SLAC-PUB-1103
(TH)
August 1972

INELASTIC STRUCTURE FUNCTION RELATIONS FROM SUMS OF DIRECT CHANNEL RESONANCES*

F. E. Close† and F. J. Gilman
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

ABSTRACT

Relations among the cross sections for both spin dependent and spin independent scattering of electrons and neutrinos on protons and neutrons are derived by summing over direct channel nucleon resonances in the symmetric quark model. The relations previously derived in the three quark-parton model are shown to correspond in the resonance approach to a sum over a particular combination of resonances belonging to the 56 and 70 dimensional representations of SU(6) which insures the absence of exotic exchanges in the t-channel of the current-nucleon elastic scattering amplitude.

(Submitted to Phys. Rev.)

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* Work supported by the U. S. Atomic Energy Commission.

† NATO Fellow 1970-1972. Present address: Daresbury Nuclear Physics Laboratory, Daresbury, Cheshire, England.

I. INTRODUCTION

The experiments on inelastic electroproduction[1], and in particular the scaling behavior of the observed inelastic structure functions, have led to great interest in parton models[2] where the electron interacts with point constituents inside the nucleon in the incoherent impulse approximation. With the further assumption that the partons can be identified with quarks, many relations between the structure functions in both electromagnetic and weak interactions can be derived.[2,3]

A separate and apparently unrelated development has been the s-channel resonance approach to inelastic electron scattering.[4] Here the forward virtual Compton amplitude, the imaginary part of which is proportional to the inelastic structure functions, is considered as a sum of resonances in the direct channel. If the resonances have appropriate excitation form factors which are functions of s/q^2, then the resulting inelastic structure functions will also be functions of s/q^2, i.e. they will scale. By duality considerations the sum of resonances should then correspond to the non-diffractive part of the virtual Compton amplitude. That the behavior of deep inelastic electron-nucleon scattering is empirically closely related to the electroproduction of the prominent nucleon resonances has been made manifest by the work of Bloom and Gilman.[5]

A prime motivation of the quark model of hadrons has previously been its relevance for classifying resonances and predicting their excitations, i.e. concerned with the coherent aspects of a composite nucleon containing three quarks. It is of interest to see if any of the results of the quark-parton model, which emphasizes the incoherent aspect of the electron-nucleon interaction, can be obtained from a quark model of resonance excitation and so provide some feeling for the interrelation between the parton and resonance model approaches to the inelastic structure functions. In reference 6 we in fact found a set of nucleon resonances in the symmetric quark model such that their electromagnetic excitation gives the same results for the ratio of neutron to proton inelastic electron scattering and for the polarization asymmetry on proton and neutron targets as does the naive three quark-parton model. In this paper we consider the relation of the two approaches in more detail, and in Section II we extend the calculations of reference 6 to the weak excitation of this same set of resonances at large q^2. We explain why the particular weighting of resonances found in reference 6 is to be chosen. Upon summing over these resonances we obtain the relations among the weak and electromagnetic structure functions which have been previously derived in the naive three quark-parton model using the incoherent impulse approximation. Assuming the validity of the Adler sum rule[7] we are able to obtain a charge-squared sum rule of the quark-parton model as well as the Gross-Llewellyn Smith sum rule.[8] A summary and discussion are to be found in Section III.

II. STRUCTURE FUNCTION RELATIONS IN THE SYMMETRIC QUARK MODEL OF RESONANCE EXCITATION

We begin with only the vector current part of the weak interaction. As in reference 6, we consider the symmetric quark model of baryons with the nucleon consisting of three quarks in a symmetric s-wave ground state. In our discussion of the nucleon's electromagnetic excitation in reference 6 we took only the interaction of the photon with the magnetic moments of the quarks, neglecting terms arising from their orbital motion, inasmuch as the magnetic terms dominate at large q^2 in various versions of the symmetric quark model.[9] In any case this forces the photon-nucleon interaction to be transverse, in agreement with theories of deep inelastic scattering containing spin $\frac{1}{2}$ partons and as suggested experimentally.[10]

The computation of electromagnetic and weak nucleon resonance excitation in the symmetric quark model is discussed in detail in reference 11 and is not reproduced here. We simply note that as we have a symmetric ground state and interactions with one quark at a time, only totally symmetric or mixed symmetry final states can be obtained, corresponding to the 56 and 70 dimensional representations of SU(6) respectively. We denote by $\sigma_{1/2}$ and $\sigma_{3/2}$ the total cross sections for "γ" + N → hadrons when the "photon" and nucleon spins are antiparallel and parallel respectively (i. e. when the net spin projection along the initial "photon" direction is 1/2 and 3/2). The vector current-nucleon cross sections[12] then receive contributions[9] as listed in Table I from the resonances in the various octets and decuplets which make up the 56 and 70 dimensional respresentations of SU(6).

Setting A = B as in reference 6, and defining $\sigma = (\sigma_{1/2} + \sigma_{3/2})/2$ as the spin-averaged total cross section, we sum over the resonance contributions in Table I and obtain the following relations for the s-channel resonance component of the vector current-nucleon cross sections:[12]

$$\sigma^{en}/\sigma^{ep} = 2/3 , \tag{1a}$$

$$A^{ep} = 5/9 , \tag{1b}$$

and

$$A^{en} = 0 , \tag{1c}$$

where

$$A = (\sigma_{1/2} - \sigma_{3/2})/(\sigma_{1/2} + \sigma_{3/2}) ;$$

$$\sigma^{\nu n}/\sigma^{\nu p} = 2 \tag{2a}$$

$$A^{\nu p} = -1/3 \tag{2b}$$

$$A^{\nu n} = 2/3 \; ; \tag{2c}$$

$$(\sigma^{ep} - \sigma^{en}) = \frac{1}{3}(\sigma^{\nu n} - \sigma^{\nu p}) \; , \tag{3}$$

$$(\sigma^{ep} + e^{en}) = \frac{5}{9}(\sigma^{\nu n} + \sigma^{\nu p}) \; . \tag{4}$$

These are exactly the relations found in the most naive three quark-parton model using the incoherent impulse approximation.

The condition A = B that was imposed on the excitation amplitudes of Table I and the resulting relations in Eqs. (1)-(4) above could have been obtained in a more direct way as follows. Consider the nucleon as a member of a 56 dimensional representation of SU(6) and assume the current is in a 35 plet. In general $\underline{35} \times \underline{56} = \underline{56} + \underline{70} + \underline{700} + \underline{1134}$, but with our symmetric three quark ground state and excitation of one quark at a time only the $\underline{56}$ and $\underline{70}$ are excitable. Allowing only the nonexotic $\underline{1}$ and $\underline{35}$ representations in the t-channel constrains the relative excitations of the $\underline{56}$ and $\underline{70}$ in the s-channel. The result of this constraint may be computed[13] by means of the crossing matrix and is that A = B in the normalization of Table I. We simply chose this to be the case in reference 6, but now understand it as the means of eliminating t-channel exotics.

The introduction of the axial-vector current adds little additional complication. A direct calculation of the symmetric quark model shows that if the weak quark current is of V-A form, then at large q^2 where we are interested, resonance excitation proceeds equally through the vector and axial-vector currents. Furthermore there is maximal interference between them. Thus for the axial-vector current alone one obtains exactly the same relations as in Eqs. (1) through (4). Assuming scaling (with $\omega = 2M_N \nu/q^2$) and using the conventional structure functions[14] we then have that for the combined vector and axial-vector excitation that

$$F_1(\omega) = \frac{\omega}{2M_N} F_2(\omega) , \tag{5}$$

since the interaction is transverse in both cases. The asymmetries in Eqs. (1) and (2) remain unchanged, but the right hand sides of Eqs. (3) and (4) double so that in terms of structure functions they become

$$F_2^{ep}(\omega) - F_2^{en}(\omega) = \frac{1}{6}\left(F_2^{\nu n}(\omega) - F_2^{\nu p}(\omega)\right), \tag{6}$$

and

$$F_2^{ep}(\omega) + F_2^{en}(\omega) = \frac{5}{18}\left(F_2^{\nu n}(\omega) + F_2^{\nu p}(\omega)\right), \tag{7}$$

while for the interference term $F_3(\omega)$:

$$18\omega\left(F_2^{ep}(\omega) - F_2^{en}(\omega)\right) = -\left(F_3^{\nu n}(\omega) + F_3^{\nu p}(\omega)\right) = 3\left(F_3^{\nu n}(\omega) - F_3^{\nu p}(\omega)\right). \tag{8}$$

Equations (1) through (8) are exactly the cross section or structure function relations of the most naive quark-parton model where the nucleon is composed of three point quarks. Note that Eqs. (1) through (8) do not set the absolute magnitude of any of the cross sections. In the parton model this magnitude is set by certain sum rules.[2] We cannot derive these sum rules in our approach, and so we arbitrarily take the Adler sum rule[7] to set the scale. In terms of the structure functions in the scaling limit above, the Adler sum rule reads:

$$\int_1^\infty \frac{d\omega}{\omega}\left(F_2^{\nu n}(\omega) - F_2^{\nu p}(\omega)\right) = 2\ . \tag{9}$$

Assuming this to be true, Eq. (6) gives us

$$\int_1^\infty \frac{d\omega}{\omega}\left(F_2^{ep}(\omega) - F_2^{en}(\omega)\right) = \frac{1}{3}\ , \tag{10}$$

which is a charge-squared rule in parton models, since in these models the right hand side is given by the sum of the squared charges of the partons in the proton minus those in the neutron (which equals 1/3 for quark-partons). Similarly, Eq. (8) plus the Adler sum rule and Eq. (6) yields

$$\int_1^\infty \frac{d\omega}{\omega^2}\left(F_3^{\nu p}(\omega) + F_3^{\nu n}(\omega)\right) = -6, \tag{11}$$

which was previously derived in the framework of the quark parton model by Gross and Llewellyn-Smith.[8]

The relations among cross sections or structure functions above which involve differences between neutron and proton targets or which involve $F_3(\omega)$ are unmodified by the addition of a nonresonant, diffractive component of the current-nucleon scattering amplitude, for such a component gives a contribution with the properties:

$$\begin{aligned} \sigma^{en} &= \sigma^{ep}\ , \\ \sigma^{\nu n} &= \sigma^{\nu p}\ , \\ A &= 0\ , \\ F_3 &= 0\ . \end{aligned} \tag{12}$$

Relations for sums such as Eq. (7) will of course be modified. Assuming SU(3) invariance and the usual octet classification of the photon, then the ratio of isovector to isoscalar diffractive cross sections should be 3 to 1. Thus for the diffractive component we expect instead of Eq. (7):

$$\frac{3}{4}\,(F_2^{ep}(\omega) + F_2^{en}(\omega) = \frac{1}{4}\,(F_2^{\nu n}(\omega) + F_2^{\nu p}(\omega)) \tag{13a}$$

or

$$F_2^{ep}(\omega) + F_2^{en}(\omega) = \frac{6}{18}\,(F_2^{\nu n}(\omega) + F_2^{\nu p}(\omega)) \tag{13b}$$

This is to be compared with Eq. (6) where 6/18 is replaced by 5/18. If both contributions are present, the coefficient of $(F_2^{\nu n}(\omega) + F_2^{\nu p}(\omega))$ will lie between these two numbers.[15] Such a difference would be rather hard to discern experimentally.

III. SUMMARY AND DISCUSSION

We have extended the calculations of reference 6 to include weak excitations of nucleon resonances. Upon summing over a particular set of s-channel resonances we reproduce the relations among structure functions which hold in the naive three quark-parton model. The particular set of s-channel resonances chosen corresponds to insuring the absence of exotic exchanges in the t-channel of the current-nucleon elastic scattering amplitude. Using the Adler sum rule for normalization we are then able to deduce several quark-parton model sum rules. The addition of a diffractive contribution can be easily handled and leads to obvious modifications of some of the relations. Scaling of the resultant structure functions is not forced in such an approach, but must be imposed, unlike in the parton models. On the other hand, there is no need to excuse the absence of partons or their decay products appearing in the hadronic final state in such an approach — the decay products of the nucleon resonances are simply the usual low mass hadrons.

Of course the electroproduction structure functions actually observed are difficult to explain with the naive quark-parton approach for which $\sigma^{en}/\sigma^{ep} = 2/3$ in the absence of diffraction and is greater than 2/3 in its presence. In the s-channel resonance approach with arbitrary weighting of the $\underline{56}$ and $\underline{70}$, $\sigma^{en}/\sigma^{ep} \geq 3/5$ if we keep the SU(6) symmetry of the model. Experimentally[1] this ratio is less than 1/2 in some regions so that neither simple approach can hope to explain the observed behavior. The s-channel approach does, however, permit the easy study of the effect upon the relations among the structure functions caused by various patterns of SU(6) and SU(3) breaking, as certainly occurs in nature. Conversely, experimental knowledge of the structure functions and asymmetries could shed considerable light on the actual pattern of s-channel resonances present in nature at large q^2.

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12. The total cross sections, which are functions of s and q^2 or equivalently the more conventional variables ν and q^2, are defined as in reference 6 except that we divide out a factor of e^2 in the electromagnetic current case and $(G/\sqrt{2})^2$ in the weak current case respectively.

13. The simplest way to obtain this result is to calculate the resonance sum for the particular process "γ^+" $+\Sigma^- \rightarrow$ "γ^-" $+\Sigma^+$ which has $I = 2$ in the t-channel and demand that the amplitude vanishes. The vanishing arises only if $A = B$. We thank Haim Harari for suggesting this method of showing that $A = B$ for the absence of t-channel exotics.

14. See the definitions of the structure functions W_1, W_2 and W_2 in C. H. Llewellyn Smith, reference 3. We take $F_1 = W_1$, $F_2 = \nu W_2$, $F_3 = \nu W_3$. Note that there is a factor of two between the weak and electromagnetic structure functions simply because of the conventional definition of the weak isospin raising current.

15. Note also that isospin invariance, CVC, and equal vector and axial-vector contributions to the inelastic scattering implies the inequality

$$\left(F_2^{ep}(\omega) + F_2^{en}(\omega)\right) \geq \frac{1}{4}\left(F_2^{\nu n}(\omega) + F_2^{\nu p}(\omega)\right) = (4.5/18)\left(F_2^{\nu n}(\omega) + F_2^{\nu p}(\omega)\right).$$

TABLE I

Contributions to $\sigma_{1/2}$ and $\sigma_{3/2}$ in the quark model for vector-current absorption by proton and neutron targets coming from the various octets and decuplets which make up the 56 and 70 dimensional representations of SU(6). A and B are dynamical factors related to the 0(3) structure of the supermultiplet wave function and S is the total quark spin. The absence of exotic exchanges in the t-channel constrains A = B and reproduced the quark-parton model results of the text.

	$\underline{56}$		$\underline{70}$		
	$\underline{8}$	$\underline{10}$	$\underline{8}$	$\underline{8}$	$\underline{10}$
	S = 1/2	S = 3/2	S = 1/2	S = 3/2	S = 1/2
$\sigma_{1/2}^{ep}$	2A	(4/9)A	2B	0	(2/9)B
$\sigma_{3/2}^{ep}$	0	(4/3)A	0	0	0
$\sigma_{1/2}^{en}$	(8/9)A	(4/9)A	(2/9)B	(2/9)B	(2/9)B
$\sigma_{3/2}^{en}$	0	(4/3)A	0	(6/9)B	0
$\sigma_{1/2}^{\nu p}$	0	(4/3)A	0	0	(6/9)B
$\sigma_{3/2}^{\nu p}$	0	4A	0	0	0
$\sigma_{1/2}^{\nu n}$	(50/9)A	(4/9)A	(32/9)B	(2/9)B	(2/9)B
$\sigma_{3/2}^{\nu n}$	0	(4/3)A	0	(2/3)B	0