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ON THE Q^2-DEPENDENCE OF INCLUSIVE ELECTROPRODUCTION OF HADRONS*

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ABSTRACT

Mueller's Regge analysis is combined with the generalized scaling laws of the parton model to determine the Q^2-dependence of inclusive electroproduction of hadrons.

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I. INTRODUCTION

Mueller's Regge analysis is, at the moment, one of the most interesting frameworks to understand inclusive interactions. Due to the belief that the photon behaves in many regards like a hadron, it is hoped for that this analysis will also provide a valid framework for inclusive electroproduction. It is, however, clear from the beginning that the Q^2-dependence of the single particle distribution has to be found by considerations outside Mueller's analysis. This is also true for the Q^2-dependence of the Regge residues in deep inelastic electron nucleon scattering. In the latter case, the dependence is determined from the scaling behavior of the structure functions.[2] In the case of inclusive electroproduction, the problem is partially solved by assuming factorization for the Regge residues. This will fix the Q^2-dependence in the proton fragmentation region and in the central region.[3,4] To go further and determine the Q^2-dependence in the photon (or current) fragmentation region, we appeal to the generalized scaling laws of the parton model.[5] We will show that these assumptions imply that the π^+/π^- asymmetry will increase as Q^2 becomes larger for fixed ν and all other relevant variables fixed. The behavior of the multiplicity will not be determined by our considerations because the integral over the photon fragmentation still involves an undetermined function.

In Section II we define our notations and conventions. In Section III we present the Mueller analysis of inclusive electroproduction and determine the Q^2-dependence in the three different regions. In Section IV we discuss the π^+/π^- asymmetry and look at the behavior of the average multiplicity for large values of Q^2 and ω.

II. KINEMATICS AND NOTATIONS

The kinematics for inclusive electroproduction are depicted in Fig. 1. The metric we choose is (+---). All our states will be normalized as:

$$\langle \vec{p} \,|\, \vec{p}\,' \rangle = (2\pi)^3 \, 2E \, \delta^3(\vec{p} - \vec{p}\,')$$

We introduce the following variables:

$$\nu = \frac{p \cdot q}{m_p} \qquad \nu_1 = \frac{p_1 \cdot q}{m_1} \qquad \kappa_1 = \frac{p \cdot p_1}{m_p} \tag{1}$$

$$\omega = \frac{2p \cdot q}{Q^2} \qquad \omega_1 = \frac{2p_1 \cdot q}{Q^2} \tag{2}$$

where $m_p (m_1)$ is the mass of the nucleon (particle 1) and, as usual, $Q^2 = -q^2$.

The Lorentz-invariant structure functions $\mathcal{W}_1$ and $\mathcal{W}_2$ are defined as:

$$\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) \mathcal{W}_1(q^2, \nu, \nu_1, \kappa_1)$$

$$+ \frac{1}{m_p^2}\left(p_\mu - \frac{p \cdot q}{q^2} q_\mu\right)\left(p_\nu - \frac{p \cdot q}{q^2} q_\nu\right) \mathcal{W}_2(q^2, \nu, \nu_1, \kappa_1)$$

$$= \frac{1}{2} \sum_{\substack{\text{nucleon} \\ \text{spin}}} \sum_F \int \frac{d\nu_1 d\kappa_1}{4\pi m_p} \, \delta\left(\nu_1 - \frac{p_1 \cdot q}{m_1}\right) \delta\left(\kappa_1 - \frac{p_1 \cdot p}{m_p}\right)$$

$$(2\pi)^4 \, \delta^4(p+q-k-p_F) \langle p | J_\mu(0) | p_1, p_F \rangle \langle p_1, p_F | J_\nu(0) | p \rangle \tag{3}$$

In terms of $\mathcal{W}_1$ and $\mathcal{W}_2$, the differential cross section is:

$$\frac{d^4\sigma}{dQ^2 d\nu d\kappa_1 d\nu_1} = \frac{4\pi\alpha^2}{q^4} \frac{E'}{E} \left[2 \sin^2 \frac{\theta}{2} \mathcal{W}_1(q^2, \nu, \nu_1, \kappa_1)\right.$$

$$\left. + \cos^2 \frac{\theta}{2} \mathcal{W}_2(q^2, \nu, \nu_1, \kappa_1)\right] \tag{4}$$

where θ is the angle between the incoming and outgoing electron momenta in the laboratory system of the nucleon.

The longitudinal and transverse differential cross sections are defined as:

$$\frac{d^2\sigma^T}{d\nu_1 d\kappa_1} = \frac{8\pi^2 m_p \alpha}{q^2 + 2m_p\nu} \mathcal{W}_1(q^2, \nu, \nu_1, \kappa_1) \tag{5}$$

$$\frac{d^2\sigma^L}{d\nu_1 d\kappa_1} = -\frac{8\pi^2 m_p \alpha}{q^2 + 2m_p\nu} \left[\mathcal{W}_1(q^2, \nu, \nu_1, \kappa_1) + \frac{\nu^2 - q^2}{q^2} \mathcal{W}_2(q^2, \nu, \nu_1, \kappa_1)\right] \tag{6}$$

A useful relation is:

$$p_1^o \frac{d\sigma}{d^3 p_1} = \frac{(\nu^2 + Q^2)^{1/2}}{m_1} \frac{d\sigma}{d\nu_1 d\kappa_1 d\phi} \tag{7}$$

The momentum of the final state hadron p_1 will be parametrized as follows:

$$p_1^\mu = m_{1\perp}\left(\cosh y_1, \frac{p_{1x}}{m_{1\perp}}, \frac{p_{1y}}{m_{1\perp}}, \sinh y_1\right) \tag{8}$$

where

$$m_{1\perp}^2 = p_{1x}^2 + p_{1y}^2 + m_1^2 \,. \tag{9}$$

We also introduce the photon's rapidity Y by:

$$q^\mu = Q(\sinh Y,\ 0,\ 0,\ \cosh Y) \tag{10}$$

In terms of the rapidities y_1 and Y, the invariants (1) can be written as:

$$\nu = Q \sinh Y$$

$$\nu_1 = \frac{Q m_{1\perp}}{m_1} \sinh (Y - y_1) \tag{11}$$

$$\kappa_1 = m_{1\perp} \cosh y_1 \,.$$

In the Bjorken limit ($Q^2 \to \infty$, ω fixed) one has, approximately:

$$Qe^{Y} \simeq 2\nu$$
$$Qe^{-Y} \simeq \frac{M}{\omega} \tag{12}$$

From energy and momentum conservation, it follows that the rapidity y_1 of the observed hadron is bounded by the following limits:

$$\ln \frac{2m_{1\perp}\nu}{s} < y_1 < \ln \frac{2\nu}{m_{1\perp}} \tag{13}$$

The total length, L_{tot}, of the rapidity plot will thus be given by:

$$L_{tot} = \ln \frac{s}{m_{1\perp}^2} \tag{14}$$

i.e., as in the purely hadronic case. We notice that L_{tot} shrinks when the mass of the virtual photon increases.

III. MUELLER ANALYSIS

The extension of the original Mueller analysis from hadronic reactions to inclusive electroproduction gives us the following expressions for[6]

$$\frac{2(2p\cdot q+q^2)}{4\pi\alpha}\, p_1^o \frac{d\sigma}{d^3p_1} \tag{15}$$

in the three different regions[7] (see Fig. 2):

1. Nucleon fragmentation region ($\nu \to \infty$, ν_1/ν finite, κ_1 finite):

$$\sum_i \nu^{\alpha_i} \beta_{\gamma\alpha_i}(Q^2)\, \beta_{pp_1}^{\alpha_i}\left(\frac{\nu_i}{\nu}, \kappa_1\right) \tag{16}$$

2. Central region ($\nu \to \infty$, $\nu_1\kappa_1/\nu$ finite, $\kappa_1 \to \infty$, $\nu_1 \to \infty$)

$$\sum_{i,j} \nu_1^{\alpha_j} \kappa_1^{\alpha_i} \beta_{\gamma\alpha_j}(Q^2)\, \beta_{p_1}^{\alpha_i\alpha_j}\left(\frac{\nu_1\kappa_1}{\nu}\right) \beta_{p\alpha_i} \tag{17}$$

3. Photon fragmentation region ($\nu \to \infty$, κ_1/ν finite, ν_1 finite)

$$\sum_i \nu^{\alpha_i} \beta_{\gamma p_1}^{\alpha_i} \left(\frac{\kappa_1}{\nu}, \nu_1, Q^2\right) \beta_{p\alpha_i} \tag{18}$$

Here, α_i, is the intercept of Regge trajectory $\alpha_i(t)$ at $t=0$, while the β's are residue functions. The requirement of scaling for the deep inelastic structure functions $m_p W_1(q^2, \nu)$ and $\nu W_2(q^2, \nu)$ leads to the following Q^2-dependence of the photon-Reggeon coupling[2]:

$$\beta_{\gamma\alpha_i}(Q^2) \propto \left(\frac{1}{Q^2}\right)^{\alpha_i} \tilde{\beta}_{\gamma\alpha_i} \tag{19}$$

where $\tilde{\beta}_{\gamma\alpha_i}$ is constant.

We can thus rewrite the contributions (16) and (17) as (absorbing some constant factors in $\tilde{\beta}_{\gamma\alpha_i}$):

Nucleon fragmentation:

$$\sum_i \omega_1^{\alpha_i} \tilde{\beta}_{\gamma\alpha_i} \beta_{pp_1}^{\alpha_i} \left(\frac{\nu_1}{\nu}, \kappa_1\right) \tag{20}$$

Central region:

$$\sum_{i,j} \omega_1^{\alpha_j} \kappa_1^{\alpha_i} \tilde{\beta}_{\gamma\alpha_j} \beta_{p_1}^{\alpha_i\alpha_j} \left(\frac{\nu_1\kappa_1}{\nu}\right) \beta_{p\alpha_i} \tag{21}$$

The factorization of Regge residues, thus, does not determine the Q^2-dependence of the inclusive cross section over the whole phase space. A complete arbitrariness subsists in the photon fragmentation region. If we would like this last region to behave in a way similar to the nucleon fragmentation region, we would impose[7]:

$$\beta_{\gamma p_1}^{\alpha_i} \left(\frac{\kappa_1}{\nu}, \nu_1, Q^2\right) \xrightarrow[Q^2 \to \infty]{} \left(\frac{1}{Q^2}\right)^{\alpha_i} \tilde{\beta}_{\gamma p_1}^{\alpha_i} \left(\frac{\kappa_1}{\nu}, \nu_1\right) \tag{22}$$

This is however arbitrary and even incompatible with other approaches to inclusive electroproduction.[5] Instead of (22) we will impose a behavior consistent with the generalized scaling laws of the parton model[5]:

$$m_p \nu^2 \mathscr{W}_1(q^2, \nu, \nu_1, \kappa_1) \xrightarrow[\substack{\nu \to \infty \\ \omega \text{ fixed} \\ \omega_1 \text{ fixed} \\ \frac{\kappa_1}{\nu} \text{ fixed}}]{} \mathscr{F}_1\left(\omega, \omega_1, \frac{\kappa_1}{\nu}\right) \tag{23}$$

This is obtained if we require:

$$\lim_{Q^2 \to \infty} \beta^{\alpha_i}_{\gamma p_1}\left(\frac{\kappa_1}{\nu}, \nu_1, Q^2\right) = \left(\frac{1}{Q^2}\right)^{\alpha_i + 1} \tilde{\beta}^{\alpha_i}_{\gamma p_1}\left(\frac{\kappa_1}{\nu}, \omega_1\right) \tag{24}$$

which has an extra factor $1/Q^2$ compared to the nucleon fragmentation function.

We thus obtain finally:

Photon fragmentation region:

$$\sum_i \frac{\omega^{\alpha_i}}{Q^2} \tilde{\beta}^{\alpha_i}_{\gamma p_1}\left(\frac{\kappa_1}{\nu}, \omega_1\right) \beta_{p\alpha_i} \tag{25}$$

In the next section we will look at the consequences of this behavior for the π^+/π^- asymmetry and the average multiplicity.

IV. CONSEQUENCES

(a) π^+/π^- Asymmetry: If we fix the value of the incoming energy ν and increase Q^2, we see from (25) that the Pomeron contributions will decrease more rapidly than the contributions coming from lower-lying Regge trajectories. This implies that the π^+/π^- asymmetry, which is due to the isospin carrying lower lying Regge trajectories, will increase as a function of Q^2 for fixed values of ω_1 and κ_1/ν. This is supported by preliminary experimental results.[8]

(b) Multiplicities: The average multiplicity $n(\nu, q^2)$ is defined as:

$$\int \frac{d^3 p_1}{p_{10}} \frac{d\sigma}{d^3 p_1 / p_{10}} = n(\nu, q^2)\, \sigma(\nu, q^2) \tag{26}$$

In order to be able to insert directly the Mueller and Regge expansion respectively in right- and left-hand sides, we multiply (26) by the flux factor $2(q^2+2q\cdot p)$ and divide by the coupling constant $4\pi\alpha$. We will also make use of relation (7) and integrate implicitly over ϕ (only Pomeron contributions are being considered).

We thus first write (26) as:

$$\int \frac{d^3 p_1}{p_1^o} \left[\frac{2(q^2+2q\cdot p)}{4\pi\alpha} \frac{d\sigma}{d^3 p_1 / p_{11}^o}\right] = n(\nu, q^2) \left[\frac{2(q^2+2q\cdot p)}{4\pi\alpha}\, \sigma(\nu, q^2)\right] \tag{27}$$

Changing variables to ν_1 and inserting the Mueller and Regge expansions, we arrive at:

$$\int d\nu_1\, d\kappa_1 \frac{1}{\nu}\, \omega\, \tilde{\beta}_{\gamma \mathbb{P}} \beta^{\mathrm{P}}_{pp_1}\left(\frac{\nu_1}{\nu}, \kappa_1\right)$$

$$+ \int d\nu_1\, d\kappa_1 \frac{1}{\nu}\, \omega\, \tilde{\beta}_{\gamma \mathrm{P}} \beta^{\mathbb{P}}_{p_1}\left(\frac{\nu_1 \kappa_1}{\nu}\right) \beta_{p\mathbb{P}}$$

$$+ \int d\nu_1\, d\kappa_1 \frac{1}{\nu Q^2}\, \omega\, \tilde{\beta}^{\mathbb{P}}_{\gamma p_1}\left(\omega_1, \frac{\kappa_1}{\nu}\right) \beta_{p\mathbb{P}}$$

$$\sim\ n(\nu, q^2)\, \omega\, \tilde{\beta}_{\gamma \mathbb{P}} \beta_{p\mathbb{P}} \tag{28}$$

Or, equivalently:

$$\int d\left(\frac{\nu_1}{\nu}\right) d\kappa \, \tilde{\beta}_{\gamma \mathbb{P}} \beta^{\mathbb{P}}_{pp_1} \left(\frac{\nu_1}{\nu}, \kappa_1\right)$$

$$+ \int d\left(\frac{\nu_1 \kappa_1}{\nu}\right) \frac{d\kappa_1}{\kappa_1} \tilde{\beta}_{\gamma \mathbb{P}} \beta^{\mathbb{P}}_{p_1} \left(\frac{\nu_1 \kappa_1}{\nu}\right) \beta_{p \mathbb{P}}$$

$$+ \int d\omega_1 \, d\left(\frac{\kappa_1}{\nu}\right) \beta^{\mathbb{P}}_{\gamma p_1} \left(\omega', \frac{\kappa_1}{\nu}\right) \beta_{p \mathbb{P}}$$

$$\sim n(\nu, q^2) \, \tilde{\beta}_{\gamma \mathbb{P}} \beta_{p \mathbb{P}} \tag{29}$$

The first term on the left-hand side of (29) represents the contribution coming from the nucleon fragmentation region. Since this term occupies only a finite portion of phase space, we assume, in analogy with the purely hadronic interaction case, that this term gives a finite contribution to the multiplicity. In the second term, $\nu_1 \kappa_1 / \nu$ is directly related to m_1^2 while $d\kappa_1/\kappa_1$ becomes dy in the central region. Making the additional assumption that the integral over $\nu_1 \kappa_1 / \nu$ is finite, we see that the contribution of the second term to the multiplicity is proportional to the length of the central region L_c. There is no a priori reason to expect the third term in (29) to give a finite contribution and no definite conclusion can be drawn for it.

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FIGURE CAPTIONS

1. Kinematics for inclusive electroproduction of hadrons.

2. (a) Mueller diagram for the nucleon fragmentation region

   (b) Idem for the central region

   (c) Idem for the photon fragmentation region.

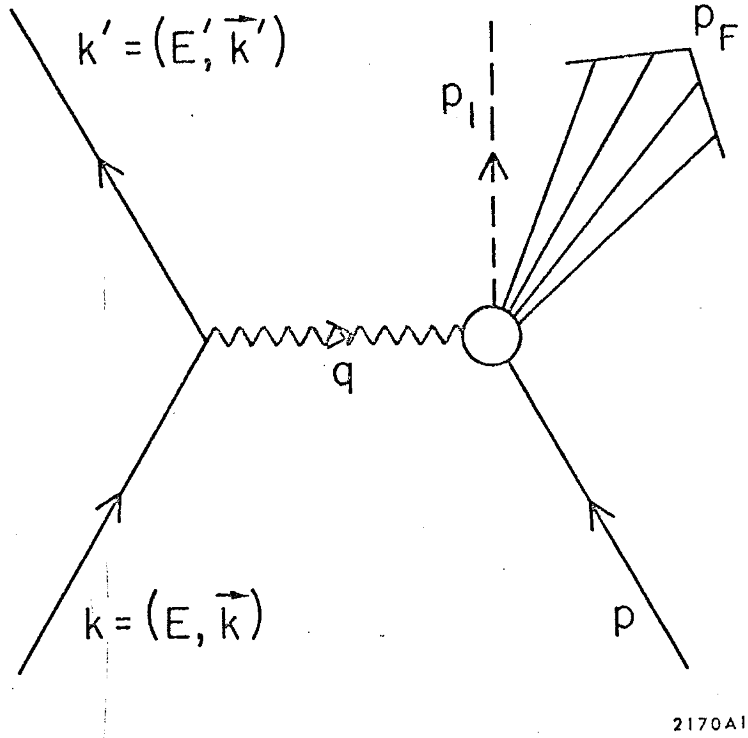

Fig. 1

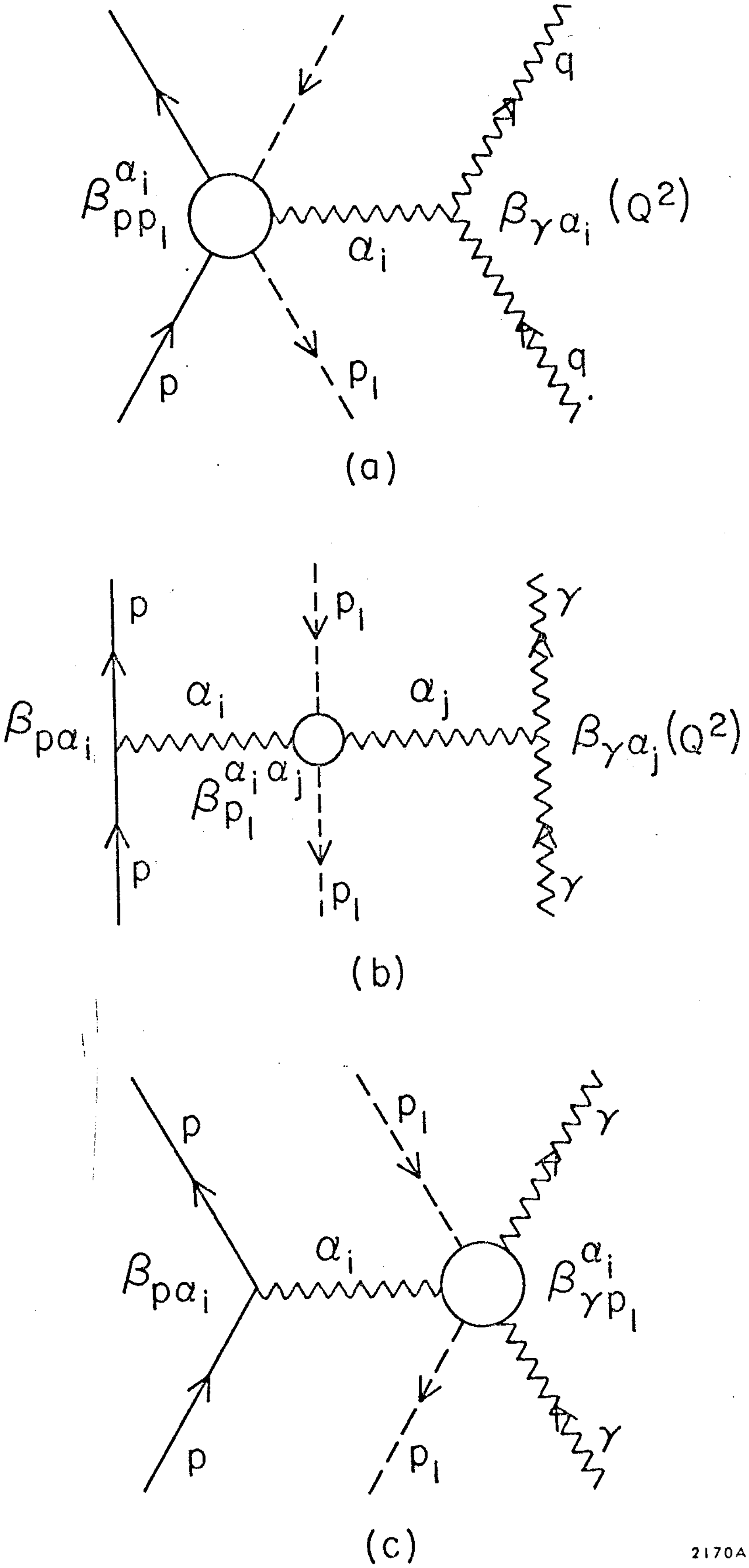
$\beta_{pp_1}^{\alpha_i}$
$\beta_{\gamma\alpha_i}(Q^2)$
α_i
q
q
p
p_1
(a)
$\beta_{p\alpha_i}$
α_i
α_j
$\beta_{p_1}^{\alpha_i\alpha_j}$
$\beta_{\gamma\alpha_j}(Q^2)$
p
p
p_1
p_1
γ
γ
(b)
$\beta_{p\alpha_i}$
α_i
$\beta_{\gamma p_1}^{\alpha_i}$
p
p
p_1
p_1
γ
γ
(c)

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Fig. 2