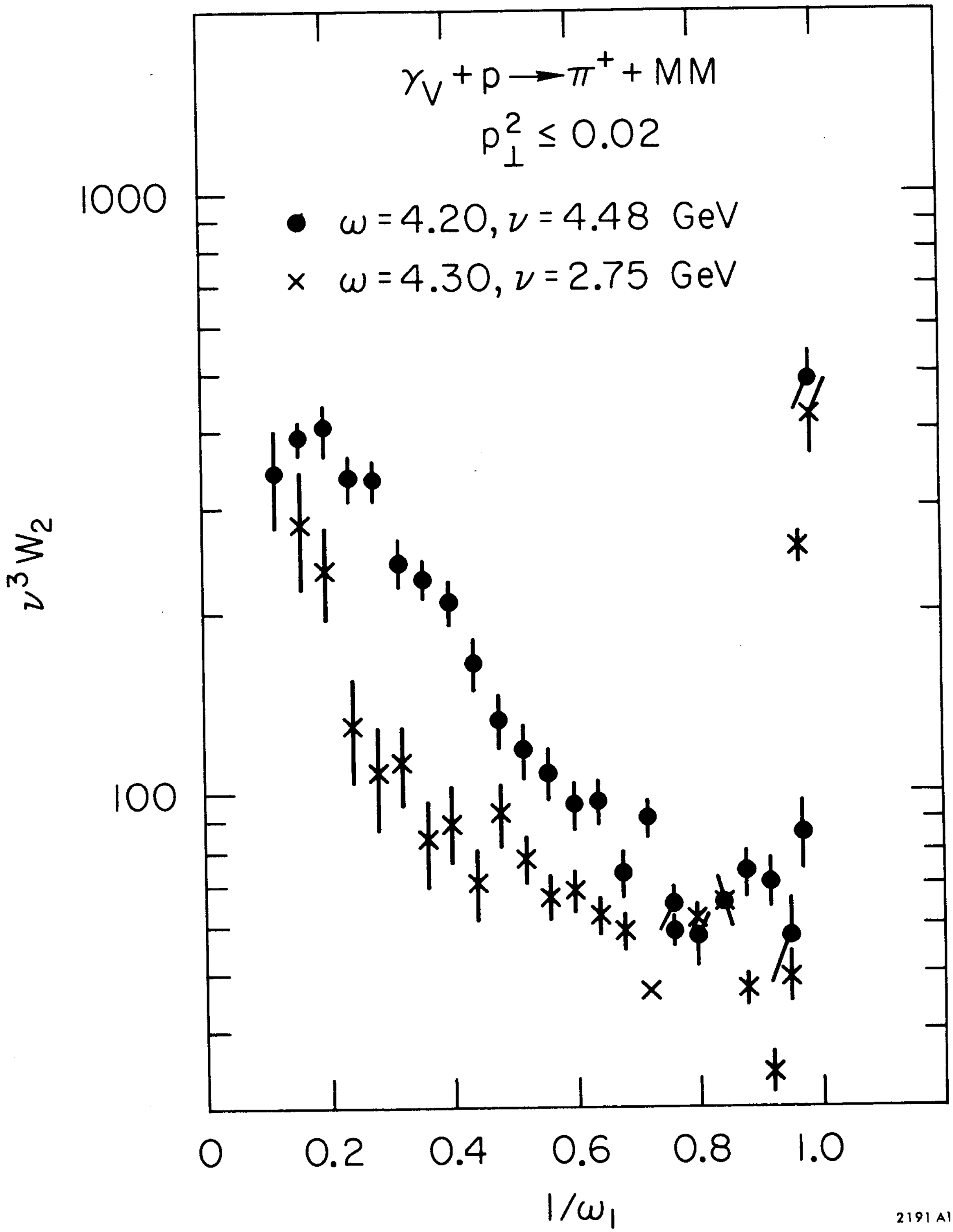
References

1. S. D. Drell and T. M. Yan, Phys. Rev. Letters 24, 855 (1970).
2. R. P. Feynman, Photon Hadron Interactions, to be published by W. A. Benjamin, New York, 1972, and talk presented at the "Neutrino '72" Conference, Balatonfüred, Hungary, June 1972 (CalTech preprint). Feynman has suggested the quantum numbers of the parton are measured by the average quantum numbers found in the parton fragmentation region.
3. S. Berman, J. D. Bjorken, and J. Kogut, Phys. Rev. D4, 3388 (1971).
4. M. Gronau, F. Ravndal, and Y. Zarmi, CalTech preprint CALT-68-367 (to be published).
5. For the distinction between the parton and photon fragmentation regions, see J. D. Bjorken, "Hole Fragmentation in Deep-Inelastic Processes," SLAC-PUB-1114 (to be published).
6. Using the variables u_1 and $P_\perp^2$ which are finite in both parton and proton fragmentation regions, reference (4) defines structure functions which scale with the same powers of ν as those of purely inclusive electroproduction. This scaling behavior is equivalent to Eq. (5) and (6). (The correct scaling behavior is also implicitly contained in references (2) and (3).)
7. These scaling relations can also be derived from naïve dimensional arguments.
8. C. J. Bebek, C. N. Brown, C. A. Lichtenstein, M. Herzlinger, F. M. Pipkin, K. Sisterson, D. Andrews, K. Berkelman, D. C. Cassel, D. L. Hartill, and N. Hicks, "Preliminary Report on a Study of Scaling in the Inclusive Electroproduction Reactions $e^- + p \rightarrow e^- + \pi^\pm + X$," submitted to the XVI International Conference on High Energy Physics, Batavia, Illinois, September 6-13, 1972.

Figure Captions

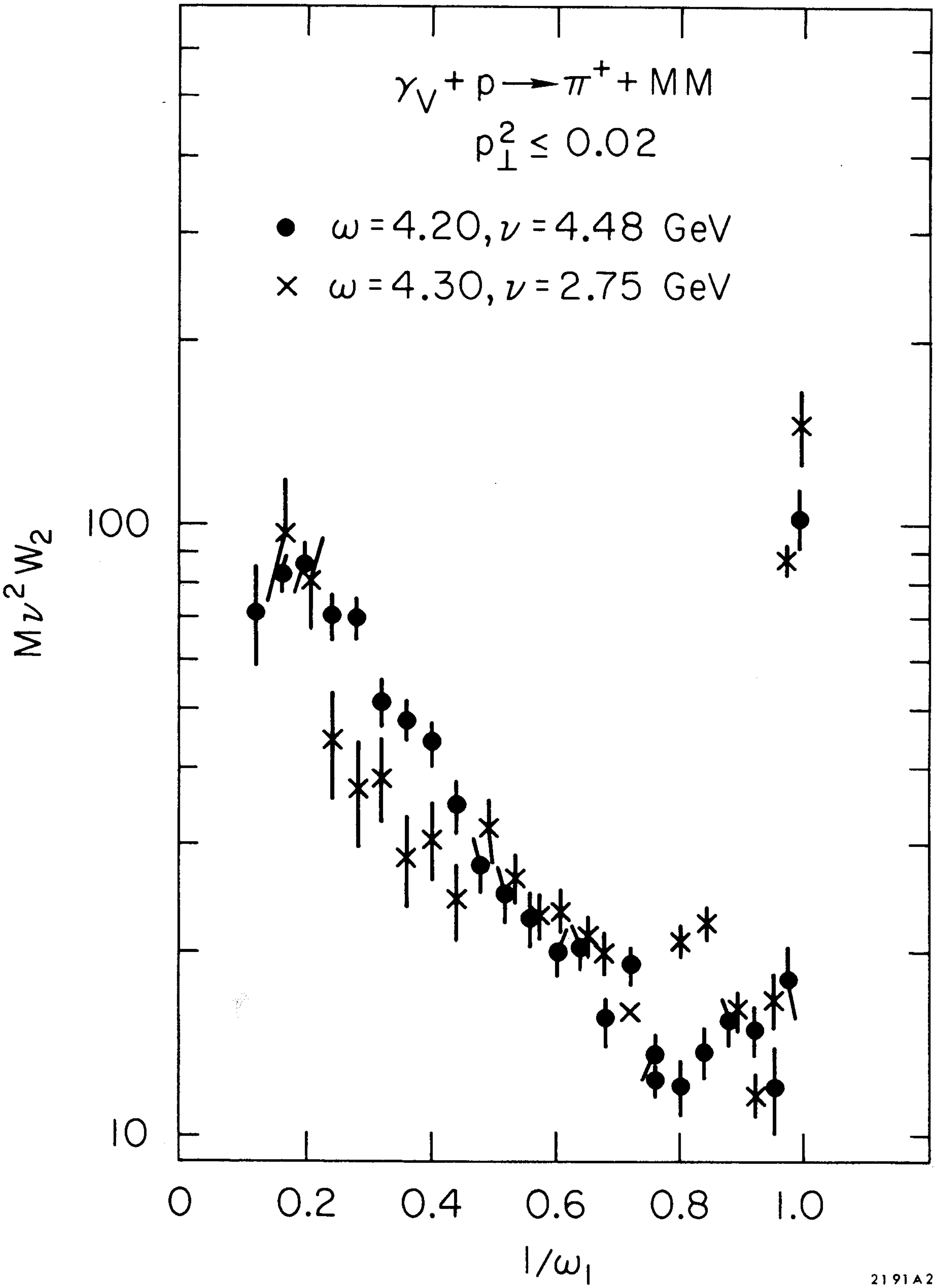
1(a). A plot of $\nu^3 \mathscr{W}_2$ versus $1/\omega_1$ taken from reference (8).

(b). A plot of $M\nu^2 \mathscr{W}_2$ versus $1/\omega_1$ for the same data.



$\gamma_V + p \rightarrow \pi^+ + MM$
$p_\perp^2 \leq 0.02$
1000
100
● $\omega = 4.20, \nu = 4.48$ GeV
× $\omega = 4.30, \nu = 2.75$ GeV
$\nu^3 W_2$
0
0.2
0.4
0.6
0.8
1.0
$1/\omega_1$
2191 A1

Fig. 1A



$\gamma_V + p \rightarrow \pi^+ + MM$
$p_\perp^2 \le 0.02$
● $\omega = 4.20, \nu = 4.48$ GeV
× $\omega = 4.30, \nu = 2.75$ GeV
$M\nu^2 W_2$
100
10
0
0.2
0.4
0.6
0.8
1.0
$1/\omega_1$
2191A2

Fig. 1B