SLAC-PUB-1111
(TH) and (EXP)
September 1972

REGGE AND PARTON MODELS FOR PHOTON-PHOTON ANNIHILATION*

T. F. Walsh†

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

We discuss some of the questions which can be investigated in the two-photon part of $e^- + e^- \rightarrow e^- + e^-$ + hadrons when the electrons are scattered at finite momentum transfer. Discrimination of Regge and parton models for this process is possible in principle by studying the transverse helicity amplitudes. In some parton models the amplitudes are predicted exactly for limits where $s = (q_1 + q_2)^2$ and $|q_1^2|$ and $|q_2^2|$ all approach infinity with constant ratios; this is a new sort of scaling limit, inaccessible in inelastic electron-proton scattering.

(Submitted for publication.)

---

*Work supported in part by the U. S. Atomic Energy Commission.

†Permanent address: II Institut für Theoretische Physik, Universität Hamburg, 2 Hamburg 50, Luruper Chaussee 149, Germany

1. Introduction

The subject of this paper is the somewhat novel reaction (the absorptive part of $\overset{*}{\gamma}(q_1) + \overset{*}{\gamma}(q_2) \rightarrow \overset{*}{\gamma}(q_1) + \overset{*}{\gamma}(q_2)$)

$$\overset{*}{\gamma}(q_1) + \overset{*}{\gamma}(q_2) \rightarrow \text{hadrons} \tag{1}$$

where two photons with large $|q_1^2|$, $|q_2^2|$ annihilate into an unanalyzed hadron system. The subject of hadron production by nearly real photons has been of some interest recently[1], and the process in which it can be experimentally realized,

$$e^{\pm}(p_1) + e^{-}(p_2) \rightarrow e^{\pm}(p'_1) + e^{-}(p'_2) + \text{hadrons} \tag{2}$$

lends itself directly, via electron scattering with large momentum transfer, to the study of (1), where both photons are far off mass shell. We shall not have much to say about reaction (2); we are principally interested in reaction (1), and we will confine ourselves almost wholly to it. For cross section estimates for (2) in the region of large q_i^2, we refer the reader to references 2 and 3. General cross section formulas have been worked out by a number of authors.[4, 6]

Our aim is to examine the relative regions of applicability in reaction (1) of Regge pole and parton model ideas, and to find tests which can discriminate between the two. This is complicated - and made interesting - by the variety of ways in which the independent variables q_1^2, q_2^2 and $s = (q_1 + q_2)^2$ can approach infinity; there is no single, obvious, "scaling" limit for (1).

We shall take it for granted that Regge-pole notions can be taken over to (1)[5,7]. It will turn out that the applicability of such ideas can be tested by comparing reaction (1) with $\overset{*}{\gamma} + p \rightarrow$ hadrons, relying on factorization. The predictions from the assumption that factorizable Regge poles with $\alpha > 0$ dominate (1) are quite strong, and the violation of such expectations at realistic values at q_1^2, q_2^2 and s would have interesting implications for theories of inelastic electron-proton scattering.

The parton model, or the bilocal algebra of Fritzsch and Gell-Mann,[8] formally describes a different kinematic limit of reaction (1) than the Regge-pole theory. It would then be of interest to see experimentally if there is evidence for such contributions away from the kinematic limit where they should be dominant. Again, violations of such parton model predictions would presumably reflect back on theoretical descriptions of ep scattering.

In the next section we discuss kinematics and kinematical limits, and in the following one we present some predictions from factorizable Regge-pole theory; in section 4 we do the same for the parton model. The last section is an assembly of remarks.

2. Kinematics

The relevant amplitude for reaction (1) can be written (summing over all hadrons)

$$W_{\mu\nu\mu'\nu'} = \int d^4x\, d^4y\, d^4z \exp -i\, Q(x-y) - i\, Pz \,.$$
$$.\langle 0 \left[T^* J_\mu^{em}\left(\frac{x}{2}\right) J_\nu^{em}\left(\frac{-x}{2}\right), \quad \tilde{T}^* J_{\mu'}^{em}\left(\frac{y}{2}+Z\right) J_{\nu'}^{em}\left(-\frac{y}{2}+Z\right)\right] | 0\rangle \tag{3}$$

where T^* ($\tilde{T}^*$) is the covariant time ordering (anti-time ordering) operation, $P \equiv q_2 + q_1$ and $Q = (q_2 - q_1)/2$. Alternatively, we can write (3) as

$$W_{\mu\nu\mu'\nu'} = (2\pi)^4 \sum_H \delta(q_1 + q_2 - P_H)\, T_{\mu\nu}(H)\, T^*_{\mu'\nu'}(H)$$

where

$$T_{\mu\nu}(H) = i\int dx\, e^{iQ\cdot x} \langle H |\, T^* J_\mu^{em}\left(\frac{x}{2}\right) J_\nu^{em}\left(-\frac{x}{2}\right) | 0\rangle .$$

It is useful to introduce helicity amplitudes

$$W_{\lambda'_2\lambda'_1;\,\lambda_2\lambda_1} = e^\mu(\lambda_1)\, e^\nu(\lambda_2)\, e^{\mu'}(\lambda'_1)^*\, e^{\nu'}(\lambda'_2)^* W_{\mu\nu\mu'\nu'} \tag{4}$$

defined in the $\gamma\gamma$ CM frame. There are eight independent amplitudes [4-6]

$$\begin{matrix} W_{11;\,11} & W_{1-1;\,1-1} & W_{-1-1;\,11} \\ W_{11;\,00} & W_{10;\,10} & W_{01;\,01} \\ W_{0-1;\,10} & W_{00;\,00} & \end{matrix} \tag{5}$$

Since we will be interested in large q_1^2, q_2^2 and $s \equiv p^2$, we need to consider the various limits when these all become large. Most obvious is the limit when q_1^2, q_2^2 and s form fixed dimensionless ratios and all three variables become large. We shall call this the S-limit,

$$-q_1^2/s = x_1 \ ; \ -q_2^2/s = x_2 \text{ and } s \to \infty \tag{6}$$

It will be more useful to write this limit in terms of

$$\xi' \equiv (q_2^2 - q_1^2)/(q_2^2 + q_1^2) ; \ \omega' \equiv 1 - s/(q_2^2 + q_1^2). \tag{6'}$$

This is the obvious analogue of the Bjorken limit $-q^2/s$ fixed, $s \to \infty$ in the reaction $\overset{*}{\gamma}(q) + p \to$ hadrons. The difference, as remarked in reference [2], is that it is not possible to show light-cone dominance of (3) in the limit (6), using the standard stationary phase argument. It has been argued recently that light cone dominance hold for (3) in the ordered limit [9] (L-limit)

$$\text{limit } s \to \infty \ (\text{limit} -q_1^2, -q_2^2 \to \infty) \tag{7}$$

where the order of the limits is important. It will be helpful in the following to consider the relatively artificial limits [10]

$$-q_1^2/s^\beta \text{ fixed}; \ -q_1^2/s^\beta \text{ fixed } s \to \infty . \tag{8}$$

When $\beta \leq 1/2$ we shall call this the R_β limit, and when $\beta > 1/2$ the S_β limit. The cases $\beta = 0$ and 1 correspond to the Regge limit proper ($s \to \infty$ at fixed q_1^2, q_2^2) and to the S-limit.

Finally, we want to consider a small kinematical fact with large consequences. Consider the diagram of Fig. 1, where the hadrons are grouped as shown. The squared four momentum transfer $t_j \equiv (q_1 - p_1 - \ldots - p_j)^2$ turns out to have a lower bound in the S-limit, namely,

$$2t_j/s \leq -(1 + x_1 + x_2) + \left[(1 + x_1 + x_2)^2 - 4x_1x_2\right]^{1/2} \tag{9}$$

and thus $t_j \to -\infty$ as $s \to \infty$ for any j. It is easy to convince ourself that $|t_j|$ is unbounded in the S_β limit, can be finite in the R_β limit with $\beta = 1/2$, and is bounded from below only by zero for $\beta > 1/2$. This is strikingly different from virtual photo-absorption, where t_{jmin} is a constant in the Bjorken limit (e.g., $x_1 \to 0$ in (9)).

3. Regge Models

We consider the Regge description for the absorptive part $W_{\lambda'_2\lambda'_1;\lambda_2\lambda_1}$, using the following assumption (i) The reaction $\gamma^*(q_1) + \gamma^*(q_2) \to$ hadrons can be described by factorizable Regge poles with $\alpha_R(0) > 0$. (ii) The coupling of the Pomeron (which we take to be factorizable) is ensured in the photon reactions at $t = 0$ by some mechanism which respects factorizability. (iii) The Regge contributions to $\gamma^* + p \to$ hadrons satisfy Bjorken scaling (are function of q^2/s only), with small σ_L/σ_T. It will be evident that some of the above are less critical to specific results than others.

It follows that we can write

$$W_{\lambda'_2\lambda'_1;\lambda_2\lambda_1} = \sum_R \beta_R^{\lambda'_2\lambda'_1;\lambda_2\lambda_1}(q_1^2, q_2^2)\,(s/s_0)^{\alpha_R(0)} \tag{10}$$

and, from factorization and scaling,

$$\beta_R^{\lambda'_2\lambda'_1;\lambda_2\lambda_1} = \gamma_R^{\lambda_1\lambda_1}\,\gamma_R^{\lambda_2\lambda_2}\,\delta^{\lambda'_1,\lambda_1}\,\delta^{\lambda'_2,\lambda_2}\,(q_1^2)^{-\alpha_R(0)}\,(q_2^2)^{-\alpha_R(0)} \tag{11}$$

where the presence of the Kroneker delta follows from a very useful result of Fox and Leader[11], and is expected to hold for the known trajectories with $\alpha_R(0) > 0$. The $\gamma_R^{\lambda_i\lambda_i}$ should be constants. If we describe longitudinal amplitudes by the small parameter $R \simeq 0.2$ (it may vanish in some limit), we have[7, 5]

$$W_{11;11} = W_{1-1;1-1} = \sum_R (\gamma_R^{11})\left(\frac{s\, s_0}{q_1^2\, q_2^2}\right)^{\alpha_R(0)} \tag{12}$$

$$W_{-1-1;11} = 0$$

and the (for us) less interesting relations

$$W_{11;00} = W_{0-1;10} = 0$$

$$W_{10;10} = W_{01;01} = RW_{11;11} \tag{12'}$$

$$W_{00;00} = R^2\, W_{11;11}$$

From now on we shall leave the longitudinal amplitudes out of the discussion. Our interest will be mainly in the sharp distinction between relations (12) and the corresponding ones which follow from a parton model with spin 1/2 partons, for which the longitudinal amplitudes are asymptotically negligible.

It is evident that the above Regge predictions can hold only in the R_β class of limits; this depends on assumption (iii). There is, in fact, no compelling reason to expect Regge contributions outside the R_β class of limits. One even has, in the S-limit, a bounded $\cos\Theta_t = \omega'/(1-\xi'^2)^{1/2}$. We can also consider two conceivable models for the leading (pomeron) Regge term. In one of these, the multiperipheral model[12], the produced hadrons are linked by t-channel propagators. We can apply the relation (9) to this case by grouping produced hadrons so as to show that the squared four momentum along <u>any</u> t-channel propagator is unbounded. The model assumes cutoffs in t, so the production amplitudes presumably vanish rapidly in the S_β limits. A second model, which goes under various names[13], assumes that the hadrons are in two clusters linked by a Reggeon (the pomeron itself). Again, the presence of a cutoff in t requires the production amplitudes to vanish rapidly in the S_β limits. Of course, it must be admitted that such models may be entirely inapplicable to reaction (1). Evidently, the rapid disappearance of the above model production amplitudes is connected with the presence of a transverse momentum cutoff. This feature may be more general than the models. From all this we would, however, like to draw the inference that Regge pole ideas are at best applicable in the R_β class of limits. This can, in principle, be checked by experiment.

4. Parton Models

Parton models are conceptually quite opposed to Regge pole models. It is therefore worthwhile to see what the former imply for $\overset{*}{\gamma}\overset{*}{\gamma}$ annihilation. As a simple model, we consider the absorptive part of the box graph for forward off-shell $\overset{*}{\gamma}\overset{*}{\gamma}$

scattering with massless fermions (quarks). The motivation is the fact that in the L-limit this term is supposed to give the entire amplitude[9, 2]. We consider the helicity amplitudes in the S-limit; direct calculation gives (with quark charges e_i)

$$W_{\pm 1\ \pm 1;\,11} = \frac{\sum e_i^4}{2\pi} \int_{-1}^{1} dZ \frac{(1 - Z^2)\left[(\xi'^2 + \omega' - \xi)^2 Z^2 \pm \xi'^2 \omega'^2\right]}{\left[\omega'^2 - (\omega'^2 + \xi'^2 - 1) Z^2\right]^2} \qquad (13)$$

$$W_{1\ -1;\,1\ -1} = 0$$

The longitudinal amplitudes are zero, and one can easily check that the limit $\omega' \to 1$ ($s << |q_2^2 + q_1^2|$) reproduces the results in the L-limit following from the bilocal algebra of Fritzsch and Gell-Mann. Interestingly, the magnitude of the W's can test various quark charge assignments - if the bilocal algebra is correct, including its disconnected parts[14].

It is quite possible that the S-limit yields different scaling functions than the above, which are strictly only applicable in the L-limit. It is interesting that there exists a model in which (13) does hold in the S-limit. This is the parton model of Landshoff and Polkinghorne[15]. In this model one has, besides the box graph, terms which a parton-parton scattering amplitude enters (Fig. 2)[10]. It is a basic assumption of the model that such amplitudes vanish rapidly as the virtual $(\text{mass})^2$ of a parton line becomes large. We can apply relation (9) to this set of diagrams, however, to show that they must vanish relative to the box graph in the S (actually the S_β) limit[16].

The contrast between (13) and (12) clearly offers the possibility of a test to distinguish parton and Regge models for this process. A large $W_{-1-1;\,11}$ term would be evidence in favor of parton models- or at least against Regge models. It should be pointed out that helicity amplitudes which are constants or functions of ξ' and ω' in the S-limit correspond to a photon-photon cross section which falls like s^{-1}.

There are several further possible tests of the parton model as opposed to the Regge model for (1). Consider, for example, the reaction where one hadron is observed in the final state. A recent discussion of processes of this sort, following the generalized Regge analysis of Mueller[17], has been given by Abarbanel and Gross[18]. The latter authors discuss inelastic e p scattering, and generalize the result of Fox and Leader to the forward three-to-three particle amplitude where one of the particles is an off-shell photon. We can take over the assumptions of ref.[18] to the case at hand and test them. For example one finds from the vanishing of s-channel helicity flip vertices that of the twenty terms in the combined lepton-hadron angular distribution for reaction (2)[19], sixteen vanish. A further interesting point, this time involving the parton model, lies in the dominance of two parton states coupled to two photons in the L limit. If partons have isospin $\leq 1/2$, we would conclude that the final hadron state resulting from reaction (1) has isospin ≤ 1. This can be tested by using isospin inequalities derived for, ray, pion states with $I \leq 1$[20]. Considering further the final hadron states in off-shell photon-photon annihilation, it is worth noting that if the angular distribution of the hadrons follows even roughly that of the partons, then the hadron distributions in the S limit should be much more uniform in angle than for hadron-hadron collisions or presumably for annihilation of on-shell photons for large s, where one expects a transverse momentum cutoff to be present. If a jet structure for $e^+ e^- \rightarrow$ hadrons is found, it is further possible that jets will also be present in $\gamma^* \gamma^*$ annihilation with the same distribution of hadrons along the jet axis as for e^+e^- annihilation. The detailed study of hadron distributions is, however, outside the limits of this paper.

5. Conclusion

So far we have not said anything about the measurability of (1) in reaction (2), so a few general remarks are in order. With unpolarized lepton beams one can, in reaction (2), measure $1/2\ (W_{11;11} + W_{1-1;1-1})$ and $W_{-1-1;11}$[4-6]. Measurement of $1/2\ (W_{11;11} - W_{1-1;1-1})$ requires that both lepton beams be longitudinally polarized, and is surely remote. It might be feasible to study (2) in the region of $|q_i|^2$ and s less than a few $(GeV)^2$ with machines now being constructed. Measurement of $W_{-1-1;11}$ at small $|q_i^2|$ might be interesting to see if any large non-Regge term is present; in this case we can make use of small electron scattering angles to enhance the counting rate. One can also study exclusive channels like $\gamma^* + \gamma^* \to \pi^+ \pi^-$ for a partonlike contribution[21].

There is a background of C = -1 hadron states in (2) which must be subtracted before one can extract cross sections for process (1). Fortunately this sort of calculable (in terms of $\sigma(e^+ e^- \to$ hadrons)) background is not a serious problem.

We have seen that there are two points where Regge models and parton models for $\gamma^* + \gamma^* \to$ hadrons can be discriminated. First, the spin structure is quite different- a feature which makes no appearance in inelastic electron proton scattering and, second, they appear to dominate in different limits. The parton model leads to helicity amplitudes at fixed s which are functions of a scaling variable $\xi = Q \cdot p/Q^2$ or $\xi' = (q_2^2 - q_1'^2)/(q_2^2 + q_1^2)$, but independent of q_i^2 and s. This is even the case in the resonance region[2] and possibly for exclusive reactions near threshold[21]. The Regge model leads to helicity amplitudes for large s which increase for those limits where $ss_0/q_1^2\, q_2^2$ increases. In testing the models at moderate values of s and q_1^2, q_2^2 the different spin structures (equation (12) and (13)) obviously play an important role.

There is an interesting model-that of Landshoff and Polkinghorne-which contains as separate pieces Regge-like[22] and parton terms. The Regge-like terms contain a transverse momentum cutoff and vanish in the S-limit. The parton term, without the transverse momentum cutoff, survives in the S-limit and gives helicity amplitudes which are known dimensionless functions of two scaling variables ξ' and $\omega' = \xi - s/(q_2^2 + q_1^2)$. In this model one can ask about the relative importance at finite s, q_1^2 and q_2^2 of the two sets of terms. A question of definite interest- which transcends this particular model- is whether one or the other terms is missing. The best place to check for the parton terms is the $W_{-1-1;11}$ amplitude.

It appears likely that the inclusive reaction $\overset{*}{\gamma}(q_1) + \overset{*}{\gamma}(q_2) \rightarrow$ hadrons can be used to gain insight into the question of scaling behavior in electromagnetic processes and may be able to shed light on the behavior of inelastic electron proton scattering.

I would like to thank S. Drell for the hospitality extended to me at SLAC.

REFERENCES

1. See S. J. Brodsky, Electron-Photon Symposium, Cornell University 1971 and references there.

2. T. F. Walsh and P. Zerwas, Nucl. Phys. B41, 551 (1972).

3. H. Terazawa, Phys. Rev. D5, 2259 (1972); Z. Kunszt, Phys. Lett. 40B, 220 (1972).

4. V. M. Budnev and I. V. Ginzburg, Sov. J. Nucl. Phys. 13, 198 (1971).

5. R. W. Brown and I. J. Muzinich, Phys. Rev. D4, 1496 (1971).

6. C. E. Carlson and W-K Tung, Phys. Rev. D4, 2873 (1971).

7. A. V. Efremov and I. V. Ginzburg, Phys. Lett 36B, 371 (1971).

8. H. Fritzsch and M. Gell-Mann, Caltech rept. CALT-68-297 (1971).

9. D. J. Gross and S. B. Treiman, Phys. Rev. D4, 2105 (1971).

10. R. L. Kingsley, Nucl. Phys. B36, 575 (1972).

11. G. C. Fox and E. Leader, Phys. Rev. Lett. 18, 628 (1967).

12. D. Amati et al., Nuovo Cim. 26, 896 (1962)

13. M. Jacob and R. Slansky, Phys. Rev. D5, 1847 (1972), and references there.

14. H. Suura, T. F. Walsh and B-L Young, Lett. al Nuovo Cim. 4, 505 (1972).

15. P. V. Landshoff, J. C. Polkinghorne and R. D. Short, Nucl. Phys. B28, 225 (1971).

16. This has also been noticed by R. L. Kingsley (private communication).

17. A. H. Mueller, Phys. Rev. D2, 2936 (1970).

18. H. D. I. Abarbanel and D. J. Gross, Phys. Rev. D5, 699 (1972).

19. See equation A.9 in reference 6.

20. C. H. L. Smith and A. Pais, Rockefeller preprint C00-3505-21.

21. S. J. Brodsky, F. E. Close and J. F. Gunion, Phys. Rev. D6, 177 (1972).

22. In reference (10), the relation $W_{11;11} = W_{1-1;1-1}$, reminiscent of the Regge

result follows in the $R_{1/2}$ limit for the diagrams with a parton-parton scattering amplitude in them.

FIGURE CAPTIONS

1. Production of hadrons in $\gamma^* \gamma^*$ annihilation.
2. Diagrams for forward $\gamma^* \gamma^*$ scattering in the Landshoff-Polkinghorne model[10]. The solid lines are partons.

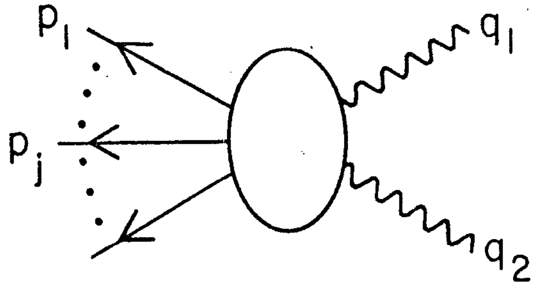

Fig. 1

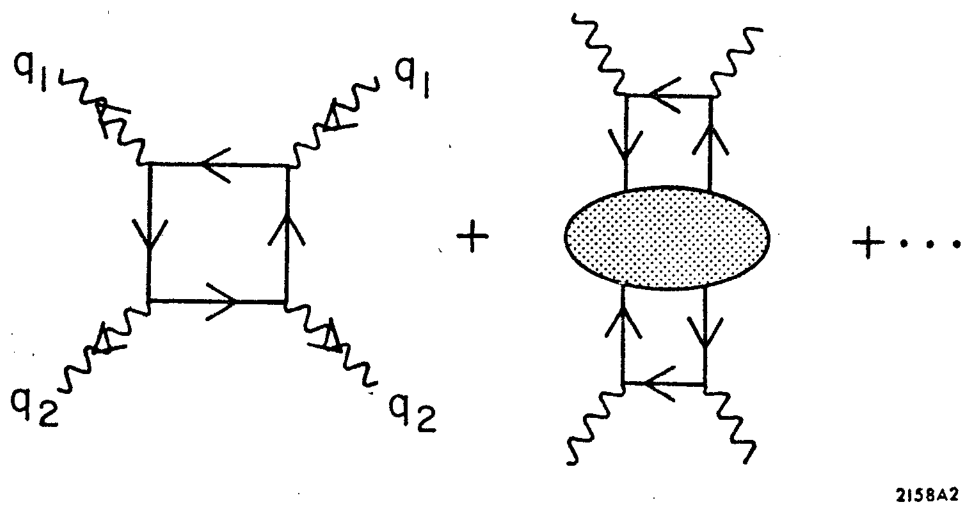

Fig. 2