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COMMENTS ON MUELLERISM AND MEAN MULTIPLICITIES*

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ABSTRACT

It is shown that the Mueller picture of inclusive reactions, in its naive form, leads to a value of the mean multiplicity for a species c in the process $a+b \to c +$ anything of the form

$$\langle n_c \rangle \sigma_{ab} = A \log s + B + C s^{-1/2} \log s + C' s^{-1/2} + \text{higher order in } s \quad .$$

Formal expressions for A, B, C, and C' are given. No terms proportional to $s^{-1/4}$ or $s^{-1/4} \log s$ occur.

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INTRODUCTION

It is well known that the Mueller picture of inclusive reactions leads to a mean multiplicity for species c in $a+b \rightarrow c+$anything of the form A log s+B.[1] In this note we isolate unambiguously the coefficients A and B and the coefficients of the next two terms. Since the approach to asymptopia in the central region is expected to go as $s^{-1/4}$,[2] *a priori* it might be anticipated that the first correction to the mean multiplicity beyond the constant term would be $s^{-1/4}$ or $s^{-1/4} \log s$. As shown below, this is not the case.[3]

FORMAL CALCULATION OF MEAN MULTIPLICITY

We shall calculate $\langle n_c \rangle \sigma_{tot}$ by integrating the cross section from a lab rapidity of $y_{min} = \log (m_a/m_c)$ up to Y/2 where Y is the lab rapidity of particle b, and then adding a piece with a and b interchanged. For simplicity we shall consider only two trajectories: a Pomeron with $\alpha_P(0)=1$, and a non-Pomeron with $\alpha(0) < 1$. The cross section integrated over transverse momentum for the fragmentation of particle a into particle c, which process we denote by (a:c |b), can be expressed as

$$\frac{d\sigma}{dy}(y, Y) = \beta_P^b F_P^{a:c}(y) + e^{-\Delta\alpha(Y-y)} \beta_R^b F_R^{a:c}(y) \tag{1}$$

where $\Delta\alpha=1-\alpha(0)$. For large values of y we can use a double 0(2, 1) expansion, i.e., the $a\bar{a}$ channel Reggeizes as well as the $b\bar{b}$ channel. Thus we have

$$\frac{d\sigma}{dy}(\text{y large, } Y) = \beta_P^b \beta_P^a F_{PP}^c + \beta_R^b \beta_P^a F_{RP}^c e^{-\Delta\alpha(Y-y)} + \beta_P^b \beta_R^a F_{PR}^c e^{-\Delta\alpha y} + \beta_R^b \beta_R^a F_{RR}^c e^{-\Delta\alpha Y} \tag{2}$$

In particular we have the limiting expressions

$$\frac{d\sigma}{dy}(y,\infty) = \lim_{Y\rightarrow\infty} \frac{d\sigma}{dy}(y, Y) = \beta_P^b F_P^{a:c}(y) \tag{3a}$$

and

$$\frac{d\sigma}{dy}(\infty,\infty) = \lim_{y\to\infty}\ \lim_{Y\to\infty}\ \frac{d\sigma}{dy}(y,Y) = \beta_P^b\,\beta_P^a\,F_{PP}^c \tag{3b}$$

We next write a formal identity:

$$\begin{aligned}\int_{y_{min}}^{Y/2} dy\,\frac{d\sigma}{dy}(y,Y) = &\int_{y_{min}}^{0} dy\,\frac{d\sigma}{dy}(y,\infty) + \int_{y_{min}}^{0} dy\left[\frac{d\sigma}{dy}(y,Y) - \frac{d\sigma}{dy}(y,\infty)\right] \\ &+\int_0^{Y/2} dy\,\frac{d\sigma}{dy}(\infty,\infty) + \int_0^{\infty} dy\left[\frac{d\sigma}{dy}(y,\infty) - \frac{d\sigma}{dy}(\infty,\infty)\right] \\ &-\int_{Y/2}^{\infty} dy\left[\frac{d\sigma}{dy}(y,\infty) - \frac{d\sigma}{dy}(\infty,\infty)\right] \\ &+\int_0^{Y/2} dy\left[\frac{d\sigma}{dy}(y,Y) - \frac{d\sigma}{dy}(y,\infty)\right] \quad .\end{aligned} \tag{4}$$

The six integrals are easily evaluated in terms of Eqs. 1 - 3. For the integrals I_i (i=1, 6) we have

$$I_1 = \int_{y_{min}}^{0} dy\ \beta_P^b\,F_P^{a:c}(y)$$

$$I_2 = e^{-\Delta\alpha Y}\int_{y_{min}}^{0} dy\ \beta_R^b\,F_R^{a:c}(y)\ e^{\Delta\alpha y}$$

$$I_3 = Y/2\ \ \beta_P^b\,\beta_P^a\,F_{PP}^c$$

$$I_4 = \int_0^{\infty} dy\ \beta_P^b\left[F_P^{a:c}(y) - \beta_P^a\,F_{PP}^c\right]$$

$$I_5 = -\ \frac{\beta_P^b\beta_R^a}{\Delta\alpha}\,F_{PR}^c\ e^{-\Delta\alpha\,Y/2}$$

$$I_6 = \int_0^{Y/2} dy\, \beta_R^b\, e^{-\Delta\alpha(Y-y)}\, F_R^{a:c}(y)$$

$$= \int_0^{Y/2} dy\, \beta_R^b\, e^{-\Delta\alpha(Y-y)} \left[\beta_P^a F_{RP}^c + \beta_R^a F_{RR}^c\, e^{-\Delta\alpha y}\right]$$

$$+ \int_0^{\infty} dy\, \beta_R^b\, e^{-\Delta\alpha(Y-y)} \left[F_R^{a:c}(y) - \beta_P^a F_{RP}^c - \beta_R^a F_{RR}^c\, e^{-\Delta\alpha y}\right]$$

$$- \int_{Y/2}^{\infty} dy\, \beta_R^b\, e^{-\Delta\alpha(Y-y)} \left[F_R^{a:c}(y) - \beta_P^a F_{RP}^c - \beta_R^a F_{RR}^c\, e^{-\Delta\alpha y}\right] \quad (5)$$

The final integral in I_6 from Y/2 to infinity is of higher order than the two preceding ones. We have then for I_6

$$I_6 = e^{-\Delta\alpha Y}\, \beta_R^b \beta_P^a F_{RP}^c \frac{(e^{\Delta\alpha\, Y/2} - 1)}{\Delta\alpha}$$

$$+ e^{-\Delta\alpha Y}\, \frac{Y}{2} \beta_R^a \beta_R^b F_{RR}^c + e^{-\Delta\alpha Y} \int_0^{\infty} dy\, \beta_R^b \left[F_R^{a:c}(y) - \beta_P^a F_{PR}^c - \beta_R^a F_{PR}^c\, e^{-\Delta\alpha y}\right] e^{\Delta\alpha y}$$

$$+ \text{higher order} \quad (6)$$

It is easy to see that the integral in (6) converges. To find $\langle n_c \rangle \times \sigma_{tot}$ we add the other half of the rapidity distribution. This contribution is given by interchanging a and b in Eq. (6). When this is done, I_5 cancels against a term in I_6 and we have

$$\langle n_c \rangle\, \sigma_{tot} = AY + B + CY\, e^{-\Delta\alpha Y} + C'\, e^{-\Delta\alpha Y} + \text{higher order} \quad (7)$$

with

$$A = \beta_P^a \beta_P^b F_{PP}^c$$

$$B = \left\{ \int_{y_{min}}^{0} dy\, \beta_P^b F_P^{a:c}(y) + \int_0^{\infty} dy\, \beta_P^b \left[F_P^{a:c}(y) - \beta_P^a F_{PP}^c\right] \right\} + (b \leftrightarrow a)$$

$$C = \beta_R^a \beta_R^b F_{RR}^c$$

$$C' = -\beta_R^b \beta_P^a F_{PR}^c/\Delta\alpha + \int_{y_{min}}^{0} dy\, \beta_R^b F_R^{a:c}(y)\, e^{\Delta\alpha y}$$

$$+ \int_0^\infty dy\, \beta_R^b \left[F_R^{a:c}(y) - \beta_P^a F_{PR}^c - \beta_R^a F_{RR}^c\, e^{-\Delta\alpha y}\right] e^{\Delta\alpha y} + (b \leftrightarrow a) \qquad (8)$$

The coefficient A is simply the height of the central plateau. The difference between the scaled distribution and the central plateau extended down to $y=0$ gives the coefficient B. We may think of C as the height of the central plateau due to terms in the double Regge expansion which have non-Pomerons in both links. The coefficient C' is due to the non-scaling term with the leading terms for large y extracted out. The most notable feature is the absence of a term proportional to $\exp(-Y/2)$, i.e., $s^{-1/4}$. This general result of course obtains as well in multiperipheral models.

Since we have ignored cuts throughout, and since we have assumed the $b\bar{b}$ channel in (a:c |b) has Reggeized even for small missing masses, Eq. (7) is best regarded as a formal result. It does provide an appropriate starting point for phenomenological analysis.[4]

REFERENCES

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