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A PARTIAL-WAVE ANALYSIS OF THE REACTION $\pi N \rightarrow \pi\pi N$ IN THE c.m. ENERGY RANGE 1300 - 2000 MeV*

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ABSTRACT

A partial-wave analysis of the reaction $\pi N \rightarrow \pi\pi N$ has been carried out in the energy region 1300-2000 MeV. Two continuous solutions have been found; they are very similar in regions where data exists, but differ in the continuation of amplitudes through a gap between our low and high energy data. The second solution ("B") gives a much better fit to the data. These new partial wave amplitudes provide important information on the inelastic couplings of the nucleon resonances to the $N\rho$, $N\epsilon$, and $\Delta\pi$ channels. A new resonance, $D_{13}(1700)$, long predicted by the quark model, has been observed coupling to two inelastic channels--ϵN and $\Delta\pi$, and the existence of a $P_{13}(1700)$ state is corroborated. Our preferred solution indicates a second new resonance, $P_{33}(1700)$, coupled strongly to the $\Delta\pi$ channel.

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Contents

Supplement

Tables of Partial Wave Amplitudes. Tables and punched cards, for Solution B, including 28×28 and 60×60 error matrices, are available on request.

The Data-Summary Tape of the actual π^- events is also available from LBL, SLAC, or Saclay. The π^+ events for $\sqrt{s} = 1810$ MeV and above may be requested from the UCR-LBL Collaboration (see Table I).

I. INTRODUCTION

Elastic phase shift analyses have provided us with an impressive list of resonances, which is both the essence of our understanding of baryon spectroscopy and also the main testing ground for many of the ideas on the dynamics of hadronic processes. The agreement among the many independent groups is very impressive,[1-3] and gives confidence in the resulting scattering amplitudes.

Corresponding investigation of the inelastic scattering reactions has not kept pace with that of elastic reactions. This derives not only from the lack of data (with high statistics, and systematically spread in energy), but also from the complexity of the phenomenological analysis. However, the study merits the effort. As can be seen from Fig. 1, the inelastic cross section represents a very substantial fraction of the total πN cross section, even at 1.0 GeV/c, and it is therefore intrinsically interesting to understand the scattering process. In addition, the inelastic decays of N^* are a very specific signature of the state and its properties, and therefore an important study in their own right. Finally, for resonances with very small coupling to the elastic channel [e.g. $D_{13}(1700)$] these studies are the only effective means of investigating the resonance in a formation experiment.

In the resonance region the principle inelastic reaction is

$$\pi N \rightarrow \pi\pi N. \qquad (1.1)$$

We have therefore made a detailed study of this channel in the c.m. energy range 1300-2000 MeV. In a previous analysis of this data in the range 1640-1760 MeV we attempted to isolate the reaction

$$\pi^- p \rightarrow \pi^+ \Delta^-$$

from

$$\pi^- p \rightarrow \pi^+ \pi^- n \qquad (1.2)$$

by selecting only events with $1.14 < M(\pi^- n) < 1.320$.[4] This study enabled us to extract values for the F_{15} and D_{15} coupling constants to the $\pi\Delta$ channel. Another group has used a similar technique to analyze[5] the reaction

$$\pi^+ p \rightarrow \pi^0 \Delta^{++} \qquad (1.3)$$

to obtain the isospin-3/2 amplitudes in the c.m. energy range 1820-2090 MeV. (They have also contributed their data to the present study.)

In reactions of type (1.1) there is the possibility of producing many resonances which overlap strongly in the final states, particularly at lower c.m. energies. The interference effects associated with these overlaps are not removed by the Δ-selection techniques described above, and hence are an inherent limitation of such analyses. At higher energies the increased phase space and the possibility of using the mass conjugation technique[6] improve the situation. Nevertheless, the interference effects are still a problem. These effects led to fitting the reactions in their entirety using the isobar model and its extension;[7] these take into account the effects associated with the many strong final-state interactions present. These methods have only been used currently at c.m. energies below 1560 MeV.[8-10]

We have extended this latter approach by including many more intermediate final states (and partial waves) and using the maximum likelihood technique in confronting the data with theory. These extensions have allowed us to successfully apply this method throughout the energy range considered, 1300-2000 MeV. The data covers the regions 1300-1540 MeV and 1640-2000 MeV with a 100-MeV gap between the two regions. We presented one solution at the 1972 Batavia Conference.[11] The energy-independent partial wave analysis now yields two continuous solutions over the whole energy range. The solutions are very similar in the two regions where we have data, but have different continuations through the 100-MeV gap in the data. We favor Solution B, but no fundamental ambiguity exists in the partial wave analysis--when data in the gap region (1540-1640) become available, a clear choice between our two solutions will emerge. We include plots of our earlier Solution A for historical reasons and also to demonstrate the stability of many of our conclusions concerning the partial wave amplitudes. This analysis provides for the first time information on 50 inelastic couplings of the nucleon isobars and essentially accounts for all of the $\pi N \rightarrow \pi\pi N$ cross section in this energy range.

II. THE DATA

In the energy range we consider, single pion production reactions can be unambiguously identified in the bubble chamber, resulting in effectively bias-free data. We have gathered data from several large bubble chamber experiments[8-10,12,13] leading to a total sample of 200,000 events covering the reactions

$$\pi^- p \rightarrow \pi^- \pi^0 p, \tag{2.1}$$

$$\pi^- p \rightarrow \pi^+ \pi^- n, \tag{2.2}$$

$$\pi^+ p \rightarrow \pi^+ \pi^0 p, \tag{2.3}$$

$$\pi^+ p \rightarrow \pi^+ \pi^+ n, \tag{2.4}$$

at energies 1300-2000 MeV. These experiments are listed in Table I.

The major features of Reactions (2.1)–(2.3) can be observed in Figs. 2 and 3. The Dalitz plots of Fig. 2 demonstrate the presence of Δ(1236) production in all cases although its contribution decreases at the higher energies. Indeed, at these higher energies the major final state interaction is due to the ρ meson and its effects clearly should not be ignored even at the lower energies. The presence of these obvious resonance bands was the motivation for the quasi-two-body analyses of Refs. 4 and 5, but while the present solutions contain their qualitative features, they differ by many standard deviations from the earlier quantitative conclusions.

The variations in the structure of the production angle, θ, of the nucleon (see Fig. 3) are indicative of the presence of rapidly varying transition amplitudes. We can therefore anticipate that many partial waves will be necessary and that these will change rapidly with energy as expected from the presence of the structure in the inelastic cross sections of Fig. 1.

III. THE EXTENDED ISOBAR MODEL

In this section we summarize the ingredients of our extended isobar model and give the final formulae and partial waves we use in our fits. A detailed discussion and derivation of all formulae may be found in Herndon, Söding, and Cashmore.[14]

A. Ingredients of Isobar Model

(i) We assume that the reaction proceeds through three quasi-two-body channels

$$\begin{aligned} \pi N &\rightarrow \pi\Delta\,(1236) \\ &\rightarrow N\rho\,(760) \\ &\rightarrow N\epsilon \end{aligned} \qquad (3.1)$$

where ϵ represents the strong s-wave $\pi\pi$ final state interaction at around 650 MeV.

(ii) The reaction can proceed through a large number of partial waves. We then have a transition amplitude for each angular momentum and isospin state which we write as $T^c_{\alpha\mu}(\omega)$, where

- c represents the charge channel, e.g. $\pi^+\pi^-n$, $\pi^-\pi^0p$, etc.;
- α is the group of quantum numbers (F, L, L', I, J) describing the reaction. These are summarized in Fig. 4, and the notation is spelled out below Eq. (3.2);
- μ represents μ_i, μ_f, the initial and final nucleon helicities;
- ω describes the four kinematical quantities necessary to describe an event: we choose these to be ω_1^2, ω_2^2 — the Dalitz plot variables — and

  $\cos\theta$, ϕ - where θ, ϕ are the angles of the incident π in our final-state coordinate system.[14] Our z-axis lies along the direction of the outgoing nucleon, and the y-axis is perpendicular to the production plane.

We note at this point that different charge channels differ only in isospin Clebsch-Gordan coefficients.

(iii) We can now explicitly develop some of the factors contained in $T^{c}_{\alpha\mu}(\omega)$;

$$T^{c}_{\alpha\mu}(\omega) = \mathbf{A}_{\alpha} D_{\alpha\mu}(\omega) B_{\alpha}(\omega) W_{F}(\omega) C^{c}_{\alpha} \equiv \mathbf{A}_{\alpha} X^{c}_{\alpha\mu} , \tag{3.2}$$

where $D_{\alpha\mu}(\omega)$ contains all spherical harmonic factors (D-functions) associated with the angular momentum decomposition;

$B_{\alpha}(\omega)$ contains the centrifugal barrier factor b_L, associated with the decay of the intermediate J^P state;

$W_F(\omega)$ represents the Watson factor for the final state interaction;

$\mathbf{A}_{\alpha}$ represents the amplitude for the particular wave and is assumed to be only dependent on the total mass of the system and not on any of the submasses. It is these complex parameters A_{α} which we vary during a fit;

C^{c}_{α} the appropriate isospin Clebsch-Gordan coefficients for channel c.

(iv) The final transition amplitude to a given final state is then obtained by making a coherent sum of these individual amplitudes,

$$T^{c}_{\mu}(\omega) = \sum_{\alpha} T^{c}_{\alpha\mu}(\omega) . \tag{3.3}$$

This summation implies some double counting of the amplitudes, which had been thought to be a small effect.[15] However, prompted by Aaron and Amado and others,[16] the Illinois group[17] and we are both estimating corrections to the amplitudes.[18]

(v) The differential cross section is then given by

$$d\sigma^{c}(\omega) = \lambda\!\!\!^{-2} \sum_{\mu} \left| T^{c}_{\mu}(\omega) \right|^{2}, \tag{3.4}$$

where we sum over initial and final nucleon helicities as we are neither working with polarized targets nor observing the final polarization.

This construction of the final state amplitude and cross section allows easy fitting to all single-pion production channels and hence allows the partial-wave decomposition of the scattering amplitude.

(vi) Cross sections are expressed in terms of the conventional T-matrix elements T_{α} (called $T_{\pi N}$ for elastic scattering, $T_{\Delta\pi}$ for Δ production, etc.). The unitary circle has unit diameter, and if there were only a single channel present, the total cross section would be

$$\sigma_{\alpha} = 4\pi \lambda\!\!\!^{-2} \, (J + \tfrac{1}{2}) \left| T_{\alpha} \right|^{2}. \tag{3.5}$$

Note that by convention T carries no subscript μ as in Eq. (3.4); i.e.,

$$\left| T_{\alpha} \right|^{2} = \int \sum_{\mu} \left| T_{\alpha\mu} \right|^{2} d\Omega. \tag{3.6}$$

The nucleon helicity states are uniquely related by Clebsch-Gordan coefficients, i.e. $T_{\alpha\mu} \propto C(J, L', \mu) T_{\alpha}$ and so the actual amplitude T_{α} is well defined.

In practice however a single incoming partial wave can feed several values of α (i.e. several channels, like $\pi\Delta$, ρN, ϵN). This may result in substantial interference effects which are observable in the integrated cross section [see Eq. (3.7)]. The cross section is written as in (3.4),

$$\sigma_{in}(J^P) = \lambda^2 \int d\Omega \sum_{\mu} \{ \sum_{\alpha} |T_{\alpha\mu}|^2 + \text{interference terms} \},$$

which in the spirit of Eqs. (3.5) and (3.6) we define as

$$\sigma_{in}(J^P) = 4\pi\lambda^2 (J+\tfrac{1}{2}) \{ \sum_{\alpha} |T_{\alpha}|^2 + \text{overlap integrals} \}. \tag{3.7}$$

Equation (3.7) defines the normalization of the Argand plots. The overlap integrals are important and are taken into account at all stages of the program.

Different incoming partial waves never interfere, so we can write

$$\sigma = \sum_{J^P} \sigma(J^P). \tag{3.8}$$

B. The Final State Interaction, W_F

This could be parametrized as a Breit-Wigner factor. However, rather than attempting to represent the Δ, ρ or ϵ by Breit-Wigner forms we chose to use the Watson Final State Interaction to describe the effects of these strong interactions by a factor

$$W_F = \frac{e^{i\delta_F} \sin\delta_F}{q^{\ell+1}}.$$

Here δ_F is the appropriate elastic phase for the strongly interacting particles, and q is the momentum of each particle in the isobar rest frame.

The actual values of δ_F are summarized in Fig. 5. The only uncertain phase shift was for the ϵ ($I = 0$, $J = 0$ $\pi\pi$) but our parametrization is in close agreement with recent analyses[19] except near 1000 MeV which fortunately is at the extreme limit of phase space for our highest beam energy. For more discussion, see Herndon.[20]

C. The Centrifugal Barrier, $b_{L'}$

We should include centrifugal barriers in both the incident state and the quasi-two-body intermediate state. However, the first of these is constant for a given partial wave at a given energy and hence we have ignored it. (It's inclusion would just result in a rescaling of A_a, which is unimportant--see below).

The barrier factor in the intermediate state has been introduced as

$$b_{L'} = Q^{L'} \tag{3.10}$$

where Q is the isobar momentum in the c.m. and L' the orbital angular momentum.

Equation (3.10) is only the low-QR limit of the standard Blatt-Weisskopf factor,[21] which we meant to use. We inadvertently started with (3.10) and caught the mistake only after it was inconvenient to change. In Appendix C we show that (3.10) is adequate for our purpose, but urge the use of the correct form for all future partial-wave analyses.

D. The Partial Waves Used in Our Fits

As discussed above we only considered three final state interactions in our analysis

(i)	the $\Delta(1236)$	- intermediate state $\pi\Delta$
(ii)	the $\rho(760)$	- intermediate state $N\rho$
(iii)	the I=J=0 $\pi\pi$ interaction	- intermediate state $N\epsilon$.

Since our decomposition of the amplitude is essentially an LS representation [where $\underline{S} = \underline{S}$ (diparticle spin) + $\underline{S}$ (bachelor spin)], in the case of the ρ meson we have two transition amplitudes. These are obtained because

$$\underline{S}_\rho + \underline{S}_N = 1/2 \text{ or } 3/2, \tag{3.12}$$

and are denoted as ρ_1 or ρ_3 waves.

In order to make the fitting problem tractable we have limited ourselves to orbital angular momenta in the incident and final states, $\ell \leqslant 3$ and total angular momentum to $j \leqslant 7/2$. In Table II we list the 60 waves with which we began the analysis. As described in the later sections the actual number of waves required increased with c.m. energy, the maximum being 28 at our higher energies.

IV. THE FITTING PROGRAMS

A. Choice of Method

The choice of fitting procedure is clearly dictated by the number of events available in a given experiment. To exploit the correlations that exist in the data in the case of limited statistics the most powerful approach is that of the maximum likelihood technique.[22]

At the time we began our analysis, none of the existing maximum-likelihood fitting programs could handle either the amount of data or the number of parameters (up to 120) in a reasonable amount of computer time, so we developed a new program (RUMBLE)[23] which handles any problem in which the parameters appear bilinearly in the probability density (see below). It took the equivalent of 400 hours on the CDC 7600 to perform the analysis described in this paper, including random starts, studies of uniqueness, etc.

B. Likelihood Formulation

1. Likelihood in Each Charge Channel

The full expression of $\frac{d\sigma^c}{d^4\omega}$ (in charge channel c) in terms of the search parameters, A_α, is given by (3.2) through (3.4) as

$$\frac{d\sigma^c}{d^4\omega}(\omega_i) = \lambda^2 \sum_\alpha |T_{\alpha\mu}|^2$$

$$\frac{d\sigma^c}{d^4\omega}(\omega_i) = \lambda^2 \sum_\mu \Big| \sum_\alpha A_\alpha X^c_{\alpha\mu}(\omega_i) \Big|^2 . \tag{4.1}$$

For this formulation we rewrite $\frac{d\sigma}{d^4\omega}$ as $p(A_\alpha)$ to emphasize that it is a cross section predicted by the A_α, i.e.,

$$p^c(\omega_i, A_\alpha) = \lambda^2 \sum_\mu \Big| \Sigma_\alpha A_\alpha X^c_{\alpha\mu}(\omega_i) \Big|^2 . \tag{4.2}$$

We write the predicted total cross sections as

$$R^c(A_\alpha) = \int d^4\omega\, p^c(\omega_i). \tag{4.3}$$

The normalized probability for each event is then

$$P(\omega_i, A_\alpha) = \frac{p^c(\omega_i, A_\alpha)}{R^c(A_\alpha)}, \tag{4.4}$$

and the likelihood L^c describing the shape of the distribution is

$$L^c_{shape}(\omega_i, A_\alpha) = \prod_{i=1}^{N^c} p^c = \frac{1}{(R^c)^{N^c}} \prod_{i=1}^{N^c} p^c(\omega_i, A_\alpha). \tag{4.5}$$

If we dealt with only one charge channel, common sense would tell us to adjust the scale of the A_α by setting $R^c(A_\alpha)$ equal to the measured channel cross section σ^c, but for several channels the correct scaling is more complicated. Hence we must next formulate the generalized likelihood L^c more precisely.

The path length ℓ^c may differ from channel to channel. If each experiment reports N^c events and a cross section σ^c, then

$$\ell^c = \frac{N^c}{\sigma^c} \text{ (events/}\mu\text{b)} \tag{4.6}$$

and the predicted number of events ν^c is given by

$$\nu^c = \ell^c \, R^c(A_a) = \frac{N^c}{\sigma^c} R^c(A_a). \tag{4.7}$$

The Poisson probability of observing ν events when N are expected is

$$P(\nu, N) = \frac{\nu^N e^{-\nu}}{N!} . \tag{4.8}$$

Using (4.7) this becomes $P(\nu, N) = \frac{1}{N!}(\frac{N}{\sigma})^N R^N e^{-\frac{NR}{\sigma}}$, i.e., inserting the superscript c,

$$P(\nu^c, N^c) = \frac{1}{N^c!}(\frac{N^c}{\sigma^c})^{N^c}(R^c)^{N^c} e^{-\frac{N^c R^c}{\sigma^c}} . \tag{4.9}$$

We now form the generalized channel likelihood,[24] $L^c = P(\nu^c, N^c) L^c_{shape}$ where P is given by (4.9) and L^c_{shape} by (4.5). The R^c factors cancel, to give

$$L^c = \frac{(N^c/\sigma^c)^{N^c}}{N^c!} \cdot \exp\{-\frac{N^c R^c}{\sigma^c}\} \cdot \prod_1^{N^c} p^c(\omega_i, A_a) . \tag{4.10}$$

Finally the multichannel likelihood L is the product of each L^c

$$L = \prod_c [\frac{(N^c/\sigma^c)^{N^c}}{N^c!}] \exp\{ -\frac{N^c R^c}{\sigma^c}\} \prod_1^{N^c} p^c(\omega_i, A_a). \tag{4.11}$$

2. Analytic Scaling of the A_α

We have already said that if we had only one charge channel we could set $R^c = \sigma^c$. Let us check this for single channel Eq. (4.10) by introducing a scale factor, s, e.g.

$$A_\alpha = s\, A^0_\alpha . \qquad (4.12)$$

Dropping factors which do not contain s, Eq. (4.10) becomes

$$L^c(s\,A^0_\alpha) = \exp\{-\frac{N^c s^2 R^c(A^0_\alpha)}{\sigma^c}\}\,(s^2)^{N^c} \prod_1^{N^c} p^c(\omega_i, A^0_\alpha) \qquad (4.13)$$

$$\ln L^c\,(s\,A^0_\alpha) = -\frac{N^c s^2 R^c(A^0_\alpha)}{\sigma^c} + N^c \ln s^2 + \text{constant} \qquad (4.14)$$

$$\frac{\partial \ln L^c(sA^0_\alpha)}{\partial s} = -\frac{2N^c\, s\, R^c(A^0_\alpha)}{\sigma^c} + 2N^c \frac{1}{s} . \qquad (4.15)$$

Setting this to zero gives

$$\frac{1}{s^2} = \frac{R^c(A^0_\alpha)}{\sigma^c} , \qquad (4.16)$$

which as we guessed earlier indeed gives

$$R^c(A_\alpha) = s^2 R^c(A^0_\alpha) = \sigma^c$$

and $$L^c(s^{max} A^0_\alpha) = e^{-N^c}\left[\frac{\sigma^c}{R^c(A^0_\alpha)}\right]^{N^c} \prod_1^{N^c} p^c(\omega_i, A^0_\alpha). \qquad (4.17)$$

This expression is manifestly independent of the magnitude of the vector A^0_α.

Next we can apply the same procedure to the multichannel L of Eq. (4.11). Details are given in Appendix A. This time the equivalent of (4.16) is easily found to be

$$s^2\Big|_{L_{max}} \equiv s^2 = \frac{\Sigma N^c \equiv N}{\Sigma \frac{R^c N^c}{\sigma^c}} . \tag{4.18}$$

It is also shown in Appendix A that inserting this into (4.11) yields a multichannel likelihood

$$L(s_{max} A_\alpha) = B\, e^{-N} N^N \left[\Sigma \frac{N^c R^c(A_\alpha)}{\sigma^c}\right]^{-N} \prod_c \prod_{i=1}^{N^c} p^c(\omega_i, A_\alpha). \tag{4.19}$$

The quantity which we actually maximize is the average log of (4.19) premaximized with respect to the scale of A_α,

$$F = \frac{1}{N} \ln L = \text{constant} - \ln\left[\sum_c \frac{N^c R^c(A_\alpha)}{\sigma^c}\right] + \frac{1}{N} \sum_c \sum_{i=1}^{N^c} \ln p(\omega_i, A_\alpha). \tag{4.20}$$

C. Unitarity and the Agreement with Elastic Phase-Shift Predictions

Using unitarity and the elastic amplitude from elastic partial-wave ("phase shift") analysis (EPSA), one gets upper bounds for $\sigma(N\pi\pi, IJ^P)$. The partial waves used were those available in 1970, when we started this analysis.[20] For most of our fits we did not need (or impose) these constraints. Even in those cases in which we predicted more cross section than allowed, we were within two standard deviations of the upper bound. To correct this, we added to $F(A_\alpha)$ the χ^2-like terms F_{EPSA}

$$F_{EPSA} = -\frac{1}{N} \sum_{IJP} a_{IJP} \left[\frac{\sigma^{IJP} - \sigma^{IJP}_{EPSA}}{\delta\,\sigma^{IJP}_{EPSA}}\right]^2 \theta(\sigma^{IJP} - \sigma^{IJP}_{EPSA}), \tag{4.21}$$

where σ^{IJP} is defined by (3.7) and σ^{IJP}_{EPSA} is an average over the different EPSA, and $\delta\sigma^{IJP}_{EPSA}$ are the external errors on σ^{IJP}_{EPSA}.

By using the step function, θ, F_{EPSA} only affected the likelihood when the fitting parameters predicted more cross section than allowed. These additional terms had very little effect on the fitted parameters. Of those amplitudes affected, the modulus was slightly reduced but the phases never changed.

D. Stepping to a Maximum

Maximizing procedures are of three general categories. At each step they evaluate

(i) only the function F, or

(ii) F and the first derivative vector ∇F or

(iii) F, ∇F, and the second derivative matrix,* $\nabla\nabla^T F$. For convenience we define the variance matrix $\underline{V} = (\nabla\nabla^T F)^{-1}$.

We have found the most efficient fitting technique is a combination of types (ii) and (iii).

In both types of fitting programs, one "step" in the parameters is given by $\Delta \vec{A} = -\underline{V}' \cdot \nabla F$ where $\underline{V}'$ is a negative definite approximation to $\underline{V}$ (for some cases $\underline{V}' = \underline{V}$). We use two methods of updating $\underline{V}'$, the Davidon method[25] and a modified form of the Newton-Raphson method.[26]

1. The Davidon Method

The Davidon method belongs to category (ii). The initial $\underline{V}' = \underline{V}'_1$ is chosen to be diagonal, and at each subsequent step V'_1 is modified by the addition of a rank-one matrix. A rank-one matrix is one having just one eigenvector with a nonzero eigenvalue. A typical example is the outer product of a vector with itself, $M_{ij} = v_i v_j$ or in matrix notation $\underline{M} = \vec{v}\vec{v}^T$. If $\vec{v}$ has unit magnitude, then $\underline{M}$ is called a projector.

*The superscript T means transpose.

Davidon showed that at the i^{th} step $\underline{v}'$ should be modified by

$$\underline{V}'_{i+1} = \underline{V}'_i + \lambda [\underline{V}'_i \cdot \nabla F_i] [\underline{V}'_i \cdot \nabla F_i]^T . \qquad (4.22)$$

The number λ is calculated from $\underline{V}'_i, \nabla F_i$, and ∇F_{i-1}. To ensure that the step is towards a maximum, rather than a minimum or saddle-point, λ is adjusted if necessary to keep $\underline{V}'_i$ negative definite. This adjustment to V'_i reflects the additional knowledge about the curvature of F gained from knowing the new first derivative ∇F_i. If the 2n-dimensional $\vec{A}$ space is quadratic, Davidon showed that after 2n steps $\underline{V}'_{2n} = \underline{V}'$.

2. The Original Newton-Raphson Method

This method chooses $\underline{V}' = \underline{V}$ and recalculates the entire matrix at each step. The modified method, which we use, calculates a negative definite approximation to $\nabla\nabla^T F$ and takes $\underline{V}'$ to be the inverse of this approximate matrix. The approximation is such that if our model exactly predicts the event distribution for some vector $\vec{A}'$, then $\underline{V}' = \underline{V}$ at $\vec{A}'$.

3. Our Procedure

Our fitting procedure was to calculate and invert the approximate second derivative matrix every twenty steps. In between these steps we used the Davidon technique to update the matrix. We also found it more efficient to take a few (in our case 5) Davidon-type steps initially before inverting the second derivative matrix.

4. Redundant Parameters

Since F is invariant to scale changes in $\vec{A}$ and to an overall phase change in $\vec{A}$, there are two redundant parameters, and the second derivative matrix is singular. These singularities are usually eliminated in problems similar to ours by permanently freezing the phase and modulus of one A_α and reducing by two the dimension of $\nabla\nabla^T F$. We find, however, that the maximizing procedure takes less than one-third the number of steps to reach a maximum if all the A_α are permitted to vary. In this case, the second derivative matrix has two eigenvectors with zero eigenvalues. The desired V' is the inverse of $\nabla\nabla^T F$, restricted to the space spanned by the set of eigenvectors with nonzero eigenvalues. For details on how the fitting program calculates the proper V', see Ref. 23 . A two-dimensional example is given in Appendix B.

E. Errors in Amplitudes on Argand Plots

The error matrix E is given by

$$E = \langle \delta \vec{A} \cdot \delta \vec{A}^T \rangle = \frac{1}{N} \cdot V' \tag{4.23}$$

where $V' \approx \nabla\nabla^T F$ as described above, and the factor $\frac{1}{N}$ arises because $\ln L = NF$.

In the neighborhood of any local maximum at A^{max} with likelihood L^{max}, we can expand L as

$$L = L^{max} \exp\{-\frac{1}{2}\chi^2\} \tag{4.24}$$

and an error hyperellipsoid may be defined by a hypersurface in **A** space labelled by $\Delta\chi^2 = 1$, i.e. by $L = L^{max} e^{-1/2}$.

To plot error ellipses on our Argand plots we project this hypersurface on the complex plane representing a single T_α (or A_α). However, the probability that a result will be within this ellipse is only 40%. In order to increase this probability we have conservatively doubled our estimated error, thereby raising the χ^2 contour to 4, and enclosing 87% of the probability. In summary, all the errors plotted or tabulated in this paper and in our previous publications are twice those calculated by the program RUMBLE which uses (4.23).

F. Relative Likelihood of Competing Solutions

We were bound to encounter two different sorts of competing solutions:

i) The number of parameters (waves) is the same for both solutions, as in the case where two starting values lead to competing maxima

ii) The number of parameters differs, as when we have cast out a wave from solution A with likelihood L_A, found a new maximum B, and wonder if the new L_B is significantly worse than L_A.

For the discussion below, assume that hypothesis A is the right one and that after a fit we find a likelihood ratio L_A/L_B which we call L_{AB}. A standard approach is then to say that solution B is ruled out to 3σ if $\ln L_{AB} = NF_{AB}$ is greater than 4.5 (or 2σ of $NF_{AB} > 2$). We now carry this test one step further and take into account the error δF_{AB} in F_{AB}. Specifically we wish not to eliminate solution B unless $F_{AB}/\delta F_{AB}$ is safely larger than unity. (More sophisticated versions of this test are discussed by P. H. Eberhard, A. H. Rosenfeld, and M. Tabak).[27] We apply the same numerical test to both sorts of competing solutions i) and ii) mentioned above.

Of course without generating Monte Carlo events we do not know δNF_{AB} but we can estimate it from the 10,000 events in hand at each energy. For each event i we form F^i_{AB} and can then evaluate.

$$(\delta' F_{AB})^2 = \frac{\Sigma(F^i_{AB})^2}{N} - \left(\frac{\Sigma F^i_{AB}}{N}\right)^2 . \qquad (4.25)$$

Figure 6 is a scatter plot of typical values of $\delta' F_{AB}$ vs. F_{AB}. The ln-likelihood-ratios are of course NF_{AB}, not F_{AB}, but we plot F_{AB} so that we can show that all points scatter about the dashed line, even though the number of events differs from energy to energy. For easy interpretation we label the scale as 10,000 δF_{AB} and 10,000 F_{AB}, corresponding to ln-likelihood-ratios for typical 10,000-event samples. The points seem to scatter around the dashed line independently of:

i) the number of events (as just mentioned),

ii) the number of parameters,

iii) whether the events are real or Monte Carlo.

We have erected a vertical line at a ln-likelihood ratio of 3, and the standard test would say that to the right of this line solution B would begin to be ruled out. But consider the "X" plotted as far right as 4.5; its error 10,000 $\delta' F_{AB}$ is estimated to be 8.5, so that the value 4.5 is not an adequate reason for eliminating one hypothesis.

Our extra test agrees well with our studies (in Section V. C.) of actually casting out waves and refitting. Thus on Table V we show the results of removing a wave (PP_{13}) whose amplitude $|T|$ appeared to be only 2 to 3 standards deviations from zero. After refitting, χ^2 went up only 11 (for Monte Carlo events) or 14 (for real events) but ln-L decreased by 30 for both sets of events. We do not believe that the solution with the PP_{13} wave is e^{30} more likely than the solution without. But Fig. 6 shows that when $\Delta \ln L$ is 30, its error is about 10, and the significance test $F_{AB}/\delta' F_{AB} = 30/10 = 3$ seems quite reasonable.

In any case this extra test ($\delta' F_{AB} < F_{AB}$) offers a convenient numerical way to relate the result of several studies to the frequently-occurring question "can we throw out this solution?" and it is sufficiently conservative that we use it with considerable faith.

Finally we should point out the large circles plotted higher on Fig. 6. They are not $\delta' F_{AB}$, but just $\delta' F_A$ or $\delta' F_B$. These fluctuations in the ln-likelihood are of course much larger than the fluctuations in its ratio, but it is reassuring to note that as hypotheses A and B become significantly different, $\delta' F$ approach $\delta' F_{AB}$. We plot these δF dots for two reasons:

i) We know that some older programs have used $\delta' F$ rather than $\delta' F_{AB}$ to compare competing solutions, and we want to warn that δF is too coarse.[28]

ii) Sometimes one knows that the two competing solutions are really very different, and it is of course easier to calculate $\delta' F$ rather than $\delta' F_{AB}$ since a program does not need to know about both hypotheses at the same time. In this case it is convenient to know that $\delta' F$ is an upper limit to $\delta' F_{AB}$.

G. Program Tests

We made several checks on our programs:

1) To insure we were calculating that $X^c_{\alpha\mu}$ correctly in Eq. (3.2), we obtained the programs from three other analyses.[7,9,10] In all cases we got agreement[29] among the different programs.

2) To test the fitting program itself, we generated artificial data from eleven "known" amplitudes and then fit for the amplitudes. We generated 7583 events (4733 $N\pi^+\pi^-$ and 2850 $p\pi^-\pi^0$) at $\sqrt{s}$ = 1690 MeV. To obtain a set of reasonable starting values, we generated

2000 sets of random amplitudes and kept the 20 sets with the highest likelihood. These 20 sets coalesced to five distinct solutions after fitting. Any wave among the 60 for which the modulus was within one standard deviation of zero in at least three solutions, was considered statistically insignificant and eliminated. After elimination, we refit with fewer waves and again eliminated waves. This was repeated a total of four times until no further waves could be removed. All five solutions coalesced into one solution and the number of waves dropped from 60 to 24. These 24 waves consisted of the original 11 waves and another 13 waves, each of which had $|T| < 0.05$. Figure 7 shows the initial 11 waves (as dots) and the corresponding 11 fitted amplitudes. The agreement is very reassuring, and gives us confidence that we can select the important waves for an original 60-wave hypothesis. However, in the case of real data, we are concerned with the extra uncertainties of fitting with an imperfect model, so we prefer to quote a "sensitivity" of $T = 0.1$.

We also tried to break up the 7583 events into smaller samples of the order of 1000 events, but we found we needed all the events generated to get a good 60-wave fit. From this we decided to work with at least 10,000 events at each energy in order to make fits to 60 waves.

This completes our discussion of program tests made before we started fitting real events. More tests of realistic amplitudes (from fitting real events) are described in Sec. VC.

H. Limits for the Observation of Partial Waves

From the tests above, and from our experience to be described in Sec. V, we estimate that we are sensitive to any inelastic partial wave for which

$$T_\alpha > 0.1, \tag{4.26}$$

or, in terms of branching fractions $X_\alpha = \Gamma_\alpha/\Gamma$,

$$T_\alpha = \sqrt{X_{el} X_\alpha} > 0.1.$$

V. FITTING PROCEDURES, TESTS, AND QUALITY OF THE FINAL SOLUTIONS

A. Obtaining The 1972 24-Wave Solution "A"

The number and distribution of the events used in the analysis are given in Table III. The continuous distribution of events was binned into c.m. energy intervals of 30 or 40 MeV, except between 1560 and 1630 MeV, where no data were available. In this section, we give an outline of the procedure used in obtaining our final solution. This procedure involved several distinct stages.

The data were fitted in three parts: below 1560 MeV, 1630-1830 MeV, and above 1830 MeV.

i) 1310-1560 MeV

In this region (9 bins) we reduced our 60 waves to 36 by removing all waves with $J \geq 5/2$. In each of the six bins between 1380 and 1560 MeV, we generated 2000 random sets of amplitudes to use as starting values to the fitting program and kept the top ten.

Any wave within two standard deviations of zero was considered statistically insignificant and removed. In this manner we were able, at each energy, to reduce to one solution. To look for continuity in energy we used each solution as a starting value for the neighboring energy bins. This new starting value always converged to the existing solution at that energy. Moreover our $\vec{A}(E)$ varied smoothly with energy providing a continuous solution. Below 1380 MeV, there were too few events for the fitting program to be able to distinguish between different solutions. In this region we propagated the solution from the bin above, removing all unnecessary waves.

ii) 1630-1830 MeV

In each of these five energy bins, we again generated 2000 random sets of amplitudes and this time kept the 20 with the highest likelihoods. From these twenty sets, we fit with as many sets (10-15) as necessary to generate five distinct 60-wave solutions. We then made a list of all waves whose modulus was within 2.0 standard deviations of zero in three of the five solutions. These waves were removed and the solutions were refit. This continued until no further waves could be removed. At this point, the five original solutions at each energy had reduced to two or three at that energy. However, the removed waves were different at each energy!

We then returned to the 60-wave solution and removed all those waves which had been eliminated in at least three energy bins. Then we remaximized. We continued to remove waves that were unnecessary in at least three bins. In this way we reduced from 60 to 30 waves, but still had several competing solutions at each energy.

iii) 1830-1990 MeV

Instead of using the full 60 waves, we started with the 30 waves from the lower energies together with the eight F_{35} and F_{37} waves which had been removed at lower energy. We generated 2000 random starts in this 38-wave set and kept only enough of the highest sets to give four separate solutions after fitting. Where possible we again removed the unnecessary waves.

iv) Continuity in Energy Above 1630 MeV

To look for continuity, we now used each solution as a starting value in the energy bin above and below its own. In more than half the cases, these starting values led to already existing solutions. If a new solution had a comparable likelihood ($\Delta F/\delta\Delta F < 1$), we not only kept it, but also used it as a starting value above and below its energy; and of course we checked that waves needed at $\sqrt{s}$ = 1540 were present above the energy gap. In looking for smooth energy behavior, we found that some waves had discontinuous (almost random) behavior. We removed these waves even though the fitting program felt they were necessary. In 1972 we found only one chain of solutions over the entire energy range 1620-1999 MeV. This chain consisted of 20 waves from 1620 to 1710 MeV, 23 waves from 1710 to 1750 MeV, and 24 waves from 1750 to 1990 MeV. Figure 8 gives the waves needed at each energy. This is the point at which our analysis stood at the end of 1972 and this solution will be referred to as solution A.

B. The 1973 28-Wave Solution B

As we will discuss in Sec. VI C, the 1972 solution had some 'undesirable' theoretical properties. In an effort to see if this solution was completely stable to the inclusion of new partial waves, we added some further amplitudes to our 1972 set (those marked "*" in Fig. 8), and repeated the fitting process. We found a new continuous solution in each energy range which differed dramatically from our previous solution only in the P_{11} waves and the new waves. However, as we shall see this has allowed us a further degree of flexibility in relating the partial-wave amplitudes in our two regions of analysis. In this case the final set contained 28 waves and we will refer to this as solution B.

C. Tests and Quality of our Fits

1. Four-dimensional χ^2

To compare our fits with the data, we consider four-dimensional (or less) binning of our data. A theoretical bin population can be calculated by binning Monte Carlo events weighted by $p(\omega_i, \vec{A})$. The solid lines in Fig. 9 were generated for Solution A in this manner. From these theoretical bin populations we can also calculate various χ^2 for the fit. Results are given in Table IV; 24 Dalitz plots corresponding to Fig. 9 are given in Ref. 20.

The purpose of using the maximum likelihood technique is to take advantage of the many correlations that exist in the data and thus the real test of our solutions lie in the ability to account for such correlations. As can be seen from Fig. 9 and the χ^2 values, we observe excellent reproduction of those correlations. This constitutes a major justification for our solutions.

2. Agreement with Total and Partial-Wave Inelastic Cross Sections

In Fig. 10 we demonstrate that the single-pion production cross sections in the channels we fit are well reproduced. However, we can compare our prediction for

$$\pi^- p \rightarrow n\pi^0\pi^0$$

$$\pi^+ p \rightarrow n\pi^+\pi^+$$

which are not included in our analysis. This is done in Fig. 11 where we observe excellent agreement for $\sigma(n\pi^0\pi^0)$ but a large discrepancy for $\sigma(n\pi^+\pi^+)$. This latter fact may well have a simple physical interpretation and we return to this point in Sec. VII.

We can make even more stringent tests by comparing our inelastic partial-wave cross sections with the predictions from EPSA. At the lower energies where single-pion production is the major inelastic channel we find excellent agreement as demonstrated in the next section. However, at high energies, other inelastic channels begin to appear and the prediction from EPSA only becomes an upper bound on the single-pion production cross section and this bound is satisfied at all energies.

3. Removing Waves, Monte-Carlo Checks

At 1530 MeV we tested the final fit to see if our model was adequately describing the data. From the final amplitudes (R for "Real") we generated an equivalent number of Monte Carlo events. By fitting these Monte Carlo events, we determined a set of amplitudes T_{MC} which gave the best fit (within the errors we indeed found $T_{MC} = T_R$). From both fits T_R and T_{MC} we then removed a wave, refit, and compared the changes in the likelihoods and in χ^2. The χ^2 was calculated by binning each variable into four bins ($4^4 = 256$ bins total). We did this for the four waves shown in Table V. The top wave listed, DS_{13} ($\rho_3 N$) is a "large" wave having $|T|/\delta|T| \approx 10$ and its removal caused the other fitted T_α to move by many standard deviations. The $PP_{33}(\Delta\pi)$ and $PP_{13}(\rho_1 N)$ are medium waves with $|T|/\delta|T| \approx 6$ and 3 respectively. The $DS_{33}(\rho_3 N)$ is a "small" wave with $|T|/\delta|T| \approx 1/2$. Table V shows the results of these changes, which seem reasonable, and suggests that our program behaves properly. Since the results are the same for both the real and Monte Carlo events we believe our model fits the data adequately.

We also removed waves at higher energies: the $DD_{15}(\Delta\pi)$ wave at 1650 and the $FP_{15}(\Delta\pi)$ wave at 1690. The $DD_{15}(\Delta\pi)$ is the only D_{15} at 1650 and is quite large. Indeed ln L dropped by 584 when it was removed. The $FP_{15}(\Delta\pi)$ wave however, is one of three F_{15} waves at 1690 and is only "medium" in size. Here Δ ln L was 186. So again the program seems adequately sensitive to "medium" waves.

4. Checks with a 60×60 Error Matrix

After finding a best fit at each energy, we found it useful to calculate the full 60×60 error matrix, even though a satisfactory fit had been obtained with only, say, 20 waves.

The idea behind inspection of the full error matrix is as follows: Suppose some untried wave, e.g. No. 59, is highly correlated with a wave used in the fit, e.g. No. 19. Then we had better try No. 59: we will probably find a new maximum with No. 59 as large as No. 19, and with No. 19 considerably changed. The clue that some wave is highly correlated with No. 19 can already be found in the diagonal elements of the error matrices, i.e., by comparing the error in a 19 as computed in the 60×60 error matrix and in the 20×20 matrix. If the 60×60 error is, say, twice as big as the 20×20 one, we should look among the off-diagonal elements for large correlation coefficients like $\langle \delta x\ \delta y \rangle_{19,59}$, where x and y can each be either amplitude or phase. A geometrical interpretation of the relation between the off-diagonal correlation and the increase in the diagonal error can be found in Fig. A1 of Appendix 1 of Ref. 20.

A 60×60 error matrix is too large to reproduce here (it is available in the supplement), however, the diagonal elements only, for both 20×20 and 60×60 matrices, are compared in Table VI. We see that the 60×60 errors are only 30% larger than their 20×20 counterparts, reassuring us that no highly correlated waves have been omitted.

Another use for the 60×60 error matrix is in making the transformation from partial-wave amplitudes, e.g. $\rho_1 N(FF_{15})$, $\rho_3 N(FP_{15})$, to helicity amplitudes, i.e., $\rho N(F_{15})$, $\lambda = 1/2$, and 3/2. One or two of the ρN partial waves may not have been needed in the fit, but clearly in making the transformation, the errors in these untried waves must be propagated along with the errors in the waves actually used.

D. Comparison of Solutions A and B

In the above sections we have discussed the origin of Solutions A and B and demonstrated that either corresponds to a good representation of the data available to date. We now make a quantitative comparison of the two solutions.

In Table VII a summary of the differences in likelihoods of Solutions A and B is given at nine energies between (1640-2000) MeV (evaluated for a standard sample of 9000 events). For each energy, Solution B gives the higher likelihood, and the difference is around 100. Since Solution B involves four additional partial waves compared to Solution A, we must determine whether this increase of 25 per wave in the likelihood function is significant. Reference to Table V, where we discuss the sensitivity of our solutions to the removal of waves, indicates that such a change in likelihood corresponds to that expected for the inclusion of three or four waves with $|T|/\delta T \sim 3.0$ (i.e., quite significant, moderately large waves). Thus we conclude that the four new waves are really required and that Solution B provides a substantially better representation of the data than the original solution A.

E. Summary

We find two solutions which possess all the following properties:

(i) At each energy the solution parameters correspond to a maximum in the likelihood function and have a high likelihood (usually the highest of the competing solutions).

(ii) The solution at each energy propagates to the solution at the adjacent energies above and below.

(iii) Qualitatively it has no discontinuous motion between adjacent energies.

(iv) It possesses good agreement with the EPSA predictions for the inelastic cross sections.

The crucial step was the selection of a good subset of waves. Our two final subsets were the only ones we found that had solutions satisfying all the above requirements. Due to cost of computer time we cannot be certain that we have found the only two subsets. But we believe that the larger waves are uniquely determined. We cannot be as certain in the case of the smaller waves, the amplitudes of which are never more than two or three standard deviations from zero. Furthermore, with 30-40 MeV energy bins, our emphasis on continuity clearly biases us against very narrow resonances.

VI. THE PARTIAL WAVE AMPLITUDES—DESCRIPTION AND DISCUSSION

At each energy the solution of any fits to inelastic data are only defined up to an overall phase. Thus, in order to give Argand diagrams of the partial-wave amplitudes we must determine this phase. Of the variety of methods by which this can be achieved, we have linked the inelastic amplitudes to published elastic amplitudes (which have known phase) via a K-matrix fit at energies where both elastic and inelastic amplitudes are large,[30] -- this turns out to be in the region of a prominent resonance. This has been done for both Solutions A and B.

In Figs. 12 and 13 we present summary Argand diagrams for Solutions A and B and, in Fig. 14 both Argand diagrams and partial-wave cross sections for Solution B. The equivalent complete figures for Solution A are contained in Ref. 11. A summary of the major characteristics of each partial wave is given in Table VIII, together with comments on resonance interpretations.

Next we discuss the unambiguous results in our two solutions and contrast the differences between them. Before entering into too great detail we should note that the major difference between the two solutions lies in the relative orientation of the low-energy (1300-1540 MeV) and high-energy (1630-2000 MeV) amplitudes. This flexibility is due solely to the fact that we have not been able to analyze data in the "gap", and clearly the correct solution will be identified when this is done reliably. It is important to point out that within each energy regime, Solutions A and B are essentially identical, except in the P_{11} waves and the new waves added. All of the major features remain the same providing one does not try to link the two energy regions.

A. $I = \frac{1}{2}$ States

The considerable amount of motion in these plots is not surprising since most of the structure observed may be associated with the existence of already established resonant states.

1. P_{11}

In both solutions, the $P_{11}(1470)$ is clearly observed decaying strongly into $\pi\Delta$ and $N\epsilon$. A higher-mass P_{11} is also present in both solutions although there are distinct differences in shape in this case.

2. $N\rho$ Couplings

We observe strong $N\rho$ couplings of the $P_{13}(1700)$, $D_{13}(1520)$ and $F_{15}(1680)$ resonances. This is not surprising, as the last two resonances are strongly seen in photoproduction and application of the Vector Dominance Model would imply this result. In the case of the new $P_{13}(1700)$, $N\rho$ is the major decay channel.

3. $D_{13}(1700)$

This state is observed in our solutions. Its presence is predicted[31] by the L excitation quark model and is the last remaining N* or Δ state of the $(70, 1^-)$ supermultiplet of negative parity baryon states to be identified. It decays into the $\pi\Delta(DS_{13})$ and $N\epsilon(DP_{13})$ channels, but its mass and width are difficult to estimate and probably differ between our Solutions A and B.

4. D_{15} and F_{15}

These two resonances are strongly observed in our analysis. This verifies our previous result that the D_{15} couples exclusively (within errors) to the πN and $\pi\Delta$ channels. Moreover, we have now determined unambiguously the relative sign of the F_{15} and D_{15} $\Delta\pi$ couplings and accounted for all the F_{15} inelasticity. Thus we have made a significant improvement over our previous analysis.[4]

5. Nε Decay Modes

In our analysis the ε is a slowly varying effect over the Dalitz plot. Hence Nε decays may also be interpreted as direct three-body ππN decays of a given J^P.

B. I = 3/2 States

1. For E < 1540, all of the I = 3/2 amplitudes are small, whereas for E ⩾ 1650 MeV (where we again have data) the S_{31}, D_{33} and P_{33} amplitudes are already large. This means that we cannot observe the complete anti-clockwise motion in these channels.

2. The presence of two low-energy P_{11} states (~ 1410 and ~ 1730 MeV) implies the need for two P_{33} states in most schemes, while the (56, $L = 2^+$) supermultiplet requires yet a third. In terms of partial-wave analyses, the situation is confused--the two EPSA's disagree; likewise our Solution A and B. We compare the CERN and Saclay parameters:

	Mass (MeV)	Γ (MeV)	γ	Author's own "Crude" estimate of resonance quality
CERN[1]	1680	220	0.1	"D"
Saclay[2]	1900	204	0.19	"***"

Our Solution A does not show these resonances, but our preferred Solution B is consistent with the existences of both.

3. F_{35}

In both Solutions A and B we have evidence for an F-wave πΔ system, as do Mehtani et al.,[5] although one might expect, on kinematical barrier-factor arguments, the P-wave would dominate. In our solution the decay into Nρ dominates, and this final state allows us to saturate the EPSA inelastic cross section prediction.

4. $\underline{F_{37}}$

This resonance is observed both in the $\pi\Delta$ and $N\rho$ channels. However, these two channels do not saturate the predicted inelastic cross section (only ~ 60% is accounted for). The strong $N\rho$ decay might well be expected again as this resonance is a dominant feature of photoproduction.

5. $\underline{P_{31}, D_{35}}$

These waves fall short of the EPSA predictions. In particular we find no need for any D_{35} interactions i.e.,

$$\sigma_{in}\ (D_{35}) = 0.$$

Thus, if there are any low-lying D_{35} resonances they do not couple to the $\pi\Delta$ or $N\rho$ channels. However, EPSA does predict a large inelastic cross section from the wave (~ 4 mb) and this is a shortcoming of our results, although it probably has a simple interpretation. [See E(i) below.]

C. The Origin of Solution B and the New Waves Added

We were motivated to study the stability of Solution A by the observation that some of the relative coupling signs between resonances in our lower energy region and resonances in our upper energy region (in Solution A) conflicted with the predictions of broken $SU(6)_W$ [32] and Melosh transformations,[33] although within each energy region there was consistency with theory. Thus we studied several ways in which another continuation of the partial-wave amplitudes across the energy gap might be achieved. A second solution--our

Solution B--was found in the following manner. We added many new waves (admittedly suggested by theory) and re-performed our whole fitting process. This led to the new solutions in the two energy regions which allow a different continuation through the energy gap and hence the new Solution B. This solution is an even better fit to the data than Solution A, and furthermore the signs of the DD_{13}(1520) and DD_{15}(1660) are now in good agreement with the theoretical expectations.

The four new waves are: SD_{11} in $\pi\Delta$, DD_{33} in $\pi\Delta$, FF_{15} in $\pi\Delta$, PP_{11} in $N\rho_1$.

From the previous discussion of this section it is clear that within the energy regions in which we have data there is little difference between these solutions. The important test lies in the "gap" through which we have to make a K-matrix extrapolation. Thus this result emphasizes the need for the free availability of complete data sets or at least a reliable partial-wave analysis in the energy region 1540-1630 MeV.

D. Resonance Parameters

We have resisted the temptation to quote any resonance parameters in the paper. The problem of extracting these quantities is difficult and is the subject of the companion papers.[30,34-37]

E. Further Comments

We would like to call attention to some conclusions we may draw from the absence of certain waves (D_{35}), the failure to reach EPSA predictions (P_{31}, F_{37}), the poor fits to the $\pi^+\pi^+n$ state, and the general deterioration of the fits at the higher energies.

1. $\pi^+\pi^+n$ and $N^*_{1/2}$ final state interactions

Our predictions for $\sigma(\pi^+\pi^+n)$ are a factor of 2 too small (~ 2-3 mb unaccounted) at the higher energies. This is not surprising as only the $\pi\Delta$ intermediate states of our isobar model connect with this final state and from inspection of the π^+n mass spectra it is clear that $N^*_{1/2}$ isobars (P_{11}, D_{13}, F_{15}, D_{15}) may be present. Furthermore, if one considers low angular momenta in the $\pi N^*_{1/2}$ system one finds:

$\pi N^*(D_{13})$ in a P wave
$\pi N^*(F_{15})$ in an S wave } are derived from D_{35}
$\pi N^*(P_{11})$ in an S wave would be derived from P_{31}

which suggests that our previously-noted failure to reach the P_{31} and D_{35} predictions is associated with not including N^* final-state interactions. This suggestion can be further substantiated if we note the

$$\sigma_{missing}(P_{31}+D_{35}+F_{37}) \simeq \sigma_{missing}(\pi^+\pi^+n) + \sigma(\pi\pi\pi N).$$

Finally, the inclusion of these waves would have an appreciable effect in the $\pi^+\pi^+n$ final state whereas Clebsch-Gordan coefficients reduce their effect in the other single-pion production reactions so that our analysis of those channels would probably show little change.

2. Peripheral nucleon

The deterioration of the fits at higher energies is generally associated with being unable to entirely account for the onset of peripheral processes. This probably indicates the necessity for inclusion of π exchange in the production of the $N\rho$ final state, so that higher partial waves are generated.

VII. CONCLUSIONS

Elastic phase-shift analyses only relate to one aspect of the πN interaction, and the analysis described here represents a substantial progress in providing complimentary information on the inelastic channels.

This analysis has demonstrated that isobar-model partial-wave analyses of inelastic final states can reproduce the detailed nature of the data. This then allows the observation of resonances in inelastic channels.

We have evidence for the existence of a $D_{13}(1700)$, long predicted by the quark model. The existence of a $P_{13}(1700)$ is corroborated through a strong ρN coupling. Our preferred solution, B indicates a $P_{33}(1700)$ resonance. Furthermore we have strengthened the interpretation of many resonances due to our observation of them in the inelastic channels $N\rho$, $N\epsilon$, and $\pi\Delta$.

Earlier analyses have produced limited $\pi\Delta$ Argand plots, but this analysis presents the first full Argand plots for all three channels, $N\rho$, $N\epsilon$, and $\pi\Delta$. These begin to allow a complete picture of the πN interaction accounting for almost all of the inelastic cross section.

Finally, we are at present using these amplitudes to study resonance parameters and couplings and their relations to other theories of hadron interactions.[30, 34-37]

VIII. ACKNOWLEDGMENTS

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APPENDIX A
MAXIMIZING THE SCALE PARAMETER IN THE LIKELIHOOD FIT

In Eq. (4.11) remember that $N^c R^c/\sigma^c$ is the number of events ν^c predicted by the parameters $\vec{A}$, so write

$$\Sigma \frac{N^c R^c}{\sigma^c} = \Sigma\, \nu^c(\vec{A}) \equiv \nu(\vec{A}) \ .$$

For this discussion the first product in Eq. (11) is only a constant B which does not depend on $\vec{A}$, so Eq. (4.11) becomes

$$L = B\, e^{-\nu(\vec{A})} \prod_{i=1}^{N} p_i(\vec{A}) \ . \qquad \text{(A1)}$$

Introducing the scale factor of Eq. (4.12),

$$\vec{A} = s\, \vec{A}^0 \qquad \text{(A2)}$$

we have

$$\nu(s\vec{A}^0) = s^2 \nu(\vec{A}^0) \equiv s^2 \nu^0$$

and

$$\Pi p_i(s\vec{A}^0) = s^{2N}\, \Pi p_i(\vec{A}^0) \ .$$

Equation (A1) becomes

$$L(s\vec{A}^0) = B e^{-s^2 \nu^0}\, s^{2N}\, \Pi p_i(\vec{A}^0) \ , \qquad \text{(A3)}$$

and the last factor Πp_i is independent of s. Combining it with B, we have

$$L(s\vec{A}^0) = B'\, e^{-s^2 \nu^0}\, s^{2N} \ . \qquad \text{(A4)}$$

Then

$$\ln L(s\vec{A}_0) = \ln B' - s^2\nu^0 + 2N \ln s$$

and

$$\frac{\partial}{\partial s} \ln L(s\vec{A}_0) = -2s\nu^0 + \frac{2N}{s} . \qquad (A5)$$

To find $s^2|_{L\,max} \equiv s_m^2$, set (A5) = 0, and we get Eq. (4.18), i.e.,

$$s_m^2 = \frac{N}{\nu_0} = \frac{N}{\Sigma \frac{N^c R^c(\vec{A}_0)}{\sigma^c}} \qquad (A6), (4.18)$$

Equation (A6) is derived in Miller's thesis,[23] but then s^2 is written by mistake as s. We now insert our result (A6) into (A4),

$$L(s_m A^0) = B' e^{-N} \left(\frac{N}{\nu_0}\right)^N , \qquad (A4_m)$$

and this in turn into (A3)

$$L(s_m \vec{A}^0) = Be^{-N}\left(\frac{N}{\nu^0(\vec{A}_0)}\right)^N \prod_{i=1}^{N} p_i(\vec{A}^0) . \qquad (A3_m)$$

Now that s is maximized, we see that $(A3_m)$ is independent of the overall magnitude of $\vec{A}^0$, and we can drop its superscript zero and call it $\vec{A}$. For the purpose of hunting in $\vec{A}$ space for L_{max} we emphasize factors containing $\vec{A}$:

$$L(s_m\vec{A}) = Be^{-N} N^N \left[\Sigma \frac{N^c R^c(\vec{A})}{\sigma^c}\right]^{-N} \prod_{i=1}^{N} p_i(\vec{A}) \qquad (A5)$$

$$\ln L(s_m\vec{A}_\alpha) = \text{const.} - N \ln \left[\Sigma \frac{N^c R^c(\vec{A})}{\sigma^c}\right] + \sum_{i=1}^{N} \ln p_i(\vec{A}) .$$

We actually work with the average of this premaximized L, which we call $F(\vec{A})$

$$F(\vec{A}) = \frac{1}{N} \ell n \, L(s_m \quad \vec{A}) , \tag{A6}$$

$$F(\vec{A}) = \text{const.} - \ell n \left[\sum_c \frac{N^c R^c(\vec{A})}{\sigma^c} \right] + \frac{1}{N} \sum_{i=1}^{N} \ell n \, p_i(\vec{A}) \tag{A7}$$

The factor $1/N$ in (A6) makes F roughly independent of N, so it is easier to compare fits at different energies where there are different numbers of events. Equation (A7) is the same as Eq. (4.20) of the text.

APPENDIX B

EXAMPLE OF STEPPING PROCEDURE WHEN $\nabla\nabla^T f$ MATRIX IS SINGULAR

As an example of the method discussed in Section IV.D.4, consider the function $f(z) = -z^2 + 2z + 1$, but let $z = x + y$, i.e.,

$$f(x, y) = -(x+y)^2 + 2(x+y) + 1 \quad , \tag{B1}$$

where f has a maximum at $z = x + y = +1$. In this case, we clearly have a redundant parameter. Now

$$\nabla f = 2\left[1 - (x+y)\right]\begin{pmatrix}1\\1\end{pmatrix} \qquad \text{and}$$

$$\nabla\nabla^T f = \begin{pmatrix}-2 & -2\\ -2 & -2\end{pmatrix}. \tag{B2}$$

The two normalized eigenvectors of $\nabla\nabla^T f$ are $\vec{v}_1 = \begin{pmatrix}1/\sqrt{2}\\ 1/\sqrt{2}\end{pmatrix}$ and $\vec{v}_2 = \begin{pmatrix}1/\sqrt{2}\\ -1/\sqrt{2}\end{pmatrix}$, with eigenvalues -4 and 0. Note that ∇f (hence $\nabla\nabla^T f$) = 0 along $\vec{v}_2$. Now

$$\nabla\nabla^T f = -4\, \vec{v}_1\vec{v}_1^{\,T} + 0\, \vec{v}_2\vec{v}_2^{\,T} \quad . \tag{B3}$$

Let

$$\underline{M} = \nabla\nabla^T f - \vec{v}_2\vec{v}_2^{\,T} = -4\, \vec{v}_1\vec{v}_1^{\,T} - \vec{v}_2\vec{v}_2^{\,T} \quad . \tag{B4}$$

Clearly

$$\underline{M}^{-1} = -\frac{1}{4}\, \vec{v}_1\vec{v}_1^{\,T} - \vec{v}_2\vec{v}_2^{\,T} \quad . \tag{B5}$$

Now let

$$\underline{V} = \underline{M}^{-1} + \vec{v}_2\vec{v}_2^{\,T} = -\frac{1}{4}\, \vec{v}_1\vec{v}_1^{\,T} ,$$

$$= -\frac{1}{4}\begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} = -\frac{1}{8}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \tag{B6}$$

Assume a starting value x_0, y_0. Then the first step is

$$-\underline{V} \cdot \nabla F_0 = +\frac{1}{8}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \cdot 2[1 - (x_0 + y_0)]\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 1 - (x_0 + y_0) \\ 1 - (x_0 + y_0) \end{bmatrix} .$$

Thus

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} - \underline{V} \cdot \nabla F_0 \tag{B7}$$

$$= \begin{pmatrix} 1/2 \\ \\ 1/2 \end{pmatrix} + \begin{pmatrix} \frac{x_0 - y_0}{2} \\ \\ \frac{y_0 - x_0}{2} \end{pmatrix} , \tag{B8}$$

and $x_1 + y_1 = \frac{1}{2} + \frac{x_0 - y_0}{2} + \frac{1}{2} - \frac{x_0 - y_0}{2} = 1$ and we are at the minimum in one step, as we should be since F is quadratic.

Thus the problem in a multidimensional case reduces to finding the eigenvectors with zero eigenvalues, subtracting the rank-one matrices generated by these eigenvectors, inverting the resulting matrix, and finally adding back the rank-one subtractions.

In the case of likelihood problems such as ours, the eigenvector for scale changes is $\underline{A}$ and the eigenvector for phase changes is $i\vec{A}$. Thus for us, we automatically had the necessary two eigenvectors.

APPENDIX C

CHOICE OF ANGULAR MOMENTUM BARRIER FACTORS AND OF THE N^* RADIUS

Unfortunately three different forms of barrier-penetration factors $B_L(QR)$ are now common in partial-wave analyses. This appendix compares them in Fig. C1, points out that for $L \geq 2$ they may differ, and makes a plea for using the standard Blatt-Weisskopf factors, which have recently been given considerable support by von Hippel and Quigg.

In the text, Eq. (3.10), we used the notation $b_{L'}$ for the barrier factor for the amplitude T; here we use the more standard notation $B_L (= |b_{L'}|^2)$ for the barrier factor for the intensity $|T|^2$. The "industry standard" for B_L is taken from Eq. 5-8, page 361, of Blatt and Weisskopf[21] (where they are called v_ℓ)

$$B_0 = 1, \quad B_1 = \frac{x^2}{1+x^2}, \quad B_2 = \frac{x^4}{9+3x^2+x^4}, \quad \cdots \tag{C1}$$

where $x = QR$. These B_L have the property that their inflection point is at about $QR = L$. For $x << L$, they start off as

$$B_L \xrightarrow{x \to 0} \frac{x^{2L}}{[(2L-1)!!]^2} \tag{C2}$$

and for $x >> L$, $B_L \to 1$. Figure C1 gives B_L vs QR for $L = 1$ to 7.

There is general agreement that the B_L should start out as x^{2L} and should approach unity for large L; but before the 1972 paper of von Hippel and Quigg,[39] the detailed form C1 was regarded with some skepticism by particle physicists because it was derived by assuming that the region of interaction was a square well. This is reasonable when the intermediate state is a nucleus, which is what Blatt and Weisskopf had in mind, but less convincing when it is an N^*. Accordingly many physicists just used the low-QR form (C2), or perhaps a form somewhat like that for L = 1, which has the correct limiting properties at both ends

$$B_L \approx \frac{x^{2L}}{\text{Const} + x^{2L}} , \tag{C3}$$

but in general does not have its inflection point near QR = L.

Von Hippel and Quigg have now shown that form C1 can be rederived using only the properties of the radial wave function $U_L(x) \propto x h_L(x)$ where h_L is a Hankel function. For more discussion and an approximate form, see Ref. 40.

Figure C1 shows not only the Blatt-Weisskopf form of B_L, but the small-QR approximation for L = 1 to 3. As one expects, both approximation fail badly by QR = L. This brings us to our plea. Now that the Blatt-Weisskopf form has been well justified, why not use it? Then, once most analyses use the same barrier, we can go on to gather experience on the best value for the radius R.

Next we take up the question of what value to chose for the radius parameter R. Barbaro-Galtieri[41] has done a thorough analysis of the $\Delta(1336)$ peak and finds that R should be about 1F. Yet when R is used in a far more indirect way to do SU(3) fits, it turns out that values of 0.2 F,[41] or even zero,[42] work best.

The radius question, specifically as it applies to our reaction $\pi N \to \pi\pi N$, has recently be studied at Saclay by J. Dolbeau[43] in his isobar analysis in the $\sqrt{s}$ range 1390-1740 MeV. These fits are very sensitive to the radius, particularly near ρ threshold around 1700 MeV. Dolbeau finds a best radius of ~ 1/4 F, i.e., in between the large value favored by the $\Delta(1236)$ and the small value favored by SU(3). Specifically he finds that $1/R$ should be 750 ± 250 MeV/c.

Finally a comment on what we ourselves did. Because of a programming mistake, we started our fits using the small-QR approximation C2, and did not catch the mistake until so late that it was expensive to refit. But now Dolbeau's finding that the best value of R is only 1/4 F keeps QR so low that it does not pay for us to refit. Specifically, consider the kinematics of our reaction even at our highest energy $\sqrt{s} = 1970$ MeV, and set $R = 1/750$ MeV/c. Then for

$$\pi N \rightarrow \Delta(1236)\pi, \; Q_\Delta = Q_\pi \equiv Q = 393 \text{ MeV/c}, \; QR = 0.52$$

$$\rightarrow \rho\,(770)\,N, \; Q_\rho = Q_N \equiv Q = 500 \text{ MeV/c}, \; QR = 0.67.$$

This means that near the resonance bands of our Dalitz plots, where we find most of our data, the small-QR approximation is quite good, and we have decided not to refit. In our K-matrix programs for energy-dependent fits we have always used the standard Blatt-Weisskopf form C1.

FOOTNOTES AND REFERENCES

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TABLE I: Experiments used in this analysis.[a]

Beam particle	Laboratory (reference)	c.m. Energy range (GeV)		Number of events	
		Low	High	$\pi^+\pi^-n$	$\pi^-\pi^0p$
	SLAC-LBL (12)	1.47	1.50	1010	648
		1.64	1.97	41175	27946
π^-	Oxford (10)	1.31	1.54	18502	5892
	Saclay (8)	1.39	1.53	13340	7314
	Total	1.31	1.97	74027	41800
				$\pi^+\pi^0p$	$\pi^+\pi^+n$
	Oxford (9)	1.43	1.56	7262	1374
π^+	Riverside-LBL (13)	1.82	2.09	41412	17255
	Saclay (8)	1.64	1.97	11522	3382
	Total	1.43	2.09	60196	22011

[a]The events in this Table, in the form of 16 full BCD Data Summary Tapes, are available on request. The π^+ events at or above $\sqrt{s} = 1820$ MeV must be requested from the UCR-LBL collaboration, care of Prof. Anne Kernan, U. C. Riverside; the remainder from LBL, SLAC, or Saclay.

TABLE II: The 60 waves with angular momenta L, L', ℓ each ≤ 3. There are two nucleon-rho terms in the isobar model, indicated by ρ_3 and ρ_1, where the subscript indicates the coupling between the spin of the $\rho(\ell = 1)$ and the spin of the outgoing nucleon. See Fig. 4 for more complete explanation of the notation. We never used more than 28 of these 60 waves.

Isospin	Incident wave	$\pi\Delta$	$N\rho_3$	$N\rho_1$	$N\epsilon$
	S_{11}	SD_{11}	SD_{11}	SS_{11}	SP_{11}
	P_{11}	PP_{11}	PP_{11}	PP_{11}	PS_{11}
	D_{13}	DS_{13} DD_{13}	DS_{13} DD_{13}	DD_{13}	DP_{13}
I = 1/2	P_{13}	PP_{13} PF_{13}	PP_{13} PF_{13}	PP_{13}	PD_{13}
	D_{15}	DD_{15}	DD_{15}	DD_{15}	DF_{15}
	F_{15}	FP_{15} FF_{15}	FP_{15} FF_{15}	FF_{15}	FD_{15}
	F_{17}	FF_{17}	FF_{17}	FF_{17}	
	S_{13}	SD_{31}	SD_{31}	SS_{31}	
	P_{31}	PP_{31}	PP_{31}	PP_{31}	
	D_{33}	DS_{33} DD_{33}	DS_{33} DD_{33}	DD_{33}	
I = 3/2	P_{33}	PP_{33} PF_{33}	PP_{33} PF_{33}	PP_{33}	
	D_{35}	DD_{35}	DD_{35}	DD_{35}	
	F_{35}	FP_{35} FF_{35}	FP_{35} FF_{35}	FF_{35}	
	F_{37}	FF_{37}	FF_{37}	FF_{37}	

TABLE III: Number of events for the energy bins used in the fits. For availability of these events, see note at bottom of Table I.

c. m. energy		Range (MeV)	$\pi^- p \to \pi^+\pi^- n$	$\pi^- p \to \pi^-\pi^0 p$	$\pi^+ p \to \pi^+\pi^0 p$
i	1310	1300-1330	1069	151	
	1340	1330-1360	1664	11	
	1370	1360-1380	2471	2	
	1400	1380-1410	5049	964	78
	1440	1430-1460	4918	1802	359
	1470	1460-1480	3252	1629	175
	1490	1480-1510	5555	3197	1523
	1520	1510-1530	3241	2588	795
	1540	1530-1560	3905	3285	1114
ii	1650	1630-1670	6061	3757	2467
	1690	1670-1710	5901	3689	1139
	1730	1710-1750	3455	2630	4061
	1770	1750-1790	3214	2352	2853
	1810	1790-1830	2447	1541	3855
iii	1850	1380-1870	3931	3183	6372
	1890	1870-1910	5072	3170	12690
	1930	1910-1950	5817	4080	4298
	1970	1950-1990	5277	3544	7744
Total		1300-1990	72299	41575	49523

TABLE IV: χ^2 at each energy based on four-dimensional bins. The four-dimensional space is subdivided into $4^4 = 256$ bins, but some have no population, hence the number tabulated below is <250.

$\sqrt{s}$(MeV)	$\chi^2_{n\pi^+\pi^-}$	Bins	$\chi^2_{p\pi^-\pi^0}$	Bins	$\chi^2_{p\pi^+\pi^0}$	Bins
1370	279	228				
1440	243	235	216	233	90	160
1530	328	229	253	236	209	216
1690	526	237	378	236	182	208
1970	864	236	601	233	907	235

TABLE V: Effect of removing waves of varying importance at $\sqrt{s} = 1530$ MeV, 9000 events. Each box is arranged [$\Delta \ell n L$ for real events/$\Delta \ell n L$ for Monte-Carlo], and the same format holds for $\Delta \chi^2$.

		$\ell n L$	χ^2/D.F.
Original 18-wave fit	Real	0.2817	790/681
	Monte Carlo	0.3198	624/788
	$\lvert T \rvert / \delta T$	$\Delta \ell n L$	$\Delta \chi^2$
18 waves minus $DS_{13}(\rho_3 N)$	11 / 10	-320 / -390	173 / 215
18 waves minus PP_{33} $(\Delta\pi)$	6 / 6	-60 / -80	53 / 102
18 waves minus $PP_{13}(\rho_1 N)$	2 / 3	-30 / -30	11 / 14
18 waves minus $DS_{33}(\rho_3 \pi)$	1 / 1/2	-1 / -1	3 / 2

TABLE VI: Square roots of diagonal elements of a 60×60 and a 20×20 (Solution A) error matrix at 1690 MeV.

$\Delta\pi$ Wave	Error 20	Error 60	$\rho_3 N$ Wave	Error 20	Error 60	$\rho_1 N$ Wave	Error 20	Error 60	ϵN Wave	Error 20	Error 60
ΔSD_{11}	----	.083	$\rho_3 SD_{11}$	----	.079	$\rho_1 SS_{11}$	.054	.069	ϵSP_{11}	.066	.087
ΔPP_{11}	.061	.076	$\rho_3 PP_{11}$	----	.077	$\rho_1 PP_{11}$	----	.080	ϵPS_{11}	.067	.084
ΔPP_{13}	----	.083	$\rho_3 PP_{13}$	----	.056	$\rho_1 PP_{13}$	.038	.053	ϵPD_{13}	----	.064
ΔPF_{13}	----	.053	$\rho_3 PF_{13}$	----	.048	$\rho_1 DD_{13}$	----	.057	ϵDP_{13}	.042	.054
ΔDS_{13}	.045	.063	$\rho_3 DS_{13}$	.040	.053	$\rho_1 DD_{15}$	----	.041	ϵDF_{15}	----	.047
ΔDD_{13}	.044	.058	$\rho_3 DD_{13}$	----	.056	$\rho_1 FF_{15}$	----	.040	ϵFD_{15}	.027	.032
ΔDD_{15}	.052	.068	$\rho_3 DD_{15}$	----	.050	$\rho_1 FF_{17}$	----	.033			
ΔFP_{15}	.042	.054	$\rho_3 FP_{15}$	.033	.042	$\rho_1 SS_{31}$	.067	.081			
ΔFF_{15}	----	.052	$\rho_3 FF_{15}$	----	.043	$\rho_1 PP_{31}$	----	.090			
ΔFF_{17}	----	.052	$\rho_3 FF_{17}$	----	.041	$\rho_1 PP_{33}$	----	.060			
ΔSD_{31}	.062	.075	$\rho_3 SD_{31}$	----	.075	$\rho_1 DD_{33}$	----	.057			
ΔPP_{31}	.063	.086	$\rho_3 PP_{31}$	----	.083	$\rho_1 DD_{35}$	----	.045			
ΔPP_{33}	.062	.096	$\rho_3 PP_{33}$	----	.069	$\rho_1 FF_{35}$	----	.040			
ΔPF_{33}	----	.052	$\rho_3 PF_{33}$	----	.047	$\rho_1 FF_{37}$	----	.038			
ΔDS_{33}	.049	.075	$\rho_3 DS_{33}$	.042	.057						
ΔDD_{33}	----	.052	$\rho_3 DD_{33}$	----	.060						
ΔDD_{35}	----	.065	$\rho_3 DD_{35}$	----	.053						
ΔFP_{35}	----	.048	$\rho_3 FP_{35}$	----	.041						
ΔFF_{35}	----	.050	$\rho_3 FF_{35}$	----	.046						
ΔFF_{37}	----	.053	$\rho_3 FF_{37}$	----	.046						

TABLE VII: Difference in likelihood, L (Solution B) - L (Solution A) for a standard sample of 9000 events. Solution B is always better.

s (MeV)	$\Delta \ln L$ (9000 events)
1650	113
1690	122
1730	139
1770	107
1810	84
1850	57
1890	15
1930	153
1970	19

	$M \sim 1710$ $\chi_{inel} \sim 0.9$	Clear resonant motion in ϵN and $\pi\Delta$, but distinctly broader than EPSA width of 100 MeV	πN, ϵN, $\pi\Delta$ First unambiguous observation of resonant behavior in this region
D_{15}	$M = 1660$ $\chi_{inel} \sim 0.6$	The $\pi\Delta$ channels show strong resonant behavior, saturating the unitary bound near the accepted resonant mass	πN, $\pi\Delta$
F_{15}	$M = 1680$ $\chi_{inel} \sim 0.4$	This resonance is observed in ϵN, ρN, and $\pi\Delta$ with comparable strength	πN, ϵN, ρN, $\pi\Delta$
S_{31}	$M = 1620$ $\chi_{inel} \sim 0.70$	We lack the experimental data which would reveal the behavior of this wave in the resonance region. The present points above 1650 show a smooth behavior which is compatible with the accepted resonance mass	πN, $\pi\Delta$, ρN
P_{31}	$M = 1790$ $\chi_{inel} \sim 0.85$	No evidence for resonant behavior	πN
P_{33}	Suggestion of resonance with $M \sim 1900$ $\chi_{inel} \sim 0.8$	Fast $\Delta\pi$ motion across gap suggests resonance near 1700, i.e., below EPSA value of 1900. (See discussion under B(2) below.)	πN, $\pi\Delta$
D_{33}	$M \sim 1720$ $\chi_{inel} \sim 0.85$	Our analysis is consistent with a resonance interpretation for $\pi\Delta$ and ρN.	πN, $\pi\Delta$, ρN
F_{35}	$M \sim 1870$ $\chi_{inel} \sim 0.85$	Strong resonance behavior seen in ρN channel smaller coupling to $\pi\Delta$	πN, ρN, $\pi\Delta$
F_{37}	$M = 1930$ $\chi_{inel} \sim 0.6$	Clear resonance behavior is apparent in ρN and $\pi\Delta$ channels	πN, ρN, $\pi\Delta$

FIGURE CAPTIONS

Fig. 1. Total and inelastic $\pi^- p$ cross section vs energy. The σ (inelastic) curve comes from Ref. 2 ("EPSA") as in Eq. 4.21; the σ (tot) data comes from Lovelace et al., LBL Report No. 63.

Fig. 2.(a) Dalitz plots for the state $n\pi^-\pi^+$ at four c.m. energies: 1490, 1650, 1770, 1930 MeV. The side of the little squares is proportional to the predicted density of our fits. On the projected distributions, the dotted line is the experimental data, while the solid histogram is the result of the fit. The scales are linear in $(\text{mass})^2$, but the tiny numbers are actually in MeV.

Fig. 2.(b) Dalitz plot for the state $p\pi^-\pi^0$. For details see caption to Fig. 2.(a).

Fig. 2.(c) Dalitz plot for the state $p\pi^+\pi^0$. For details see caption to Fig. 2.(a).

Fig. 3. Distribution of the angle of the final nucleon with respect to the incident pion in the center of mass. The histograms are given at four c.m. energies: 1490, 1650, 1770, 1930 MeV for the same channels as in Fig. 2. The dotted line is the data, the solid histogram the result of the fit.

Fig. 4. Schematic representation of the isobar model and definition of the partial-wave notation. F describes the final-state particles, $\Delta\pi$, etc.

Fig. 5. Phase shifts and modulus of Watson factor $|W| = \sin\delta/q^{\ell+1}$ for Δ, ρ, and ϵ diparticles. Dashed line on the ϵ plot corresponds to the later analysis of Protopopescu;[19] all phase shifts used are given in Herndon's thesis.[20]

Fig. 6. Scatter plot of $\delta' F_{AB}$ vs F_{AB} used when trying to reject one of two competing local maxima A or B. F_{AB} is the average log likelihood ratio; $\delta' F_{AB}$ is its error. More precisely, $F_{AB} = (1/N)\,\ell n[L(A)/L(B)]$, and $\delta' F_{AB}$ is calculated using Eq. (4.25). Key to symbols:

+, competing 18-wave fits to the 9000 Monte-Carlo events of Table V;

X, 60-wave fits to 21,000 real events at 1890 MeV;

dots, 20-wave fits to 10,700 events at 1690 MeV. For comparison, we have also plotted, as symbols with circles, the errors in the ℓn-likelihoods themselves, δF.

Fig. 7. Results of a fit to 7583 Monte Carlo events generated at 1690 MeV to test TRIANGLE/RUMBLE. The small square covers the area where we indeed reconstructed the four smallest waves, but also 13 "noise" waves. So we are insensitive to the area inside the square.

Fig. 8. Waves used in 1973 Solution B. In Solution A waves not used are indicated with an *, and D_{15} started later (at $\sqrt{s}$ = 1650 MeV).

Fig. 9. Solution A fits to the reaction $\pi^- p \rightarrow \pi^+ \pi^- n$ at a c.m. energy of 1690 MeV. The figure contains $\cos\theta$ vs ϕ plots for individual regions of the Dalitz plot where $\cos\theta$ and ϕ are the angles of the incident pion in a coordinate system defined by the final state. The z axis lies along $\vec{p}_N$ and the y axis lies along $p_{\pi^-} \times p_{\pi^+}$. The plots outside the Dalitz plot are the sums of the corresponding plots within the boundary.

Fig. 10. Solution B cross sections for the fitted channels $n\pi^-\pi^+$, $p\pi^-\pi^0$, and $p\pi^+\pi^0$. Crosses correspond to our fits; tiny dots with vertical error bars are experimental, from Ref. 38.

Fig. 11. Solution B cross sections predicted for the channels $n\pi^0\pi^0$ and $n\pi^+\pi^+$. Crosses are predicted from our fits to the other channels of Fig. 10; dots with vertical bars are from Ref. 38.

Fig. 12. Summary $N\pi\pi$ Argand plots for Solution A (1972). Nominal resonance energies come from the CERN 1972 EPSA (Ref. 1). For more details see caption to Fig. 14.

Fig. 13. Summary $N\pi\pi$ Argand plots for Solution B (1973). Nominal resonance energies come from the Saclay 1973 EPSA (Ref. 2). For more details see caption to Fig. 14.

Fig. 14(a-e). Argand diagrams and partial-wave cross sections for the elastic and inelastic channels. The elastic solutions are from CERN 1972 (Ref. 1). The inelastic channels are our 1973 solution B.

On the Argand plots the nominal resonance energies come from Saclay 1973 (Ref. 2). Arrowheads are spaced every

20 MeV, with a wider arrowhead at integral hundreds of MeV. Lower-ℓ waves are plotted starting at $\sqrt{s} = 1400$ MeV, higher-ℓ start where introduced into the fit. Last arrowhead is always at 1940 MeV. To show the gap in our data the straight line joining the five arrows in the gap has been deleted.

The + or - signs at the upper left of each circle show how to transform from our sign conventions to the "Baryon-first" convention. "New" indicates one of the four waves used in solution B only. Nine ρ_1 and ϵ signs were changed 5 July 1974.

Facing each inelastic Argand diagram of T_α, we plot $|T_\alpha|^2$ vs $\sqrt{s}$. Facing the top (elastic) Argand plot, the total $\sigma_{inelastic}(\propto 1-\eta^2)$, derived from EPSA, is compared with the sum of the σ_{in} from each isobar-model channel plotted below. The small interference terms between channels are taken into account, see Eq. (3.7). Note that σ, as plotted, are labelled by I-spin, not charge, so to get $\sigma(\pi^- p \to N\pi\pi)$ one must use $2/3\ \sigma(I = 1/2) + 1/3\ \sigma(I = 3/2)$.

Fig. C1. Blatt-Weisskopf barriers (Eq. C1) for L = 1 through 7, compared with their approximate forms (Eq. C2) for low QR.

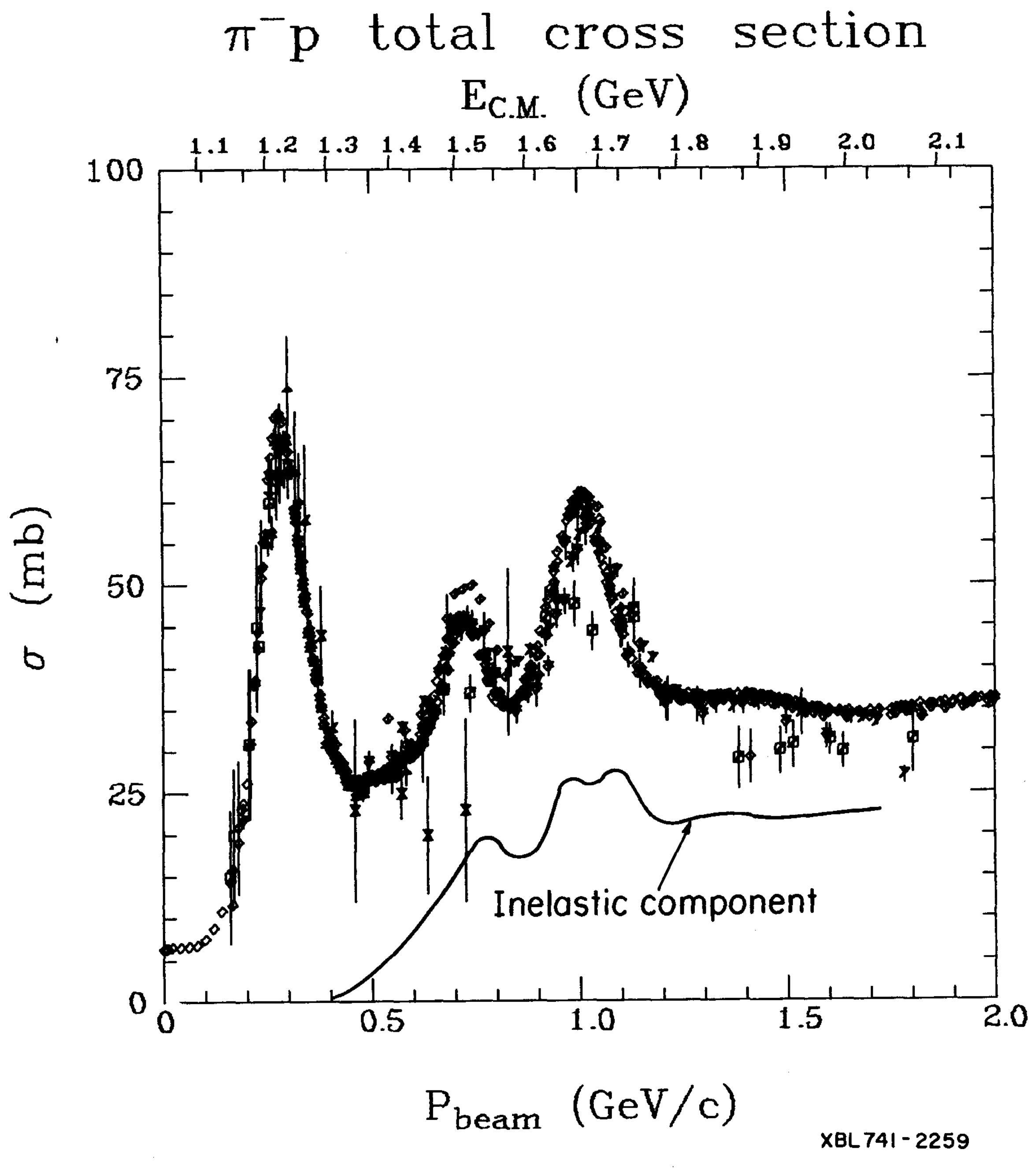

π⁻p total cross section
E_C.M. (GeV)
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
2.1
100
75
50
25
0
σ (mb)
Inelastic component
0
0.5
1.0
1.5
2.0
P_beam (GeV/c)
XBL741-2259

FIG. 1

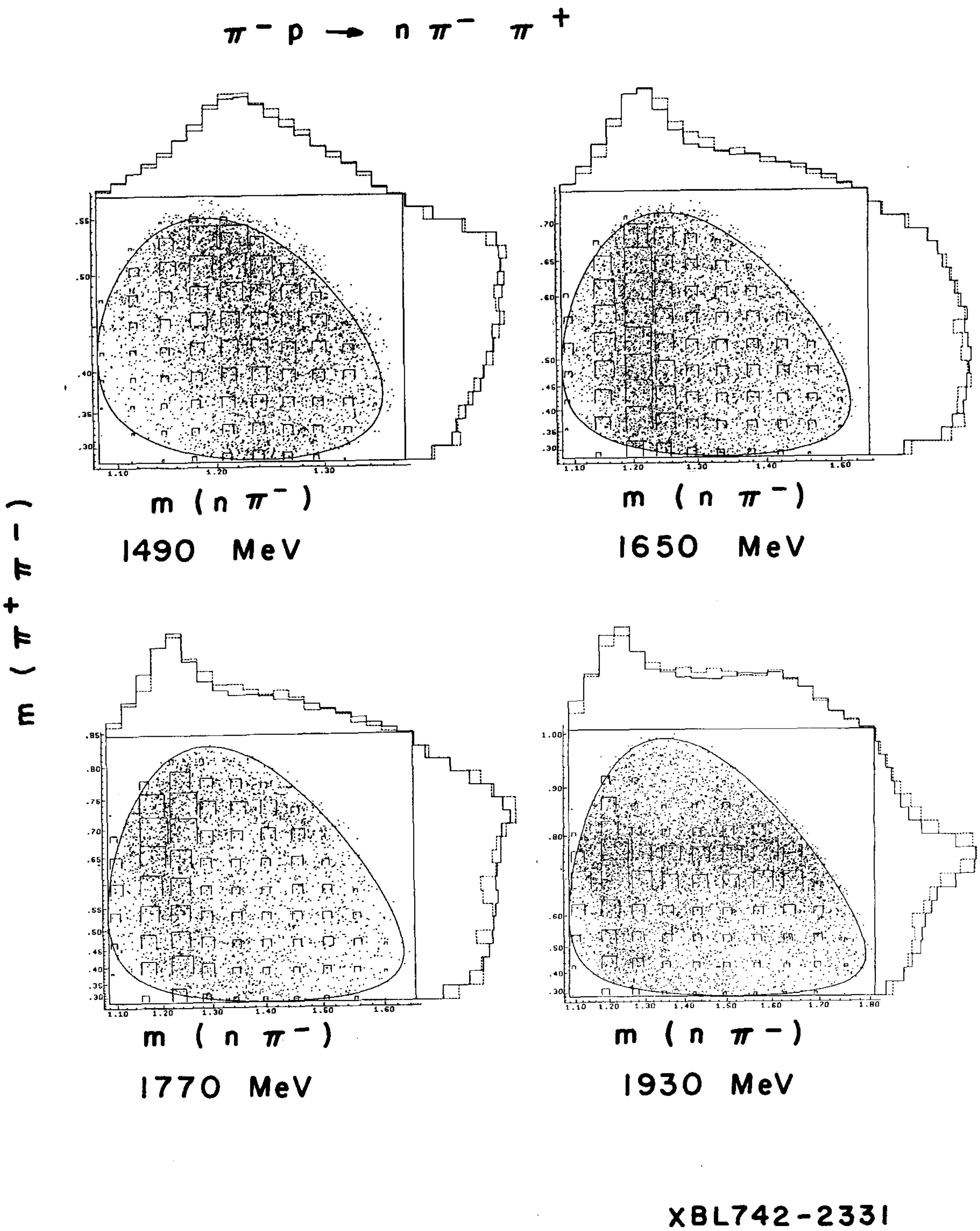

$\pi^- p \rightarrow n \pi^- \pi^+$
$m(\pi^+\pi^-)$
m (n π⁻)
1490 MeV
m (n π⁻)
1650 MeV
m (n π⁻)
1770 MeV
m (n π⁻)
1930 MeV
XBL742-2331

FIG. 2a

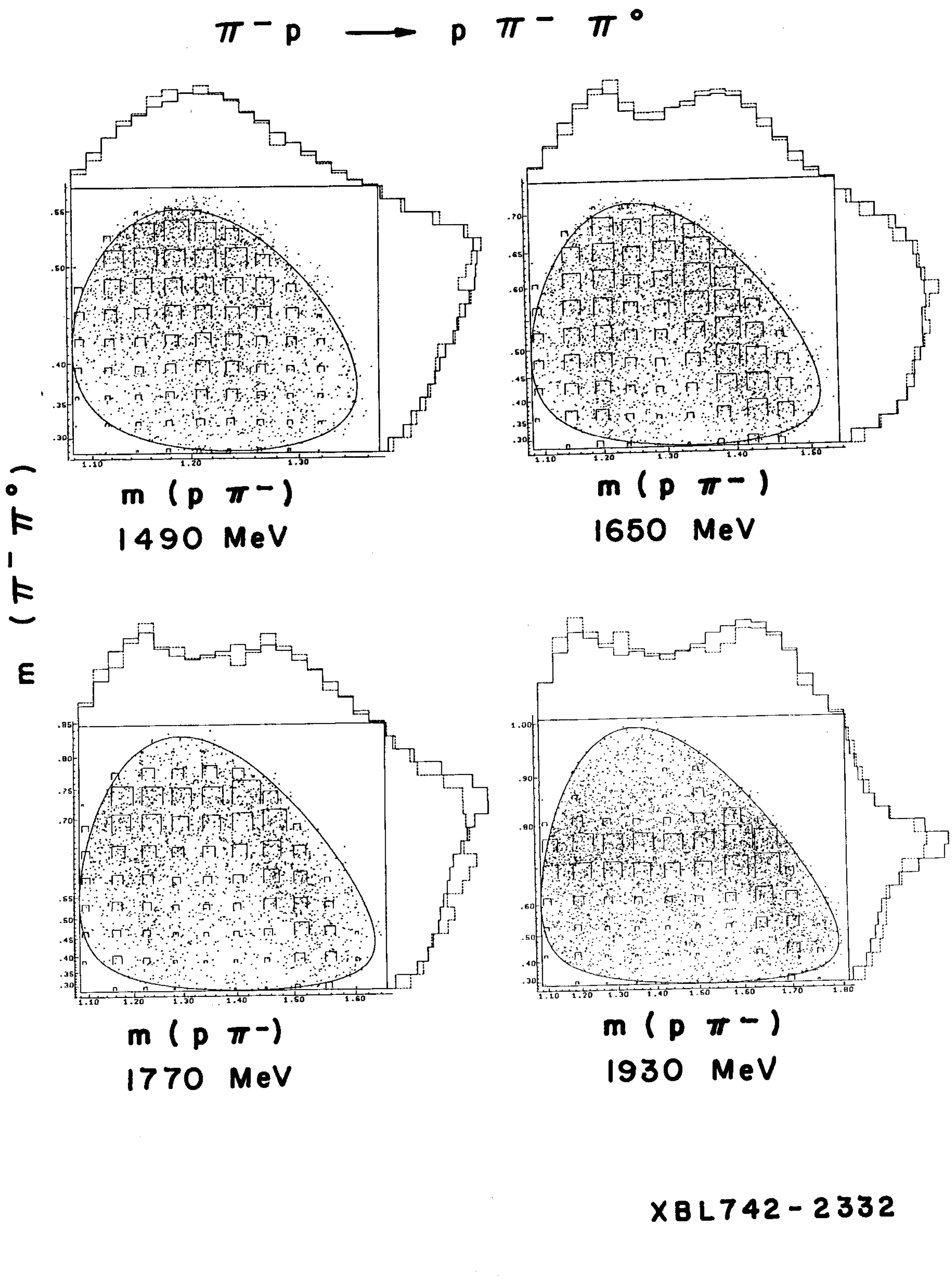

π⁻ p ⟶ p π⁻ π°
m (π⁻ π°)
m (p π⁻)
1490 MeV
m (p π⁻)
1650 MeV
m (p π⁻)
1770 MeV
m (p π⁻)
1930 MeV
XBL742-2332

FIG. 2b

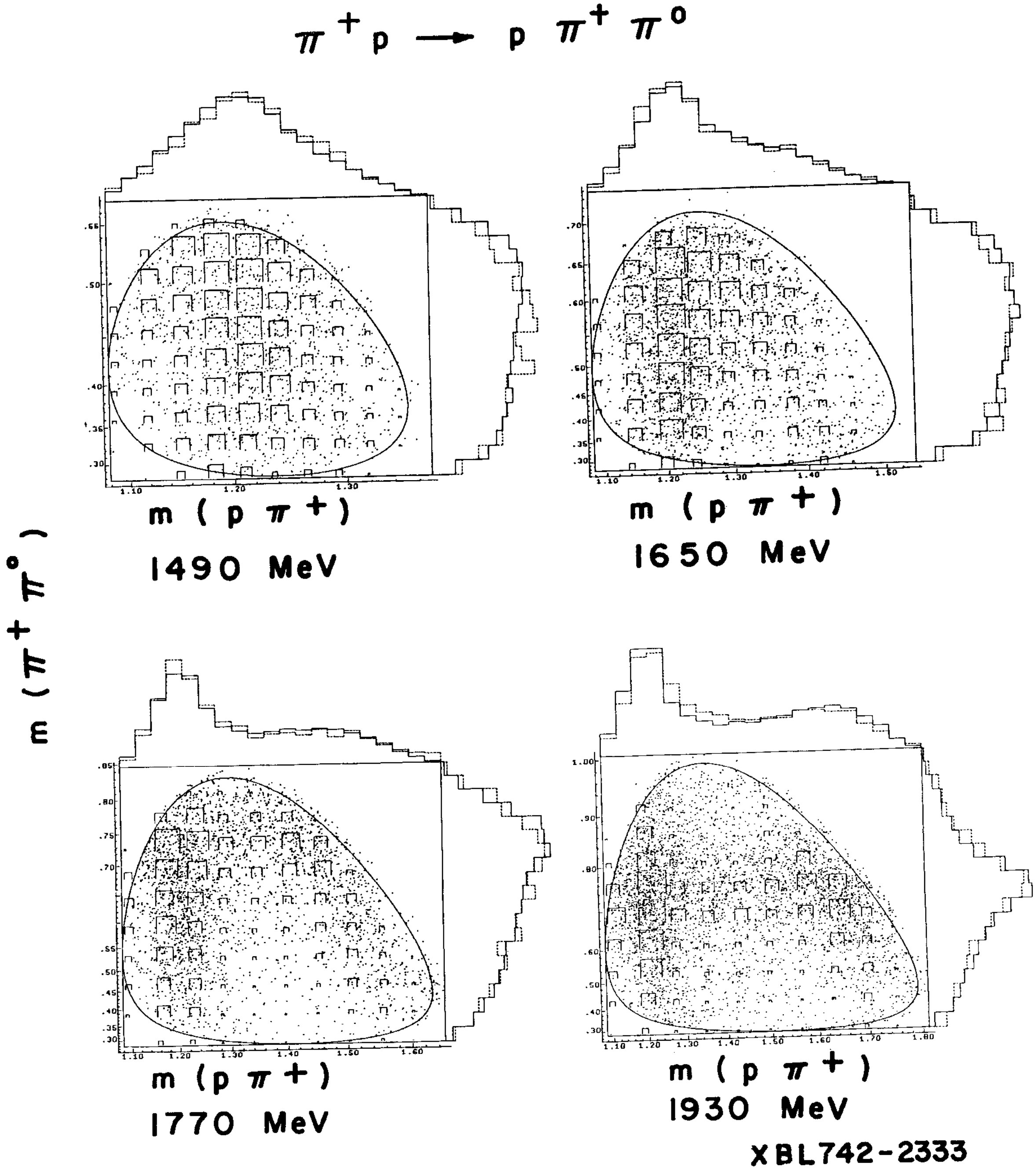
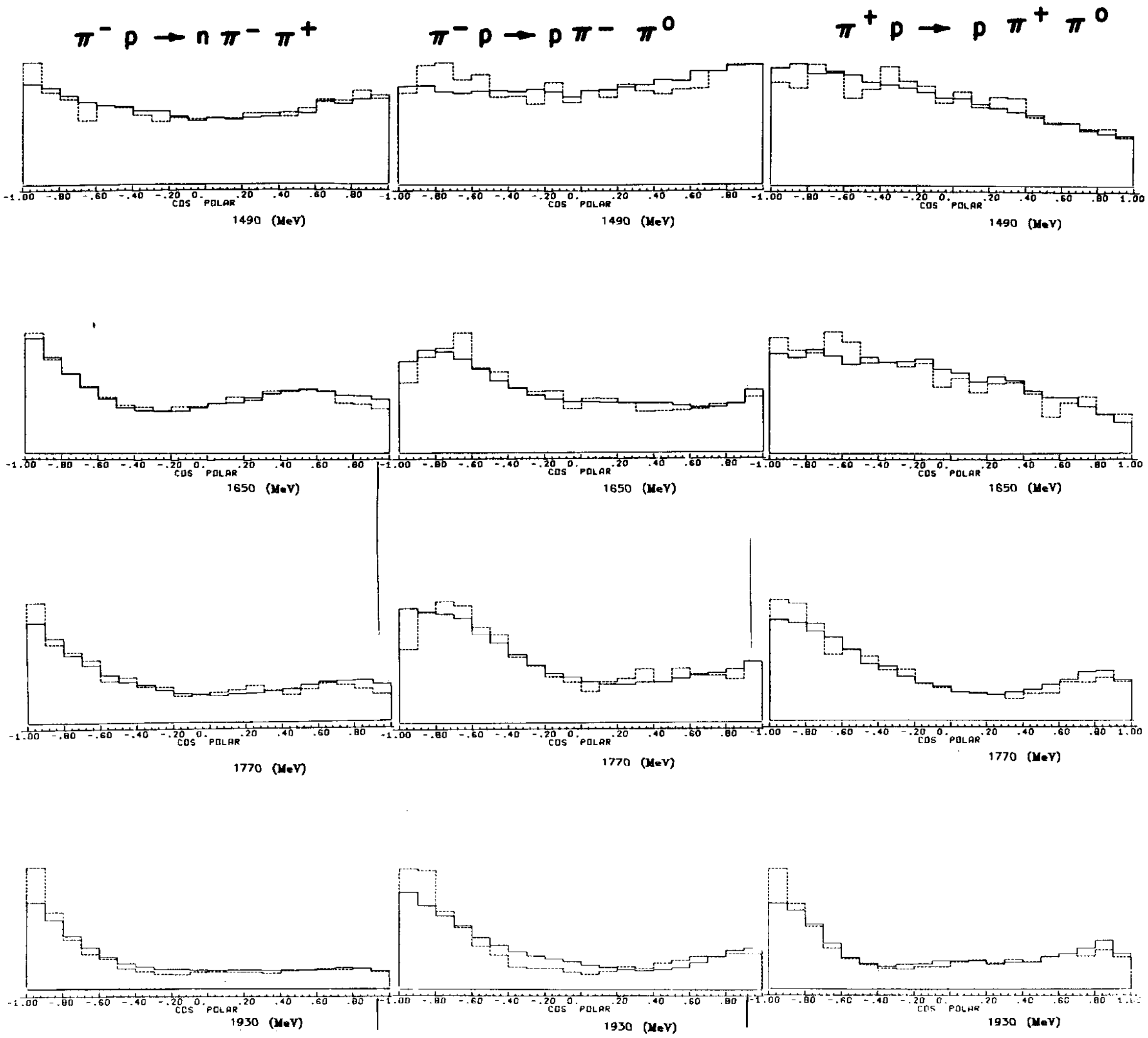

$\pi^+ p \longrightarrow p\, \pi^+ \pi^0$
m ($\pi^+ \pi^0$)
m (p π +)
1490 MeV
m (p π +)
1650 MeV
m (p π +)
1770 MeV
m (p π +)
1930 MeV
XBL742-2333

FIG. 2c



XBL742-2335

FIG. 3

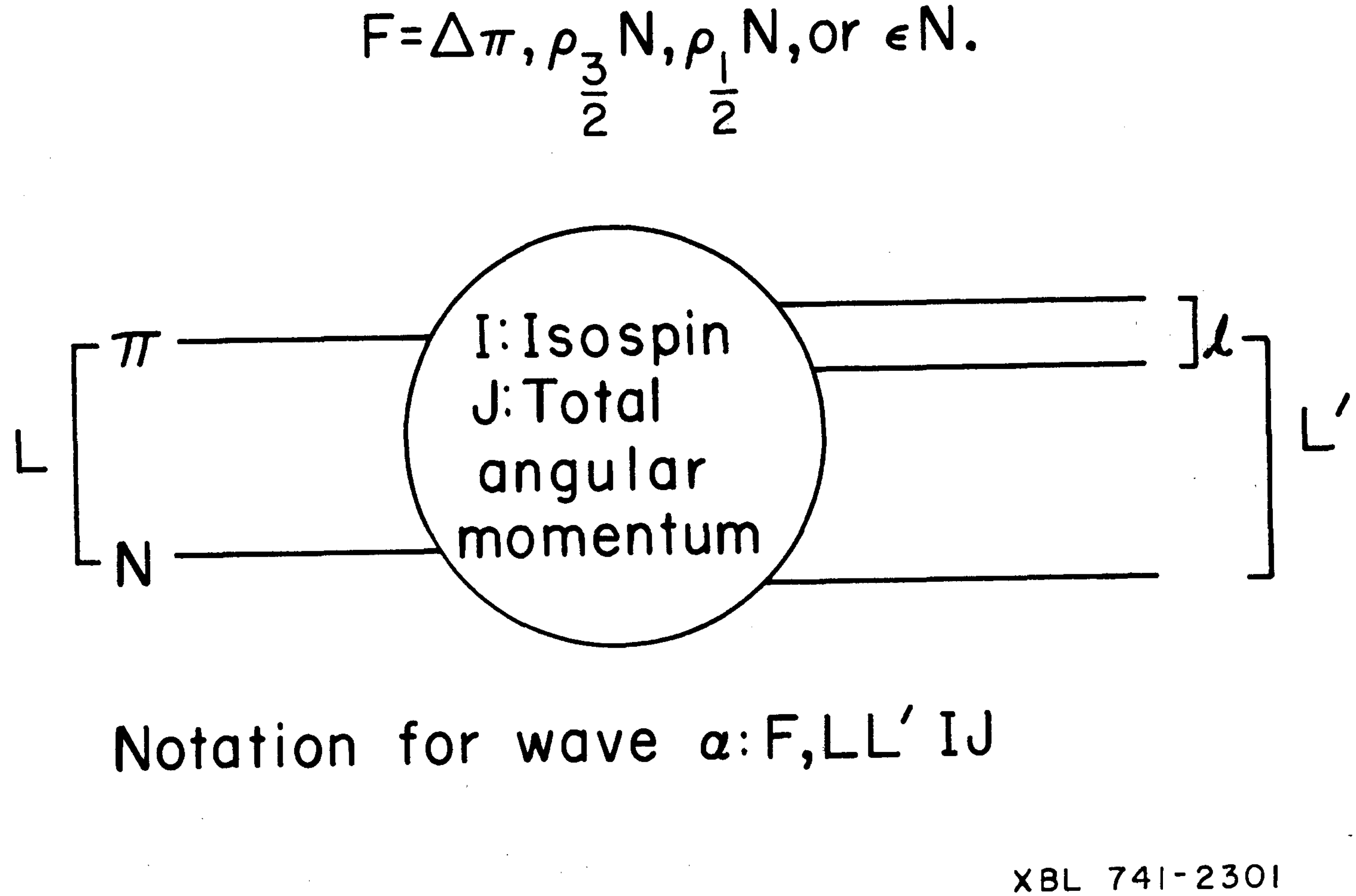

F=Δπ, ρ3/2N, ρ1/2N, or εN.
π
N
L
I: Isospin
J: Total angular momentum
ℓ
L′
Notation for wave α: F,LL′ IJ
XBL 741-2301

FIG. 4

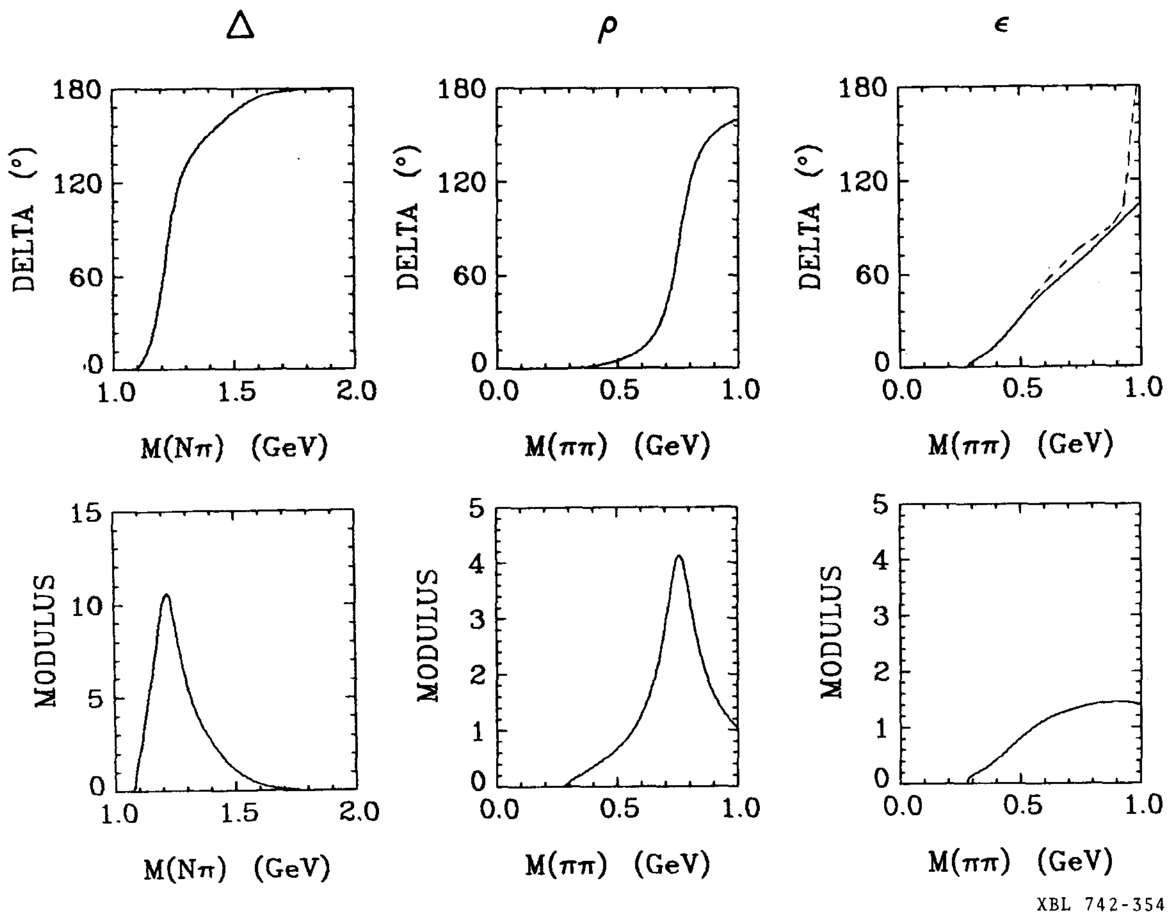


FIG. 5

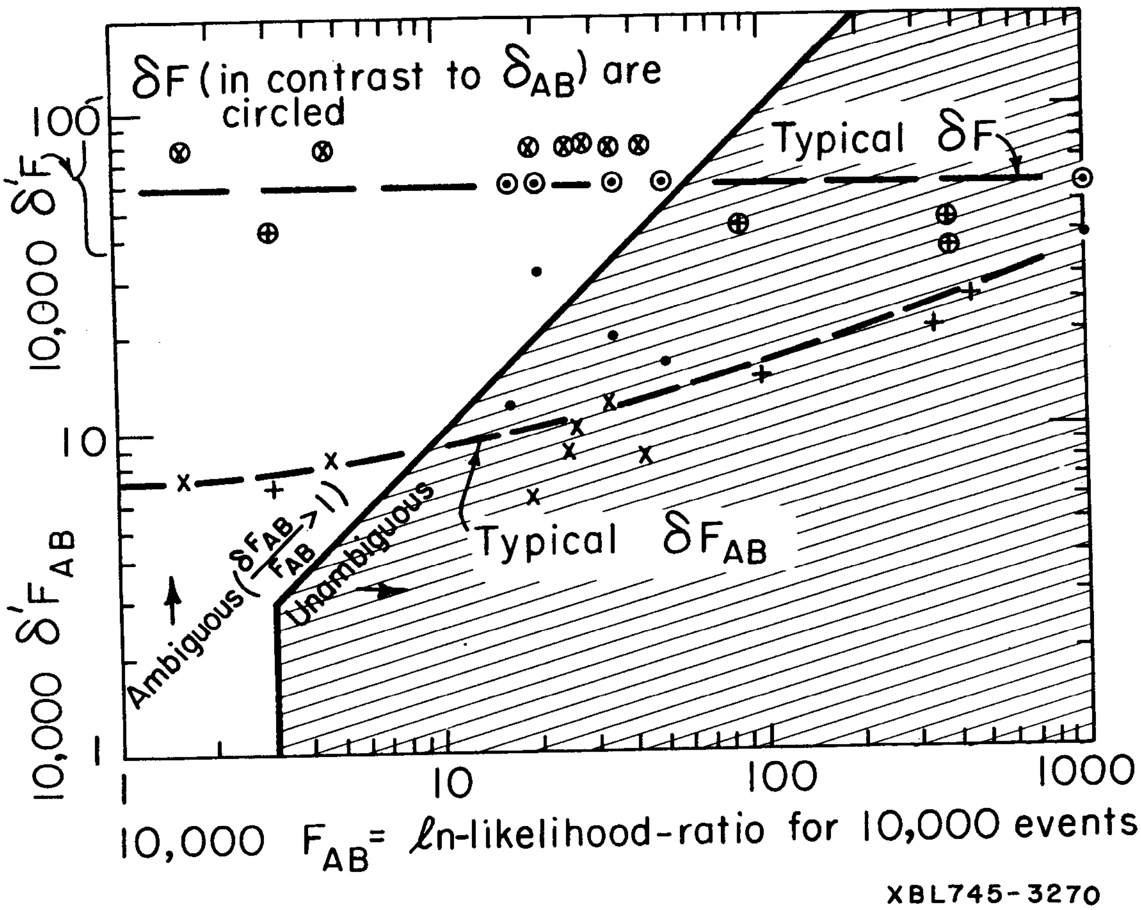
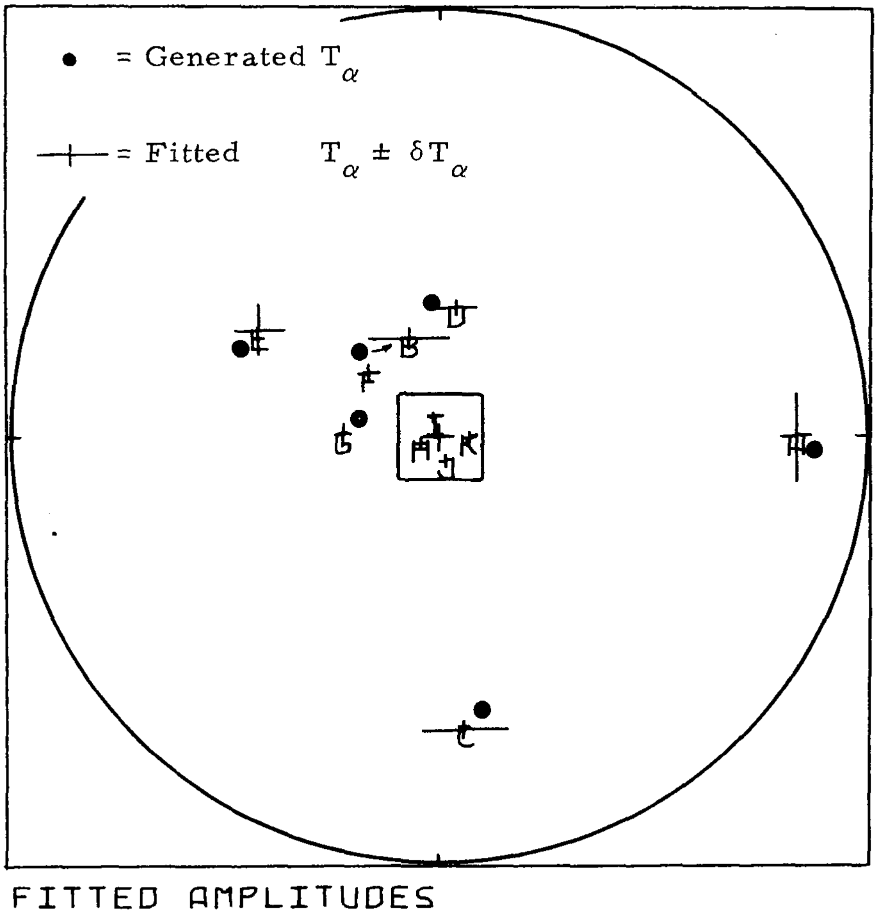

δF (in contrast to δ_AB) are circled
Typical δF
Typical δF_AB
Ambiguous (δF_AB / F_AB > 1)
Unambiguous
10,000 δ'F
10,000 δ'F_AB
100
10
1
1
10
100
1000
10,000 F_AB = ℓn-likelihood-ratio for 10,000 events
XBL745-3270

FIG. 6



XBL 726-934

FIG. 7

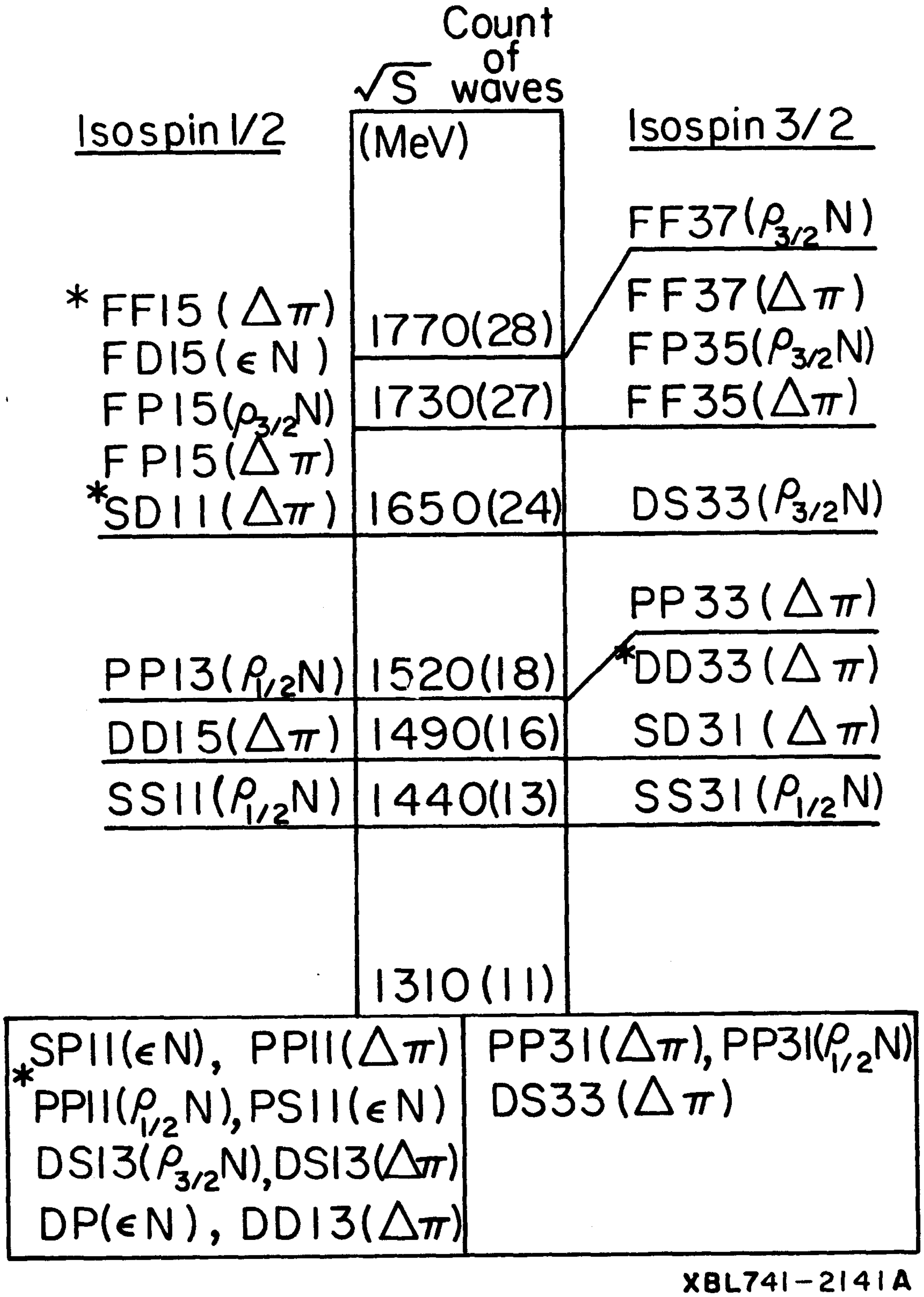
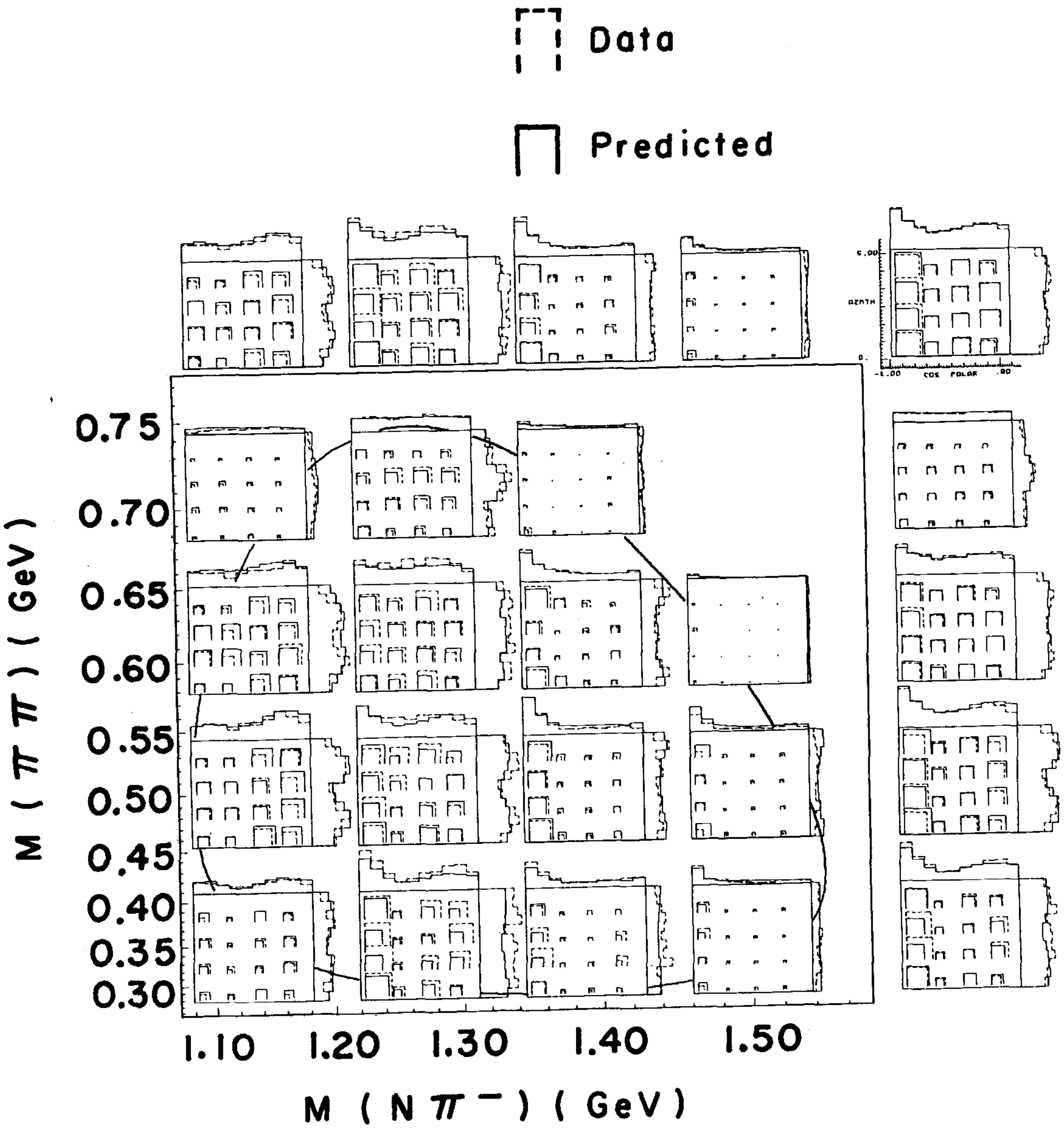
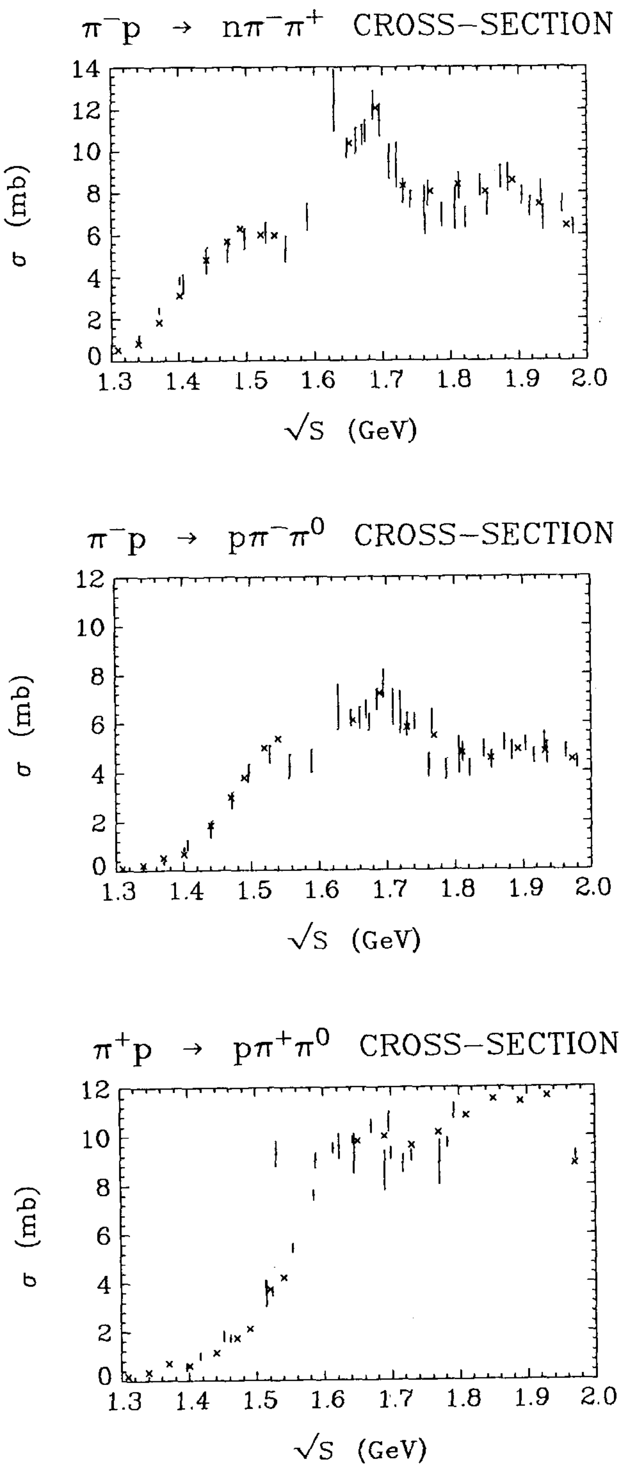

Count of waves
√S (MeV)
Isospin 1/2
Isospin 3/2
FF37(ρ3/2N)
FF37(Δπ)
FP35(ρ3/2N)
FF35(Δπ)
*FF15(Δπ)
FD15(εN)
FP15(ρ3/2N)
FP15(Δπ)
*SD11(Δπ)
1770(28)
1730(27)
1650(24)
DS33(ρ3/2N)
PP33(Δπ)
*DD33(Δπ)
SD31(Δπ)
SS31(ρ1/2N)
PP13(ρ1/2N)
DD15(Δπ)
SS11(ρ1/2N)
1520(18)
1490(16)
1440(13)
1310(11)
SP11(εN), PP11(Δπ)
*PP11(ρ1/2N), PS11(εN)
DS13(ρ3/2N), DS13(Δπ)
DP(εN), DD13(Δπ)
PP31(Δπ), PP31(ρ1/2N)
DS33(Δπ)
XBL741-2141A

FIG. 8



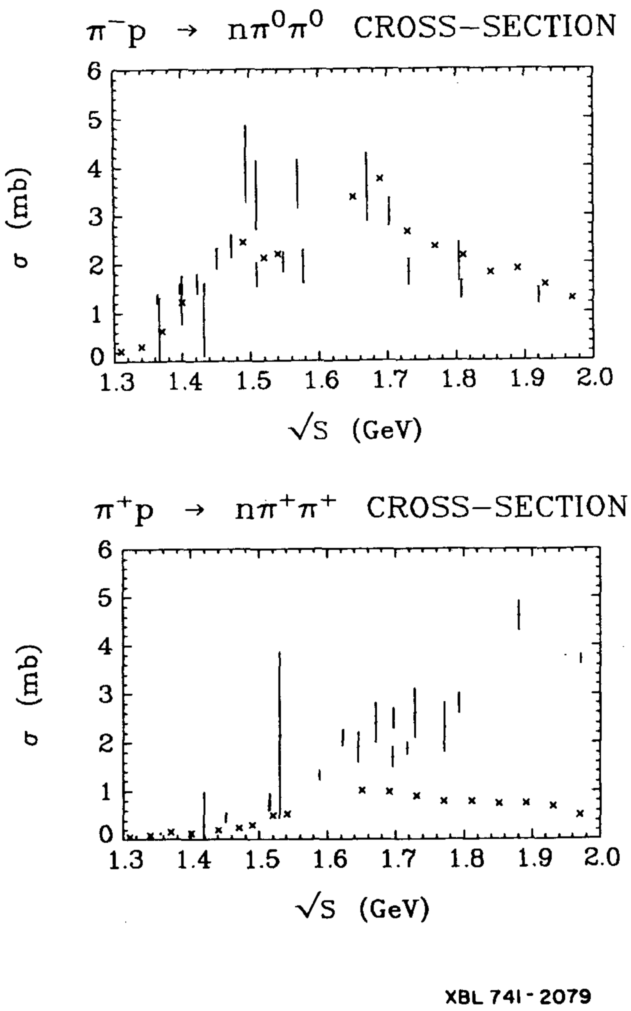
XBL742-2334

FIG. 9



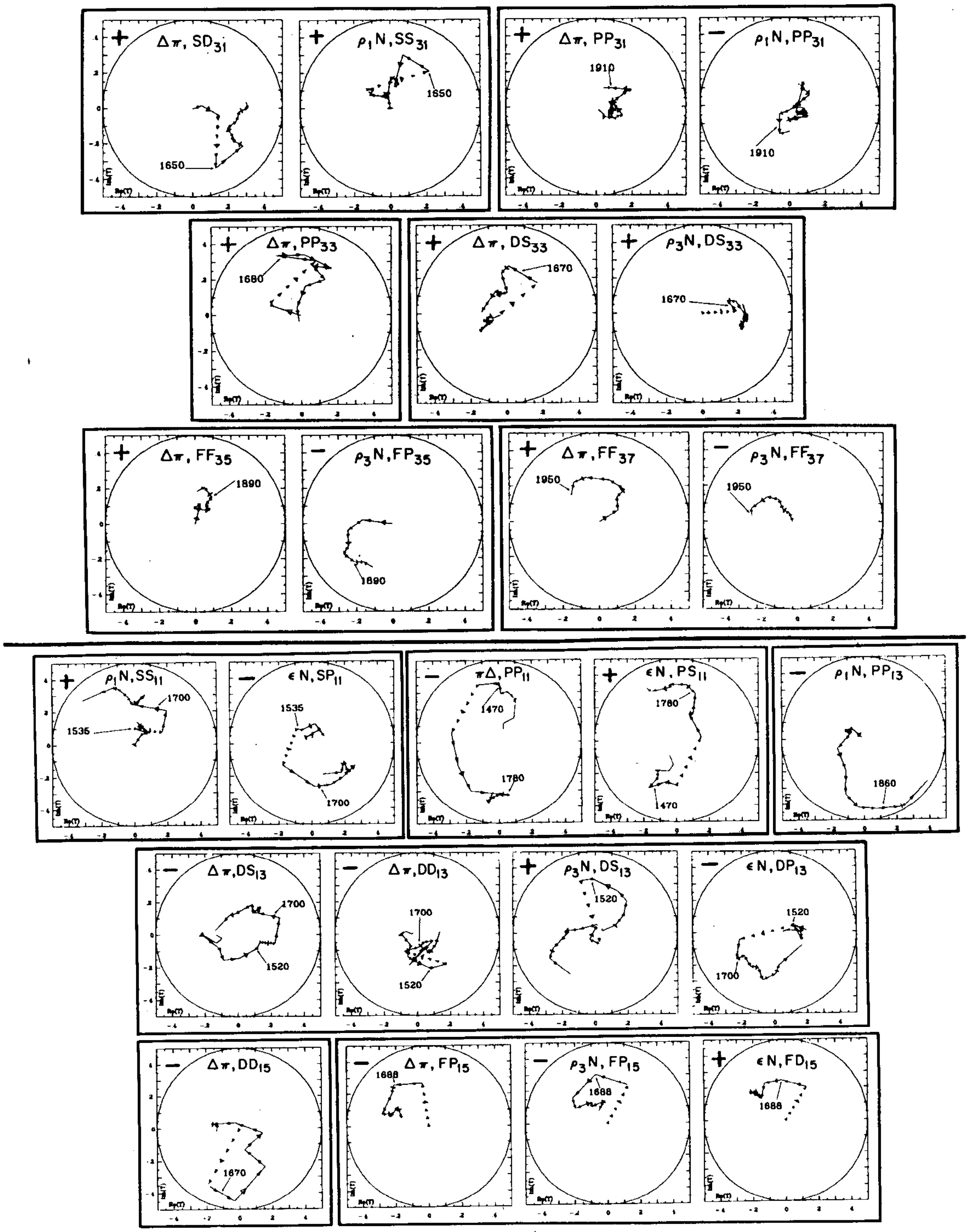
XBL 741-2080

FIG. 10



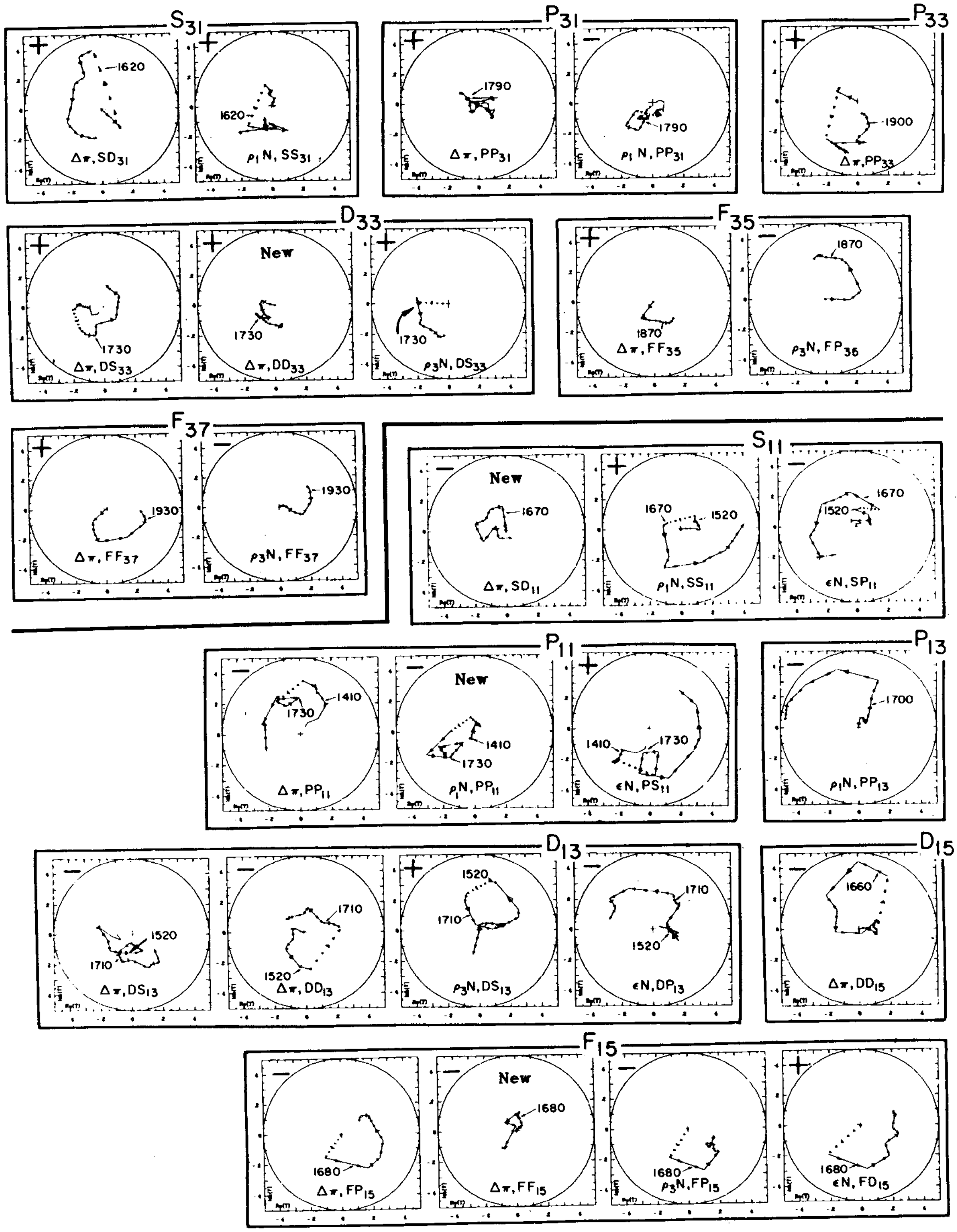
$\pi^-p \rightarrow n\pi^0\pi^0$ CROSS-SECTION
σ (mb)
√S (GeV)
$\pi^+p \rightarrow n\pi^+\pi^+$ CROSS-SECTION
σ (mb)
√S (GeV)
XBL 741-2079

FIG. 11



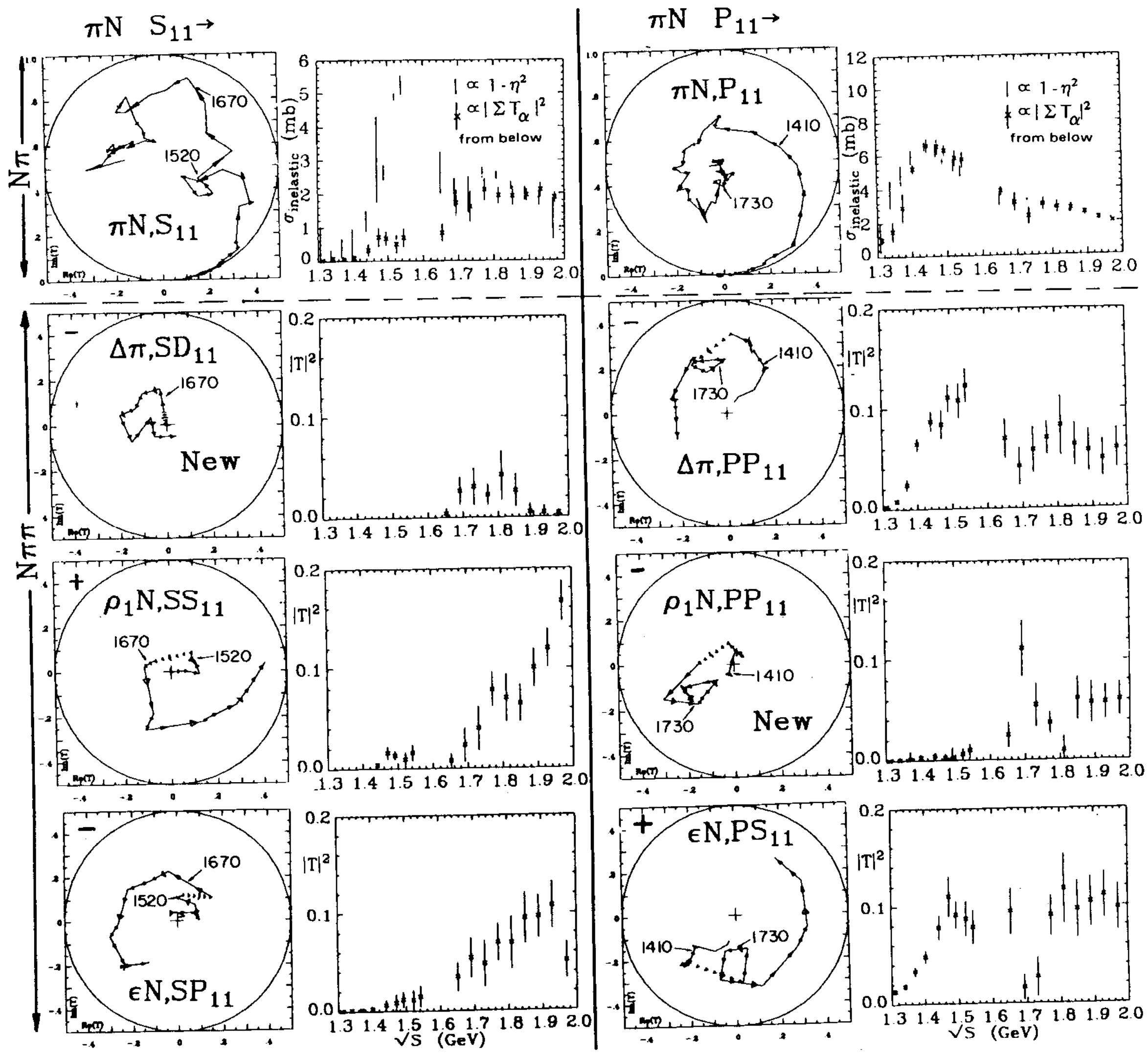
XBL 745-778A

FIG. 12



XBL 745-777A

FIG. 13



XBL 745-944 A

FIG. 14a

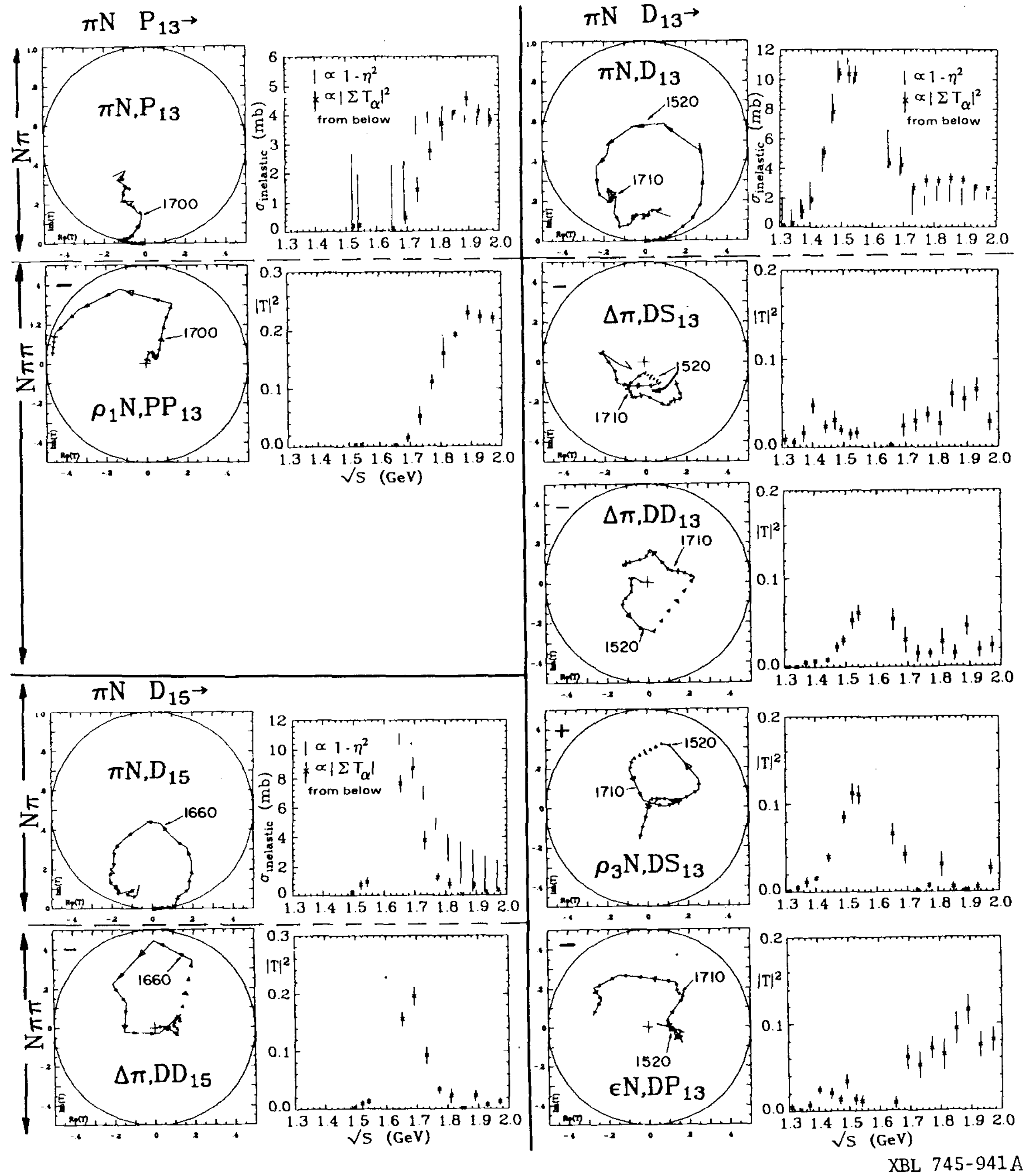
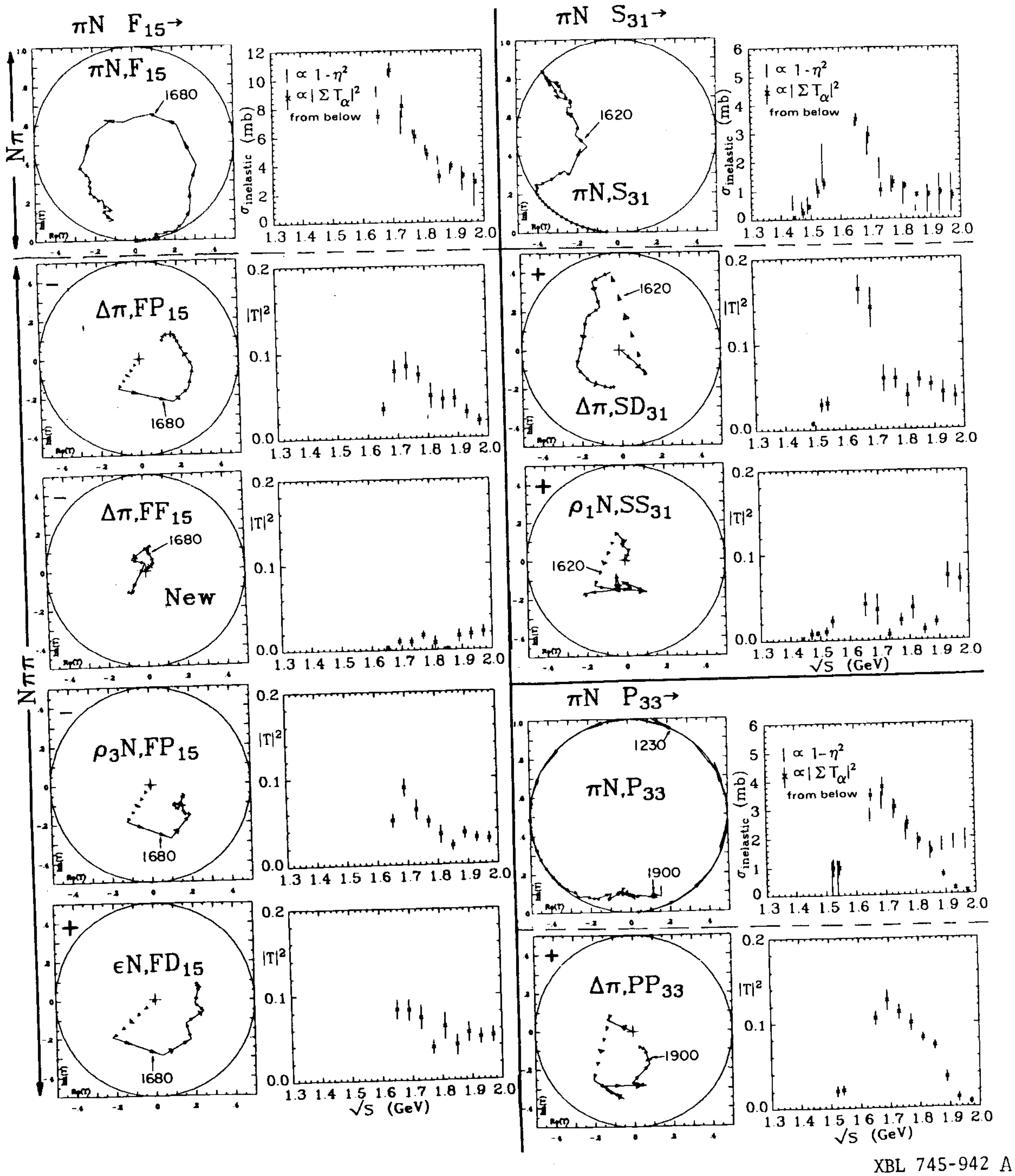
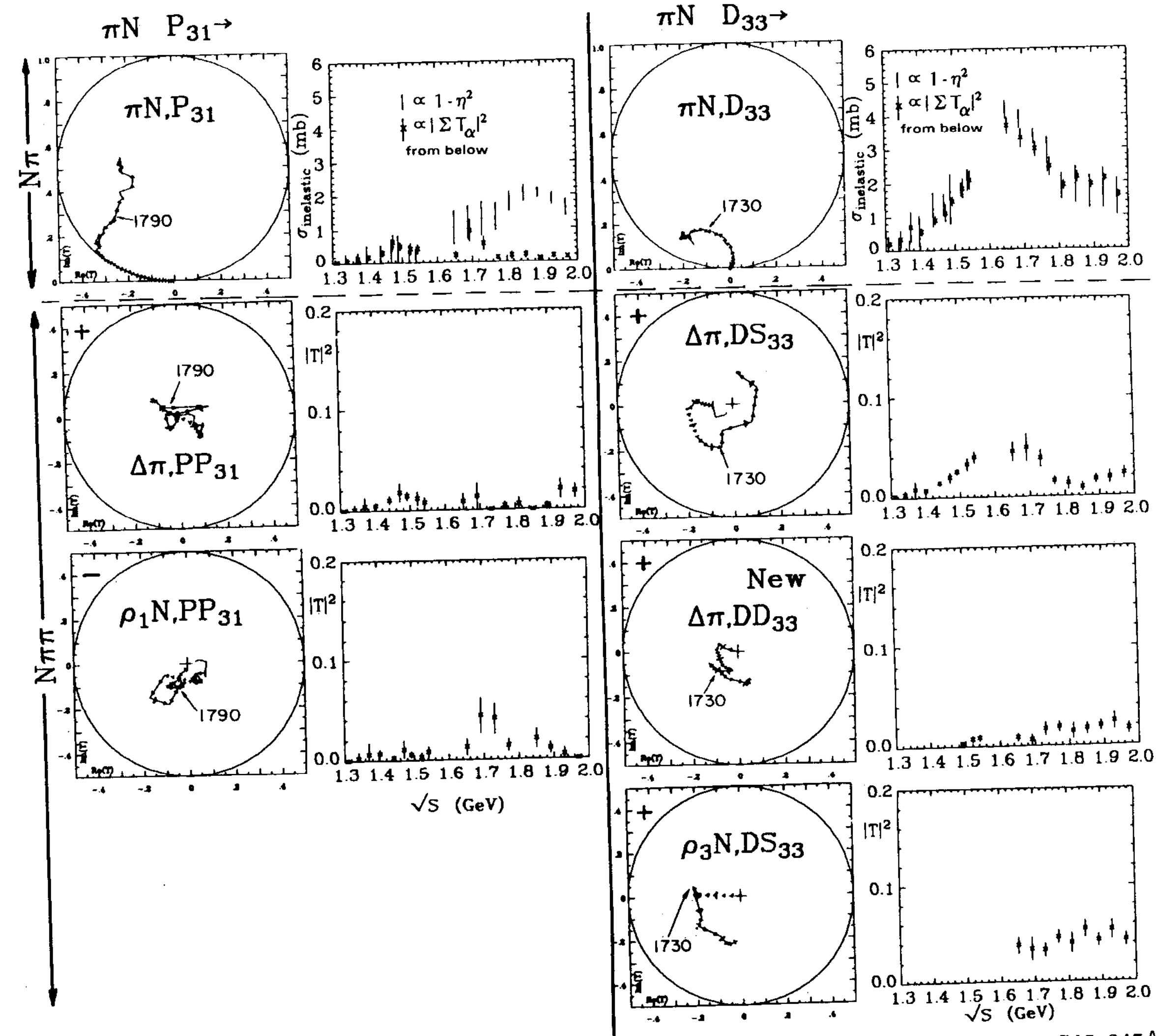

πN P13→
πN D13→
πN D15→
Nπ
Nππ
πN,P13
1700
ρ1N,PP13
πN,D13
1520
1710
Δπ,DS13
Δπ,DD13
ρ3N,DS13
εN,DP13
πN,D15
1660
Δπ,DD15
| ∝ 1 - η²
∝ |Σ Tα|²
from below
σ inelastic (mb)
|T|²
√S (GeV)
XBL 745-941A

FIG. 14b



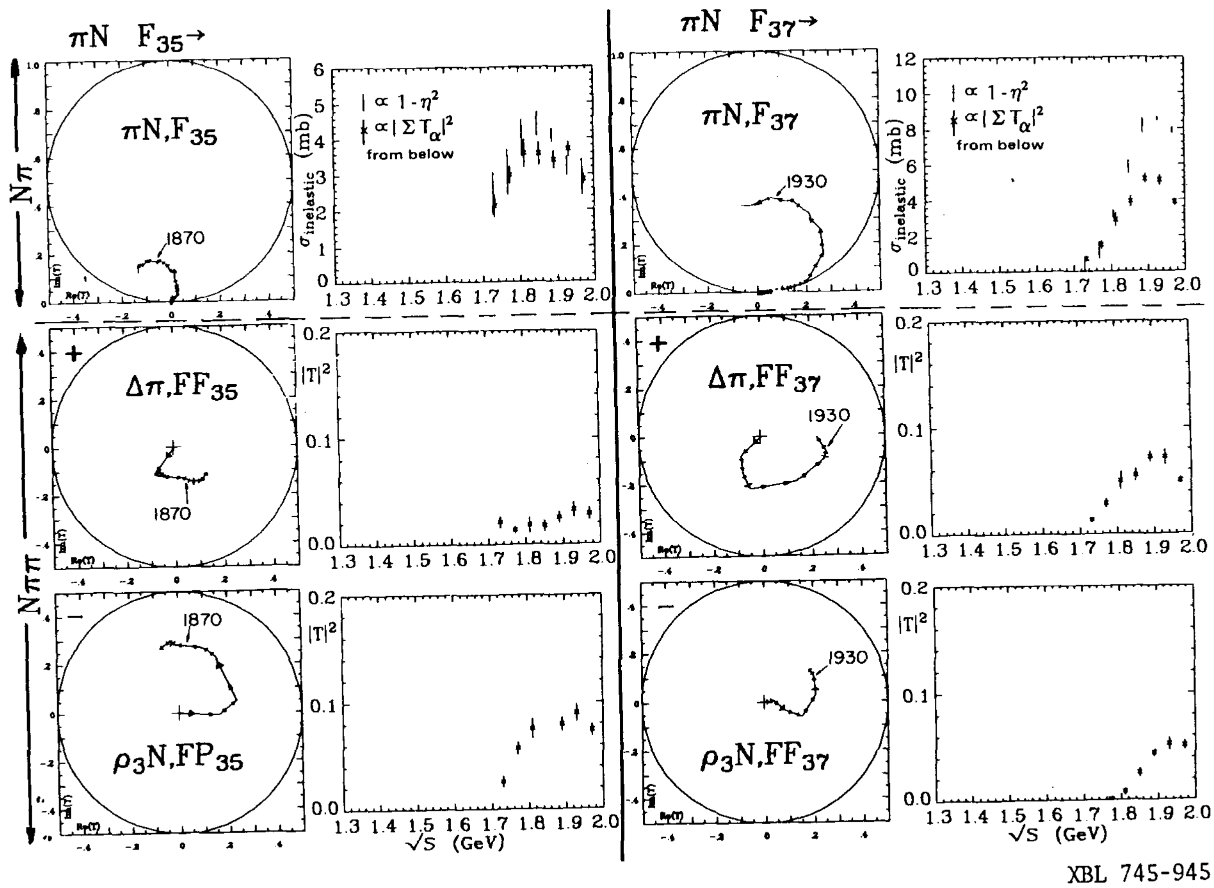
πN F15→
πN,F15
1680
Nπ
Nππ
Δπ,FP15
Δπ,FF15
New
ρ3N,FP15
εN,FD15
πN S31→
1620
πN,S31
Δπ,SD31
ρ1N,SS31
πN P33→
1230
πN,P33
1900
Δπ,PP33
| ∝ 1-η²
∝ |Σ Tα|²
from below
σinelastic (mb)
|T|²
√s (GeV)
XBL 745-942 A

FIG. 14c



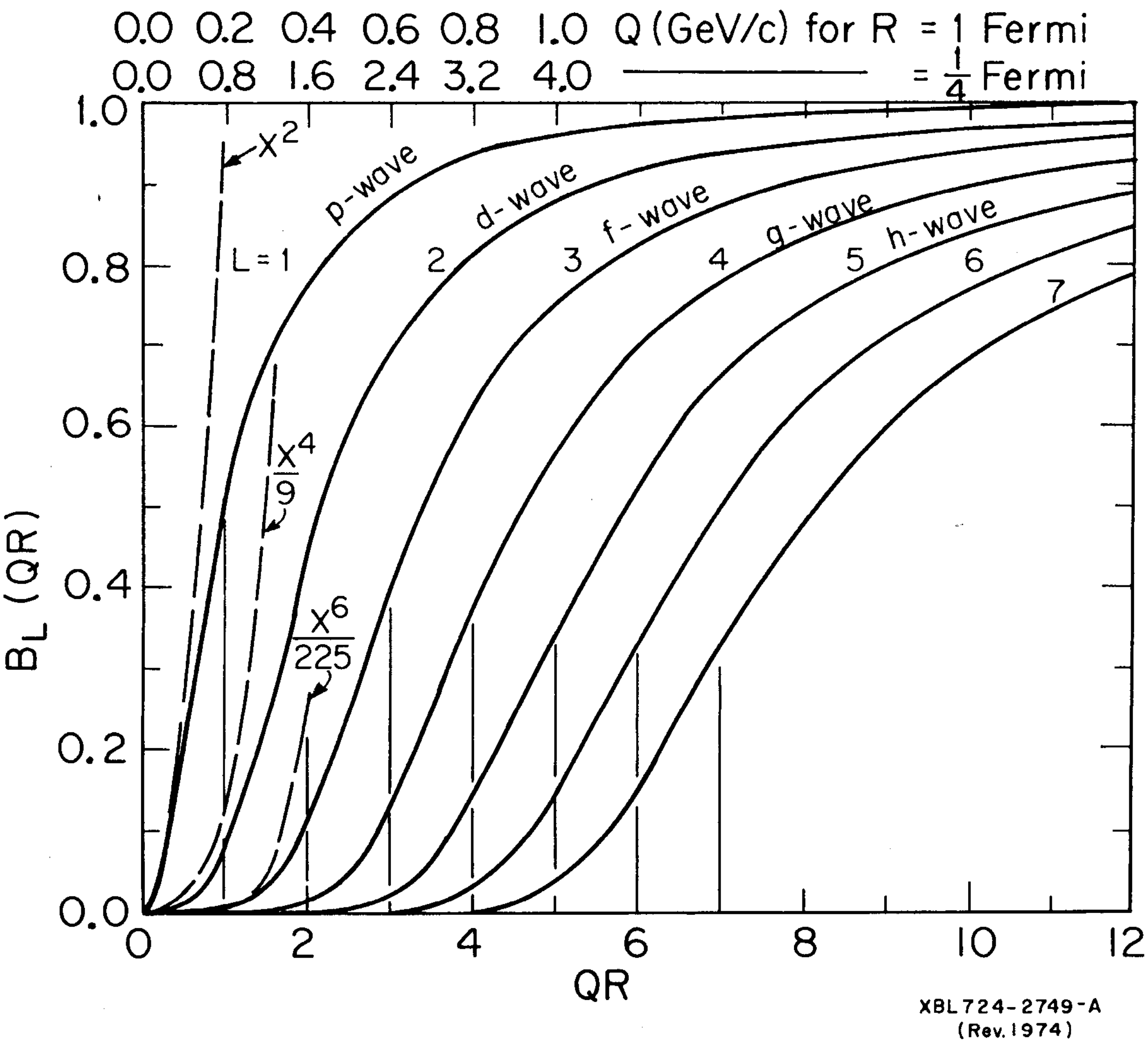
πN P31→
πN D33→
Nπ
Nππ
πN,P31
1790
| ∝ 1-η²
∝ |Σ T_α|²
from below
σ inelastic (mb)
πN,D33
1730
+
Δπ,PP31
|T|²
Δπ,DS33
−
ρ1N,PP31
New
Δπ,DD33
ρ3N,DS33
√s (GeV)
1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0
XBL 745-943A

FIG. 14d



πN F35→
πN F37→
πN,F35
1870
πN,F37
1930
| ∝ 1-η²
∝ |ΣTα|²
from below
σ inelastic (mb)
Nπ
Nππ
Δπ,FF35
Δπ,FF37
ρ3N,FP35
ρ3N,FF37
|T|²
√S (GeV)
XBL 745-945

FIG. 14e



0.0 0.2 0.4 0.6 0.8 1.0 Q (GeV/c) for R = 1 Fermi
0.0 0.8 1.6 2.4 3.2 4.0 = 1/4 Fermi
1.0
0.8
0.6
0.4
0.2
0.0
B_L (QR)
X^2
$X^4/9$
$X^6/225$
L = 1
p-wave
2
d-wave
3
f-wave
4
g-wave
5
h-wave
6
7
0
2
4
6
8
10
12
QR
XBL 724-2749-A
(Rev. 1974)

FIG. C1

LBL-1065 Rev.
SLAC-PUB-1108 Rev.
Supplement

Tables of Partial Wave Amplitudes*

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CONTENTS

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INTRODUCTION

A deck of 367 punched cards and a library of 16 magnetic tapes are both available to supplement Herndon et al., LBL-1065 Rev. This printed supplement documents those cards and tapes and also reproduces some computer printout of partial wave amplitudes at four representative energies.

Convention on Scaling Errors

We quote here a paragraph from Section IV.E of the main text:

"An error hyperellipsoid may be defined by a hypersurface in $\vec{A}$ space labelled by $\Delta\chi^2 = 1$, i.e. by $L = L^{max}\, e^{-1/2}$. To plot error ellipses on our Argand plots we project this hypersurface on the complex plane representing a single T_α. However, the probability that a result will be within this ellipse is only 40%. In order to increase this probability we have conservatively doubled our estimated error, thereby raising the χ^2 contour to 4, and enclosing 87% of the probability.[11] In summary, all the errors plotted or tabulated in this paper and in our previous publications[11,20,22] are twice those calculated by the program RUMBLE,[11] which uses Eq. (4.23)."

Accordingly, the cards with the data on amplitudes and errors (Sect. I of this Supplement) are not the original cards punched by RUMBLE, but have had their errors expanded by two. On the other hand the reproductions of RUMBLE printout in Section II of this Supplement have their errors unscaled.

I. ARGAND AMPLITUDES ON PUNCHED CARDS

We have prepared a deck of 367 punched cards, which is available from A. H. Rosenfeld on the Particle Data Group at LBL or M. Neveu, Saclay. The cards are listed below, and there should be enough comment cards that they can be understood with only the following remarks:

At each energy there are listed 28 waves (complex numbers occupying two words, i.e. two columns of printout); but of course at the lower energies some of these amplitudes are written as 0,0.

To read the data cards use the following formats:

Amplitude cards: 8F10.5

Error Matrix cards: 9F8.X [They were written 3(2F8.5, F8.2).]

The phases of the Argand amplitudes have been rotated by a K-matrix fit and correspond to our published Argand plots (Fig. 13 of the document of which this is a supplement) or Fig. 2 of Rosenfeld et al., LBL-2633 submitted to Phys. Rev. Letters.

```
*   ARGAND PLOT AMPLITUDES GIVEN AS T-REAL AND T-IMAG..  ERRORS ARE GIVEN AS
*   ERROR T-REAL, ERROR T-IMAG. AND CORR. ANGLE.  IF A WAVE WAS NOT USED IN FIT
*   AT A GIVEN ENERGY, T AND ERROR ARE PRINTED ZERO.  LIST OF ORDER OF WAVES AS
*   FOLLOWS.
*   DELTA - PP11,SD11,DS13,DD13,FP15,DD15,FF15,PP31,SD31,DS33,PP33,DD33,FF35,FF37
*   RHO 3/2 - DS13,FP15,DS33,FP35,FF37
*   RHO 1/2 - SS11,PP11,PP13,SS31,PP31
*   EPSILON - PS11,SP11,DP13,FD15
```

```
●   ARGAND PLOT   1970      SAMPLE
------------------------------ DELTA -----------------------------------------
    -.21661    -.11124     .04326    -.04658     .14334    -.09130    -.12584     .09006
     .11377     .08405     .11645    -.00765    -.09652    -.10779    -.11538     .06531
    -.03288    -.19034     .02910     .14647     .02030    -.07872     .04677    -.11954
                                               I----- RHO 3/2 -----------------------
     .13066    -.10624     .22412    -.00474 I -.03911    -.16026     .15576    -.06952
------- RHO 3/2 -------------------------I                         I- RHO 1/2 --------
    -.02147    -.20708    -.07883     .26012     .18138     .13496 I  .40659     .03748
---------- RHO 1/2 ------------------------------------------------I
    -.22656    -.09667    -.46791     .04626    -.19741    -.17905    -.00442    -.01767
----------------------- EPSILON ---------------------------------------------------
     .18003     .25602    -.12859    -.18609    -.27740     .06450     .20785     .09370
*   ERRORS  1970      SAMPLE
-------------------------- DELTA -------------------------------------------------
   .04030  .04409    60.93  .04265  .04389    64.36  .02687  .02957   58.78
   .02908  .03090    58.43  .02518  .02171    96.69  .03088  .02310  -51.09
   .02813  .02363    93.49  .02704  .02755   -68.98  .03139  .03116  109.99
   .01886  .02023  -105.73  .01597  .02261    51.92  .02229  .01966  -53.96
                                                  I-------- RHO 3/2 -------------
   .01509  .01990    48.50  .01208  .01742    56.80 I.02651  .02460  100.43
----------- RHC 3/2 -----------------------------I
   .01713  .02539    54.91  .01927  .01695  -116.88  .01114  .01650 -100.28
                           I---------------- RHO 1/2 ---------------------------
   .01195  .01000   106.92 I.02979  .03062    66.76  .04338  .03353   79.75
----- RHO 1/2 -----------I
   .01184  .02013    58.26  .03440  .02617  -126.36  .03138  .02758 -120.73
------------------------------- EPSILON ------------------------------------------
   .03746  .03846   114.85  .04290  .03956   102.71  .02416  .02866   51.84
   .02258  .02021    87.36
```

Sample

```
●   ARGAND PLOT   1310
     .03015     .04695    0.00000    0.00000    -.09601     .00520    -.02225     .00644
    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000     .02221     .00701
    0.00000    0.00000    -.02694    -.03345    0.00000    0.00000    0.00000    0.00000
    0.00000    0.00000    0.00000    0.00000     .03028     .01847    0.00000    0.00000
    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000
    -.00394     .01144    0.00000    0.00000    0.00000    0.00000     .02975     .01061
    -.02718    -.11173    -.00414     .01668     .06801     .00708    0.00000    0.00000
*   ERRCRS  1310
   .02829  .00859    90.82 0.00000 0.00000   -90.00  .01867  .02292  119.75
   .00980  .01048   116.00 0.00000 0.00000   -90.00 0.00000 0.00000  -90.00
  0.00000 0.00000   -90.00  .02980  .02428    99.10 0.00000 0.00000  -90.00
   .01863  .00978    96.71 0.00000 0.00000   -90.00 0.00000 0.00000  -90.00
  0.00000 0.00000   -90.00 0.00000 0.00000   -90.00  .01433  .01583  118.22
  0.00000 0.00000   -90.00 0.00000 0.00000   -90.00 0.00000 0.00000  -90.00
  0.00000 0.00000   -90.00 0.00000 0.00000   -90.00  .01756  .01310  103.36
  0.00000 0.00000   -90.00 0.00000 0.00000   -90.00  .02058  .02317  115.72
   .01896  .01145   104.68  .02428  .01681  -126.25  .02854  .01631 -132.16
  0.00000 0.00000   -90.00
●   ARGAND PLOT   1340
     .04814     .07498    0.00000    0.00000    -.06789    -.03484    -.01987     .00218
    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000     .03581     .01135
    0.00000    0.00000    -.03753    -.04661    0.00000    0.00000    0.00000    0.00000
```

0.00000	0.00000	0.00000	0.00000	.05376	.04320	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
-.00879	.02573	0.00000	0.00000	0.00000	0.00000	.04499	.01603
-.03713	-.12982	.03122	.02345	.03733	.01964	0.00000	0.00000

* ERRORS 1340

.05600	.01176	88.85	0.00000	0.00000	-90.00	.04254	.03140	78.43
.01984	.01141	92.66	0.00000	0.00000	-90.00	0.00000	0.00000	-90.00
0.00000	0.00000	-90.00	.04719	.04501	88.20	0.00000	0.00000	-90.00
.03581	.03110	84.53	0.00000	0.00000	-90.00	0.00000	0.00000	-90.00
0.00000	0.00000	-90.00	0.00000	0.00000	-90.00	.03378	.02335	83.34
0.00000	0.00000	-90.00	0.00000	0.00000	-90.00	0.00000	0.00000	-90.00
0.00000	0.00000	-90.00	0.00000	0.00000	-90.00	.03269	.04478	122.10
0.00000	0.00000	-90.00	0.00000	0.00000	-90.00	.05064	.03272	-126.25
.01934	.01189	105.49	.02212	.01430	78.80	.03274	.02433	80.56
0.00000	0.00000	-90.00						

ARGAND PLOT 1370

.11675	.10240	0.00000	0.00000	-.12585	-.01075	-.06735	.02978
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.06580	.00171
0.00000	0.00000	-.07599	-.05348	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	.10188	.04293	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
-.00378	.03539	0.00000	0.00000	0.00000	0.00000	.07933	.00494
-.08927	-.15983	-.00034	.04027	.08640	.00249	0.00000	0.00000

* ERRORS 1370

.06545	.01766	87.74	0.00000	0.00000	-90.00	.04115	.03835	71.00
.02388	.02412	-67.93	0.00000	0.00000	-90.00	0.00000	0.00000	-90.00
0.00000	0.00000	-90.00	.05662	.05617	74.68	0.00000	0.00000	-90.00
.04379	.04265	68.60	0.00000	0.00000	-90.00	0.00000	0.00000	-90.00
0.00000	0.00000	-90.00	0.00000	0.00000	-90.00	.04042	.02929	75.21
0.00000	0.00000	-90.00	0.00000	0.00000	-90.00	0.00000	0.00000	-90.00
0.00000	0.00000	-90.00	0.00000	0.00000	-90.00	.05521	.10430	-92.40
0.00000	0.00000	-90.00	0.00000	0.00000	-90.00	.04275	.07779	-98.92
.02052	.01581	108.55	.02817	.02042	-122.80	.03076	.03109	66.79
0.00000	0.00000	-90.00						

ARGAND PLOT 1400

.16803	.19127	0.00000	0.00000	-.21239	.05294	-.08066	.02561
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.05998	-.03885
0.00000	0.00000	-.08730	-.00104	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	.12532	.02780	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
-.02488	.04835	0.00000	0.00000	0.00000	0.00000	.07165	-.04284
-.17621	-.13271	-.03391	.03791	.15651	-.02063	0.00000	0.00000

* ERRORS 1400

.03494	.01198	91.70	0.00000	0.00000	-90.00	.01979	.01719	72.54
.01185	.01473	59.47	0.00000	0.00000	-90.00	0.00000	0.00000	-90.00
0.00000	0.00000	-90.00	.02135	.02368	-75.24	0.00000	0.00000	-90.00
.00920	.01284	56.59	0.00000	0.00000	-90.00	0.00000	0.00000	-90.00
0.00000	0.00000	-90.00	0.00000	0.00000	-90.00	.01028	.01455	52.98
0.00000	0.00000	-90.00	0.00000	0.00000	-90.00	0.00000	0.00000	-90.00
0.00000	0.00000	-90.00	0.00000	0.00000	-90.00	.01520	.01532	113.02
0.00000	0.00000	-90.00	0.00000	0.00000	-90.00	.02343	.01906	95.06
.02682	.01437	82.74	.02191	.01847	-130.25	.01351	.01302	106.31
0.00000	0.00000	-90.00						

ARGAND PLOT 1440

.12099	.27036	0.00000	0.00000	-.14913	-.04019	-.07995	-.04157
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.07776	-.06811
0.00000	0.00000	-.11996	.00197	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	.19352	.05078	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.06260	.00376
.00722	.06557	0.00000	0.00000	.01747	.04981	.03192	-.04398
-.20080	-.19531	.06947	.03968	.13159	-.06141	0.00000	0.00000

* ERRORS 1440

.04755	.01658	88.53	0.00000	0.00000	-90.00	.02828	.01704	85.12
.01845	.01326	75.10	0.00000	0.00000	-90.00	0.00000	0.00000	-90.00
0.00000	0.00000	-90.00	.01961	.02436	-92.38	0.00000	0.00000	-90.00
.01194	.01135	-66.39	0.00000	0.00000	-90.00	0.00000	0.00000	-90.00
0.00000	0.00000	-90.00	0.00000	0.00000	-90.00	.01790	.01270	-53.91
0.00000	0.00000	-90.00	0.00000	0.00000	-90.00	0.00000	0.00000	-90.00
0.00000	0.00000	-90.00	.02884	.01752	-51.05	.01876	.02624	48.75
0.00000	0.00000	-90.00	.02378	.01606	95.01	.02062	.02158	-85.09
.03354	.02391	75.33	.03115	.02890	-125.02	.01523	.02336	50.31
0.00000	0.00000	-90.00						

ARGAND PLOT 1470

.10548	.27132	0.00000	0.00000	-.13970	-.10738	-.12227	-.09012
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.09890	-.09113
0.00000	0.00000	-.14055	.01022	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	.25687	.08725	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.12594	-.01055
.03872	.03999	0.00000	0.00000	-.02673	.08635	.05863	-.08846
-.25013	-.21894	.09611	.00351	.11715	.00273	0.00000	0.00000

* ERRORS 1470

.05445	.02406	85.04	0.00000	0.00000	-90.00	.04080	.02540	89.92
.02432	.02034	71.58	0.00000	0.00000	-90.00	0.00000	0.00000	-90.00
0.00000	0.00000	-90.00	.03136	.03455	-97.81	0.00000	0.00000	-90.00
.02179	.01970	-63.96	0.00000	0.00000	-90.00	0.00000	0.00000	-90.00
0.00000	0.00000	-90.00	0.00000	0.00000	-90.00	.02903	.01567	-54.00
0.00000	0.00000	-90.00	0.00000	0.00000	-90.00	0.00000	0.00000	-90.00
0.00000	0.00000	-90.00	.03604	.02488	-59.61	.02831	.04668	48.41
0.00000	0.00000	-90.00	.03349	.03335	111.71	.02998	.03409	61.05
.03809	.03861	67.19	.03607	.03436	-114.59	.02519	.02569	-68.22
0.00000	0.00000	-90.00						

ARGAND PLOT 1490

.10519	.31658	0.00000	0.00000	-.07107	-.12001	-.12369	-.12266
0.00000	0.00000	.05246	.00691	0.00000	0.00000	.07598	-.09227
.06489	-.06192	-.15640	.02815	0.00000	0.00000	-.05814	.01342
0.00000	0.00000	0.00000	0.00000	.25763	.13853	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.10554	.05029
.03558	.03064	0.00000	0.00000	-.00098	.09442	.01794	-.07604
-.20844	-.21947	.07667	.07821	.16547	-.08143	0.00000	0.00000

* ERRORS 1490

.03989	.02018	84.97	0.00000	0.00000	-90.00	.02433	.01919	105.32
.02524	.01443	84.13	0.00000	0.00000	-90.00	.01705	.01695	67.78
0.00000	0.00000	-90.00	.01706	.02182	-99.98	.01603	.01815	-85.71
.00831	.01706	-82.65	0.00000	0.00000	-90.00	.00921	.01639	-84.77
0.00000	0.00000	-90.00	0.00000	0.00000	-90.00	.01249	.02362	45.54
0.00000	0.00000	-90.00	0.00000	0.00000	-90.00	0.00000	0.00000	-90.00
0.00000	0.00000	-90.00	.01691	.04047	49.51	.02292	.03682	51.36
0.00000	0.00000	-90.00	.01611	.01662	114.20	.01774	.01901	127.19
.03167	.02400	76.20	.02582	.03612	-87.94	.01577	.02404	47.77
0.00000	0.00000	-90.00						

ARGAND PLOT 1520

.08814	.31749	0.00000	0.00000	.02614	-.12007	-.04175	-.22555
0.00000	0.00000	.09959	-.02370	0.00000	0.00000	.10278	-.03839
.12026	-.12233	-.18089	-.01008	-.13004	.06205	-.07974	.03501
0.00000	0.00000	0.00000	0.00000	.11431	.31531	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.08652	.04624
-.00671	.07939	.00936	.05988	-.01096	.10257	.02626	-.05493
-.21749	-.20063	-.00106	.10760	.11788	-.00045	0.00000	0.00000

* ERRORS 1520

.05196	.02546	82.22	0.00000	0.00000	-90.00	.03010	.03294	116.49
.03398	.02153	100.66	0.00000	0.00000	-90.00	.02214	.02391	120.61
0.00000	0.00000	-90.00	.02942	.03655	-87.75	.02893	.02666	-115.89
.01878	.02498	-74.95	.02560	.02621	-111.75	.01740	.02798	-89.11
0.00000	0.00000	-90.00	0.00000	0.00000	-90.00	.04091	.01718	87.94

0.00000	0.00000	-90.00	0.00000	0.00000	-90.00	0.00000	0.00000	-90.00
0.00000	0.00000	-90.00	.04014	.02446	78.64	.04744	.02841	88.16
.02452	.01704	77.33	.03139	.02360	90.94	.03468	.03119	98.22
.03834	.03229	72.45	.03616	.03317	-63.47	.02310	.02769	51.74
0.00000	0.00000	-90.00						

ARGAND PLOT 1540

.02486	.35062	0.00000	0.00000	.07013	-.10673	.02615	-.24486
0.00000	0.00000	.11157	-.03969	0.00000	0.00000	.07755	-.04309
.12137	-.12770	-.19945	-.01483	-.11809	.09058	-.09403	.00998
0.00000	0.00000	0.00000	0.00000	.07495	.32461	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	.09347	.08530
-.02209	.09701	.02524	.06468	-.04418	.14459	.03190	-.08597
-.17237	-.22257	.02439	.12058	.10625	-.01866	0.00000	0.00000

* ERRORS 1540

.05261	.02329	85.42	0.00000	0.00000	-90.00	.02687	.03108	118.21
.03136	.01946	100.63	0.00000	0.00000	-90.00	.01893	.02331	125.59
0.00000	0.00000	-90.00	.03164	.02838	-122.39	.03112	.02345	-132.04
.01523	.02683	-83.93	.02922	.01924	-132.69	.01714	.02399	-97.42
0.00000	0.00000	-90.00	0.00000	0.00000	-90.00	.03372	.01634	84.48
0.00000	0.00000	-90.00	0.00000	0.00000	-90.00	0.00000	0.00000	-90.00
0.00000	0.00000	-90.00	.03297	.02405	74.31	.04219	.02804	81.91
.02081	.01698	73.22	.02987	.02253	96.29	.03207	.02726	94.43
.03751	.03104	75.12	.03383	.03882	-76.40	.03044	.02343	-50.37
0.00000	0.00000	-90.00						

ARGAND PLOT 1650

-.15112	.21659	-.00709	.06468	-.00853	-.05156	.22910	.02816
-.10738	-.15333	.19364	.34374	.04178	.04061	-.08158	.04038
-.04670	.40219	-.15457	-.14529	-.20691	-.25167	-.06798	-.06597
0.00000	0.00000	0.00000	0.00000	-.07429	.24593	-.12059	-.19120
-.19711	.00952	0.00000	0.00000	0.00000	0.00000	-.07625	.04532
-.15742	.00088	.05010	.02362	-.16056	-.13129	-.10297	-.04868
.02947	-.30647	.15345	.10670	.09046	.04056	-.21566	-.19230

* ERRORS 1650

.03762	.03953	64.88	.03602	.04721	51.11	.02929	.03115	63.52
.02465	.03360	-93.57	.01760	.03147	45.43	.02108	.02025	-66.77
.02573	.01943	-58.43	.03089	.03984	55.20	.02400	.03245	61.33
.02482	.02708	-70.44	.01438	.02110	-73.57	.02611	.02182	-61.67
0.00000	0.00000	-90.00	0.00000	0.00000	-90.00	.03289	.02245	86.20
.02073	.02153	-69.08	.03097	.02172	-132.52	0.00000	0.00000	-90.00
0.00000	0.00000	-90.00	.04430	.03104	79.65	.03560	.04080	-90.46
.02400	.02360	90.51	.03382	.03653	-72.73	.03498	.03931	-100.20
.03882	.03774	-56.42	.03922	.05171	-100.17	.02964	.03288	-70.58
.01709	.01067	99.07						

ARGAND PLOT 1690

-.08489	.18307	-.02238	.16665	-.09056	-.12180	.16570	.05700
.16888	-.22537	.00658	.44363	.01655	.09718	.12809	.04231
-.14058	.35522	-.11230	-.18380	-.05388	-.35115	-.02345	-.08539
0.00000	0.00000	0.00000	0.00000	-.08668	.19216	.09863	-.27591
-.18941	.00531	0.00000	0.00000	0.00000	0.00000	-.11464	.02235
-.30850	-.14934	.06758	.08888	-.03325	-.15387	-.17284	-.14926
.05215	-.14788	-.03819	.22711	.17779	.16379	.03454	-.29009

* ERRORS 1690

.04333	.04554	65.47	.06348	.03851	84.92	.04198	.02980	89.10
.03285	.03054	70.46	.02781	.02566	109.44	.02132	.02638	63.33
.02515	.02477	67.90	.04180	.03940	90.73	.06043	.02999	86.29
.02929	.03942	51.63	.01780	.03358	57.09	.03051	.03390	64.36
0.00000	0.00000	-90.00	0.00000	0.00000	-90.00	.03409	.02342	92.27
.02784	.01842	87.13	.02998	.03214	121.79	0.00000	0.00000	-90.00
0.00000	0.00000	-90.00	.04816	.03771	104.58	.03510	.04067	-76.30
.02892	.02686	95.87	.05098	.04014	94.95	.03165	.04251	-78.06
.04311	.05643	55.52	.04466	.04890	62.30	.03405	.02779	-47.71
.01663	.02016	-103.11						

ARGAND PLOT 1730

-.00005	.24160	-.10063	.14288	-.07007	-.15862	.10669	.06252
.24543	-.15595	-.20575	.22366	.00968	.09860	.00900	.01740
-.10415	.22108	-.05136	-.18941	-.13573	-.30813	-.09718	-.08572
-.08468	-.11672	-.08538	-.07882	-.01971	.03540	.19903	-.15386
-.18280	-.30104	.16180	-.00817	0.00000	0.00000	-.07676	-.18522
-.15746	-.17313	.08936	.21162	-.04929	-.06711	-.08826	-.18192
-.04747	-.15861	-.11704	.18617	.14296	.17743	.17824	-.20596

* ERRORS 1730

.05879	.04580	90.79	.06050	.04886	-130.74	.03556	.03420	109.52
.03321	.03552	116.91	.02957	.03123	-108.59	.02651	.03210	118.56
.02992	.02834	109.80	.04025	.03407	101.98	.03688	.03509	109.93
.02535	.02739	64.55	.01801	.01871	66.86	.02577	.02457	68.93
.02196	.02064	69.96	.01030	.02239	49.98	.03254	.03323	-109.22
.02682	.02403	-117.95	.02100	.03058	-90.95	.01661	.02005	-105.72
0.00000	0.00000	-90.00	.05062	.05116	-68.42	.04947	.04439	-45.00
.03113	.03444	-79.42	.03242	.03836	-87.82	.03678	.03615	-62.27
.05033	.05865	121.84	.06375	.05051	-131.61	.03584	.03276	76.81
.02429	.01840	-54.80						

ARGAND PLOT 1770

-.13279	.22967	-.12947	.07669	-.06249	-.18260	.09483	.07932
.26894	-.04480	-.13477	.12642	.02676	.12913	-.06187	.01171
-.17261	.17188	-.04994	-.11401	-.14915	-.28006	-.12817	-.05042
-.05580	-.10307	-.08156	-.14962	.08547	.00541	.16251	-.15162
-.21052	.04866	.23464	.05088	.03382	.00823	-.11202	-.25698
-.13480	-.13766	.12878	.30710	-.05154	-.14381	-.04698	-.10894
-.07287	-.29152	-.22007	.14768	.13426	.23229	.15460	-.12015

* ERRORS 1770

.05820	.04520	89.75	.05134	.05893	131.34	.03581	.03604	66.60
.03883	.03734	72.20	.03087	.02735	-117.02	.03086	.03120	66.73
.03152	.03495	57.30	.04074	.04289	57.45	.04141	.03898	106.41
.02675	.03336	-83.56	.02922	.01755	-58.70	.02762	.02870	-68.98
.03013	.02119	-50.42	.02479	.01664	-57.60	.03560	.03680	-90.86
.02744	.02725	112.10	.03519	.02553	-127.32	.01773	.01879	-110.99
.01669	.02262	-100.88	.05736	.04414	-58.13	.04971	.05058	-68.60
.03299	.03456	-69.93	.03737	.04615	-64.24	.04460	.04620	-110.25
.04595	.05800	49.96	.04351	.05994	125.81	.03381	.03379	-67.44
.02588	.02788	-83.04						

ARGAND PLOT 1810

-.14405	.24915	-.19923	.04710	-.01080	-.16473	.01701	.16797
.21959	.03616	-.15010	-.02129	-.07268	.05478	-.04059	-.06066
-.19339	.04540	.08214	-.07853	.06095	-.28103	-.07940	-.09003
-.05228	-.12356	-.04992	-.21853	.17451	.03540	.15847	-.09828
-.17627	-.09590	.14920	.23123	.08187	-.03491	.14802	-.22006
-.07111	-.06560	-.12926	.38061	.10512	-.16424	-.07230	-.08073
.11502	-.32300	-.26338	-.00697	-.00048	.25396	.24548	-.05756

* ERRORS 1810

.07238	.05040	89.38	.06132	.06101	-112.80	.04475	.04142	78.61
.04281	.04475	50.50	.03391	.03573	-107.94	.03408	.03462	66.18
.03747	.03748	112.62	.04180	.03800	90.48	.04285	.03429	-128.86
.02780	.02976	61.40	.01524	.01185	71.48	.02525	.03544	48.84
.02453	.02886	57.72	.01659	.02848	52.72	.04059	.03793	-129.05
.02862	.03127	116.91	.02468	.03479	-85.43	.02349	.01944	-62.14
.01657	.01887	-99.68	.07404	.04447	93.88	.05867	.06243	62.50
.03887	.03608	70.42	.04135	.03600	73.79	.04698	.04508	-64.98
.06280	.05305	75.51	.06033	.05756	-114.49	.03918	.04058	65.27
.03126	.02698	-45.00						

ARGAND PLOT 1850

-.24323	.06512	-.14718	-.07105	.10781	-.22141	-.00928	.12350
.18669	.09951	.02404	-.03045	-.00089	.03689	-.00211	-.00440
-.22505	-.08427	.09530	.01300	-.01529	-.26941	-.05228	-.11872
.03172	-.12899	.13639	-.19045	.06212	.04797	.10849	-.10016

-.19086	-.13597	.10135	.26873	.15022	-.05875	.14696	-.20819
-.22708	-.09997	-.33642	.28158	-.02647	-.10855	-.09773	-.10520
.27436	-.14530	-.30019	-.06798	-.14822	.26960	.20105	-.01966

ERRORS 1850

.05238	.04141	91.00	.05009	.04637	-128.11	.03263	.03267	67.38
.03597	.03417	-53.17	.02893	.02617	-124.19	.03054	.02382	-55.67
.02965	.02710	83.01	.03125	.02973	-131.33	.02027	.03328	127.73
.02363	.02201	-122.91	.01110	.01792	-76.27	.02259	.02150	-64.15
.01998	.02115	63.71	.01272	.02328	47.76	.02848	.02722	70.51
.02435	.01924	77.79	.02460	.01917	-123.04	.01454	.02067	-98.13
.00758	.00896	65.02	.02610	.03705	-35.02	.03317	.03582	-106.27
.04638	.04961	61.31	.03984	.05645	133.26	.03474	.03057	-56.60
.02660	.03000	-90.50	0.00000	0.00000	-90.00	0.00000	0.00000	-90.00
0.00000	0.00000	-90.00						

ARGAND PLOT 1890

-.22483	.08196	-.06234	.03891	.12695	-.19540	-.10763	.09746
.17604	.12230	.13430	.07089	-.07400	-.10490	-.05755	.02460
-.15670	-.16618	.10551	.07628	.08002	-.17201	.02126	-.13846
.06685	-.14422	.24964	-.09974	.01354	.04328	.13587	-.13181
-.09544	-.18383	-.01909	.28136	.20771	.02933	.28057	-.14656
-.17718	-.16106	-.45383	.15683	.02868	-.14265	-.02097	-.10337
.31805	-.05166	-.23092	-.20628	-.27261	.20518	.22289	.07680

ERRORS 1890

.04688	.04396	72.29	.04530	.04621	-73.12	.03171	.03068	71.27
.03418	.03281	86.73	.02699	.02495	-129.71	.02728	.02607	70.83
.02886	.02668	-49.97	.02410	.02448	65.30	.02368	.02247	-114.56
.01833	.01716	-119.07	.01817	.01697	70.36	.01775	.01820	65.89
.01919	.01440	88.77	.01900	.01020	84.30	.03288	.02342	87.39
.02626	.01665	85.17	.01508	.02061	-83.94	.01460	.01274	-62.72
.00848	.01191	126.64	.04871	.02844	93.98	.04252	.03919	98.17
.03656	.01398	91.86	.02628	.02229	-61.23	.02803	.02645	-62.66
.04827	.03947	99.25	.03947	.04044	-111.49	.02822	.02924	65.68
.02443	.02395	-58.88						

ARGAND PLOT 1930

-.22196	.01927	-.04955	-.04820	.17838	-.18371	-.10155	.09634
.12472	.11966	.08999	.01337	-.06748	-.11710	-.11309	.08480
-.06292	-.19639	.02464	.13467	.05353	-.09448	.04054	-.14927
.10877	-.14384	.26057	-.06488	-.01595	-.07390	.16804	-.04962
-.06355	-.22334	-.03399	.29876	.20080	.11035	.33869	-.07035
-.22304	-.09145	-.46052	.10532	-.21709	-.16745	-.04471	-.05594
.29598	.15228	-.25937	-.20122	-.23408	.14338	.21446	.06771

* ERRORS 1930

.03934	.04567	60.86	.04167	.04462	-80.18	.02593	.03179	53.85
.03010	.03301	48.81	.02349	.02486	127.64	.02264	.02738	59.78
.02528	.02457	-62.52	.03061	.04121	45.55	.03205	.03727	-98.24
.02538	.02404	-62.73	.02429	.02304	69.07	.02439	.02957	51.10
.02026	.02300	61.69	.02152	.01457	77.98	.03114	.02322	87.92
.02754	.01560	88.60	.02081	.02803	-98.11	.01426	.01922	-78.85
.01739	.01227	-131.69	.04540	.02743	87.02	.04075	.03578	106.25
.02918	.01299	83.27	.03675	.02789	-122.10	.03561	.04231	-97.01
.03703	.04260	120.14	.04221	.03568	-132.40	.02816	.02911	64.18
.02143	.02253	118.78						

ARGAND PLOT 1970

-.21661	-.11124	.04326	-.04658	.14334	-.09130	-.12584	.09006
.11377	.08405	.11645	-.00765	-.09652	-.10779	-.11538	.06531
-.03288	-.19034	.02910	.14647	.02030	-.07872	.04677	-.11954
.13066	-.10624	.22412	-.00474	-.03911	-.16026	.15576	-.06952
-.02147	-.20708	-.07883	.26012	.18138	.13496	.40659	.03748
-.22656	-.09667	-.46791	.04626	-.19741	-.17905	-.00442	-.01767
.18003	.25602	-.12859	-.18609	-.27740	.06450	.20785	.09370

* ERRORS 1970

.04030	.04409	60.93	.04265	.04389	64.36	.02687	.02957	58.78

.02908	.03090	58.43	.02518	.02171	96.69	.03088	.02310	-51.09
.02813	.02363	93.49	.02704	.02755	-68.98	.03139	.03116	109.99
.01886	.02023	-105.73	.01597	.02261	51.92	.02229	.01966	-53.96
.01509	.01990	48.50	.01208	.01742	56.80	.02651	.02460	100.43
.01713	.02539	54.91	.01927	.01695	-116.88	.01114	.01650	-100.28
.01195	.01000	106.92	.02979	.03062	66.76	.04338	.03353	79.75
.01184	.02013	58.26	.03440	.02617	-126.36	.03138	.02758	-120.73
.03746	.03846	114.85	.04290	.03956	102.71	.02416	.02866	51.84
.02258	.02021	87.36						

II. SOLUTIONS AT FOUR ENERGIES

On the following pages are reproductions of RUMBLE printout for four representative energies. These are single-energy fits, whose overall phase is undetermined, so the phase of $\Delta\pi(PP_{11})$ is arbitrarily set to zero.

The printout contains:

1st: Amplitudes and error, annotated.*

2nd: An error matrix dimensioned equal to the number of waves used in the fit.

3rd: A 60×60 error matrix. However, this involves repeating the amplitudes and error table in order to specify the numbers of waves and to give the the diagonal elements. Further, because of the size of the 60×60 matrix, the format is different. See annotated example on page 2.4.

*An explanation of the columns headed $\frac{2}{3}$ SIGMA INEL and $\frac{4}{3}$ SIGMA INEL:

These are merely sums of the σ (charge channel) at right. The first three of these channels are neutral ($n\pi^0\pi^0$, $n\pi^-\pi^+$, $p\pi^-\pi^0$) and must come from π^-p. The last two ($p\pi^+\pi^0$, $n\pi^+\pi^+$) came from π^+p. So the output gives fit predictions for:

I	3 neutral modes	2 charge^{++} modes
1/2	$\pi^-p \rightarrow N^*_{1/2} \rightarrow N\pi\pi,\ \frac{2}{3}\ \sigma_{\frac{1}{2}}$	0
3/2	$\pi^+p \rightarrow N^*_{3/2} \rightarrow N\pi\pi,\ \frac{1}{3}\ \sigma_{3/2}$	$\pi^+p \rightarrow \Delta \rightarrow N\pi\pi,\ \sigma_{3/2}$

The leftmost I = 1/2 column being a sum of $2/3\ \sigma_{\frac{1}{2}}$ for three modes is then labelled $\frac{2}{3}\ \sigma_{\frac{1}{2}}$.

The leftmost I = 3/2 column similarly must be labelled $\frac{4}{3}\ \sigma_{3/2}$. We are sorry if the notation on the summed columns is confusing, but it arises because they are just sums of the more physical cross sections for the individual charge channels.

Radial and Azimuthal Components of Error in T **2.2**

SOLN B 1490 MeV

	WAVE	RE A (Fitting Param. A)	E(RE) ($\delta(A)$)	D(RE) ($\frac{1}{\sqrt{D_{ii}}}$ *)	IM A ($Im(A)$)	E(IM) (δ)	D(IM) ($\frac{1}{\sqrt{\ }}$ *)	PHASE (Degrees)	E	RE T	IM T	E(AMP) (T)	E(ANG)	T**2	E(T**2)	WAVE
Δπ	P33 PP11	1.00	.03	.02	0.00	.09	.04	0	4	.3336	0.0000	.0101	.0209	.111304	.006844	1
	P33 DS13	-.09	.01	.30	.02	.01	.00	168	6	-.1363	.0296	.0104	.0137	.019450	.003002	2
	P33 DD13	-1.98	.09	.37	1.00	.11	.10	153	4	-.1554	.0787	.0081	.0126	.030323	.002904	3
	P33 DD15	.29	.10	.39	-.60	.12	.11	-64	10	.0231	-.0476	.0087	.0091	.002799	.001002	4
	P33 PP31	-.20	.04	.33	-.32	.03	.02	-122	5	-.0636	-.1012	.0088	.0111	.014292	.002194	5
	P33 SD31	-.48	.11	.39	-1.02	.10	.08	-115	6	-.0383	-.0811	.0083	.0089	.008052	.001552	6
	P33 DS33	-.02	.01	.30	.11	.00	.00	98	4	-.0226	.1573	.0041	.0097	.025257	.001309	7
	P33 DD33	-.07	.11	.38	.76	.05	.05	95	8	-.0056	.0594	.0045	.0085	.003561	.000563	8
ρN	RH3 DS13	1.81	.08	.24	-1.71	.08	.04	-43	2	.2127	-.2008	.0063	.0118	.085536	.003700	9
	RH1 SS11	.69	.13	.37	-.72	.15	.08	-46	11	.0810	-.0843	.0086	.0215	.013665	.002084	10
	RH1 PP11	1.41	.52	.36	-.84	.58	.38	-31	23	.0403	-.0241	.0115	.0192	.002209	.001210	11
	RH1 SS31	.76	.07	.25	.26	.08	.06	19	6	.0893	.0307	.0073	.0096	.008924	.001438	12
	RH1 PP31	-2.30	.32	.22	-1.92	.34	.27	-148	7	-.0665	-.0410	.0088	.0095	.006100	.001456	13
εN	SIG PS11	-1.95	.08	.16	.71	.13	.07	155	3	-.2740	.1286	.0115	.0184	.091612	.007107	14
	SIG SP11	2.81	.52	.36	-1.37	.37	.31	-26	7	.0984	-.0481	.0171	.0142	.011990	.004026	15
	SIG DP13	-.71	.28	.21	-5.20	.30	.16	-98	3	-.0251	-.1827	.0110	.0094	.034001	.004165	16

*See appendix 1. of Herndon's Thesis LBL 544, reproduced here as pp 2.17 & 2.18, 2.19, 2.20.

Seldom Used!

L,J	SIGMA-TOTAL (mb)		CHANNEL- 1 $n\pi^0\pi^0$ (Predictions)	2 $n\pi^-\pi^+$	3 $p\pi^-\pi^0$	4 $p\pi^+\pi^-$	5 $n\pi^+\pi^+$ (Prediction)
P1	4.849	Sum of Channels at right	1.278	2.642	.424	.445	.061
D3	8.844		1.062	3.287	3.055	1.221	.219
D5	.145		.034	.080	.031	0.000	0.000
S1	1.145		.074	.253	.263	.429	.027
TOTAL	14.083		2.447	6.263	3.773	2.095	.307

INELASTIC $I=\frac{1}{2}$

	L,I,J	ETASQUARE (Pred. from the Nππ fit)	$\frac{2}{3}$ SIGMA-~~TOTAL~~ (mb) (← sum of charge channels)			CHISQ	$\frac{2}{3}\sigma(n\pi^0\pi^0)$ 1 $n\pi^0\pi^0$	$\frac{2}{3}\sigma(+)$ 2 $n\pi^+\pi^-$	$\frac{2}{3}\sigma(+)$ 3 $p\pi^-\pi^0$	4 $p\pi^+\pi^0$	5 $n\pi^+\pi^+$
$I=\frac{1}{2}$	P11	.030609	4.178	+-	.106	5.7	1.207	2.586	.385	0.000	0.000
	D13	.196989 (See Text p. 2.1)	6.922	+-	.128	.1	1.077	3.365	2.480	0.000	0.000
	D15	.988805	.145	+-	.048	.4	.034	.080	.031	0.000	0.000
	S11	.897377	.442	+-	.068	.1	.070	.215	.157	0.000	0.000
	TOTAL THIS ISPIN (1/2) →		11.698				2.388	6.246	3.054	0.000	0.000
			$\frac{4}{3}\sigma$(inelastic) $I=\frac{3}{2}$				$\frac{1}{3}\sigma_{3/2}$ ($\pi^-p\to n\pi^0\pi^0$)	$\frac{1}{3}\sigma_{3/2}$	$\frac{1}{3}\sigma_{3/2}$	$\sigma_{3/2}$	$\sigma_{3/2}$
$I=\frac{3}{2}$	P31	.922349	.659	+-	.072	.0	.007	.096	.061	.445	.061
	S31	.930053	.613	+-	.062	.9	.003	.093	.052	.429	.027
	D33	.888325	1.955	+-	.089	.0	.024	.274	.186	1.221	.219
	TOTAL THIS ISPIN (3/2) →		3.198				.034	.463	.300	2.095	.307

Zero for $I=\frac{1}{2}$ (channels 4 and 5).

This is $\sigma(\pi^-p\to n\pi^0\pi^0)$ i.e. $\frac{1}{3}\sigma\ N^*_{3/2}\to n\pi^0\pi^0$)

These are fitted predictions for actual σ(mb) for $\pi^-p\to n\pi^0\pi^0$, $n\pi^+\pi^-$, $p\pi^-\pi^0$ or $\pi^+p\to p\pi^+\pi^0$, $n\pi^+\pi^+$

16x16

ERROR MATRIX Λ (AMP(I) AMP(J) PHA(I) ~~PHA~~ AMP(J) / AMP(I) PHA(J) PHA(I) PHA(J)). RAD = .006186 HYPERVOLUME = 1.6531E-67

I → (columns), J ↓ (rows)

J \ I	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1	A0	A2	0.	1A	A0	.	01	1	0.	00	0.	..	F2	21	20
	1	05	A4	10	0	00	05	.1	A2	0	3C	1	00	06	31	2A
2	A0	.0	2C	0A	A0	B	0B	0B	00	.A	A1	00	.0	0C	CB	F4
	05	0	C5	01	.	.0	01	1	E2	A1	0C	0	.0	05	24	3D
3	AA	2C	.0	AA	A	.A	0B	.C	0B	.A	.1	AA	0	0B	AC	B4
	24	C5	0	2.	0	01	1	A1	A3	01	D	01	00	04	32	3C
4	01	00	A2	.8	0	00	00	.	A0	A0	0B	0	.	1	.0	00
	.0	A1	A.	8	00	00	0	.0	01	01		.0	0	00	.0	1A
5	10	A	0	0	.B	B1	A.	A1	.0		0A	0A	04	.A	0.	0A
	A	0.	A	00	B	00	03	.A	21	0A	11	01	B2	A0	0B	AA
6	A0	.	.0	00	B0	.A	0A	2B	.1	.1	.0	B0	0A	0.	00	0
	00	B0	A1	00	10	A	A3	A3	11	1	.0	13	11	A0	.0	0A
7	0	00	0	00	A0	CA	.	A0	00	0B	0	.A	B1	00	0A	A.
	02	B1	B1	0	03	A3	.	.4	32	2B	10	04	1	B1	B	AA
8	0.	0	.A	.	A.	2A	A.	.	0	01	0.	B.	0A	0.	A.	01
	11	B1	C1	.0	1A	B3	04	.	10	2.	00	22	A	A1	0.	0A
9	A	0E	0A	A0	.2	.1	03	01	.	2B	.1	A2	11	AB	0C	B2
	12	02	B3	01	01	11	02	0	.	03	A0	A1	0A	01	10	2B
10	0	.A	.0	A0	0	.1	02	02	20	.B	1	A3	0.	0.	AB	A2
	.0	A1	A1	01	A	1	BA	1.	B3	B	CA	0.	0B	2A	.3	4B
11	03	A0	.	0	01	..	1	00	.A	1C	.1	01	00	02	1A	A0
	0C	1C	1D	8	A1	00	07	.0	10	A	1	0	00	1D	BD	C0
12	0	0C	A0	.	00	B1	.0	B2	AA	A0	00	.0	B1	D0	00	A
	.1	0	A1	00	A1	03	A4	02	21	3.	10	0	1B	A0	0B	01
13	.0	..	00		0B	01	B	0A	10	00	00	B1	.	0.	.0	00
	.0	00	0	.0	42	A1	11	A	1A	.8	00	1B	.	.B	0A	AC
14	F0	00	00	0	.A	0A	00	0A	A0	02	01	0A	0.	.B	C.	A.
	26	C5	B4	10	A0	.0	01	.1	B1	.A	2D	00	.B	B	41	2.
15	23	C2	A3	..	00	0.	0	A0	01	A.	1B	00	.0	C4	.2	1B
	11	B4	C2	00	.B	00	AB	..	C0	B3	AD	0B	0A	.1	2	5E
16	22	F3	B3	01	0A	00	A4	00	B2	A4	AC	0	0A	A2	15	.C
	0A	4D	4C	0A	AA	A	.A	1A	2B	2B	00	A1	0C	..	BE	C

RUMBLE NOTES: Key for Error Matrix.

SYMBOL	RANGE		SYMBOL	RANGE
(BLANK)	-.01 TO .01			
.	.01 TO .03	Small	,	-.01 TO -.03
0	.03 TO .05		O	-.03 TO -.05
1	.05 TO .15		A	-.05 TO -.15
2	.15 TO .25		B	-.15 TO -.25
3	.25 TO .35		C	-.25 TO -.35
4	.35 TO .45	Large	D	-.35 TO -.45
5	.45 TO .55		E	-.45 TO -.55
6	.55 TO .65		F	-.55 TO -.65
7	.65 TO .75		G	-.65 TO -.75
8	.75 TO .85		H	-.75 TO -.85
9	.85 TO .95		I	-.85 TO -.95
+	.95 TO 1.05		J	-.95 TO -1.05
*	1.05 TO 5.00		X	-1.05 TO -5.00
S	5.00 AND ABOVE		Y	-5.00 AND BELOW

Whole Matrix is symmetric. Each box is not. Each box reads

$$\begin{array}{c|c} \dfrac{\langle \delta T_i\, \delta T_j \rangle}{|\delta T_i||\delta T_j|} & \dfrac{\langle \delta\varphi_i\, \delta T_j \rangle}{|\delta\varphi_i||\delta T_j|} \\ \hline \dfrac{\langle \delta T_i\, \delta\varphi_j \rangle}{|\delta T_i||\delta\varphi_j|} & \dfrac{\langle \delta\varphi_i\, \delta\varphi_j \rangle}{|\delta\varphi_i||\delta\varphi_j|} \end{array}$$

where $T(\text{complex}) \equiv T e^{i\varphi}$.

Disregard normalized diagonal, read δT, $\delta\varphi$ above.

2.3

This page same format as annoted 16-wave format on page 2.3 except that here all 60 waves are defined.

SOLN B 1490 MeV *repeated with 60-waves Defined,*

WAVE	RE A	E(RE)	D(RB)	IM A	E(IM)	D(IB)	PHASE	E	RE T	IM T	E(AMP)	E(ANG)	T**2	E(T**2)	WAVE
P33 PP11	1.C0	.04	.02	0.00	.11	.04	0	6	.3355	0.0000	.0128	.0373	.112580	.008736	1
P33 SD11	.CC	.24	.12	0.00	.31	.15	0	999	.0000	0.0000	.0184	.0241	.000000	.000339	2
P33 DS13	-.10	.01	.00	.01	.01	.00	175	8	-.1421	.0117	.0134	.0207	.020336	.004010	3
P33 PP13	.C0	.C7	.03	C.00	.09	.03	0	999	.0000	0.0000	.0220	.0275	.000000	.000486	4
P33 DD13	-2.06	.15	.07	.68	.25	.11	162	6	-.1622	.0532	.0126	.0190	.029140	.004474	5
P33 PF13	.C0	.57	.33	0.00	.65	.44	0	999	.0000	0.0000	.0113	.0128	.000000	.000128	6
P33 FP15	.C0	.03	.01	0.00	.04	.02	0	999	.0000	0.0000	.0114	.0146	.000000	.000131	7
P33 DD15	.40	.17	.C9	-.49	.19	.12	-51	19	.0314	-.0385	.0122	.0161	.002472	.001368	8
P33 FF15	.C0	.51	.28	0.00	.76	.42	0	999	.0000	0.0000	.0100	.0149	.000000	.000101	9
P33 FF17	.00	.42	.30	0.00	.62	.45	0	999	.0000	0.0000	.0083	.0123	.000000	.000069	10
P33 PP31	-.17	.05	.03	-.36	.03	.02	-116	7	-.0551	-.1152	.0108	.0165	.016314	.002863	11
P33 SD31	-.28	.17	.C9	-1.07	.15	.08	-105	9	-.0224	-.0858	.0121	.0136	.007864	.002297	12
P33 DS33	-.C3	.01	.00	.10	.01	.00	105	6	-.0412	.1521	.0105	.0164	.024834	.003427	13
P33 PP33	.00	.05	.02	0.00	.05	.02	0	999	.0000	0.0000	.0185	.0167	.000000	.000344	14
P33 DD33	-.18	.17	.08	.73	.11	.05	104	13	-.0141	.0574	.0081	.0134	.003493	.001017	15
P33 PF33	.00	.37	.27	0.00	.35	.23	0	999	.0000	0.0000	.0074	.0070	.000000	.000055	16
P33 FP35	.00	.03	.02	0.00	.02	.01	0	999	.0000	0.0000	.0079	.0063	.000000	.000062	17
P33 DD35	.C0	.14	.C8	0.00	.11	.06	0	999	.0000	0.0000	.0108	.0086	.000000	.000117	18
P33 FF35	.C0	.48	.25	0.00	.33	.18	0	999	.0000	0.0000	.0097	.0066	.000000	.000094	19
P33 FF37	.00	.39	.27	0.00	.33	.19	0	999	.0000	0.0000	.0079	.0066	.000000	.000063	20
RH3 PP11	.00	.88	.27	0.00	.98	.36	0	999	.0000	0.0000	.0255	.0282	.000000	.000649	21
RH3 SD11	.00	2.20	1.14	C.00	3.09	1.44	0	999	.0000	0.0000	.0179	.0251	.000000	.000322	22
RH3 DS13	2.C6	.12	.04	-1.35	.15	.04	-33	4	.2432	-.1590	.0095	.0208	.084419	.005582	23
RH3 PP13	.00	1.11	.31	0.00	1.12	.35	0	999	.0000	0.0000	.0318	.0322	.000000	.001013	24
RH3 DD13	.00	2.66	.76	0.00	2.73	.83	0	999	.0000	0.0000	.0215	.0222	.000000	.000464	25
RH3 PF13	.C0	4.31	2.59	0.00	5.69	3.10	0	999	.0000	0.0000	.0106	.0140	.000000	.000113	26
RH3 FP15	.C0	.38	.15	0.00	.50	.17	0	999	.0000	0.0000	.0111	.0143	.000000	.000123	27
RH3 DD15	.00	2.60	.84	0.00	2.85	.96	0	999	.0000	0.0000	.0211	.0231	.000000	.000444	28
RH3 FF15	.C0	7.42	2.21	0.00	8.03	2.34	0	999	.0000	0.0000	.0183	.0198	.000000	.000335	29
RH3 FF17	.00	5.72	2.34	0.00	5.17	2.61	0	999	.0000	0.0000	.0141	.0127	.000000	.000199	30
RH3 PP31	.CC	.68	.23	0.00	.68	.21	0	999	.0000	0.0000	.0197	.0197	.000000	.000389	31
RH3 SD31	.00	1.71	.79	0.00	1.50	.78	0	999	.0000	0.0000	.0141	.0124	.000000	.000200	32
RH3 DS33	.00	.10	.03	0.00	.11	.04	0	999	.0000	0.0000	.0120	.0128	.000000	.000143	33
RH3 PP33	.00	.85	.21	0.00	.71	.21	0	999	.0000	0.0000	.0246	.0206	.000000	.000607	34
RH3 DD33	.00	1.80	.53	0.00	1.48	.50	0	999	.0000	0.0000	.0148	.0122	.000000	.000221	35
RH3 PF33	.00	3.28	1.91	0.00	3.23	1.67	0	999	.0000	0.0000	.0083	.0081	.000000	.000068	36
RH3 FP35	.00	.23	.12	0.00	.26	.11	0	999	.0000	0.0000	.0067	.0075	.000000	.000045	37
RH3 DD35	.00	2.10	.57	0.00	1.53	.63	0	999	.0000	0.0000	.0173	.0126	.000000	.000298	38
RH3 FF35	.00	4.84	1.54	0.00	4.50	1.50	0	999	.0000	0.0000	.0122	.0114	.000000	.000149	39
RH3 FF37	.00	3.02	1.57	0.00	4.10	1.76	0	999	.0000	0.0000	.0076	.0104	.000000	.000058	40
RH1 SS11	.95	.21	.C7	-.29	.28	.09	-17	18	.1125	-.0341	.0171	.0372	.013813	.004300	41
RH1 PP11	.CC	1.04	.34	0.00	1.17	.40	0	999	.0000	0.0000	.0299	.0337	.000000	.000897	42
RH1 PP13	.C0	.68	.21	0.00	.81	.27	0	999	.0000	0.0000	.0196	.0233	.000000	.000384	43
RH1 DD13	.00	2.63	.82	0.00	3.26	.96	0	999	.0000	0.0000	.0214	.0265	.000000	.000458	44
RH1 DD15	.00	1.69	.62	0.00	1.88	.77	0	999	.0000	0.0000	.0137	.0153	.000000	.000189	45
RH1 FF15	.C0	6.15	2.17	0.00	7.09	2.54	0	999	.0000	0.0000	.0152	.0175	.000000	.000230	46
RH1 FF17	.C0	4.03	1.81	0.00	3.79	2.20	0	999	.0000	0.0000	.0100	.0094	.000000	.000099	47
RH1 SS31	.75	.13	.05	.35	.17	.06	25	12	.0886	.0412	.0151	.0200	.009543	.003176	48
RH1 PP31	-1.87	.79	.22	-1.83	.86	.26	-136	17	-.0544	-.0531	.0258	.0223	.005784	.004591	49
RH1 PP33	.00	.48	.15	0.00	.50	.18	0	999	.0000	0.0000	.0140	.0145	.000000	.000196	50
RH1 DD33	.C0	1.88	.55	0.00	1.95	.54	0	999	.0000	0.0000	.0155	.0161	.000000	.000240	51
RH1 DD35	.C0	1.36	.44	0.00	1.13	.52	0	999	.0000	0.0000	.0112	.0094	.000000	.000126	52
RH1 FF35	.C0	4.03	1.55	0.00	4.73	1.53	0	999	.0000	0.0000	.0102	.0119	.000000	.000103	53
RH1 FF37	.00	2.39	1.26	0.00	3.18	1.43	0	999	.0000	0.0000	.0060	.0080	.000000	.000037	54
SIG PS11	-1.96	.12	.06	.81	.22	.07	158	6	-.2776	.1142	.0171	.0316	.090104	.010571	55
SIG SP11	2.19	.79	.36	-.44	.58	.31	-9	12	.0984	-.0156	.0279	.0206	.009917	.006332	56
SIG DP13	-.25	.42	.21	-5.50	.44	.15	-93	4	-.0090	-.1942	.0157	.0145	.037783	.006345	57
SIG PD13	.00	1.97	.93	0.00	1.46	.70	0	999	.0000	0.0000	.0199	.0147	.000000	.000397	58
SIG FD15	.00	1.70	.72	0.00	1.44	.48	0	999	.0000	0.0000	.0172	.0145	.000000	.000294	59
SIG DF15	.00	3.84	2.51	0.00	2.34	1.77	0	999	.0000	0.0000	.0120	.0073	.000000	.000143	60

60 × 60 Error Matrix, page 1

2.4

ERROR MATRIX (AMP(I) AMP(J)) i.e. $\langle \delta T_i\, \delta T_j \rangle$
(AMP(I) PFA(J)) $\langle \delta T_i\, \delta \varphi_j \rangle$

RAD = .009008 HYPERVOLUME = 3.5748-246

I ---
J
v

SOLN B 1490 MeV

```
    1 2 3 4 5 6 7 8 9101112131415161718192021222324252627282930313233343536373839404142434445464748495051525354555657585960
 1  +1BA003,  01B0AA00A10000  00A000AA  0001000AB0000.1,00  .0F22010
    0,CAA0000000A1000  000  100.A11  0011,1A  .     C0  2  .  A01  .0,130A2,
 2  1+AB4A1A1   .0A  0   ,0,31.0A110A  AA0A  00A10,000  0,  0A.00000B0.A0
    12A1.BA  0.0800A,  00   A0020A0.0BB000.000040CA0A010..0,    00  B0
 3  BA+120A00,  0.00,A1.,A01,2.000011A,10A00  .1.0000000.,  00AF  00
     AC0C0000.C0000000000  10E0A   0,   0.1000  ,,AA010  .  01.,0013.A10
 4  AB1+A0E  A0,,0100000000000301A0A00  00.1  A1110200  ,0,  188010  280
    ,C00,00.00,0.00,  00  11  A0110AAC,00  ,,,00BA0,01A0.00.0  03.AA2A
 5  042A+0AB,  100A00,01,22  1  100A,000A,A,0100  1A1.00AA.0000BB0A0
    22C,BA1010AA0  A,0,0  0AA0   ,0,0AA.0.0  ,0,00A0000,01,,0,031A10
 6  CA00C+1AE000  00A100,AA,00,A  101100,.  0A00.000A.000  01,AB2210
    0AA0B1010100  0,0,10000..0000000000..0.00  0001,000100.,  00  011
 7  31AEA1+,1000.AA10  A,0  0BA00,1,....   0..0CAA0  1  0,0  010C02A4A
    10A  B020  0.  .A0  00..00,0,A0,....000100,0,..  01  0011  0.A,1  00
 8  ,A0  BA,+11,,0,,0,A  .0.00.0000  .0,1,000A000A,B.0010  00.00  .,A
    00A1BA0B0A000.   0A0000,  ,00A000.  0,  000,1001A,0,000,.00  0010
 9   10A,E11+10,,00.A0C  11,0001000AA0A..1A10  0A.,1  1000A00  1AA1
    111.2A0A0A10.,.,00C00AA  0.1A00AA.00,0,.  100A00   A0.,0,AA00A0
10  0  ,0  0011+0.  0..,00B00,  00  1,000  00  0000  000001.  ..00100,1A,
    0,.  ..    ,2  ,,0,0.  ,0.,000   00  .00.  ,00  .     00  00   0,00000  .00
11  1   ,100,00+AAAA,1  1..0000,0,00010000...0A0,    .B   00,0000  0.
    A0ACAAC.20A0.1A0  0B01A2  000.  .,1,  000...0000.0.A.0    ,.02B00,
12  B.C,C0C,,.A+A  40A0,.0A  00  0,0  13  0800  ,,01  0  0  B00   .01.,0..
    0.C0C00,1  CA0A,00B0,1010  0A  0,BB10001A10000.0.,4A00AA0A10  0.
13  00.C0  .0,  AA+020U0A10A0.000A.,,0  G0A  A,00,100..,A1,A0  00,A.0,
    ..CCA0..0,00,01  A.0  1A21  .00000,,1B00  .,,0010A0A01A.0001001,
14  AA01A0A,00A  0+.001,10A,110000  00181 0B0000.000A  0,BA00A00,1
    00AC0  ..  0A.A  0.A  .00  ,0,AC0000013  A1,11  0  0A.02040   1000  .,
15  A  0C00A,0.A42.+0,00,A  000,    0.01A0C,00  .010,.0.0AA.0A000  00.
    00B  BA,  1,  AB,010B001001.0  0   BC300  1A101A01A,.3A  0AA0  010.0
16  C0,00A10...,0000+AUBA0A  00  A0.  11.0  110  ,00000011.A0100..  00
    0000CA0A00.00  10  A0A.  A.00000000,.  A02,,A00   0,.,010  00.01A000
17  0  AC,10,A,1A00,A+B1,,  00000,.000100,.0A,00  ,,A.0A0.0A.00201,
    .010.0,.A0AA2,0  CA20A  ,..0A0,0A  A  ,0B,0000.A.  ,0  00  0,0.A0A
18  A,1CCU  A00  0A100B+.0..0,,.100,000.00A1000  00  0,000,1101  A00
    0  010,,A0,C..00.A100  0  00000,.  0000  0A   ,,00.,00  0,C000  A..0
19  10.C10A  C01,1,0B1.+100  1,  A,0  AA000AA  0,0010,,,AA,0,  ,0,.  A
    00,,1  00.01B,BA,1.0,01..0.0.0  00000000000A0  00  A0100,  00.00.
20  0,,0,,,.  B..01,A,01+,    0  ,0,01..   00A,.00  0,00  000  .,B  .0,.
    .000  .  00000,   0100,0,     .0    .0  1.0000,0,,.0,,  0000103.  00.
21  03AC2A0010.0A0A0,.0,+104013B  1,A10001000  F43B000.,0,,03,0000
    A.2C000000000001,.A000AA1A20B10000,,00B00,12AA1  ,000     020A1A0
22  01002A  .100A0A  A  .0  1+01B,10AAABA,,0,010BB2002A  A.10,  0B0  ,0
    B1,C1.0  ,0C0000,1  0  15.000200,,0A0,  A,00201A000.000    ,2012  ,
23  0.10  ,00,,0  .,0  00   00+03.00001  A0  0.00,1200000A20000  A0C.A.
    .1CCC0     0A10.00C..0180011  ,,0,001A00  ,,20A   A0A01..0000  01
24   0,310B00  0001000,10410+E0,0220.0,000000AE74CB1000  AA00002A0
    00,2.0A0  0,0000A0  1,AB.0100,0,A,00  .0,     23,,00,0000000.20  100
25  0A20  0A.00C001000,,  0B3E+106BB10.0   000035CG52A00  000000A.A
    0110CA0.00C000,.  .0,000000,800  0  00.000000000AA0.0  .000A2B,A
26  01.11,0000,  00,  0.  ,1,.01+01C.  000,B,0100,,A20000A  0..0   C21
    01000000  ,C  0.000.101.0,1400B000.000A    00.1A1A0,0A00.,0  AC23
27  A10A0A001  00A0  A01A0310,00+A11100.0A,0.  .020BA0A  A010  1   A20
    1A10,0  A0A.A00.AC  ,0B001.000.,00,0010,,,A02,00  00  100  B00A11
28  00000  ,001,,.0  0,0,,B00061A+ED..0000,0    ,30F74C,  ,00,,,.  000
    00  0.,0  0    ,,00.000  1111BA0A1000,        ,..0011A0  ,.  .  0  .00000
29  0A0AA1100,00,00..000  A02BC1E+611  1.0,1A00034DG5A00A1  .0.0100
    .A.C0.00     0  .00C.00AA00.B  ,2A0.0,,01    0..010  A0  0    ,  00.1A0
30  0  00,0,  00C  ,  .  0,  11A02B.106+10,0  000011A22CF70000  01,,00.0
      00,  .   00    ,0.0,000AA  A20.0AA0,0  .0    ,,010A01A.0    .0  0.  00,
31  AA1001..A0C1000100A.,A101  1.11+4B5C010A,0000.00C42001,  000  ,
    0AACA0C10    A2.100AA.0,  0.AA..  0AA10210,01  AA.  .15200010,  000
32  AA1  01.0A013  01100A.AB  .000.104+  2A1A2BAA000.00C  0A10001,1.0
    0,001.  000101A00100002AA,000  01  01ABA.  000  A,00BA  11,  0A0A.0
33   0A000.,0  0  G1A.100  1AA0.000  ,B  +A200010AA010.  2CA0.A0103  0.
    0  1.0,0A,.AA0A,00010A1100.00A0,A0,10A,,  110000,3B,000,00A00.
34  0A,0A0.1A0000B000.0  0,0,00.01052A+E  ,A  11  00A,0C4730  0  0.0,0
     .0CA000A0C01C,02A.  ,00,00.,    0.00010A00100,,0  0411000     0  0
35  0  1.,,  ,.00BA1C  00000,  0  ,00.  CA2E+2B6DB  00,00  2BDE43A.  0  1,
    0AA,00,0  001103.,,000,0,000,11,1.0,AA0,00.000A00A0A1.00,1  0,
36  C001A.  0.  C0  0,  ,0A00000  BA000010  2+,1BA0  A  00,.1110,  00A20
    C000000,,,.,0,..1,  00..0010,00A0AA   000  .00,  A  00A0.1000000A
37  10A  ,  0010000AB0I.AAA1,.00,,,,01A0,B,+A200001,103111BA1A00,0.
    0,1C.0A  0  ,    00B,  10A000,10,,00  A000CA10001000.0B,1A,.000  0
38  0A0A00.0A0.  ,00101  ,0000000010020A61A+HE.0000A  A0CG86C..0000
    ,00C.0C.0001.10,A  01  0.00A0.1C00,  00.0000.0,  A0B,0G00...,  ,.
39  01011A.A10.,00  0A00.010001.  A0AB1  DB2H+600  0  002025FF4  ,0,00
    00,  0  ,  ..  .A100....  A  0  00  10001.,,1  00  000,A.100.0A0000  ,,
```

40 00 1000000.,0C. ,0,000,000 01,A01BA0E6\.000 A200130F50 .,
0, ,, 00 000..A000 .,,.11,0. 00 01..0 .A10,. 001 . 0,0

41 A,.100C0 C0,00,0000 B1A30.,010AA1 00.0.+2AA,A0 10. , 1B01A
.101C C0.0C00 0 .A0,0BBA200,0.0A0,.0 ,..5.AA 0.1, 000 .B2,..

42 BC10 .A000A11.100 0 FB2E5,030A00A 0 00002+0020AA 0,0.000A0A0
A0101000. C... 0,,0,1103A.0A .000.,00 .A01000.0 0.,0 BAA0,0

43 0C.210AAA00 0000 0104207C,2D32000000A00 0AD+3DC1A000 .,10, 01
.,02,0 A.B,,0.0A0.10A ,0002A10A0 0.0000..130AA ,00000 0.,A0.

44 0000A00,.0,000,0,00,3004GA0F420010, 1000AD3+FC200 ,00001000 0
0A0AA,1.00C000.0C.0,AABA,00100,00 .00000A0A2000 .00.001 A11

45 0 0010 B,0 .0.0, ,0800C52B7DC..0A00,0 ,2DF+2C0 00 0 ,0,0,0
01..10 ,00 0.0011C1A0 A000,,0 ,,00 0001A0,.0. 0..00100

46 000 .A1.10 0.A00A0,0020B20A4GF00.,001A0AA0CC2+F10010,0000000
0C00C0 ,..,0000 000 1. A0,.011,00001. 0000002000.00,.,. ,0A

47 .,0,C. 0 1. , .0.,, 0A01A00C57C0 0 ,0 020A12CF+00,0000 0000
,00.C. 0,2C ,..0C0.AAA,A1. 0AA0,0 , ,0010A 000, 0, 0 ,..

48 1 0000001.BBA0010JA00 A000A,A0CC2C2.3A20 AA0010+CA.BA1A01.0
0,0,,0A ,AA0A1C0A.0,12000000..CC1,00 0,310A0,.1010. ,AAA,0.

49 ,00,A0,10 01,A1A0A0.A2000 004 C4B110001 00 00C+4100..0A .0
0.A.C0.01,2A0 .010A 1C,0, 0 00CA2C201. 000. 0020B, A0,0,,00

50 0A0 A0000. 0,BA.00,0,.00 AA,0020A7011C21000 00,A4+5BB1A0 00,
0 . 0000.C.0B0B0,11.0.0,12A0000A10A0A1000100000 2.1A,100 A1

51 0..1. 0.0 AA.A.,0 010 0 00A00A03E11G53.,0,010.15+FD200 000
01. 000.0C010B00,10,00.00A.AA0.A0 110000.01.101100A0001A000

52 0,B0000A00 000001,.,00A00101 01.040B8FD 0 0 00B0BF+6C,.000,
001 0.,, 0.2.00 A3,0.0. .0001011 00000 .0 0.00.B.000.A0 000.

53 .0 B011000,. 0A1A1 ,,,0A0.0, 010A 3,A6FF,..00,0A0BD6+E .A10
A.00 0C 0,0.0,0,01A 0.0 .01 A010.001 0A , 1081000.0100001

54 000C0,0.01000A0C.0,B0 00. ,.1,000A 1C45 0,0 001.12CE+, 000
,0,.C, 0.,. .00..A000 .,0.00.0 ,00..00 0 .01100.00 . 100

55 F0010AC0 0010000010 30A0001,0, 01 .0A. 01010,0 A.A0, ,+BA00.
28CAC131. A.0,0 000.00A,B0, 000.1A.1 ,0DA01 0 002.00 B41B3,

56 2BA0BB00100.,C0.0 ,.,B000 .,,0100 00., B00100 0000. B+AB..
0,01B.CA0,, A,01 .,A0B,.0A.0 1010000,...01.0000A,,00.002AA0

57 20F B22 A,C,A, .2A.000C0A 000,3.0A000 0A,0,001A 0.0AA+11A
B030 30A A0C1000AC11.0220,.0.0021B2A0B.0 1C00000C00 01018C200

58 0. 202A.A1 0.10 00 ,0 .2.CA01001 0 2,0,.10 0000. 000A00B1+CC
01.2 A001A.0 0.A.0,0100.0120A0AA,A010A10001A1,,. 1BA 0.AA20

59 1A0BA14,1AC.0 0010A.0,AAA2200. .0,10000,AA00,000,000100.1C+1
1B0AB25000 ,,,01000 B 000AA11011000000,,B0A0.100,0010,B0211A

60 000000AA ,..,.0, 00.0 10000,0.0, .00 010000 0,0,0 ..AC1+
, .0011101 ,0 . ,0..0, 100,.. .000,0.0,,0001 ,, 000,000,,

60x60 Error Matrix Page 2

25

SOLN B 1490 MeV

Diagonal Elements of 60 x 60 Error Matrix

*** ERROR .29728121 73/08/27. 13.21.20. 1 (RUMBLE)

E	Re	Im	Corrl Coeff	Re	Im	Corr. Coef	Re	Im.	C. C.	
Wave (1)	.0128	.C373	-.0447	(2) .0184	.0241	.2847	(3) .0130	.0210	-.2396	(3)
(4)	.0220	.C275	-.0303	.0118	.0155	.0280	.0113	.0128	.1783	
	.0114	.0146	.2412	.C132	.0154	.3080	.0100	.0149	.0835	(9)
	.0083	.0123	.2943	.0164	.0109	-.2227	.0139	.0118	.0435	(12)
	.0160	.0111	.2407	.C185	.0167	-.0096	.0131	.0086	.2738	
	.C074	.0C70	-.0065	.0C79	.0063	-.0309	.0108	.0086	.1934	(18)
	.0097	.0066	.0649	.0C79	.0066	-.0100	.0255	.0282	-.1743	
	.0179	.0251	.4966	.0143	.0178	.6378	.0318	.0322	-.0759	
	.0215	.0222	.0718	.0106	.0140	.4933	.0111	.0143	.0821	(27)
	.0211	.0231	-.1318	.0183	.0198	.2055	.0141	.0127	-.1195	
	.0197	.0197	-.0856	.0141	.0124	.0095	.0120	.0128	-.0572	
	.0246	.0206	.0781	.0148	.0122	-.0105	.0083	.0081	-.0038	(36)
	.0067	.0075	-.2979	.0173	.0126	.0390	.0122	.0114	.0717	
	.C076	.0104	.1555	.0242	.0329	.7623	.0299	.0337	-.0645	
	.C196	.0233	.3437	.0214	.0265	.2088	.0137	.0153	.1267	(45)
	.0152	.0175	.2132	.01C0	.0094	.0587	.0148	.0202	-.1112	
	.C231	.0251	.1480	.0140	.0145	.0156	.0155	.0161	.0450	
	.0112	.0094	.0476	.0102	.0119	.0282	.0060	.0080	.0095	(54)
	.C175	.0314	.2895	.0280	.0205	-.0279	.0147	.0155	.3093	
	.0199	.0147	-.1230	.0172	.0145	.1114	.0120	.0073	-.0268	(60)

*** TMATRIX .29728121 73/08/27. 13.21.20. 1 (RUMBLE)

T

.3355	C.C000	.0000	0.0000	-.1421	.0117	.0000	0.0000	-.1622	.0532
.0CC0	0.C000	.0000	0.0000	.0314	-.0385	.0000	0.0000	.0000	0.0000
-.0551	-.1152	-.0224	-.0858	-.0412	.1521	.0000	0.0000	-.0141	.0574
.0000	0.00C0	.0000	C.0000	.0000	0.0000	.0000	0.0000	.0000	0.0000
.0000	0.0000	.0000	C.COC0	.2432	-.1590	.0000	0.0000	.0000	0.0000
.0000	0.00C0	.0000	0.0000	.0000	0.0000	.0000	0.0000	.0000	0.0000
.0C00	0.C0C0	.0000	C.0000	.0000	0.0000	.0000	0.0000	.0000	0.0000
.0000	0.0000	.0000	0.0000	.0000	0.0000	.0000	0.0000	.0000	0.0000
.1125	-.C341	.0000	0.0000	.0000	0.0000	.0000	0.0000	.0000	0.0000
.0000	0.0000	.0000	0.0000	.0886	.0412	-.0544	-.0531	.0000	0.0000
.0000	0.00C0	.0000	C.0000	.0000	0.0000	.0000	0.0000	-.2776	.1142
.0984	-.C156	-.0090	-.1942	.0000	0.0000	.0000	0.0000	.0000	0.0000

repeated from amplitude list.

2.6

SOLN B 1520 MeV

WAVE	RE A	E(RE)	D(RB)	IM A	E(IM)	D(IB)	PHASE	E	RE T	IM T	E(AMP)	E(ANG)	T**2	E(T**2)	WAVE
P33 PP11	1.00	.04	.03	0.00	.09	.05	0	5	.3295	0.0000	.0129	.0301	.108596	.008689	1
P33 DS13	-.08	.01	.01	-.04	.01	.01	-152	9	-.1087	-.0573	.0137	.0192	.015098	.003546	2
P33 DD13	-2.66	.12	.09	-.23	.23	.14	-175	5	-.2285	-.0201	.0104	.0197	.052617	.004858	3
P33 DD15	.04	.13	.11	-1.19	.14	.11	-88	6	.0038	-.1023	.0120	.0112	.010480	.002603	4
P33 PP31	-.03	.06	.04	-.35	.05	.03	-95	10	-.0095	-.1093	.0148	.0182	.012038	.003468	5
P33 SD31	-.99	.19	.14	-1.72	.15	.12	-120	6	-.0857	-.1486	.0120	.0172	.029425	.004256	6
P33 DS33	-.05	.01	.01	.13	.01	.01	109	5	-.0581	.1716	.0085	.0159	.032833	.003156	7
P33 PP33	.07	.05	.03	.41	.03	.02	80	7	.0250	.1419	.0111	.0172	.020773	.003312	8
P33 DD33	.15	.16	.11	1.01	.10	.09	82	9	.0124	.0862	.0089	.0139	.007578	.001621	9
RH3 DS13	2.86	.07	.04	-.22	.18	.11	-4	4	.3344	-.0258	.0086	.0207	.112487	.005870	10
RH1 SS11	.58	.10	.08	-.61	.21	.15	-46	13	.0677	-.0710	.0172	.0216	.009612	.003668	11
RH1 PP11	2.47	.47	.40	.91	.79	.61	20	16	.0747	.0277	.0159	.0227	.006352	.002789	12
RH1 PP13	1.99	.28	.24	.23	.48	.40	7	13	.0602	.0070	.0090	.0142	.003672	.001170	13
RH1 SS31	.82	.10	.09	.32	.13	.10	22	8	.0959	.0380	.0123	.0152	.010645	.002696	14
RH1 PP31	-1.51	.51	.37	-1.31	.58	.45	-139	16	-.0459	-.0400	.0161	.0170	.003708	.002224	15
SIG PS11	-1.83	.11	.09	1.14	.18	.11	148	5	-.2515	.1559	.0151	.0243	.087566	.009154	16
SIG SP11	2.83	.54	.45	.82	.45	.42	16	8	.1034	.0298	.0207	.0153	.011589	.004882	17
SIG DP13	.85	.34	.27	-3.13	.36	.22	-75	6	.0311	-.1137	.0124	.0132	.013886	.003085	18

L,J	SIGMA-TOTAL	CHANNEL- 1	2	3	4	5
P1	4.440	1.035	2.532	.451	.378	.044
D3	9.327	.891	2.570	4.033	1.557	.276
D5	.492	.109	.277	.106	0.000	0.000
S1	1.627	.077	.319	.256	.885	.090
P3	1.415	.010	.285	.144	.887	.090
TOTAL	17.302	2.121	5.982	4.990	3.707	.500

L,I,J	ETASQUARE	SIGMA-TOTAL		CHISQ	1	2	3	4	5
P11	.008270	3.879 +-	.152	3.3	1.031	2.325	.522	0.000	0.000
D13	.120667	6.878 +-	.197	.0	.788	3.092	2.999	0.000	0.000
D15	.958078	.492 +-	.115	.3	.109	.277	.106	0.000	0.000
S11	.915194	.332 +-	.087	.0	.060	.172	.100	0.000	0.000
P13	.985311	.115 +-	.034	.5	0.000	.038	.077	0.000	0.000
TOTAL THIS ISPIN		11.695			1.988	5.903	3.804	0.000	0.000
P31	.928163	.562 +-	.111	.3	.005	.083	.051	.378	.044
S31	.834340	1.296 +-	.135	1.4	.010	.194	.117	.885	.090
D33	.843473	2.449 +-	.196	.2	.031	.348	.236	1.557	.276
P33	.916906	1.300 +-	.199	.3	.010	.196	.118	.887	.090

SOLN B 1520 MeV

ERROR MATRIX (AMP(I) AMP(J) PHA(I) PHA(J) / AMP(I) PHA(J) PHA(I) PHA(J)). RAD = .011882 HYPERVOLUME = 4.9727E-70

I →
J
↓

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	+2	A1	02	,1	.0	01	01	1	01	,2	0A	BC	00	10	00	E3	10	20
	2+	45	A6	04	1	4	04	,3	,1	A5	10	0E	00	02	,1	,5	3,	1E
2	AA	+.	A	0A	,0	,B	C	08	08	80	A1	A.	0	,C	00	1A	01	E1
	15	.+	35	A3	2	03	4	01	02	E3	1.	0C	00	1	00	A5	40	0E
3	0A	B	+A	AB	0A	,A	B	0	1C	A0	A0	00	01	;0	,	00	AA	2
	26	45	A+	04	02	03	.4	,1	A1	05	00	8E	A0	01	.0	A5	30	1C
4	,0	0A	A0	+1	.A		1	A1	10	,0	,.	A0	AA		A.	1	A0	01
	14	A3	B4	1+	01	01	2	0	A0	03	0.	0A	A0	.1	0,	03	1A	1B
5	.	,	00	.0	+0	AA	1.	80	.0	00	0A	A	00	,B	A5	A	.,	,0
	01	12	A2	A1	0+	0,	01	AA	.A	01	1A	0A	,	1	AA	A2	0	00
6	0	,0	,0	0	A0	+0	C	00	10	00	A1	00	00	B	AA	00	..	0,
	14	93	A3	1	A,	0+	A3	04	,3	12	01	A0	00	23	0,	A2	10	1A
7	C0		.	1	10	CA	+0	0A	01	00	18	00	,0	0A	A,	00	A1	1.
	14	04	34	2	.1	3	0+	02	A4	03	4B	0C	A0	00	A0	B3	1,	1A
8	,	00	0,	A	3A	00	D0	+2	BA	00	00	10	0	0.	1	0	0A	
	13	31	1	10	0A	04	A2	2+	12	01	,1	0C	00	13	81	01	00	00
9	0,	00	1A	1A	..	1,	0A	B1	+A	,0	A1	0,	.0	A0	0	1A	0	01
	11	92	C1	00	0A	03	14	A2	A+	10	2	A8	00	12	0.	A0	00	1A
10	,A	BE	A0	,0	00	01	80	00	,1	+0	,1	A0	0.	,1	1	B	A0	,2
	25	03	05	03	01	02	03	01	00	0+	01	A8	88	A,	0A	A5	1A	1A
11	01	A1	A0	,0	C1	AC	14	0,	A2	,0	+F	0A	,A	00	,1	B2	,,	01
	AC	1.	00	..	AA	11	8B	01	1	11	F+	A	00	02	1A	2A	0,	0A
12	80	40	03	AC	0	0A	00	10	0A	AA	0A	+3	AA	,0	00	0	CA	A
	CE	.C	0E	0A	AA	0D	0C	0C	,B	08	A	3+	,C	0B	0C	1C	BC	0.
13	00	0	0A	AA	0,	00	,A	0	.0	0B	,0	A,	+3	.,	A	0A	.1	0A
	00	00	10	A0	0	00	00	00	00	.B	A0	AC	3+	01	.A	,,	11	B
14	10	,	,0	.	,	82	00	01	A1	,A	00	,0	.0	+1	A,	A1	00	0A
	02	01	01	1	B1	3	A0	.3	02	1,	02	0B	,1	1+	0A	A1	10	,.
15	0,	00	.	A0	AA	A0	AA	8	0	10	,1	00	.	A0	+1	00	C0	80
	01	00	,0	.,	5A	A,	,0	11	0.	A	1A	0C	AA	,A	1+	.0	.A	00
16	E,	1A	0A	0	A	0A	08	0	1A	A	82	1	0,	AA	0.	+8	81	A0
	35	A5	05	13	A2	02	03	01	A0	85	2A	0C	A,	11	00	8+	3A	1A
17	13	04	A3	A1	.0	.1	A1	00	00	A1	,C	0B	.1	01	0.	B3	+1	.C
	0,	10	A0	0A	,	.0	1,	A0	0	0A	,,	AC	11	00	0A	1A	1+	0C
18	21	E0	1	01	,0	01	11	0	01	,1	00	A0	0	0,	80	A1	.0	+A
	0B	1E	2C	18	00	,A	.A	0	1A	2A	1A	.	AB	A.	00	0A	CC	A+

18X18 Error Matrix

NEW STARTING PARAMS   ITERATION LIMIT REACHED

2.61

* .28319235 +- .0037 73/08/27. 13.13.39. 1 (RUMBLE)

A

1.000000	0.000000	0.000000	0.000000	-.082486	-.043511
0.000000	0.000000	-2.664897	-.234940	0.000000	0.000000
0.000000	0.000000	.044038	-1.193822	0.000000	0.000000
0.000000	0.000000	-.030210	-.347801	-.994751	-1.724028
-.045065	.133078	.071590	.406082	.145362	1.010670
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	2.857860	-.220679	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	.577799	-.606074	2.466730	.914596
1.994057	.232226	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	.817580	.324292
-1.507292	-1.313360	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
-1.834884	1.137288	2.834911	.816898	.854625	-3.125180
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

SOLN B 1520 MeV

WAVE	RE A	E(RE)	D(RB)	IM A	E(IM)	D(IB)	PHASE	E	RE T	IM T	E(AMP)	E(ANG)	T**2	E(T**2)	WAVE
P33 PP11	1.00	.05	.03	0.00	.13	.05	0	8	.3295	0.0000	.0173	.0443	.108596	.011704	1
P33 SD11	.00	.29	.15	0.00	.38	.18	0	999	.0000	0.0000	.0242	.0321	.000000	.000584	2
P33 DS13	-.08	.01	.01	-.04	.02	.01	-152	13	-.1087	-.0573	.0180	.0285	.015098	.004742	3
P33 PP13	.00	.08	.04	0.00	.09	.03	0	999	.0000	0.0000	.0261	.0283	.000000	.000683	4
P33 DD13	-2.66	.19	.09	-.23	.36	.14	-175	8	-.2288	-.0201	.0162	.0313	.052617	.007696	5
P33 PF13	.00	.71	.39	0.00	.75	.48	0	999	.0000	0.0000	.0163	.0173	.000000	.000267	6
P33 FP15	.00	.05	.02	0.00	.06	.03	0	999	.0000	0.0000	.0160	.0198	.000000	.000255	7
P33 DD15	.04	.20	.11	-1.19	.21	.11	-88	9	.0038	-.1023	.0179	.0167	.010480	.003788	8
P33 FF15	.00	.60	.31	0.00	.80	.44	0	999	.0000	0.0000	.0140	.0187	.000000	.000196	9
P33 FF17	.00	.49	.33	0.00	.69	.40	0	999	.0000	0.0000	.0113	.0162	.000000	.000128	10
P33 PP31	-.03	.08	.04	-.35	.06	.03	-95	14	-.0095	-.1093	.0199	.0259	.012038	.004759	11
P33 SD31	-.99	.27	.14	-1.72	.21	.12	-120	8	-.0857	-.1486	.0152	.0253	.029425	.005459	12
P33 DS33	-.05	.02	.01	.13	.02	.01	109	8	-.0581	.1716	.0189	.0246	.032833	.007213	13
P33 PP33	.07	.09	.03	.41	.07	.02	80	12	.0250	.1419	.0235	.0305	.020773	.007340	14
P33 DD33	.15	.22	.11	1.01	.17	.09	82	12	.0124	.0862	.0147	.0182	.007578	.002770	15
P33 PF33	.00	.52	.37	0.00	.46	.33	0	999	.0000	0.0000	.0122	.0108	.000000	.000149	16
P33 FP35	.00	.04	.02	0.00	.04	.02	0	999	.0000	0.0000	.0126	.0112	.000000	.000158	17
P33 DD35	.00	.19	.11	0.00	.17	.08	0	999	.0000	0.0000	.0163	.0147	.000000	.000266	18
P33 FF35	.00	.60	.35	0.00	.45	.27	0	999	.0000	0.0000	.0143	.0108	.000000	.000203	19
P33 FF37	.00	.48	.35	0.00	.46	.26	0	999	.0000	0.0000	.0113	.0110	.000000	.000127	20
RH3 PP11	.00	1.09	.32	0.00	1.40	.72	0	999	.0000	0.0000	.0330	.0422	.000000	.001090	21
RH3 SD11	.00	1.98	1.26	0.00	3.99	2.02	0	999	.0000	0.0000	.0179	.0360	.000000	.000320	22
RH3 DS13	2.86	.10	.04	-.22	.30	.11	-4	6	.3344	-.0258	.0124	.0350	.112487	.008447	23
RH3 PP13	.00	.89	.33	0.00	.87	.41	0	999	.0000	0.0000	.0271	.0265	.000000	.000732	24
RH3 DD13	.00	1.92	.83	0.00	2.37	1.14	0	999	.0000	0.0000	.0174	.0215	.000000	.000303	25
RH3 PF13	.00	4.03	2.73	0.00	7.37	4.26	0	999	.0000	0.0000	.0117	.0214	.000000	.000138	26
RH3 FP15	.00	.50	.17	0.00	.72	.30	0	999	.0000	0.0000	.0150	.0219	.000000	.000225	27
RH3 DD15	.00	1.98	.88	0.00	1.96	1.21	0	999	.0000	0.0000	.0181	.0179	.000000	.000326	28
RH3 FF15	.00	4.94	2.22	0.00	6.42	3.24	0	999	.0000	0.0000	.0144	.0187	.000000	.000208	29
RH3 FF17	.00	4.75	2.36	0.00	4.42	3.19	0	999	.0000	0.0000	.0140	.0130	.000000	.000195	30
RH3 PP31	.00	.78	.38	0.00	.94	.44	0	999	.0000	0.0000	.0237	.0284	.000000	.000559	31
RH3 SD31	.00	1.98	1.20	0.00	2.59	1.32	0	999	.0000	0.0000	.0181	.0236	.000000	.000326	32
RH3 DS33	.00	.16	.05	0.00	.19	.08	0	999	.0000	0.0000	.0183	.0227	.000000	.000336	33
RH3 PP33	.00	.88	.31	0.00	.89	.32	0	999	.0000	0.0000	.0270	.0272	.000000	.000731	34
RH3 DD33	.00	1.93	.87	0.00	1.98	.83	0	999	.0000	0.0000	.0176	.0180	.000000	.000310	35
RH3 PF33	.00	4.07	2.67	0.00	4.66	2.72	0	999	.0000	0.0000	.0120	.0137	.000000	.000143	36
RH3 FP35	.00	.36	.20	0.00	.41	.23	0	999	.0000	0.0000	.0110	.0125	.000000	.000121	37
RH3 DD35	.00	1.98	.85	0.00	1.73	.93	0	999	.0000	0.0000	.0182	.0159	.000000	.000330	38
RH3 FF35	.00	4.44	2.26	0.00	4.98	2.38	0	999	.0000	0.0000	.0131	.0147	.000000	.000172	39
RH3 FF37	.00	4.08	2.28	0.00	3.86	2.55	0	999	.0000	0.0000	.0122	.0115	.000000	.000148	40
RH1 SS11	.58	.18	.08	-.61	.39	.15	-46	24	.0677	-.0710	.0285	.0409	.009612	.006395	41
RH1 PP11	2.47	1.07	.40	.91	1.26	.61	20	29	.0747	.0277	.0298	.0402	.006352	.005633	42
RH1 PP13	1.99	.53	.24	.23	.90	.40	7	25	.0602	.0070	.0164	.0269	.003672	.002249	43
RH1 DD13	.00	2.05	.90	0.00	3.11	1.30	0	999	.0000	0.0000	.0185	.0281	.000000	.000342	44
RH1 DD15	.00	1.39	.70	0.00	1.97	1.09	0	999	.0000	0.0000	.0126	.0178	.000000	.000159	45
RH1 FF15	.00	4.51	2.29	0.00	6.07	3.20	0	999	.0000	0.0000	.0132	.0177	.000000	.000173	46
RH1 FF17	.00	3.36	1.93	0.00	4.33	3.02	0	999	.0000	0.0000	.0098	.0126	.000000	.000096	47
RH1 SS31	.82	.16	.09	.32	.24	.10	22	15	.0959	.0380	.0187	.0277	.010645	.004211	48
RH1 PP31	-1.51	.91	.37	-1.31	1.21	.46	-139	32	-.0459	-.0400	.0316	.0336	.003708	.004847	49
RH1 PP33	.00	.52	.24	0.00	.69	.30	0	999	.0000	0.0000	.0159	.0210	.000000	.000252	50
RH1 DD33	.00	2.16	.92	0.00	2.13	.90	0	999	.0000	0.0000	.0196	.0194	.000000	.000385	51
RH1 DD35	.00	1.38	.71	0.00	1.74	.84	0	999	.0000	0.0000	.0126	.0158	.000000	.000159	52
RH1 FF35	.00	4.31	2.37	0.00	3.96	2.03	0	999	.0000	0.0000	.0127	.0117	.000000	.000162	53
RH1 FF37	.00	3.32	1.87	0.00	3.60	2.22	0	999	.0000	0.0000	.0098	.0106	.000000	.000096	54
SI0 PS11	-1.83	.18	.09	1.14	.24	.11	148	7	-.2515	.1559	.0231	.0342	.087566	.014233	55
SI0 SP11	2.83	.82	.45	.82	.72	.42	16	14	.1034	.0298	.0306	.0258	.011589	.007518	56
SI0 DP13	.85	.58	.27	-3.13	.54	.22	-75	11	.0311	-.1137	.0190	.0216	.013886	.004825	57
SI0 PD13	.00	2.01	1.01	0.00	1.67	.97	0	999	.0000	0.0000	.0222	.0184	.000000	.000493	58
SI0 FD15	.00	1.75	.78	0.00	1.67	.67	0	999	.0000	0.0000	.0194	.0184	.000000	.000376	59
SI0 DF15	.00	3.68	2.02	0.00	3.26	2.36	0	999	.0000	0.0000	.0133	.0118	.000000	.000176	60

L,J	SIGMA-TOTAL	CHANNEL- 1	2	3	4	5
P1	4.440	1.035	2.532	.451	.378	.044
S1	1.627	.077	.319	.256	.885	.090
D3	9.327	.891	2.570	4.033	1.557	.276
P3	1.415	.010	.285	.144	.887	.090
F5	0.000	0.000	0.000	0.000	0.000	0.000
D5	.492	.109	.277	.106	0.000	0.000
F7	0.000	0.000	0.000	0.000	0.000	0.000
TOTAL	17.302	2.121	5.982	4.990	3.707	.500

L,I,J	ETASQUARE	SIGMA-TOTAL		CHISQ	1	2	3	4	5
P11	.008270	3.879 +-	.171	3.3	1.031	2.325	.522	0.000	0.000
S11	.915194	.332 +-	.127	.0	.060	.172	.100	0.000	0.000
D13	.120667	6.878 +-	.257	.0	.788	3.092	2.999	0.000	0.000
P13	.985311	.115 +-	.063	.5	0.000	.038	.077	0.000	0.000
F15	1.000000	0.000 +-	0.000	1.0	0.000	0.000	0.000	0.000	0.000
D15	.958078	.492 +-	.159	.3	.109	.277	.106	0.000	0.000
F17	1.000000	0.000 +-	0.000	.4	0.000	0.000	0.000	0.000	0.000
TOTAL THIS ISPIN		11.695			1.988	5.903	3.804	0.000	0.000
P31	.928163	.562 +-	.149	.3	.005	.083	.051	.378	.044
S31	.834340	1.296 +-	.178	1.4	.010	.194	.117	.885	.090
D33	.843473	2.449 +-	.277	.2	.031	.348	.236	1.557	.276
P33	.916906	1.300 +-	.290	.3	.010	.196	.118	.887	.090
F35	1.000000	0.000 +-	0.000	.4	0.000	0.000	0.000	0.000	0.000
D35	1.000000	0.000 +-	0.000	.6	0.000	0.000	0.000	0.000	0.000
F37	1.000000	0.000 +-	0.000	.7	0.000	0.000	0.000	0.000	0.000
TOTAL THIS ISPIN		5.606			.056	.821	.522	3.707	.500

60X60 Error Matrix page 1

ERROR MATRIX (AMP(I) AMP(J))
(AMP(I) PHA(J))

SOLN B 1520 MeV

RAD = .012704 HYPERVOLUME = 2.9782-228

2.62

I →
J
v

```
   1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60

 1 +0(A)001(2)0(A)A0A1000001,00,0A0B0010A0000000    ABA00010  00000F1200A
   100AB10000A1A110A.,.  AA,00010  ,00  0..,0020A01,0,00     .,130B2,

 2 0+AA5B.A1.   1010  01  11A0B02001AABA00  .00A0.100000AA00  0A018.
   121128.A2  0A0.0  10A.10.AA00B0.0B010.0A00BA01B  01001A1.00,AA0

 3 A)A+00.1A,0A000000000,00B0000,  ,01011  ..0  A00.010.01  00,1.E,00
   1000B,,000A.,1000,,000E0B00100A0000  .0.00ABA0       1.....30A0.

 4 0A0+00D  A1,,A00  ,0,0A.  2,280BA  00A000A0  000  00000  10,.00.280
   00  ,A110A00.  00.,0,  0.000100A0A00B0000001AAA0  ,AAA0.0,,1AC30

 5 0500+B0A1A00101A.  1012AAA010A0AAAA00  .00A001A.,,  00.0  0B01A0
   3  00A0  00A  01  000,,00AAA.00..0A0100AA  0ABB  0  00.1.00003,B1A

 6 1B.0B+2.E0.00000000  C0  000B11,.1.000000,000B1A.00,00   A0110,
   0AA.B1B1A1.0  .  0,0A.1,010,00  .  00000,0  01,0.  00..  .00.,A0101

 7 (2).1D02+A1000.  ,10000A00B0A  021  010  ,  01  ABA,,11000000,021A30
   1.000B1A01  0,00A000.1  0A0A3A001  11.  ,  00,A.nA100  00000.210A

 8 0AA  A.A+01  ,0.0  0.0,1A,01,0..00,  0A0  0  .100AA0000,10000A0000
   1,00BA0011A001001B00100A0  0,  .000000,  0AAB.0,0..10   002081A

 9 A)1,A1E10+   000.,008  210B001A00,A100000..00A1A100A  0  1011.A0.
   11003A0A101.000A1  0A...A  01A00  0001A00  AB0..A.0  0,A000A,00  0

10 A.01A001  +0,0.0100  B0,,0.A0000.0000010  ,2A  0  1.0.  ,,.1  1A0
   0.  0.100A  .,,0.0A  0,00000  A100000A  ,.  .,0  0A1BA  00.01A  .  0  ,

11 0  A,0,0  0+A,BA010,,,,0A1  000  B10B0A,100,0,000,AAB,100A.000
   A  00AA0020.00.0.00C  1,AA  01000001,,A..  ,00A1.000B000,,120A0.

12 A  0,000,0,A+A,200000,    0  0.   12  0A0  0000..0000C0.00.1.  010
   10B00A0011B0011  001,2A10.000000AA1A00A000BA  0003110.00.200  .

13 110A10.n00,A+A201010,10,A  0  .A0F0C,A  000000,.00.0B  0.AA0  00
   00A  AA,,1,,0,1..01000000A0001.  0010A0001010A000..A  A  0,020A0,

14 000000  .0.B,A+000,,.1,,1,,  0,.0160.0A01.A..01  00500.  10A00,
   10A0A001100.1101B1A2.000.00A,ABA202  A0CAAA  ,0  0  20000.0  A0,

15 010010,0,0A220+0001.00  ,0,0.,,.  B  B  0.  0000  .0.A1  00,,000.,0
   00B0AA.0100A00001A001A2A1000,00B01A..00000A0,0,020,0  00,11A0,

16 000  A01  ,100000+0AB  ,A0  ,  A00000100  10.000A  0.01.00A.01.,00
   000  0A  A0201AAA01210  0.000  .,00AA0100A0,100.  ,0100.00A.0..:

17 0  0,.00000101000+B1A10000.0000.AA00.00A  B0000.01A,0,B0A,1A10
   0,  0.0  A0,,0100.101.A0000.A000  .A,,0B,0,,10000,000..  0A0.00

18 0000  00.00000,0AB+01,000.,0000001.,0A,0010100.0.0.000  .0  000
   001..  0  ,0A10.20A1A  0A0000  ,00.1..0,00.00000,00,10,C0,0.0.0.

19 110,1000B  ,01,1B10+1000  ,0000000A,,000A,A.1,000A.,000.00.0A,
   000  0  01A0,A  2A0A0.00  0,.0,0  000.0.0A  00.0,0A  0000000.00

20 ,  ,00  0,  B,00..  A11+0.0,  0,0.0000  0,A0000    0,0A00..0A.0.00.
   00000  ,00,0A00A10100000  0.00.A  00  0.0,,0,00000,0,  02,  0  00

21 010A1CA120≠≠≠10≠1≠00=002A15BB10A-10  2A000E21C0120001≠20AA0-
   A0211A  .2.0A0101A00,0A0A1.010A00100  0.00,00010A0A0,  0  20A0A,

22 010.200A1,.  1,0A000.0+A0AA0100AAA0  ,A  .  00A120BA00000A0.,A
   A10010,.01,  0  0010  ,21,1A  200,00A.00  .100  100200A0.00.0B0100

23 ,AB  A  0,0,0  0,  000000A+.3  0A0,10A.  0.  ,  0  0100002  .,  0A0000
   1A000AA10AA1  100001.00AB  1,10,AA  10,AA01AAB,00,AA00,000020B10

24 0002A0B0B0A  ,1,  00  ,20.+A1BAA000,010.0AA2C4  000101  0000AA2B0
   .0.320A.  00A  0AA101A  A.A0110.0A00A0  .000A00  000  00A10.1A,0.A

25 AB0,A0010.10A,0,0.,  AA3A+  13.,1011.A  00.111E2,0  0.00,  00  0  0
   102,101,,,01.  ,000000100A0A1  00000.100.01.0A0,1  00  00A  B0.0

26 000200A,0A    ,,  .,001A  1  +,0B2,0  0000A,.0A  1,A20AA000  ,00000
   0,.0  .0..  01,0  ,0.100  0A.001A000,AA0B001A00.10.A0A1001000B22

27 B20B1B  010000  0A000,500B1,+00.2001  A10.0  A2nB10.00B00  3.0  .:
   20A  ,20A0C1A0A  A0.A0A01.01AAA,00000.0,  ,B110A0  00100,C,1A2,

28 00,0010.A00.  .00000B1AA300+.A01,0,,,021.1AE70AA0.01A0  ..0  0
   00,000020.,,0  .000.0,001,A0  B  00000,.  00102,  .B,.00.00,0A0.0

29 00  BA12.000  0,0000.B00A.B0.+3,1001013001000D200.B20,,.01.A
   A,AA,10,0000  00,,00,A0  AB0,3A00  0  0110..002.A0  1100A  010,00

30 11,A0,1000  .,,0000010,0,2.A3+,,,,,,  ..0  ,BA0AD610000.00  0.  0
   ,.  00.000  .0  ,.00,A,  A20,20000..00.1.  ,0.A200A.0A1..0  0  0A

31 0A0  A,  0,.B1A..0.0000A101,20,,+113,02BA00,1001,2220AA00.1100
   ,A  0A000A00B  0  1,0,00A101000.00.A1001.0.A1AA0,  121.0  0000.0.

32 AA10A10,A01200  0A000AA0000011,1+001012  021  0100A00A00  00,10:
   .001000  00  .00801  ,001A1A10.A,.00,00A00,A,1,,10AE00.00,10A0

33 0B00A.1  100  F1B1A1A0.AA,1  0,0.10+22  0  001A10,.,.B.1,0  212  .0
   00.  A0  0.AA01,A201,00000.,0,.,AA200AA0000A.,0.00110.00000

34 0A1AA00000B006  00.,  10.010100,302+01.B010B1001,2161A001.000:
   00.000,  A01,0DA01  00AA1B1A0.1,.000110100.10A00,020  100A00  0.

35 001000  A000ACOB00,,00  1.0  ,1,,120+.140C0  200,0001C30A0,01  .
   0  00000..  A13A3A010  00000101  10A2.0A  A.0.100  010001,00  0,

36 00  000,000A0,.  .00,  ,00A0A,0  00  1.+0A020,A  .0,0113AA20A,000
   00,  10  0A0A01B0000,,.0.001A,A0A0AAA00A000,000011181A  00A,10A

37 0  .0  0  01,  A0010A0A2A..  01,1.210.10+3  A0.00  0200A,00,A000
   ..00  000,0A,,00000,A  .00  1,,0  0  00000..A.10A0.0A.1.00  0,..0

38 0..A.0000010  A,00,00A  00A003,B2  B4A3+2B01001A0AA0E50A,,.00.
   00000  ,00,.1.  1002  0  A  1000,1A000.0,.00.1011  .A0A00.000.  0.A

39 000001  .  0000  .A0A00.,A0,.200A  0000  2+5,0001A0000,2C3  .0001
   000..  00  00,,,AA1  0,  000A001A000010221 01,0110001 0A0A  .000,,
```

60x60 Error Matrix Page 2

2.63

```
40  0   0 +  . . , 0 0 0 1 0 0   0 , 0 0   A . . 0 1 0   0 0 0 1 C 2 A B 5 + , 0 A 0 1 0 0 , 1   3 A D 5  .   , 0 ,
    .  : 0 ,  0 0 .   0   0 0 0 0 , 0 0 0 0 . 0 , . 0 0 1 0 ,     . 0 0 . . , 0 1   . 0 0 1 0   0 0 1 . 0 0 0 , 0 ,   0 0
41  A A A 0 A 0 A 1 0 2 , 0 0 . 0 0 8 1 A 0 0   0 2 1 0   . 1 , 0 2 1 0 0 0 0 0 , , + 0 1 0 0 0 0 0   0 0 .   0 . 0 0 1 A ,
    1 0 0 1 , A 0 . 0 0 0 0 0 0 0 0 2 A 0 0 2 A 1 B , . 0 A A   . C A 1 0 . 0 A , 0 0 B A   A 0 0 1 1 0 1 0 0 0 1 0 0 B 1
42  8 0 0 0 0 0 B 0 0 A 0 . 0 A 0 0 0 0 .   E 0   C 1 A A 1 0 B , 1 A B   , . 1 0 0 0 + A A 2 0 B A 0 A   0 A   B A 0 1 A .
    B 0 , A 0 0 0 0 . 0 A 0 0 1 0 0 .   A 0 4 . 0 4 A 0 0 A B 0 . 0 0 1 A 1 0 A 0 . 0 0 2 0 A 1 . 1 A 1   0 0 , 0 B A   0 .
43  A . 0 0 0 0 A 0 A   , . 0 . 0 A 0 1 1   2 0 0 4 1   2 A 0 A 1   1 1 2 A 0 0 0 A 1 A + 0 A 0 A . . . A 0 1 A 0 A , 1 A
    1   , 1 0 . 2 A 1 B 0 , 0 A . B   0 0 0 B 1 0 B 1   0 A   A 0 A 0 0 0 B A , , 0 8 0 1   A 0 0 A   0 , . 0 0 A 2 1 B 0
44  0 1 .   1 B , A 1   0 0 0 .     0 0 ,   1 A 1   E 1 0 E 0 0 0 0 0 0 0   0 0 0 0 0 A 0 + D 0 0 0 0 A , A . , 1   , . 0 .
    0 A 1 A B 0 1 , 0 A   1 0 0 , 0 .   0 . 0 0 A 0 A 1 8 1 0 0 , 0 0   A .   0 0 0 8 A A 0 1 A 1 0 0 , 0 0 0 0 0 0 0 8 2 0
45  0 0 0 0 A 1 , A A 0 0 0 , 0 . 0 0 0 0 0 C 1 0 0 2 , B 7 0 A 0 1 , 0 0 .   1 1 1 0 2 A D + A A A   0 0 0 A 0 0 0 . 0 0 .
    0 A ,   0 1 0 2 0 , ,   0 . 0 0   0 .   0 A . 0 0 1 .   0 B   0 0 0 . , 0   0 1 1 1 0 1 0 8 0 . .   . 0 0   ,   1 0 0
46  0 0 1 0 . A 1 0 1   0 0 . 1 0 . . . 0 , 0 2 0 0 , A 1 0 0 D 1 0 . 1 , 0   A A 0 0 0 0 0 A + D 0 0 1 0 A   0 0 . 0 0 . 0
    0 0   0   0 1 A , . 0 ,   0 0 0 , 0     0 0 0     . . , 0 1 .   0 0     1 , A 0 . 0 A 0   1 1 0 0 0 0   , , 0 1 A 0 A A
47  1 0 0 0 , . 1 0 0 1 , 0 0   . 0 0 0 0 1 0 0 0 0 2 0 A 2 6 , 0 , , 0 , 0 0 0 0 0 B A 0 A D + 0 , 0 , . ,   0 . . 0 .
    . 0 0 0 0 1   0 0 2   . . 0     0 0 0 A 0 . 0 0 1 0 . 1   0 0   , , 0 0 0 1 . , 0   0 A 1 0 0 0 0 0 0 0 0 0   0 0 . 0 .
48  0 0 . 0 , 0 0 0 0 . A C 0 0 A 1 1 . A A 2 B 0 1   0 . A 0 1 2 A . 2 0 0 2 A 0 , 0 A . 0 A 0 0 + . 2 0 A . 0     0 0 0 0
    0 0 A 0 0 , A , . 0 B A 0   1 0 0 A 0 0 .   1 A   . A 0 0 0 0 C C , , . , 0 , 0 B 0 A A . 0 0 1 2 0 0 . . . A . 0 0 0
49    0 0 0   0 0 0 A 0 A 0 . 0 1 . A 0 , 0 0 A 2 0 0 A 0 0 0 0 2 0 B 1 0 1 0 A 0 1   0 , 0   0 . . + 2 1 B 0 0 , B A 0 0 .
    . . A 0 0 0 , , A 0 5 A 0 B   ,   0 0   A A 1 B 1 0 0 1 2 , A 3 0 C 0 , 0 2 1 0 0 1 A   1 A . B 2 C 0 0 0 , 0 0 , .   0
50  0 A 1   0 , 0 ,   . B 0 0 5   0 , . , 0 0 0   1 . A 0 . . 0 2 0 . 6 1 1 0 0 0   0 A . A 0 1 0 2 2 + . , 0 . . , 0   , .
      0   , . 0 0   0   0 0 0 C 0 B 0 0 1 0 0 , 0 A 1 A 2 0 1   , 0   0 1 A A 0 0 0 0 0 1 0 0 0 , A 3 A , 0 0 0 0 0 0 .   0
51  0 A   1 0 0 0 1 0   , . B 0 0 0 0 0 0 . 0 0 .   0 0 B 0 B 0 0 A 1 1 C 3 A E , 3 0   A , 0 0 , 0 1 . + E 0 2 0 0 . . 0 0
    1 0 , 0 1 0 0 1 0 0 0 0 2 0 A 1 1 A 0 0   0 0 A 0 A 0 0 0 A 0 A , A   , 1 0 , 0 A . .   0 0 1   1 0 A 2 0 0 0 0 0 ,
52  0 0 0 0 . 0 0 0   , 1 0   0 0 A , 0 0 . 0 0 , 0 0 0 0 1 2 0 A 0 , A 3 A , 5 2 A . 0 0 A 0 A . A B , E + . 0 0 0 1 . . 0
    0 0 0 . 0 0 0 0 ,   , 0 0 0 0 0 A 3 . 0   A 0 1 0 , 0 0 1 A 0 .   0 0 0 . 0 0   1 , 1 1 0 . 0 , A   A . 0 0 0 . 0 . , 0
53  0 0 0 , 0   0 0 1 , 0 0 0 . , . B 0 0 0 1 0   0 , 0 0 A 0 . A 0 0 0 0 A 0 0 C D   A 1 , A   , . 0 0 0 , + 0 0 0 0 0 0 0
    , , 0 A , , 0 0 0 , 1 .     0 0 , . A   0   . A 0 , 0 0 . 0 0 . 0 0 A 1 1   0 . 0   A 0 0 0 0 A 0 0 1 0 1 0 . 0 0 0 1 0
54  0   , .     , 0 0 . 0 . .   , 0 0   . A , 0   0       0 , 0 0     0 A 2 0 A 3 5 0   A , 0 0   0 0 . 2 0 D + 0   0 0 0 0
      , 0 1 0   , ,   0   0 0 0   0 0 0 0 B . 0 , 0   0 , 0 0 0 0 0 .   0 , 1 , . . 0 .   0 0 0 0 0   1 0 0 0 0 , , 0 0 , 0
55  F 0 1 0 0 A 0 0 1 1 A 1 A 1 0 1 A . 0 . 2 A 0 0 0 , 3   , 0 0 0 2 1 0 0 , ,     . B 0 1 0 0 0   , . 0 0 0 0 + , A 0   0
    2 A 0   B 2 1 1 A 0 A . , 0 0 0 0 0 0 . A 0 C   0 0 B 2 0   0 . 1 0   . , , 0 0 1 A B A 1 , , 0 A 0 0 0 , 0 A 3 1 A 2 0
56  1 A . 0 B 0 2 A 1   . . A 0 0 . , 0 0 0 0 0 A A 0 0 . . .   . 0 1 . , A A , . . 0 A A   0 . .   B . 0 0 0   , + . C 1
      2 0 0 1 2 A 0 , 0 . 0 0 0 1 . . 0 0 C   A A 0 0 A 0 1 . 2 1 2 0 1 0 1 1 0 0 , 0 . 0 1 0 . 0 A   0 . A 0   . 2 1 A .
57  2 0 E . 0 1 1 0 .   0   0 A 0 , 1   . . A . 0 A   0 0 . 0 0 1 , 2 0 0 , 0 . 0   0 0 , , . 0 . 0 A 0 . 1 0 0 A , + 1 , 0
    A 1 0 0 3 0 A 1 , 0 0 0 0 0 1 , A 0 1 , A 0 1 0 1 0 0 1 0 , 1 0 0 0 , 0 A 0 0 0 3 1 . 0 0   0 A 0   . 0   0 1 C 0 3 B ,
58  0 1 , 2 1 1 A 0 A 1 0 0   0 . 0 A 0 0 0 A , 0 2 0 0   0 1 , 1 1   0 1 0 0 0 0 , 1 1 1 . 0 0 0 0 0   . . 0 0 0 C 1 + D A
    0 0 0 , A 0 1 0 , . 0 0 0 . 1 0 0 . 0   0 , 0 A A 0   A 0 0 0 0 0 0 1 1 1 1 1 0 0 0 A 0 . 1 0 , 0 0   , 0 1 . 0 , 0 2 0
59  0 B 0 B A 0 3 0 0 A 0 1 0 0 , 0 1 0 A 0 0 A 0 B   0 .   ,   0 0 . 0   0 0 0 0 0 A A A 0 0 , . 0 0 , 0 . 0 0   1 , 0 + 2
    1 0 0 1 A 1 3   0 0 1   . 0 A 0 . . A . B 0 0 . 1 0 0 1 1 . 1   0 A 0 0 A A 0   0 0 0 0 1 0 . A A 0 . 0 0 , A 0 1 . 1 A
60  A . 0 0 0 , 0 0 . 0   0 0 , 0   0 0 , . .   0 0 0 0 . 0 A 0 0 . 0 . . 0   . 1 , , .   . . 0   0 . . 0 0 0 0 0   0 A 2 +
    0 0 A 0 . 0   2   .     ,   0 0 , ,     0 0 0   0 0 , 0 A 0 0 0 0 . , 0 0 0 1 0 .   0 0 0 0 0 0 , 0   0 , 0 0   0 0 0 1
```

SOLN B 1520 MeV

*** ERROR .28319235 73/08/27. 13.13.39. 1 (RUMBLE)

E

.0173	.0443	.1021	.0242	.0321	.2077	.0197	.0273	-.3238
.0261	.0283	-.0106	.0166	.0311	-.1885	.0163	.0173	.1114
.0160	.0198	.1801	.0167	.0179	.0444	.0140	.0187	.1218
.0113	.0162	.0023	.0259	.0200	-.0580	.0235	.0178	-.3975
.0240	.0197	.1694	.0299	.0243	-.1817	.0183	.0145	.0277
.0122	.0108	-.1419	.0126	.0112	.1031	.0163	.0147	.1680
.0143	.0108	-.0461	.0113	.0110	-.0676	.0330	.0422	-.0919
.0179	.0360	.1319	.0123	.0351	.0550	.0271	.0265	-.1446
.0174	.0215	.0610	.0117	.0214	.0872	.0150	.0219	-.1434
.0181	.0179	-.0050	.0144	.0187	.3243	.0140	.0130	-.0655
.0237	.0284	.0680	.0181	.0236	-.0680	.0183	.0227	-.1616
.0270	.0272	-.0470	.0176	.0180	.0205	.0120	.0137	.0373
.0110	.0125	-.0708	.0182	.0159	.0649	.0131	.0147	.1818
.0122	.0115	.1397	.0209	.0453	.4978	.0324	.0382	-.2612
.0160	.0271	.0365	.0185	.0281	.0715	.0126	.0178	.1247
.0132	.0177	.1359	.0098	.0126	.0418	.0184	.0279	-.1358
.0276	.0370	-.0246	.0159	.0210	-.1095	.0196	.0194	.1257
.0126	.0158	.0174	.0127	.0117	.1497	.0098	.0106	.0615
.0242	.0335	.2720	.0300	.0264	.1131	.0210	.0196	.1404
.0222	.0184	-.0574	.0194	.0184	.1334	.0133	.0118	.1483

*** TMATRIX .28319235 73/08/27. 13.13.39. 1 (RUMBLE)

T

.3295	0.0000	.0000	0.0000	-.1087	-.0573	.0000	0.0000	-.2285	-.0201
.0000	0.0000	.0000	0.0000	.0038	-.1023	.0000	0.0000	.0000	0.0000
-.0095	-.1093	-.0857	-.1486	-.0581	.1716	.0250	.1419	.0124	.0862
.0000	0.0000	.0000	0.0000	.0000	0.0000	.0000	0.0000	.0000	0.0000
.0000	0.0000	.0000	0.0000	.3344	-.0258	.0000	0.0000	.0000	0.0000
.0000	0.0000	.0000	0.0000	.0000	0.0000	.0000	0.0000	.0000	0.0000
.0000	0.0000	.0000	0.0000	.0000	0.0000	.0000	0.0000	.0000	0.0000
.0000	0.0000	.0000	0.0000	.0000	0.0000	.0000	0.0000	.0000	0.0000
.0677	-.0710	.0747	.0277	.0602	.0070	.0000	0.0000	.0000	0.0000
.0000	0.0000	.0000	0.0000	.0959	.0380	-.0459	-.0400	.0000	0.0000
.0000	0.0000	.0000	0.0000	.0000	0.0000	.0000	0.0000	-.2515	.1559
.1034	.0298	.0311	-.1137	.0000	0.0000	.0000	0.0000	.0000	0.0000

SOLN B 1690 MeV

WAVE	RE A	E(RE)	D(RB)	IM A	E(IM)	D(IB)	PHASE	C	RE T	IM T	E(AMP)	E(ANG)	T**2	E(T**2)	WAVE
P33 PP11	1.00	.10	.07	0.00	.13	.10	0	7	.2018	0.0000	.0208	.0263	.040736	.006841	1
P33 SD11	2.06	.25	.19	-.64	.42	.33	-17	11	.1606	-.0498	.0199	.0322	.028266	.007089	2
P33 DS13	-.14	.03	.02	.25	.04	.03	119	6	-.0724	.1334	.0197	.0166	.023031	.006539	3
P33 DD13	-.23	.18	.14	-2.23	.24	.20	-96	5	-.0180	-.1743	.0194	.0140	.030702	.007161	4
P33 FP15	-1.35	.06	.04	-.29	.08	.06	-168	3	-.2755	-.0584	.0115	.0168	.079306	.006604	5
P33 DD15	3.12	.17	.10	-2.47	.26	.16	-26	3	.3997	-.1926	.0089	.0276	.196851	.007993	6
P33 FF15	2.63	.37	.30	-1.81	.56	.45	-35	10	.0812	-.0559	.0136	.0180	.009730	.002198	7
P33 PP31	-.08	.10	.08	-.68	.11	.10	-97	8	-.0155	-.1340	.0209	.0197	.018206	.006066	8
P33 SD31	4.88	.19	.19	-.29	.40	.27	-3	5	.3814	-.0219	.0149	.0310	.145930	.011598	9
P33 DS33	-.23	.03	.02	.34	.03	.02	124	5	-.1195	.1792	.0151	.0197	.046411	.006739	10
P33 PP33	-1.43	.08	.05	.95	.12	.07	146	5	-.2959	.1966	.0082	.0232	.126216	.005883	11
P33 DD33	-.87	.20	.14	.74	.27	.21	140	14	-.0676	.0572	.0139	.0218	.007839	.002661	12
RH3 DS13	1.93	.11	.08	-.02	.16	.11	-1	5	.2108	-.0022	.0117	.0171	.044457	.005092	13
RH3 FP15	-4.41	.30	.23	.86	.45	.35	175	3	-.2918	.0266	.0092	.0140	.085869	.005485	14
RH3 DS33	.80	.14	.09	1.60	.15	.11	64	5	.0845	.1696	.0152	.0160	.035886	.005977	15
RH1 SS11	.64	.17	.14	.89	.26	.19	54	12	.0685	.0946	.0225	.0246	.013638	.005769	16
RH1 PP11	-.19	.69	.53	11.25	.55	.42	91	4	-.0057	.3427	.0170	.0215	.117483	.011967	17
RH1 PP13	1.68	.43	.36	-3.18	.47	.37	-62	7	.0522	-.0967	.0143	.0135	.012474	.003405	18
RH1 SS31	-1.19	.19	.14	.90	.24	.20	143	8	-.1256	.0949	.0227	.0233	.024302	.007665	19
RH1 PP31	-2.04	.76	.61	7.14	.50	.52	106	6	-.0627	.2196	.0151	.0237	.052176	.007108	20
SIG PS11	-1.93	.30	.24	.18	.34	.24	175	10	-.1561	.0149	.0234	.0282	[illegible]	.007887	21
SIG SP11	7.52	.75	.64	-2.06	.87	.75	-15	7	.2221	-.0609	.0211	.0267	.053035	.010169	22
SIG DP13	2.50	.53	.40	-7.79	.52	.42	-72	4	.0738	-.2302	.0145	.0165	.058446	.007223	23
SIG FD15	-23.19	.90	.72	7.57	.68	.91	162	2	-.2777	.0907	.0109	.0079	.085360	.006478	24

L,J	SIGMA-TOTAL	CHANNEL- 1	2	3	4	5
P1	3.846	.217	.601	1.830	1.153	.039
S1	4.807	.190	1.574	.237	2.522	.284
D3	7.136	.601	2.011	1.257	3.046	.221
F5	7.082	1.418	3.775	1.889	0.000	0.000
D5	5.844	1.308	3.253	1.283	0.000	0.000
P3	5.243	.048	.770	.677	3.317	.432
TOTAL	33.958	3.782	11.985	7.173	10.042	.976

L,I,J	ETASQUARE	SIGMA-TOTAL		CHISQ	1	2	3	4	5
P11	.093310	2.243 +-	.095	.0	.206	.953	1.085	0.000	0.000
S11	.573354	1.056 +-	.122	0.0	.267	.619	.169	0.000	0.000
D13	.438734	2.777 +-	.140	1.9	.630	1.554	.593	0.000	0.000
F15	.045906	7.082 +-	.172	1.1	1.418	3.775	1.889	0.000	0.000
D15	.212598	5.844 +-	.239	0.0	1.308	3.253	1.283	0.000	0.000
P13	.950104	.247 +-	.063	0.0	0.000	.082	.165	0.000	0.000
TOTAL THIS ISPIN		19.249			3.830	10.236	5.184	0.000	0.000
P31	.676621	1.600 +-	.068	.0	.004	.260	.139	1.158	.039
S31	.241808	3.752 +-	.182	.5	.032	.567	.347	2.522	.284
D33	.559817	4.356 +-	.192	7.5	.025	.677	.388	3.046	.221
P33	.495135	4.990 +-	.215	2.1	.048	.736	.464	3.317	.432
TOTAL THIS ISPIN		14.705			.108	2.240	1.338	10.042	.975

2.8

24X24 Error Matrix

ERROR MATRIX (AMP(I) AMP(J) PHA(I) PHA(J) / AMP(I) PHA(J) PHA(I) PHA(J)). RAD = ,01+300 HYPERVOLUME = 2.8577E-89

I --->
J
v

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	+?	,2	BU	0A	BO	A0	0A	A	01	A0	0,	.0	,.	11	11	0.	F0	,0	00	1	C0	1,	0	00
	2+	1.	02	01	A3	A3	0	A1	01	C1	,0	.	,1	02	02	[illegible]	A0	,0	,0	11	A,	0	0	AA
2	.1	+8	00	3	10	B0	AA	UA	0A	A	0A	.A	00	10	00	A1	.0	A0	0	0,	,	A	1.	00
	20	3+	C0	3,	A0	00	.0	1	1	1	1	..	AA	10	00	0	A0	A	01	0.	0.	00	,A	0A
3	B0	0C	+B	3	0	U0	,0	CA	0A	1B	0	A	30	00	AA	0	10	.B	.	U0	,C	0	04	0
	02	0,	B+	0,	A2	11	03	0	01	,0	,1	,0	A4	A1	2	01	0.	01	0C	,1	02	0,	00	,A
4	00	33	00	+A	BA	A.	0C	0	.0	,0	0	,1	B0	00	,0	0A	A0	0	0	0	C,	01	0C	2A
	A1	,	,	A+	A2	13	A0	00	01	,2	01	11	12	01	,1	.0	02	01	01	,0	.1	00	0	0B
5	BA	1A	0A	BA	+A	AC	AB	1A	,C	0C	CC	0B	1A	AB	1A	A0	2A	00	,B	0B	,B	.A	1A	04
	03	00	2	A2	A+	A3	1	01	01	01	,2	0A	02	03	,2	.1	.1	0	01	01	A1	00	00	0C
6	AA	BC	01	A1	AA	+0	A2	0.	A0	00	A1	0	..	A0	,0	0	0	00	00	0	10	A,	0	10
	03	00	01	.3	C3	0+	01	01	,2	02	,3	00	02	03	02	12	02	1.	02	11	.1	A.	A0	0
7	0	A.	,0	0A	A	A0	+B	00	,.	0.	00	01	00		,0	00	.	0.	00	00	0	0	00	,.
	A0	A0	03	00	B1	21	B+	02	,1	A2	,2	0.	02	00	0	0.	00	,0	,1	,1	01	.0	A	B
8	AA	0	0	0	00	00	00	+.	A0	0	.	00	00	,A	0,	00	10	..	0.	GD	0,	,0	00	01
	1	A1	A0	00	A1	.1	02	.+	03	A2	,0	11	.0	,1	01	00	.1	..	01	4A	00	0	,0	,A
9	00	0	00	.0	,0	A,	,,	AC	+.	A.	A0	1	0.	,	00	0A	0	,1	E0	10	.0	,0	.	0
	11	A1	A1	01	C1	02	.1	03	.+	02	03	A+	02	.1	13	11	A1	0,	12	00	,2	0	C0	,C
10	A0		1,	,,	00	00	0A	A	A0	+B	A1	AA	A.	00	F2	12	.0	0A	11	00	01	00	.0	,
	01	A1	B0	02	C1	02	.2	02	.2	B+	02	13	01	.2	A,	,A	02	.0	01	01	02	00	C0	1C
11	0,	0	0,	00	0,	A,	0,	,,	A0	A0	+0	A0	0.	0	0.	A		A	1	,,	,,	00	0	.
	,0	A1	1	1	C2	13	02	0	03	12	0+	A1	A2	12	A2	12	A1	0,	12	,2	01	0.	C0	0C
12	.	..	,	,1	00	0	0C	01	1A	A1	AA	+B	.0	0	AA	00	1	01	B0	,0	.1	0.	..	0A
	0.	A.	A0	11	BA	00	1.	01	4	A3	01	B+	0	.1	2	1,	01	0.	1,	.0	,1	00	,,	1A
13	,,	0A	3A	B1	10	.0	00	0.	00	A0	0A	.	+0	B0	00	0B	00	A1	A0	,A	,1	.A	A2	10
	.1	0A	C4	02	A2	.2	02	0C	.2	.1	.2	00	0+	2	,2	01	A2	01	02	A2	23	01	A0	.0
14	10	11	0A	01	A0	A0	0	,,	..	0.	01	0.	B	+0	00	A,	0	BB	01	,0	.0	10	0,	C0
	12	00	01	01	B3	03	0	A1	1	02	2	1	02	0+	02	21	A3	.2	.0	11	01	0A	0	AE
15	10	00	A	,,	1,	,0	,	00	01	FA	0A	A2	0,	00	+1	BA	,0	01	0B	00	AB	0A	.A	00
	12	00	A2	0L	A2	02	0C	,1	03	2,	.2	A	02	02	1+	,1	A2	.	03	0.	A1	0A	C0	AB
16	00	A	0	0.	A.	01	00	00	01	1,	A1	01	00	A2	B,	+4	AA	2A	10	A	0	A0	00	1C
	.0	10	01	A0	01	2	0.	00	A1	2A	2	0,	B1	,1	A1	4+	A,	2B	21	0,	.1	A1	10	,A
17	FA	.A	10	A0	2.	0	0	1.	A	.0	A	0	0A	BA	,A	AA	+A	A.	0	0	AA	0A	00	,1
	00	00	0.	02	A1	02	.0	01	01	02	01	11	02	3	02	A,	A+	02	A2	..	02	05	CA	0E
18	,,	AA	.0	00	00	01	0,	..	,0	0.	A0	00	A0	B.	0	22	A0	+0	10	,0	00	A1	C.	0,
	00	0	01	1	C	0.	.0	..	1,	A0	,	1.	11	B2	1.	A0	.2	0+	B0	.	01	,A	CA	.0
19	0,	0	.0	00	,0	0C	C,	00	E1	10	1	B1	A0	0.	00	12	0A	1B	+B	0	00		C	,0
	0	01	0	1	B1	02	01	.1	02	11	12	0,	02	10	B3	01	2	00	B+	.1	01	0	0	,B
20	11	00	0,	,	00	U1	C,	G4	10	00	,,	,.	,A	,1	00	0	0.	,	0.	+0	00	..	00	A
	1	,.	01	00	B1	1	01	DA	00	01	,2	00	A2	01	0.	A,	.	0.	1	0+	0	01	00	A0
21	CA	,0	,0	0.	,A	1.	0C	C0	.,	0	,0	.,	,2	.0	AA	.	A0	00	00	0	+3	02	A0	A
	0,	,	02	,1	B1	01	1	,0	02	12	,1	11	13	01	B1	01	A2	01	01	00	3+	A.	,2	1C
22	1	A	0	00	.0	AA	.	,	,0	00	00	00	.0	10	00	AA	00	A,	U	.0	0A	+1	A0	A0
	,0	,	0,	10	A0	,.	0C	00	0	00	0.	.0	A1	0A	AA	01	A3	1A		.1	2.	1+	C.	.C
23		1,	00	0	10	0A	CA	0,	.0	.0	0	..	AA	00	.0	C1	00	0U	00	00	A,	A0	+A	C1
	00	,A	40	00	A0	0	0	C0	0	00	00	.,	20	,	A0	00	0A	.A		00	02	0.	/+	1B
24	0A	00	0	20	B0	1	,	C.	,	,1	0	01	1.	CA	0A	1,	,0	0.	,,	A	1	A.	C1	+0
	0A	A	A	AB	4C	00	.0	1A	0C	C	.C	AA	0D	0E	0B	0A	1E	,B	0B	AD	AC	0C	1B	0+

2.9

SOLN B 1690 MeV

WAVE	RE A	E(RE)	D(RB)	IM A	E(IM)	D(IB)	PHASE	E	RE T	IM T	E(AMP)	E(ANG)	T**2	E(T**2)	WAVE
P33 PP11	1.00	.13	.07	0.00	.16	.10	0	9	.2018	0.0000	.0254	.0325	.040736	.010920	1
P33 SD11	2.06	.27	.19	-.64	.49	.33	-17	13	.1606	-.0498	.0226	.0372	.028266	.008107	2
P33 DS13	-.14	.04	.02	.25	.05	.03	119	8	-.0724	.1334	.0267	.0209	.023031	.008822	3
P33 PP13	.00	.12	.06	0.00	.15	.08	0	999	.0000	0.0000	.0245	.0296	.000000	.000598	4
P33 DD13	-.23	.22	.14	-2.23	.29	.20	-96	6	-.0180	-.1743	.0232	.0172	.030702	.008650	5
P33 PF13	.00	.51	.38	0.00	.78	.55	0	999	.0000	0.0000	.0159	.0241	.000000	.000252	6
P33 FP15	-1.35	.07	.04	-.29	.10	.06	-168	4	-.2755	-.0584	.0148	.0197	.079306	.008548	7
P33 DD15	5.12	.21	.10	-2.47	.34	.16	-26	4	.3997	-.1926	.0132	.0286	.196851	.011855	8
P33 FF15	2.63	.52	.30	-1.81	.73	.45	-35	13	.0812	-.0559	.0158	.0229	.009730	.003368	9
P33 FF17	.00	.46	.24	0.00	.64	.38	0	999	.0000	0.0000	.0142	.0199	.000000	.000201	10
P33 PP31	-.08	.14	.08	-.68	.14	.10	-97	12	-.0155	-.1340	.0273	.0284	.018206	.008127	11
P33 SD31	4.88	.23	.19	-.28	.47	.27	-3	6	.3814	-.0219	.0179	.0370	.145930	.013996	12
P33 DS33	-.23	.04	.02	.34	.05	.02	124	7	-.1195	.1792	.0227	.0274	.046411	.010290	13
P33 PP33	-1.43	.14	.05	.95	.18	.07	146	7	-.2959	.1966	.0217	.0413	.126216	.015902	14
P33 DD33	-.87	.26	.14	.74	.35	.21	140	18	-.0676	.0572	.0199	.0272	.007839	.003918	15
P33 PF33	.00	.48	.36	0.00	.70	.47	0	999	.0000	0.0000	.0148	.0216	.000000	.000219	16
P33 FP35	.00	.08	.05	0.00	.10	.06	0	999	.0000	0.0000	.0158	.0185	.000000	.000250	17
P33 DD35	.00	.24	.11	0.00	.35	.17	0	999	.0000	0.0000	.0188	.0271	.000000	.000352	18
P33 FF35	.00	.48	.30	0.00	.69	.44	0	999	.0000	0.0000	.0148	.0213	.000000	.000220	19
P33 FF37	.00	.44	.25	0.00	.62	.40	0	999	.0000	0.0000	.0136	.0189	.000000	.000186	20
RH3 PP11	.00	.91	.49	0.00	1.00	.54	0	999	.0000	0.0000	.0282	.0310	.000000	.000795	21
RH3 SD11	.00	2.56	1.50	0.00	2.83	1.69	0	999	.0000	0.0000	.0287	.0318	.000000	.000823	22
RH3 DS13	1.98	.14	.08	-.02	.19	.11	-1	6	.2108	-.0022	.0148	.0204	.044457	.006459	23
RH3 PP13	.00	.70	.41	0.00	.61	.37	0	999	.0000	0.0000	.0217	.0189	.000000	.000472	24
RH3 DD13	.00	1.99	1.08	0.00	2.13	1.25	0	999	.0000	0.0000	.0224	.0239	.000000	.000500	25
RH3 PF13	.00	3.78	2.48	0.00	4.04	2.81	0	999	.0000	0.0000	.0171	.0183	.000000	.000293	26
RH3 FP15	-9.41	.37	.23	.86	.55	.35	175	3	-.2918	.0266	.0115	.0171	.085869	.006879	27
RH3 DD15	.00	1.62	.93	0.00	1.53	.92	0	999	.0000	0.0000	.0181	.0171	.000000	.000329	28
RH3 FF15	.00	4.07	2.26	0.00	4.25	2.62	0	999	.0000	0.0000	.0185	.0193	.000000	.000341	29
RH3 FF17	.00	3.34	2.13	0.00	3.12	1.99	0	999	.0000	0.0000	.0151	.0141	.000000	.000229	30
RH3 PP31	.00	.97	.53	0.00	1.06	.59	0	999	.0000	0.0000	.0298	.0326	.000000	.000891	31
RH3 SD31	.00	2.47	1.54	0.00	2.76	1.71	0	999	.0000	0.0000	.0275	.0306	.000000	.000756	32
RH3 DS33	.80	.18	.09	1.60	.19	.11	64	6	.0845	.1696	.0194	.0207	.035886	.007714	33
RH3 PP33	.00	.92	.43	0.00	.73	.42	0	999	.0000	0.0000	.0283	.0226	.000000	.000801	34
RH3 DD33	.00	2.24	1.17	0.00	2.05	1.12	0	999	.0000	0.0000	.0249	.0228	.000000	.000621	35
RH3 PF33	.00	3.84	2.62	0.00	3.83	2.63	0	999	.0000	0.0000	.0172	.0172	.000000	.000297	36
RH3 FP35	.00	.43	.28	0.00	.56	.33	0	999	.0000	0.0000	.0133	.0180	.000000	.000177	37
RH3 DD35	.00	1.90	1.03	0.00	1.61	.96	0	999	.0000	0.0000	.0211	.0179	.000000	.000445	38
RH3 FF35	.00	4.42	2.49	0.00	3.83	2.35	0	999	.0000	0.0000	.0199	.0172	.000000	.000394	39
RH3 FF37	.00	4.02	2.42	0.00	3.15	2.09	0	999	.0000	0.0000	.0180	.0141	.000000	.000325	40
RH1 SS11	.64	.22	.14	.89	.32	.19	54	16	.0685	.0946	.0264	.0317	.013638	.006866	41
RH1 PP11	-.19	.93	.53	11.05	.68	.42	91	5	-.0057	.3427	.0211	.0288	.117483	.014894	42
RH1 PP13	1.68	.59	.36	-3.18	.73	.37	-62	10	.0522	-.0987	.0221	.0191	.012474	.005418	43
RH1 DD13	.00	1.62	.97	0.00	1.64	.88	0	999	.0000	0.0000	.0181	.0184	.000000	.000329	44
RH1 DD15	.00	1.19	.80	0.00	1.49	.85	0	999	.0000	0.0000	.0133	.0167	.000000	.000177	45
RH1 FF15	.00	3.03	1.94	0.00	3.25	1.92	0	999	.0000	0.0000	.0137	.0147	.000000	.000188	46
RH1 FF17	.00	2.48	1.72	0.00	2.86	1.74	0	999	.0000	0.0000	.0112	.0130	.000000	.000126	47
RH1 SS31	-1.19	.25	.14	.90	.31	.20	143	11	-.1256	.0949	.0307	.0297	.024802	.010604	48
RH1 PP31	-2.04	1.00	.61	7.14	.69	.52	106	8	-.0627	.2196	.0198	.0318	.052176	.009435	49
RH1 PP33	.00	.70	.38	0.00	.69	.39	0	999	.0000	0.0000	.0214	.0211	.000000	.000457	50
RH1 DD33	.00	1.87	1.10	0.00	1.57	.96	0	999	.0000	0.0000	.0208	.0174	.000000	.000433	51
RH1 DD35	.00	1.46	.87	0.00	1.25	.83	0	999	.0000	0.0000	.0163	.0138	.000000	.000264	52
RH1 FF35	.00	3.69	2.27	0.00	2.86	1.97	0	999	.0000	0.0000	.0165	.0128	.000000	.000274	53
RH1 FF37	.00	3.04	1.91	0.00	2.65	1.75	0	999	.0000	0.0000	.0136	.0119	.000000	.000185	54
SIG PS11	-1.93	.36	.24	.18	.43	.24	175	13	-.1561	.0149	.0281	.0355	.024600	.009589	55
SIG SP11	7.52	.96	.64	-2.06	1.13	.75	-15	9	.2221	-.0609	.0266	.0349	.053035	.012936	56
SIG DP13	2.50	.68	.40	-7.79	.66	.42	-72	5	.0738	-.2302	.0180	.0212	.058446	.009027	57
SIG PD13	.00	1.86	1.09	0.00	2.22	1.35	0	999	.0000	0.0000	.0223	.0266	.000000	.000497	58
SIG FD15	-23.19	1.03	.72	7.57	.81	.91	162	2	-.2777	.0907	.0124	.0095	.085360	.007429	59
SIG DF15	.00	2.86	2.04	0.00	3.82	2.44	0	999	.0000	0.0000	.0147	.0197	.000000	.000217	60

60X60 Error Matrix page 1

ERROR MATRIX (AMP(I) AMP(J)) (AMP(I) PHA(J))

SOLN B 1690 MeV

RAD = .015143 HYPERVOLUME = 4.2287-219

2.10

```
I --->
J
v
   1 2 3 4 5 6 7 8 9101112131415161718192021222324252627282930313233343536373839404142434445464748495051525354555657585960

 1 +0B11BA010A0A C010002AC,13100100l 0,00,0.F .,0101,0     C10A1
   110A 2A001B1 1 ,.0000A100,180 .001   00.01A0 A 0A2100,000AAA0

 2 0+10B11AA A001 0.ACCA,0 U   0 ,. 00 01 00B00. 0,  .100 00A1000
   2BCA30BA10   ,C .100.1AA0101000O.0.000.0008,00,0 1A1  0.0000.

 3 B1+1C100  ,01 .0CA0,003010,00A..A00.0000.2,2  A00,00 ,000,A.
   0CC01CA0 A 0  ,011.00A00 UA0,1 0,00001 00A00..1 ,000, 1,0011

 4 101+AB111B00A,.A0,0A00.0C100000001 0.000 A0A1,0.0, .00,00.1,A
   1,02.,20A2 00 , 0A,02A B.0AB0,0A00, 0A ... 1A0 000.00 A00000

 5 1BCA+ABA100.0.,,1100,ABAU,000.1000',00000,B ,.00000.00000 20
   A 00A1,0B0000.0,0000001.A1CAA0.00 .0,. 0 10000 00,,, 000 000

 6 B11BA+ 0A10, 1000AC.,.1.,.CA A.0,1,0..0001, 0,0 0000 ,,C,,00
    0 101,A3CA0,.000 0.001,A0,0,  .0. ,. .0000000.0,1  1,.1000A.

 7 A101B +.B2100 ,01A1 200,0,B0A..A100 000CA21.0000A000,.0021BA
   .00,A.,A0.,0000. ,.000 0000 0,A0,A0 ,,000.10,1,00...C,0,010A

 8 0A01A0.+A0 0ACC,CA10 00AA10DA.00.21 0100..A1DA000100 000A000
   00.,0 B0000A10AC1, ,2A01100.0.1AC0  ,.,.202A2,.10,00000CB02.0

 9 1A 11ABA+0A0 C ,01B0AA.01,0020,0, ,00. 0.A00,A,01000.0000,0
   0AA1.AA2B010AA0 .0 01A,010001 .,,10, 0.A.000,1 00 ,0 0 0.00

10 0  B01200+0,0.000A001A.0.000200.000100000011A01C .0 0.0010A0
   0 ,1,00A4200 0000.0 000AA00000.00,0000 1.001 0B.0 11 000,0 0

11 AA,0001 A0+B A000,1000A101,000,01A01C0A1011A1,,20A0,01 002 A
   0AA10AB01A01AA1B01A0A0000000. C00A00A,100A0A,. 040.0,.01 0 0

12 0000.,000,B+A,20B0A1A.1A .00,0A000,0001CA0A A0.F21.00001,A ,
   0A0.00B,100A,0B01A,0.0A100000A001A01A.001A100003lA000 .0,10

13 A01AC 0A 0 A+ 0C000 0000000.00,0F 01 .002010000lA0B. 0.000 ,
   0AA000C11BB0B.2. 20.A00A 010,002B1.002 00AB000002 110000CB11

14  1 ,.1 00.A, +A0 0 00,00 0A000.0702.10, 0A0AA00.4A00,A00000
    01..0A0,00A10B 2C002CA1 000002000  0,A01010.,,2,A10,008,1,0

15 0 ..,C,0 00.20A+.A1 0011A0000,0,1B0C0A00,00B A000C0,201.01,A,0
   1A011AB.1A0AA0BB2000000000,00,00120.0000,1A.0,0,3100000,A,.00

16 000A,00,,00000.+00A0 A.0, 0.,000A,0A0 ,0000.,0 00A0,,0  0.0
   0000000000A100A10AAA0,0A00000.0,0 1011,A0001A0 011.000C.0,.

17 1.0010100000B0 A0+02B1AA. 00A003A,010A0B10A1,0.010A, B0 0010,
   00,,10 , 01BC,A,1010.0A, ,,00A0, A00A0101A2C 002A00A1 0000..

18 0AA,1AAA1A,C00100+A1B,000010,0,1B10.040.0 A000000 020001A010
    ,AA000 ,A00001,.4 2A1..001100A2,000A0A,A0BA00001A0C1.01C0.

19 00000011B01A0  A2A+0.0,.A0 .,0100.001.C0.00 ..0180.02 ,000A
   0,AA,.,A,.100 A0A1210.01.0010000.000 0111  010,0A.0000 .,. 0

20 00,A0. 00001 000B10+.0000,01.0A1,0001,.0000A0,0010000,00000
   0, 0,0.01000A 10.1020...000,0,00.01A00 ,,0A,A  0100.,201.A0.

21 2A00,,2 A10A000 1B..+A011A0 A12A,AA0.AC 1 200 11AA 0,0AA01.0
   00,00011A0,0.0A,000 2B,020A,B100 01. ,A,001,.B10 100A00,,000

22 A,00A.00AA0.0,1AA,00A+ ,D1010002 .0  .,A 0A,,10 0A,A00 010.0
   ,.0010 A00001C0. 000000100001A1,000010.00.01010.,A.,000A 1 .

23 003.B100..A1001.A0,00 +000AA0000,00 1 . 00B1A0080000   0BA00
   000.0.A10 . ,0. 0 0,,0001,,0,00,,,00 100AA0 0 1,,1 C,02,A,.0

24 , 00A.,A001A00A0.0.01,0+10021 ,0000.000,. 1A2A01AA 0,,  .10.
   .013000 ,,00 0 10002A0001AA0180.11 0,0200.,10 000,000A,20A

25 10100,0A1.0 0 0, 0A01001+A002,0A0,1 00,0A0C.A,,00000,000000
   ,.01A,100A0     0,000 0000A,00A,,100A001000.00A0,00A1 0000 0A

26 0 01,.,1,01.0 0 000,A100A+000200 .02. 0.0000A01.,00A0.0..0,0
   0 A 1.000 00.00.010000A.100102.1000 0,01 A0B..20,A0A0101000O

27 1 ,000B000,0000000l 000A000+001,1,1 ,0,..AB0.001,00.A00.20AB1
   1 AA. C00000,00    .0B1000 ,..A1 0, 0. 1A.A000 1010. 0 0,0

28 00000A000000.A0.A0.1 1A2000+10A00BA 00.0A00A50.000100  0101
   A00, 10A 0,0, .00,00,211 A,11B11000A0,00A.A000A100A.  100001

29 0 000 AA200,00,,0,,.A0012001+A 0. .0  1A0,1B10A  0A0 0 00. 0
   B01 A0A..00,00,.00 0B0001B1,0 AA.000  A 001A100A.1..0 ,.01000

30 1,A0.A..02000000000000l00 ,210A+.A.,00  A,0A00 .5A,01, ,.0 ,00
   10B00A 0000 0  .010,,,A0.20203000l0 1 10100101,20 00,010,0000

31 0..01..0,0,A,0,03,1A200,00,A .+A010000B00 00..01BB.100,A0110
   1001 0 10 2BA.B.3A1A10A.,0.0A03B112.. B0,,1. 0010A00A1A00100

32 0 .000A000000.10A101A200A0100AA+000100,0 0A,00010A0A1.010A0.
   ,0AB00A,00B2102001A 000A 00000A1B1A0A01  AA.A0 A100.1,110A00

33 10A10,1.,.10F0BA,B0,, ,00 ,0..00+01,0A,0B0, ,0  A0000A 100A
   1000 .,,A.0A3.B01A0000A00A000AA.10A0A 01 01,..01. A0 0A0 000

34  00 0102 0A0 70,01.0A.00,.1B ,100+10,10,00B0AA,A03A10. 00000
   .100001 A.0A,4A0 01000,1.0,,00102001,,0.0100 001A. 0..B00.0

35 0 00,,01,00,D0C01000A00010 A.00C11+A021A0 .00A.0.1C0AA0  0.
   1 A . .0000,A1B,CA0100A0000000A, 3A00, 000.1A00A0.0,,A0A00.0.

36 ,0..00 0010120A0.000 . 2, 0001,0A+AAB20..0,,.1002B.0 00000
   00.0,001. 10 0002 0A1 0 .0000100 0110101000000.1.A.,..0 A01.0

37 01000.000100 .A0A011. 10 .00  000,0A+11B0100100A.3,C.AA 0000
   100.0000.0,,00 03.,01A , .0A001B,00101A,000, 0.010108.00..10

38 0 000.01.000.10 04.,A. 00 ,0  00A12A1+200,00, ,0.1A5A,00AA.0
    100000,0.C0111.A1A1 1,0A 00A01B1A,110A  A0,,0031010A 0,A.1

39 ,0000000 0A1000,B0C.0,.000..1AB,,01B12+, ,,0.00A12B1C00100..
   01.0,0,0 .A00000B,A10 0,00 00AA1  A0.010000,,00,11000A  .0,0
```

60x60 Error Matrix Page 2

2.11

```
40 000 000000100,,01.00 A ,,..0A,000,A2B0,+.000.0010C1,05100000
   00.0.0..  2000,.101B,00,00,001000,.1A.A100.,,010BA 0011.02,,

41 .B.A,0A..00A2 0000.01 0.00AA000 B00000 .+A2001,10 ,.0. A .0.
   .0100 ,0001B2A0A100A11B0A0A1A,1,A000000,402,10,3AA0000 A11.

42 F020B12.A0100000A 00 00 A0B0,A 000 .1,,0A+01,0A,A., 00BA1100
   1A 0 CA0000,CA.0 A.,2 ,.0 0100.1B,0 A00,A.0010 0A0C0,CA,2A

43 0,A ,1A011A1AB01A002AB100D0100A,B..00,020+B1,01A 0., 0B020,
   00,002000A00A0.,0000B,120AA12B001.,A0.00B0A00ABA00,C 00,00

44 ..21 .101A 00 .,C A0,1ACD.AB00, 00000000C1B+A1. 0,.10.B,00,,
   00,.001,0000001.A0C0001B00A0B0.,.0AA A00AA10A2,A,, A2010000

45 , ,,C00,A1A0AA,C0.00,A2.A051 .0,A0,1,..C,1A+A.1A0000 .AC010
   ,.00001B000,00A00C.00BA1,CB000ABOA,., 00003A01,001000.. 010A

46 00 0.,0AA0,00A00.0., 10AA0000..00AA,0 C010,1A+ 0 01C01 C 0,0
   A001,0A0CCC0,C0.,00,B 0C010,00A,.0100 00 0200000....00,0000

47 1,A.0000,1,.00C 000010C0,11.A500 ,..0,00,A0.. +0.,0,. .000.0
   100..,0011 000,0 0,02A 0.B02010 000A00 1,A.11A10 0 2000 00 0

48 0 000 00002F100010101 B1,.,0 A11 A01A0A11,1 100+BB .0000 1,,
   0000.CB00,A0101 100,00000 00 .00B1,.0100.000,0.B1001A00A ,00

49 1.0,0CA01 G2A.C000B0A00A0,00 ,B0 0.0..100AA0A .B+00,00,1.B.0
   00 00A 0CD001100,00,0 A0A.01010A1 .C0110,0 0. 001001 C,A000

50 ,1, C0010.A104,AA 01AA0A0000000BAA310312C . ,00,B0+B1CC0.C0,
   00,.01,.. B,,4A 0A01,,00,,0 .A011101 ,0.0.00,0C01000.,0A0001

51 000.000000CC.BA20,C.0 ,0 00.1A1.00AC2,AB1,,0.010 0B+B100 1 .
   00, 00010,0000C00.0A0,000 0A011A0000 0A 00.C11B0 00.,0,0100

52 000.0000 ,0.00, 2000A000AA00,1A010B051,. .100,.,1B+B00,0 0
   000.00000 B000 .02,.A000. 0 , A0,0111000000 A100C00. ,A 1

53 00 , .000 01,B020,0 ,0000 0100A..AC000,000.0001B+A00000
   0.0 .,0 ,00..,C,A1.,A 00.0 0A.0.000 0., ...,0A1.1 0.00,0

54 0,,0,.00.100,.000 C0C ,,.0 0,0.0.A0A,05.0 . 1 00C00A+00,0,0
   0.1.000,0,100A0 101C0...0,..012A0,0 ,.A00.0,1 0.BA..,100 0 0

55 C0000,C000 0.A0 0,,A 00. .,0A 0 A001 B0B. .0,00000+ B000
   ,,0CB000A1001 C00000C1 . 0 ,1A0B. 0,10A.AA0001A11 C0A2 0B02

56 1A0000000001001 0100A00 0.2000A1 0 0 010AAB,A0001. ,0C +ACA0
   ,0AA10A000A10000A1A000AA 01000A0A01 001,.A A00001,000 310A02

57 010.D,2A010,CC, 0A0001B.C.010 0010 00A00 1000 0 .0100,BA+1BA
   0040010,.,0C1.0 A 00.030IAA 0A.0A,A00. 1 1 0A, 001A ,A010

58 A0,1 ,10,C2A00A010001DA1C0A0.,1A00000AC0.12000180 C00C1+.C
   00010A11A02A0.A01A1,11A A1.00008.01 00,A0100.0018.0C0A0C13A

59 10A,20B00A 0,.01A0..000,B1 01000.00..0000,1,.,., 00,0AB.+0
   00 0A 400020 0000.0,,,00A0,0A 010 . A,02 1000.0 0 0CB0100A

60 0.A00A0 CA,,000,0 0000.001 000.A0 000.0.0,,000,0 . 0000AC0+
   000A01B 0,B0001000,00110001, .,001,AC00A AA.0 0A2,,,.0.0AD05
```

SOLN B 1690 MeV

*** ERROR .23301724 73/08/28. 10.07.19. 1 (RUMBLE)

E

.0254	.0325	.1749	.0213	.0380	.0695	.0187	.0283	-.0328
.0245	.0296	.2460	.0175	.0229	.1672	.0159	.0241	.1183
.0151	.0194	-.1356	.0164	.0269	.5236	.0161	.0227	.2668
.0142	.0199	.2362	.0282	.0275	-.0429	.0177	.0370	-.0301
.0235	.0267	.2559	.0285	.0369	.5254	.0204	.0268	.2743
.0148	.0216	.1596	.0158	.0185	.1198	.0188	.0271	.4093
.0148	.0213	.2202	.0136	.0189	.2594	.0282	.0310	.2777
.0287	.0318	.0935	.0148	.0204	-.0738	.0217	.0189	.0487
.0224	.0239	-.0939	.0171	.0183	.0631	.0116	.0171	.0681
.0181	.0171	.1682	.0185	.0193	.0081	.0151	.0141	.3261
.0298	.0326	.3155	.0275	.0306	.1665	.0195	.0206	-.1251
.0283	.0226	.0868	.0249	.0228	.0576	.0172	.0172	.1141
.0133	.0180	-.0340	.0211	.0179	.1563	.0199	.0172	.1036
.0180	.0141	.1668	.0235	.0339	-.3271	.0288	.0211	.1388
.0184	.0227	-.0291	.0181	.0184	.0764	.0133	.0167	.0751
.0137	.0147	-.0599	.0112	.0130	.1014	.0267	.0333	-.0983
.0308	.0214	.2923	.0214	.0211	.0587	.0208	.0174	.0794
.0163	.0138	.0911	.0165	.0128	-.0047	.0136	.0119	.1077
.0290	.0347	.3039	.0284	.0334	.2501	.0199	.0194	.2364
.0223	.0266	.3910	.0123	.0097	-.1351	.0147	.0197	.5345

*** TMATRIX .23301724 73/08/28. 10.07.19. 1 (RUMBLE)

T

.2018	0.0000	.1606	-.0498	-.0724	.1334	.0000	0.0000	-.0180	-.1743
.0000	0.0000	-.2755	-.0584	.3997	-.1926	.0812	-.0559	.0000	0.0000
-.0155	-.1340	.3814	-.0219	-.1195	.1792	-.2959	.1966	-.0676	.0572
.0000	0.0000	.0000	0.0000	.0000	0.0000	.0000	0.0000	.0000	0.0000
.0000	0.0000	.0000	0.0000	.2108	-.0022	.0000	0.0000	.0000	0.0000
.0000	0.0000	-.2918	.0266	.0000	0.0000	.0000	0.0000	.0000	0.0000
.0000	0.0000	.0000	0.0000	.0845	.1696	.0000	0.0000	.0000	0.0000
.0000	0.0000	.0000	0.0000	.0000	0.0000	.0000	0.0000	.0000	0.0000
.0685	.0946	-.0057	.3427	.0522	-.0987	.0000	0.0000	.0000	0.0000
.0000	0.0000	.0000	0.0000	-.1256	.0949	-.0627	.2196	.0000	0.0000
.0000	0.0000	.0000	0.0000	.0000	0.0000	.0000	0.0000	-.1561	.0149
.2221	-.0609	.0738	-.2302	.0000	0.0000	-.2777	.0907	.0000	0.0000

SOLN B 1890 MeV

WAVE	RE A	E(RE)	D(RB)	IM A	E(IM)	D(IB)	PHASE	E	RE T	IM T	E(AMP)	E(ANG)	T**2	E(T**2)	WAVE
P33 PP11	1.00	.03	.07	0.00	.10	.09	0	6	.2393	0.0000	.0212	.0249	.057263	.010613	1
P33 SD11	.56	.18	.16	-.12	.17	.16	-12	18	.0719	-.0152	.0233	.0226	.005395	.003958	2
P33 DS13	-.42	.03	.03	.31	.04	.03	143	4	-.1862	.1401	.0149	.0165	.054319	.007152	3
P33 DD13	1.04	.13	.11	-.42	.13	.11	-22	7	.1345	-.0547	.0165	.0170	.021083	.005060	4
P33 FP15	-.52	.05	.05	-.73	.05	.05	-125	4	-.1235	-.1752	.0125	.0136	.045943	.005498	5
P33 DD15	-.79	.10	.08	-.88	.11	.10	-132	5	-.1019	-.1126	.0145	.0126	.023063	.004607	6
P33 FF15	.48	.20	.18	1.77	.20	.17	75	6	.0336	.1239	.0141	.0137	.016485	.003826	7
P33 PP31	.26	.05	.04	-.01	.05	.04	-3	12	.0625	-.0034	.0117	.0126	.003920	.001604	8
P33 SD31	.70	.11	.08	1.63	.08	.08	67	4	.0903	.2098	.0099	.0145	.052160	.004599	9
P33 DS33	-.16	.02	.02	-.24	.02	.02	-124	4	-.0730	-.1078	.0083	.0096	.016955	.002223	10
P33 PP33	-.56	.03	.03	.56	.04	.03	135	3	-.1341	.1342	.0083	.0103	.036006	.003207	11
P33 DD33	-.52	.07	.05	.95	.08	.06	119	4	-.0674	.1228	.0089	.0095	.019620	.002573	12
P33 FF35	-1.60	.10	.08	1.61	.14	.10	135	3	-.1122	.1126	.0084	.0085	.025259	.002751	13
P33 FF37	-3.84	.07	.05	.12	.14	.10	178	2	-.2687	.0082	.0050	.0101	.072241	.002728	14
RH3 DS13	.01	.07	.07	-.29	.11	.08	-87	15	.0021	-.0453	.0165	.0118	.002060	.001771	15
RH3 FP15	-2.50	.12	.12	1.12	.19	.16	156	4	-.1728	.0773	.0090	.0129	.035830	.003474	16
RH3 DS33	.17	.07	.05	1.31	.05	.04	83	3	.0267	.2054	.0077	.0104	.042890	.003239	17
RH3 FP35	1.65	.12	.09	-3.73	.09	.06	-66	2	.1143	-.2578	.0057	.0088	.079511	.003259	18
RH3 FF37	-11.60	.31	.24	-6.19	.35	.30	-152	2	-.1851	-.0987	.0042	.0062	.044021	.001769	19
RH1 SS11	-2.00	.09	.08	.27	.16	.13	172	4	-.3138	.0416	.0147	.0245	.100196	.009512	20
RH1 PP11	1.61	.28	.23	3.07	.31	.26	62	5	.1113	.2120	.0208	.0201	.057315	.010371	21
RH1 PP13	6.94	.10	.12	.12	.27	.24	1	2	.4801	.0081	.0070	.0185	.230585	.006733	22
RH1 SS31	-.48	.09	.07	.79	.07	.06	121	6	-.0758	.1242	.0103	.0154	.021157	.003092	23
RH1 PP31	-.23	.21	.17	1.51	.19	.15	99	8	-.0157	.1043	.0128	.0147	.011116	.002870	24
SIG PS11	-3.51	.22	.20	-.67	.28	.26	-169	4	-.3165	-.0604	.0193	.0249	.103816	.012815	25
SIG SP11	3.32	.54	.56	6.19	.42	.44	62	5	.1463	.2729	.0173	.0245	.095877	.011020	26
SIG DP13	7.40	.31	.29	-2.25	.37	.36	-17	3	.3264	-.0994	.0128	.0169	.116416	.008886	27
SIG FD15	-7.79	.52	.50	-6.32	.51	.52	-141	3	-.1831	-.1485	.0123	.0119	.055602	.005948	28

L,J	SIGMA-TOTAL	CHANNEL- 1	2	3	4	5
P1	1.821	.323	.884	.502	.107	.005
S1	2.439	.209	.629	.748	.785	.068
D3	4.700	.621	1.670	.491	1.825	.094
F5	7.130	.566	2.725	.444	3.294	.100
D5	.455	.102	.253	.101	0.000	0.000
P3	3.983	.010	1.305	1.958	.620	.091
F7	6.885	.043	1.065	.617	4.778	.383
TOTAL	27.414	1.872	8.530	4.860	11.409	.742

L,I,J	ETASQUARE	SIGMA-TOTAL		CHISQ	1	2	3	4	5
P11	-.015442	1.671 +-	.062	1.5	.339	.897	.435	0.000	0.000
S11	.209131	1.301 +-	.061	4.5	.212	.646	.444	0.000	0.000
D13	.349344	2.141 +-	.091	12.2	.607	1.321	.213	0.000	0.000
F15	.472965	2.502 +-	.099	2.0	.589	1.453	.559	0.000	0.000
D15	.907747	.455 +-	.086	0.0	.102	.253	.101	0.000	0.000
P13	.077661	3.035 +-	.085	8.4	0.000	1.012	2.023	0.000	0.000
TOTAL THIS ISPIN		11.206			1.849	5.582	3.775	0.000	0.000
P31	.954321	.150 +-	.034	0.0	.001	.024	.013	.107	.005
S31	.654297	1.138 +-	.066	2.7	.008	.175	.102	.785	.068
D33	.611208	2.559 +-	.079	37.6	.010	.406	.224	1.825	.094
P33	.856001	.948 +-	.082	0.0	.010	.138	.089	.620	.091
F35	.541367	4.528 +-	.164	0.0	.011	.734	.389	3.294	.100
F37	.476971	6.885 +-	.170	0.0	.043	1.065	.617	4.778	.383
TOTAL THIS ISPIN		16.208			.082	2.540	1.434	11.409	.742

SOLN B 1890 MeV

ERROR MATRIX (AMP(I) AMP(J) PHA(I) PHA(J) / AMP(I) PHA(J) PHA(I) PHA(J)). RAD = .011051 HYPERVOLUME = 2.6994-110

2.13

28x28 Error Matrix

I →→→
J ↓

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
1	+1 1+	00 AA	,A 1	AA ,1	A0 01	AC A1	0, A0	,C 00	0 01	,0 00	00 00	01 01	00 .1	,0 00	0, 0	0 .0	C .0	.0 .0	0 00	0 0	D0 0A	10 00	,0 ,,	0 0	E1 00	., 00	10 00	1A A
2	0A 0A	+0)+	10 00	A0 0,	A0 0	01 0	00 0.	.0 00	., 0,	00 .0	00 C0	00 1.	0. 00	.0 .0	00 A0	0 01	.. 0,	,. C.	. .	0A 2.	01 ,,	00 ,0	.0 0	, ,,	,0 ,1	A0 A0	0 .,	00 0C
3	, A1	10 00	+0 0+	AJ A	A. 2	,1 01	,0 A0	00 0	C 01	 .1	.0 01	0, 00	,0 ,1	,0 .1	21 01	01 .0	., 00	 .0	.0 0,	.U 00	0 .1	0 01	, .	 ,	00 .0	0 00	A1 8,	0, 00
4	A, A1	A0 J,	A 0A	+, ,+	,0 01	0A A0	00 10	, 0	0 00	G0 1	, 0.	.0 00	. 01	. ,0	10 01	,, 00	,. ..	0 00	. .	0 00	C. 1.	,0 .0	. ,	00 0	10 .	0 0A	C0 00	10 00
5	A0 C1	A0 ?	A .2	,0 01	+ +	0C 0C	C, A0	00 ,C	00 ,.	,, 01	C 1	0 .1	.0 00	0 .1	0A 00	., 01	00 0	,0 .0	0 00	, ,0	U ,U	.0 01	.0 0.	.. .	1A 01	. 00	00 ,	B2 C
6	AA 01	0 10	,0 11	0A A0	00 00	+1 1+	0, 81	,. 0,	,0 ,0	. 1	0C 02	,. 00	0 01	0. 2	., 10	0, 0.	.0 ,0	00 0	. .0	1 ,0	1, ,0	A0 .1	00 0	,0 0,	,0 0,	A0 00	00 0	., 0A
7	0A ,0	00 0.	,A 00	01 00	CA ,0	0B ,1	+0 0+	00 00	, .1	 01	C0 ,1	.0 ,1	0. A2	 .2	.1 00	0 02	0 0	.0 1	.0 01	C0 00	00 0	C 1	.0 ..	,, .	, .,	0A 0	00 AA	A0 B
8	,0 C0	.0 00	0 00	0 ,	0, 00	,C .,	00 00	+0 0+	02 12	A1 10	01 2A	,A 8.	00 C0	A0 10	0, 0	., .	0 G0	00 A1	.0 0,	L 00	. C,	, .C	,. .C	2A 02	, 0	,0 ,.	, .,	., 0
9	00 1	.0 ,,	00 1	00 0	0, 0.	,, 00	. ,1	01 22	+. .+	00 3	0, 01	A0 80	,0 A2	8A 13	0, ,	..	0. 10	01 01	00 .1	.0 .C	 00	.. ,1	C8 2,	0 0	0. ,	00 ,0	.. 0A	.. 0A
10	,0 00	0. 00	. 1	0 01	,0 ,1	 .1	0 1	A1 10	0 03	+0 0+	A0 .1	A1 A2	,0 B2	A0 03	,0 ..	, 0C	01 0	00 01	,. 01	,0 ,0	,0 0C	.0 .1	1A 00	2A .0	0 0,	00 .A	0. 0A	0 0A
11	00 C0	00 00	.0 01	J ,.	 01	00 02	0, 01	02 1A	C0 ,1	A. 01	+B B+	AU .3	,. 03	A, 5	, ,0	00 .	1A C0	A0 01	0C 02	00 .0	,A 01	.0 1	AA 01	B2 00	0 00	,0 A	.0 0B	.0 0B
12	00 11	01 0.	00 ,0	.0 C0	. 01	,C .C	., 01	,8 A.	A0 00	AA 12	A. 03	+B 8+	A A3	,0 A2	, C,	00 00	U1 0A	1 01	.. 01	. 0	00 C0	, .0	A0 .	,0 00	0 0,	, 0	,, A	,. 0A
13	0. 01	00 .C	,, 01	0 .1	.0 00	0 C1	0A .2	0C 0C	,A 02	,B 02	,0 .3	A A3	+B B+	DB ,4	, 0,	, 0.	.1 00	A1 00	00 12	,. .0	00 00	0 .0	0 0C	.0	,. 00	00 A	. 0A	. 0B
14	,0 00	.. 00	,. 01	, .0	. 01	0 .2	. 2	A1 00	81 A3	A0 03	A ,5	,A 02	0, 84	+1 1+	. ,	.0 0.	C C1	00 .2	A0 02	, .0	00 01	,C 1	11 01	0A .	0, ,0	0 A	0 0B	,C 0B
15	00 ,	0A 00	20 11	10 01	00 A0	.1 ,0	.0 10	0C ,	0, ,	,. 0.	,, C	0 ,,	,0 ,	. ,	+0 0+	00 0,	., 0	A .	.. 0	00 E,	11 0	.0 1,	02 0,	.0 ,	AB 0	AA 10	A. 0.	00 0
16	. 00	0 01	0. 10	,0 ,0	.0 ,1	00 ,.	0 02	. ,.	. .	0 ,0	0 0.	D0 00	,0 .	.C 0.	00 0,	+C C+	0. C	, 01	. 0A	00 ,0	1, A0	AA ,1	C 0A	0 00	00 00	00 10	. .0	80 0
17	0. 0	.0 .,	.0 ,0	,. ..	0 00	., 00	 00	0C 0	01 .0	0 10	10 A0	D0 1A	.0 10	00 1	. ,0	00 .	+A A+	D1 2	.0 01	00 A2	0 .0	.0 ,C	.C 1C	A2 20	00 0	, 00	0. AA	0 0A
18	.. 00	,C ..	. 0	0 00	,. 00	0 0C	. 01	0A 01	00 11	00 01	A0 01	10 1	A0 10	0. 02	A .	,0 1	D 12	+0 0+	C, 11	. 0	0C C1	,0 .2	0A A2	.A 81	.0 A	0 A	00 0A	., 0B
19	0 0C	. .	.0 0,	 ..	0 00	. .C	.0 01	.0 0,	0. 01	,0 .1	00 02	.0 .1	01 02	A0 02	. .0	0 .A	.0 01	C1 ,1	+A A+	.. ,.	,. 00	0 0	80 12	00 0A	. 0A	, C	.0 00	0. AE
20	 00	02 A.	.0 00	0 00	,, 0	, 10	00 00	0C 0	.. C0	,, 00	0. 00	 .0	,. .0	. ,0	0B 0,	0, 00	0A 02	. 0	., ..	+6 E+	C0 12	81 01	00 0	C. 1.	,. 0,	E0 0A	,A A0	1 A0
21	00 0A	J, 1,	0. 1	01 ..	, 00	1, ,0	0 00	.0 ,	0 0	,0 00	,0 A1	00 00	00 00	00 01	1 10	1A ,0	. 00	00 01	,0 .0	01 02	+C 0+	C, 3	0C 02	,. CA	80 ,0	00 0B	0 AA	,0 0A
22	10 00	0, 00	0 01	,. 00	.0 01	A. 01	 01	,. 0	., .1	.. 01	. 01	. ,0	. 00	, 01	.1 0,	A, A1	., 00	,. 02	 00	80 11	C ,3	+0 0+	,, 01	00 0	0 0,	1, 0A	0 AB	0. 0B
23	,, 0,	. 00	 ,.	.,	.0 0.	0 0C	.. 0.	,. .0	C2 8,	10 A0	A0 A1	A 0.	0 00	10 11	D0 2,	0 0A	.1 C0	0A A2	E1 02	00 0	0C 02	,0 ,1	+0 0+	10 8A	00 ,A	0 A	0 0A	., 0A
24	0 0	, ,,	 ,	00 0	.. .	,0 0,	, ,.	20 A2	C C	2. A0	80 20	,0 00	. 0	0. A	. 0,	0 00	A2 20	.8 A1	00 0A	01 ..	,0 .A	00 0	18 0A	+A A+	., 0C	00 1	0, ..	.,
25	E0 10	,, 01	0. 00	1 0.	10 A1	,0 0,	. ,,	, 0	0 .,	00 ,	CC 0	0 0,	,0 .0	0, ,0	A 80	0C 00	00 0	.A 0	0 .A	,0 .,	E, 00	00 ,	0, 0A	.0 ,0	+. .+	00 00	AA 00	01 0A
26	.0 ,0	AA 00	0 00	0 0A	.0 0	A0 00	0 A0	,, 0.	0, C0	G. 0A	, 0A	 ,0	0 0A	 0A	A1 A0	01 00	0 ,0	 0A	 ,C	EC 0A	C0 08	10 ,A	0 A	0 01	00 00	+0 0+	01 00	A 1C
27	10 C0	. 0,	A8 1,	C0 00	0, 0	00 0	0A 0A	. ,,	.0 .A	00 .A	.C 0B	, ,A	0 .A	C 0B	A0 ..	.. 0	CA .A	00 0A	.0 00	,A A0	0A A	0A 8	0 0A	0. ,.	A0 A0	C0 10	+0 0+	A1 10
28 -	1 AA	00 00	00 ,0	10 00	8C 2	.0 ,A	A 0B	. ,0	.0 .A	00 A	.0 0B	,0 .A	.0 8	,0 0B	0 00	8 00	0 0A	.0 ,8	0A .E	1A 0	,0 0A	00 .8	.0 ,A	. ,	00 1A	A1 C	A1 10	+ +

2.14

SOLN B 1890 MeV

WAVE	RE A	E(RE)	D(RB)	IM A	E(IM)	D(IB)	PHASE	E	RE T	IM T	E(AMP)	E(ANG)	T**2	E(T**2)	WAVE
P33 PP11	[illegible]	.13	.07	0.00	.11	.09	0	6	.2393	0.0000	.0241	.0270	.057263	.012137	1
P33 SD11	.56	.20	.16	-.12	.19	.16	-12	20	.0719	-.0152	.0256	.0254	.005395	.004412	2
P33 DS13	-.42	.04	.03	.31	.04	.03	143	5	-.1862	.1401	.0164	.0188	.054319	.007907	3
P33 PP13	.00	.08	.06	0.00	.10	.07	0	999	.0000	0.0000	.0185	.0229	.000000	.000341	4
P33 DD13	1.04	.14	.11	-.42	.15	.11	-22	8	.1345	-.0547	.0182	.0195	.021083	.005624	5
P33 PF13	.00	.26	.20	0.00	.24	.21	0	999	.0000	0.0000	.0182	.0168	.000000	.000331	6
P33 FP15	-.52	.06	.05	-.73	.06	.05	-125	4	-.1235	-.1752	.0138	.0156	.045943	.006084	7
P33 DD15	-.79	.11	.08	-.88	.14	.10	-132	6	-.1019	-.1126	.0169	.0159	.023063	.005421	8
P33 FF15	.48	.23	.18	1.77	.22	.17	75	7	.0336	.1239	.0157	.0159	.016485	.004265	9
P33 FF17	.00	.19	.12	0.00	.22	.18	0	999	.0000	0.0000	.0130	.0152	.000000	.000170	10
P33 PP31	.20	.06	.04	-.01	.06	.04	-3	13	.0625	-.0034	.0139	.0145	.003920	.001938	11
P33 SD31	.70	.12	.08	1.63	.09	.08	67	4	.0903	.2098	.0109	.0168	.052160	.005091	12
P33 DS33	-.16	.02	.02	-.24	.02	.02	-124	5	-.0730	-.1078	.0099	.0116	.016955	.002666	13
P33 PP33	-.56	.04	.03	.56	.05	.03	135	4	-.1341	.1342	.0108	.0127	.036000	.004197	14
P33 DD33	-.52	.09	.05	.95	.09	.06	119	5	-.0674	.1228	.0111	.0111	.019620	.003218	15
P33 PF33	.00	.14	.10	0.00	.16	.12	0	999	.0000	0.0000	.0096	.0111	.000000	.000092	16
P33 PF35	.00	.04	.03	0.00	.04	.03	0	999	.0000	0.0000	.0091	.0094	.000000	.000083	17
P33 DD35	.00	.06	.04	0.00	.09	.06	0	999	.0000	0.0000	.0080	.0116	.000000	.000064	18
P33 FF35	-1.60	.12	.08	1.61	.15	.10	135	3	-.1122	.1126	.0101	.0094	.025259	.003318	19
P33 FF37	-3.84	.09	.05	.12	.17	.10	178	2	-.2687	.0082	.0062	.0117	.072241	.003364	20
RH3 PP11	.00	.31	.23	0.00	.44	.31	0	999	.0000	0.0000	.0213	.0306	.000000	.000453	21
RH3 SD11	.00	.73	.50	0.00	.80	.63	0	999	.0000	0.0000	.0237	.0259	.000000	.000561	22
RH3 DS13	.01	.08	.07	-.29	.12	.08	-87	16	.0021	-.0453	.0190	.0129	.002060	.002092	23
RH3 PF13	.00	.31	.20	0.00	.37	.22	0	999	.0000	0.0000	.0217	.0254	.000000	.000470	24
RH3 DD13	.00	.50	.38	0.00	.75	.54	0	999	.0000	0.0000	.0160	.0242	.000000	.000257	25
RH3 FF13	.00	.39	.63	0.00	1.16	.92	0	999	.0000	0.0000	.0142	.0186	.000000	.000202	26
RH3 FP15	-2.50	.13	.12	1.12	.26	.16	156	5	-.1728	.0773	.0103	.0176	.035830	.003991	27
RH3 DD15	.00	.53	.38	0.00	.57	.41	0	999	.0000	0.0000	.0187	.0183	.000000	.000350	28
RH3 FF15	.00	.80	.62	0.00	1.18	.87	0	999	.0000	0.0000	.0128	.0189	.000000	.000165	29
RH3 FF17	.00	.84	.57	0.00	1.01	.67	0	999	.0000	0.0000	.0134	.0162	.000000	.000181	30
RH3 PP31	.00	.24	.16	0.00	.22	.14	0	999	.0000	0.0000	.0165	.0153	.000000	.000271	31
RH3 SD31	.00	.56	.39	0.00	.41	.30	0	999	.0000	0.0000	.0181	.0134	.000000	.000327	32
RH3 DS33	.17	.08	.05	1.31	.06	.04	83	4	.0267	.2054	.0091	.0129	.042890	.003865	33
RH3 PP33	.00	.19	.11	0.00	.19	.11	0	999	.0000	0.0000	.0130	.0128	.000000	.000168	34
RH3 DD33	.00	.34	.23	0.00	.34	.22	0	999	.0000	0.0000	.0110	.0112	.000000	.000121	35
RH3 FF33	.00	.65	.50	0.00	.57	.43	0	999	.0000	0.0000	.0104	.0091	.000000	.000108	36
RH3 FP35	1.65	.15	.09	-3.73	.12	.06	-66	2	.1143	-.2578	.0074	.0111	.079511	.004217	37
RH3 DD35	.00	.34	.17	0.00	.28	.18	0	999	.0000	0.0000	.0109	.0091	.000000	.000119	38
RH3 FF35	.00	.52	.37	0.00	.60	.37	0	999	.0000	0.0000	.0082	.0096	.000000	.000068	39
RH3 FF37	-11.60	.41	.24	-6.19	.45	.30	-152	2	-.1851	-.0987	.0058	.0079	.044021	.002454	40
RH1 SS11	-2.00	.11	.08	.27	.20	.13	172	6	-.3138	.0416	.0181	.0307	.100196	.011775	41
RH1 PP11	1.61	.40	.23	3.07	.36	.26	62	7	.1113	.2120	.0237	.0287	.057315	.011931	42
RH1 PP13	6.94	.12	.12	.12	.34	.24	1	3	.4801	.0081	.0082	.0236	.230585	.007949	43
RH1 DD13	.00	.48	.32	0.00	.65	.46	0	999	.0000	0.0000	.0157	.0211	.000000	.000246	44
RH1 DD15	.00	.40	.23	0.00	.52	.38	0	999	.0000	0.0000	.0128	.0168	.000000	.000165	45
RH1 FF15	.00	.72	.52	0.00	.99	.70	0	999	.0000	0.0000	.0115	.0158	.000000	.000133	46
RH1 FF17	.00	.63	.40	0.00	.88	.63	0	999	.0000	0.0000	.0101	.0140	.000000	.000103	47
RH1 SS31	-.48	.12	.07	.79	.09	.06	121	8	-.0758	.1242	.0124	.0197	.021157	.003772	48
RH1 PP31	-.23	.29	.17	1.51	.24	.15	99	11	-.0157	.1043	.0165	.0201	.011116	.003763	49
RH1 PP33	.00	.19	.11	0.00	.17	.10	0	999	.0000	0.0000	.0130	.0116	.000000	.000169	50
RH1 DD33	.00	.38	.25	0.00	.33	.22	0	999	.0000	0.0000	.0123	.0106	.000000	.000152	51
RH1 DD35	.00	.35	.21	0.00	.22	.16	0	999	.0000	0.0000	.0113	.0071	.000000	.000127	52
RH1 FF35	.00	.55	.40	0.00	.52	.34	0	999	.0000	0.0000	.0087	.0082	.000000	.000076	53
RH1 FF37	.00	.48	.32	0.00	.40	.28	0	999	.0000	0.0000	.0077	.0065	.000000	.000059	54
SIG PS11	-3.51	.24	.20	-.67	.32	.26	-169	5	-.3165	-.0604	.0214	.0291	.103816	.014275	55
SIG SP11	3.32	.65	.56	6.19	.50	.44	62	6	.1463	.2729	.0201	.0299	.095877	.012870	56
SIG PP13	7.40	.34	.29	-2.25	.43	.36	-17	3	.3264	-.0994	.0138	.0199	.116416	.009591	57
SIG PD13	.00	1.01	.72	0.00	.95	.73	0	999	.0000	0.0000	.0237	.0223	.000000	.000563	58
SIG FD15	-7.79	.58	.50	-6.32	.56	.52	-141	3	-.1831	-.1485	.0136	.0132	.055602	.006583	59
SIG DF15	.00	1.20	1.00	0.00	1.35	.98	0	999	.0000	0.0000	.0158	.0177	.000000	.000249	60

60X60 Error Matrix page 1

ERROR MATRIX (AMP(I) AMP(J))
(AMP(I) PHA(J))

SOLN B 1890 MeV

RAD = .011685 HYPERVOLUME = 1.3088-232

2.15

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I ...
J
v
    1 2 3 4 5 6 7 8 9101112131415161718192021222324252627282930313233343536373839404142434445464748495051525354555657585960

 1  +0,0A1A,0200000 0,01.0 ,0000  00 00 0 0.012A.1000 0 ,E 1000
    .A ..A0AAA0000000, ,00,,.0 .,00..000,0 0 ,0 0,. 0,, 0A00.
 2  0+110A001 1 000.,0, ,1 100 ,,,. 0 . 0...A . . ,0.00,
    00,0.B00010 .010..0.,0A1.0000, ,0,0,00 ,1 000 0,,,0..00.00
 3  ,1+0A0A0 000 000,.0 0,2,1 A0,00,.0, ,0 . ,00.00,.0, ....A01
    000A0 A 001 . A0,00,.A0A,A0 ,00 , 000,0..0.,.0 0.,,A ...B100
 4  000+1A..1, .00 A 00A002,1,A.1,A0 .0,.000100,A.,000 ,100101
    0A.11A0A1A.00000.00000.A.0.0 0 0.0.0000.0.B00000.0 , .00 A00
 5  AAA1+,,00A.0.001C,. ,A10000.0 00 ,...0 0000A000.0 ,1.C 1,
    A011 00A1A00.00,0.A 01 00,0.,.,0...0. ,0000A.0, ,.0, ,000.0,
 6  100A,+ 00,0 .A 10, ,00 ,0.,00.0 ,00,,.0A100 10.,.000,00 0,
    0 1A0 .10 00000, 0,00.00.A .,.0., , ,0,2.000.,,00 000000,
 7  A0A., +,C100 ,000 .00 .0.000000. .00,.0000000. 0.0 .0.00B0
    0 0A01,0AA0 , ,100000000A0 011 0..0 ,00000 01.0 . ,,00,0B0
 8  ,10.0 ,+A20 .00.000A100,0000000... 00., 000.A.0 ... 00000.,
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 9  0 100CA+0,, 000.00,A00..0 ,0 0,0.,.,. 00 000.,0.0 ..0 A0
    010A02 003,00,0.0,A 0A010A00,00 0,, 0, 0.0 0 ,0 ,,0.00 ,
10  210,A012+ .,,0, 00000.1 00A11 . 0 , 00,0,0AA20, 0. 0B 0.00
    00 1.A0A1.0 0 0000 0. ,.00 .0 ,0,... 0 0A1.B ,,00.01,0100
11  0 0 .,00, +00.01A 00 00 .0.0 ,00000,,.0,000000002AA,10 0,00,
    100 00 000.102BB00C1. 0, ,0 A 11 0,00 00.. 0,000000.00 .
12  0 0.000 ,.0+00AAC,,B000 , 0 011000.00000 . 0..0000A,000. .
    00.0.00 ,02. 0C,0AB2 0 00000 ,111 A000A.., ,. 21A .0 . 0...
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    , 0000, ,0100,300AB0,0.,0,00 01 .,0,201.,.0 . 10A2.,, .000
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    ,,.0,0,010 .1A..11A1. .00 .00,0A,.10,0000.000 , , .0,1 0.,00
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21  1,0A,,01A0 0,, 0,00+000001A010. 0.00,000,,0000.0100.0AA.00,
    A1A00.000 00 ,, .,.02010C,000 0,00...00 0A, 00000A000 010
22  .1,0A000000000 . 0 ,0+B0C112010A0.00,.0.00,A00.00 .1,,, 0.0.
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23  0 2010 00.00 0000..,0B+A00,1, 0000000A1,00100.0.0010,.,0AA,.,
    ,0100AA,00 0, , ,0A110AA0A.,, .00 B01,0 00 ,,, . 2000
24  1,20 ,.1 . ,0 00.00A+010C110,00,,000000010AA.00,A0A,0 00.1
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    ..,10A.A00.000.0.00 BA.A00A010A00101A.A01.1 A00A A00.0.,1,0
26  00 10,000000,0 ., 0 0101A+10C10.000A ..0 0 ,.B1,0000 00,-.00
    00 100 ..1. .,0 ,00. A0 0A001000.00 ..000A110A0. 0,0,0,.000.
27  0 A,00.0 0. .00.0. 11,0A1+1A10.0.,00,.000B0,,0 .100.000000B0
    0 11.00A0A.0 100000,CA02,,DA10,000 01 ,0000,,00A0B 0 .11,10.
28  0 0A..00,A00001 0,100A21C001+A000001.,0...,A 1 00100000000AA0
    0 000 ,1, ,,,0 00. 1A001000A01..0,.00 100,00. .,,0100A01B,A
29  0 ,.0,0001 .. 0 00,10CAA+ ., ,0 0 A0,00.0A10..,,000..0,0
    00A0.A01A00.0, 00,A10B0011B,01 ,A0A.010,A0120 .00.0,,,000.
30  ,01 000 1,00 ,,0,, 11 1B110 +0 00,00.0011 00 1 01A100AA000
    ,, .000000.0,,.00 0AA,.2BBA3C0. .0,0.,0010,0,0.00,0.10.00,0
31  ,0, 000 01 AA 00 .000000000.0+0 A00A00000000 0,A10 1 A0 0,0
    0...00 , ,1001A020B1..0000.0. 1001200B00,. ., ,0AA00 00..0,0
32  0,,A .0.0 010,B ,A01.A0,0..0, 0+2 BAB2A0.0, 0 0 ,0A.100 000
    00,00,0. .A0.01 01.010000 00,0A0,021A013,,000 0,02.1100 00,0
33  0..0000.,,0002E01A,1 000.000 0 2+0A1C000 . ...AAA 000.00 0
    ,, 0.0,00.,202 ,.100 00 01 .00B 0. A0.A0,10 0021A011 1A1,0
34  0 0 ..0 00,30A.AA10.00,0.0,0A 0+0.1B1A,0, 000A1.AA,0000, 0
    00 0.. 0,.,0100A 0AA ,1,. 0..A0A0,.0 12,000 ..AC0A.AA,.01 0
35  00, , .000 01 02,0.00,00,10,0BA0+,1B.0,,0..0.A0200000, 000
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36  0 .,0.0, ,.,2000.0A000, A0. 00A1.,+A000, 0 0 00AA01 00000.0
    ,, 0,A0.01000 0. 00 ,,0,.0A,A0.0B2, , . ,00A1.,00. 0. .
37  ,0.000.,,0.020A0AA0,A0, 0,00ABC11A+B.D ,,,000A01,B0A.,0000
    0,000.A0.,01,.000,00.00A00.0000010 00A1,. 1.,.BA0AA0000,.0,
38  0.0,.,0.,  .00AA1A210,.100.,0 .020BB0B+020, A..011A11 000,.
    .00.,000,0A 1,0A1B 00. 00.. 00,AA00000.1,,.0.00A011A0A .,1,0
39  ..,,,.000.000A, 100,00...A00A01.0.0+000 ..00A0 0000, 0,0
    0 0,. 0.01,0A010.1B,01A,,0,,.03.BB0B2B3 0,0 0100.12A 000,
```

```
40 0  . 0 0 . .     0 , 0 1 A 0 , . 0   A 0 . 0 0 . 0 0 . 0 0 0 0 0 A 0 0 0 2 0 +   ,   0 0 0 0 A 0 , 0 1 A 0 0     , 0 0
   0  0 0 0 0 0 , , A 0 A 0   0   0 0 , 1 0 . . , . 0 , 0 0   0 1 0 0 A 0 0 1 1 A   0 0 , 0 , 0 1 0 0 0 1 0 1   . A 0 0 ,
41 . 0  0   0 0 0   , 0 0 0 . , , 0       0 0 0 0 0   0 , , 1 0 .   , , ,   0 0   + A B A 3 0 1 0 0 0 0 0 . . . F 0 0 0 .
   A 0 0 0 0 0 1 0 0   . 0 0 0 , 0 , 0 0 0 0 0 0 1 . 1 0 A 0   0 A 0 , 0 0 , 0 0 A 0 0 , 2 1 0 0 1 1 0   0 0   0 0 0 A 0
42 0 . , 0 0 A , 0 0 0   0 0 0   . 0 . . , 0 1 0   0 0 , 0 1 0 0   0 ,   , , 0 , A + B 0 . , 0 0 . A     . 0 B   0 0 , 0
   0 1 . 0   0 0 1 0 0   . 0 0 . 0 . 0 0 0 2 0 1 A . . 0 , . . 0 0   0 . 0   0 , 0   . . 2 C A 3 0 0 0 0 0 0 . 0   A 0 0 0
43 1 . 0 1 0 1 0 0 0 , 0 . . . 0 0   ,   , , , 0 1 .   B A 0   0 , . , 0 0 ,     B B + 1 A 0 0 , , A   0 , . 0 2 0 0 . ,
   0 0 0 0 0 A 0 1 0   0 0 0 . . , . 0 , , 0 0 1 A 0 0 0 1 B 0 0 0   1 . 0 0 0 , 0 1 0 0   1 1 A A 0 1 0 A . 0 0 0 0 , 0 ,
44 2 . 0 0 0 0 0 .   0 0       , 0 0 , , 0 A 0 0 A , 0   . 0 0     .   , A   0 A 0 1 + 0 A 2 , 0 1 A , . , A 1 0 A 0 0
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45 A A . 0 0 0 0 A 0 A 0 0 , .   0 0 0 0   0 0 . A 0 . , 1 0 0   0   0 . 0 0 . . 0 3 . A 0 + 1 A 0 .   , A . 0 1 B . 1 0 .
   0 0 0 . 0 , 0 2 0 , 0   0 0 0 0 0 0 0   0 0 . 0 . .   0 0   0   .   0 , 0 1 0 . 0 0 , A 0 0 0   0 0 , , . 0 0 0 . , 0
46 .   0 , A   0 . 0 A 0 . . , , . . 0   0 0 0 0 A 0 B ,   A   0   . 0 0   0 . . 0 0 , 0 A 1 + C ,   , 0 0 A 0   , 0 1 . 0
   0 0   0 . 0 A 0 , 0 , ,   , 0 . , 0   , 1 A 0 1 0 0 0 0 1 0 , 0 0   0 0 1   0 0 . 0       0 A       . A   .     , 0 0 ,
47 1 . 0 A 0 1 0 0 0 2 0 . 0   0 , 0   ,   0 . . . A 1 0   1 1 ,   . 0 . 0 0 0 0 0 1 0 0 2 A C + , , 0 . . , A A 0 , 0 0 0
   A   . 0 1 0 0 0 . 1 0 0 , 0 0 0 , 0 . . , A , 0 1 A , 0 1 A , , , 0 0 0 0 . 0 A A 0 0 A 0 A A 0 0 . 0     . 0 0 A 0 0 0
48 0   , , 0 0 .   . 0 0 0 0 0 0 1 0 A . 1 . 0 0 0 0 ,   0 0   A 0 . A A 0 A 1 0 A 0 0 , , 0 , , + 1 . 1 1 0 1 0 0   .
   0 0 , 0 0 0 , 0 0 0 . 0 0 A 0 0 1 0 0 0 1 , 2 0 , . 1 , 0 0 , 0 0 0 , 0 B 0 A 2 0 . 0 0 0   1 0 0 1 A 2 0 A 0 . 0 0 A 0
49 0   , , 0 .     , , 2 0 2 8 , 0 . . 0 0 0 0 0 0   0 . 0 . 0 1   A 1 0 A 0 1 A 0 0 . , 0 .   , 1 + 0   0 0 1 0 0 0 0   0
   . . ,   , , 0 . , 0 0 0 0 1 1 0 0 1 A 0 0 , A . 0 0 1   0 , 0 0 B   , A 1 . . 0 0 , 0   . 0 0   2 3 A   0 . 0 0 0 . .
50 0   0 0 . , 0 . 0   A 0 0 1 2 0   0 1 A 1   1 , 0 0 1 1 . 1 0 , A . 2 A 1 A 0 , 0 A A 1   , 0 . 0 + A A , A 0   0 0 . .
   0   , , . 0 0 0 0 0   A , 1 A 1 0 0 , 0 1 , 0 0 , 0 . , , 1 B 2 . 0 A 0 A 1 0 1 0 0   0 0   0 1 A 1 A 3 0 A ,   , . 0
51     , 0 0 . . , . 0 A 0 0   1   . 0   , 0 . 0 A 0 0 0 0 , A   0 A A 0 0 , 1   0 0     A , 0 . 1   A + A A 1   0   0 0 0
       . 0 0 , 0 , 0 0 , 0 A 0 1 0 0 1 . 0 0   0 0 0 0 0 0 , 0 1 , A 0 2 A , 2 0 1   0 , . . . 0 A 0 1 A . A 0 0 , 0 0 .
52 0 .   0   0 0 . 0 . , A 1 , A 0 1 2 0   0 1 , 0 0 0 0 0 , 1 1 A   A 0 1 B 1 0 1 0   0 . A 0 . 1 0 A A + 0 B 0 0 , 0 0 0
   0     0 0 0   0 0 0 0 0 0 0 A , . 1 . 0   . , , . 0 . . ,   1 0 1 , A 1 A 0   2   , . 0 0 0 0 A 0   2 1 0 0 0   . ,   0
53     .     0     1 , 0 . . 0 A , 0   . , . A ,   . 0 0 0   . 0 , 0   0   0 A . , . , . . A , 0 0 , A 0 + A . , 0 A , 0
   .   0 . 0     0   0 A . A 1 0 0 A , 0 0 ,   0 0 , 0 0 . . , 0 , 0 0 1   1 A 3 A 0 0 0 0 0 . 0 A 0 . 0 A B 1 0 , ,   0 ,
54 ,   . . , 0 . 0   0 0 0   0 .   0 0 0 2 0 , , , 0 0 0 0 0 0 A 1 0 0 0 0 A 0 0 0 . 0 . , 0 0 A 1 1 A 1 B A +   0   , 0 0
   , .   0 0 0 ,     1 1   0 0 0 . , 0 0 0 0 0 0   ,     0 0 . 0 . 0 1 0 0 A A , . 0 ,   0 0 0 A A 0 0 B 2   , , 0 0 0 .
55 [ , . 1 1 , 0 0 . B   0 . 0 . 0   ,   0 A , 0 0 . 0 0 0 0 A 0 0 0 0 0 0 . 0 0 0 . B 0 A 1   A 0 0 0   0 .   + 0 A 0 0
   0 0 0 1 , A A A , 0 . 0 . . . 0 ,   .   0 0 B 0 0 0 A A   0     0   ,   0 0 0   0 . . 0 0 . 0 0 , A 0 A . . 0 0 . 2 1 1
56   0 . 0 . 0 . 0 .   0 0 0       0   , . A   A   0 , 0 0 . A   0 . 0 , 0 , 0 ,   F   2 1 B , 0 0 0   0 0 , 0 0 + , 0 A 0
   , 0 A 0 0 . . 0 A 0 0 0 .   . ,   . 0 0 ,   B 1 0 . . 0 0 0 0 0 0 A 0 . 0 0 0 0 . 0 , 0 0 A 0 0 , . 0 . 0 A 0 0 0 B 0 0
57 1 . A 0 C 0 0 0 0 0 , . 0 0     . . . . , . 0 A 0 0   0 0 . 0     0 0   0 0 ,     0 0 0 0 . 0 ,   0 0   , 0   A , + 0 A .
   , 0 1 1 . A 0     0 , . 0   0 ,   , 0 0 0 . 0 0 . . 0 0 . 1 0 0 0 0 0 , . 0 , 0 0 1   A 1 0 A 0 , , 0 A   . 0 0 0 1 1 0
58 0 0 0 1     0 0   . 0   0 0 0   , . . 0 0 . , 0 0 . 0 A 0 0 0 0 0 , 0 0 0 . 0 , 0 0 0 A 1 1 0 . 0 0 0 0 A , 0 0 0 + 1 2
   , 0 0 1   A . A 1 0 .     0 0 . 0 0 0 0   A 0 0 . 0 0 B 0 . 0 0 0 0 0 0 0   , 2 , 0   1 0 , . 0 1 0 A . 0 0 B 0 0 A 1
59 0 0 1 C 1 0 B . A 0 0 . . 0 . . A 0 0 0 0 0 . . 0 0 B A , 0 , 0     0 . 0   , 0 0 , . 0 0 . 0     , 0 0 , 0 0 A A 1 + 0
   0 0     A , 2 . 0 1 ,   . . 0 . 0 0   0 ,   1 0 , . 0 0 0 , 0 0   , 0 , 0 0 0 0 , 0 , 0 A 0 0 0         0 ,   . . 1 , 0 0
60 0 ,   1 , . 0 , 0 0 ,   . ,   , . 0 , , , . , 1 0 0 0 0 0   0 0 0 0 0 0 0   0 0 . 0 , 0 . 0 0   0 . 0 0 0 0   0 . 2 0 +
   1   0 B 0 1 0 1 0   , . , 0 0 , 0 0 0 . 0 0 , A   A 0 0 . A     0 0 0 1 , 0 0   A   0 1 B 0 0   0 0 . 1 0 0 A 2 0 B 0 A
```

60x60 Error Matrix page 2

2.16

SOLN B 1890 MeV

*** ERROR .43459114 73/09/15. 14.27.03. 1 (RUMBLE)

.0241	.0270	.0245	.0260	.0249	.0774	.0165	.0188	.1075
.0185	.0229	.1000	.0134	.0194	.0425	.0182	.0168	.0042
.0151	.0143	-.1129	.0147	.0180	.0406	.0161	.0155	.0391
.0130	.0152	.0267	.0139	.0145	.0223	.0160	.0121	-.3172
.0108	.0107	-.1726	.0102	.0132	.1684	.0110	.0112	.0172
.0096	.0111	.0249	.0091	.0094	.0600	.0080	.0116	.2046
.0086	.0108	-.0748	.0062	.0117	.0967	.0213	.0306	.0622
.0237	.0259	.0546	.0128	.0191	.0814	.0217	.0254	.0563
.0160	.0242	-.1054	.0142	.0186	-.1183	.0092	.0182	.1619
.0187	.0183	-.0581	.0128	.0189	-.2624	.0134	.0162	-.3397
.0165	.0153	.1004	.0181	.0134	-.0671	.0131	.0088	.1317
.0130	.0128	-.0744	.0110	.0112	-.0268	.0104	.0091	.0219
.0103	.0035	.3507	.0109	.0091	-.4018	.0082	.0096	-.2710
.0066	.0072	-.2061	.0178	.0309	.0173	.0275	.0251	-.1644
.0083	.0236	-.1323	.0157	.0211	-.0436	.0128	.0168	-.0619
.0115	.0158	-.0442	.0101	.0140	-.1430	.0183	.0143	.3690
.0200	.0166	.0611	.0130	.0116	.1818	.0123	.0106	-.1182
.0113	.0071	.1454	.0037	.0082	-.2121	.0077	.0065	.0023
.0215	.0290	-.0763	.0285	.0220	-.2928	.0148	.0192	.2673
.0237	.0223	.0643	.0137	.0131	.0188	.0158	.0177	-.1295

*** TMATRIX .43459114 73/09/15. 14.27.03. 1 (RUMBLE)

T

.2393	0.0000	.0719	-.0152	-.1862	.1401	.0000	0.0000	.1345	-.0547
.0000	0.0000	-.1235	-.1752	-.1019	-.1126	.0336	.1239	.0000	0.0000
.0625	-.0034	.0903	.2093	-.0730	-.1078	-.1341	.1342	-.0674	.1228
.0000	0.0000	.0000	0.0000	.0000	0.0000	-.1122	.1126	-.2687	.0082
.0000	0.0000	.0000	0.0000	.0021	-.0453	.0000	0.0000	.0000	0.0000
.0000	0.0000	-.1728	.0773	.0000	0.0000	.0000	0.0000	.0000	0.0000
.0000	0.0000	.0000	0.0000	.0267	.2054	.0000	0.0000	.0000	0.0000
.0000	0.0000	.1143	-.2578	.0000	0.0000	.0000	0.0000	-.1851	-.0987
-.3138	.0416	.1113	.2120	.4801	.0081	.0000	0.0000	.0000	0.0000
.0000	0.0000	.0000	0.0000	-.0758	.1242	-.0157	.1043	.0000	0.0000
.0000	0.0000	.0000	0.0000	.0000	0.0000	.0000	0.0000	-.3165	-.0604
.1463	.2729	.3264	-.0994	.0000	0.0000	-.1831	-.1485	.0000	0.0000

APPENDIX I (reproduced from D. J. Herndon, Thesis, LBL-544, 1972).

In this Appendix, we wish to review another property of the variance matrix, E, and the second derivative matrix, D, where D is E^{-1}. E (or D) is positive definite. This is necessary to ensure a maximum in the likelihood, L, rather than a saddle point or a minimum.

Defining the origin of the parameter space to be at the maximum, we can write

$$\mathcal{F} = \ln L = \ln L_0 - 1/2\,\chi^2 , \qquad \text{(A.1)}$$

and near the origin we can expand χ^2 as

$$\chi^2 = \underset{\sim}{A}^{\dagger} \cdot D \cdot \underset{\sim}{A} . \qquad \text{(A.2)}$$

The surface, enclosing the origin, defined by $1 = \underset{\sim}{A}^{\dagger} \cdot D \cdot \underset{\sim}{A}$ is known as the error ellipsoid. This surface intercepts each axis at the points $A_i^{int} = (D_{ii})^{-1/2}$. (A.3)

This is sketched, in two dimensions, in Fig. A.1. It can be shown* that the planes defining the circumscribed box intercept the axes at the points A_i^{max}, given by

$$A_i^{max} = (E_{ii})^{1/2} \equiv \delta A_i . \qquad \text{(A.4)}$$

(The plane $A_1 = A_1^{max} = \delta A_1$ is the dashed line of Fig. A.1). In A.4 we have written

$$E = \begin{pmatrix} \delta A_1 \delta A_1 & \delta A_1 \delta A_2 c_{12} & \cdots & \delta A_1 \delta A_n c_{1n} \\ \delta A_1 \delta A_2 c_{12} & \delta A_2 \delta A_2 & \cdots & \delta A_2 \delta A_n c_{2n} \\ \vdots & & & \\ \delta A_1 \delta A_n c_{1n} & \delta A_2 \delta A_n c_{2n} & \cdots & \delta A_n \delta A_n \end{pmatrix} \qquad \text{(A.5)}$$

---

* Rosenfeld, A.H. and Solmitz, F.T. Lawrence Berkeley Laboratory Group A Memo 753 (unpublished, 1972).

and $|c_{ij}| < 1$ and are the correlation coefficients. Now (as is evident from Fig. A.1)

$$(D_{ii})^{-1/2} \leq \delta A_i \qquad (A.6)$$

and the equality holds if and only if $c_{ij} = 0$ for all $j \neq i$. Thus one can get a measure of how strong the correlations are by comparing $(D_{ii})^{-1/2}$ with δA_i.

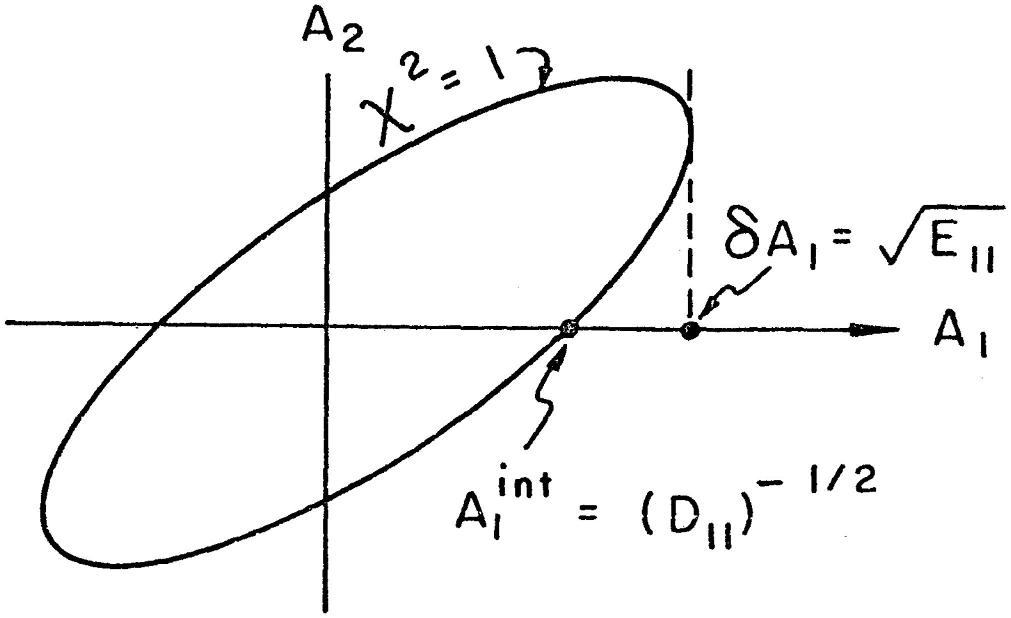
In Table AI we compare δA_i and $(D_{ii})^{-1/2}$ for all the waves at a typical energy, $\sqrt{s}$ = 1690 MeV. Note that there are three entries for $(D_{ii})^{-1/2}/\delta A_i$, which exceed 1.0, one of them by 20%! This difficulty presumably has to do with invalid approximations in the inversion of D. Remember that we fit in a space of two, too many variables; i.e., we use the real and imaginary part of all amplitudes, even though the overall amplitude and phase are undetermined. Therefore, D is singular, and we have to subtract two eigenvalues -- see Eq. 14 of Miller's thesis, Ref. 4. Apparently approximations in this procedure can introduce errors at the 10-20% level.

2.19

TABLE AI. Comparison of δA_i and $(D_{ii})^{-1/2}$ for all the waves at a typical energy, $\sqrt{s}$ = 1690 MeV.

		Re			Im		
		δA_i	$(D_{ii})^{-1/2}$	$(D_{ii})^{-1/2}/\delta A_i$	δA_i	$(D_{ii})^{-1/2}$	$(D_{ii})^{-1/2}/\delta A_i$
P_{33}	PP_{11}	0.0755	0.0630	0.834	0.0734	0.0528	0.719
	DS_{13}	0.0206	0.0162	0.786	0.0208	0.0144	0.692
	DD_{13}	0.1520	0.1281	0.843	0.1223	0.1036	0.847
	FP_{15}	0.0589	0.0382	0.649	0.0392	0.0288	0.735
	DD_{15}	0.1916	0.1021	0.533	0.1269	0.1269	0.551
	PP_{31}	0.0913	0.0745	0.816	0.0642	0.0533	0.830
	SD_{31}	0.2283	0.1944	0.852	0.1589	0.1346	0.847
	DS_{33}	0.0264	0.0194	0.735	0.0189	0.0134	0.709
	PP_{33}	0.1006	0.0689	0.685	0.0295	0.0327	> 1
	FF_{35}						
	FF_{37}						
RH3	DS_{13}	0.0973	0.0767	0.788	0.0882	0.0647	0.734
	FP_{15}	0.3184	0.2347	0.737	0.1910	0.1493	0.782
	DS_{33}	0.980	0.0742	0.757	0.0979	0.0701	0.716
	FP_{35}						
	FF_{37}						
RH1	SS_{11}	0.1111	0.0976	0.878	0.1356	0.0974	0.718
	PP_{13}	0.3066	0.2297	0.749	0.2924	0.2474	0.846
	SS_{31}	0.1549	0.1198	0.773	0.1414	0.1037	0.733
	PP_{31}	0.5790	0.4471	0.772	0.4885	0.4268	0.874
SIG	PS_{11}	0.2106	0.1559	0.740	0.1927	0.1675	0.869
	SP_{11}	0.6360	0.5678	0.893	0.4441	0.4621	> 1
	DP_{13}	0.3282	0.2855	0.870	0.3741	0.3107	0.831
	FD_{15}	0.4801	0.5859	> 1	0.6085	0.5367	0.882

2.20



XBL727-3577

Fig. A.1

III. DATA TAPES OF ALL THE $N\pi\pi$ EVENTS

All 160,000 $N\pi\pi$ events are available on 16 BCD data summary tapes, but from two different sources:

1) The π^+p events at or above $\sqrt{s}$ = 1810 MeV are available from Prof. Anne Kernan, University of California, Riverside, CA.

2) The rest of the events may be requested from LBL (Rosenfeld) or SLAC (Leith) or Saclay (Mme. M. Neveu).

The tapes consist of 132-character BCD records, two such records per event. Each record was written with an 11F11.8 format. Each event consists of 21 words including the center-of-mass 4-vectors (P_x, P_y, P_z, E) for each of the five particles in the following order,

words 1-4 : beam pion
5-8 : target proton
9-12: outgoing baryon
13-16: outgoing pion of same charge as beam
17-20: remaining outgoing pion
21: "mark number" M, specifying the final state:

$M = 2$ for $n\pi^-\pi^+_0$ $M = 4$ for $p\pi^+\pi^0$
$M = 3$ for $p\pi^-\pi^0$ $M = 5$ for $n\pi^+\pi^+$ [see 1), above].

The tape numbers and $\sqrt{s}$ bins (1 file/bin) are:

1.	DST131	1310 1340 1370 1400
2.	DST144	1440 1470
3.	DST149	1490
4.	DST152	1520

5.	DST154	1540	
6.	DST165	1650	
7.	DST169	1690	
8.	DST173	1730	
9.	DST177	1770	
10.	DST181-	1810 1850	π^- only
11.	DST189-	1890	π^- only
12.	DST193-	1930	π^- only
13.	DST197-	1970	π^- only
14.	DST181+	1810 1850	π^+ only
15.	DST189+	1890	π^+ only
16.	DST193+	1930 1970	π^+ only