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A DYNAMICAL MODEL OF HADRONS WITH FERMION QUARKS*

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ABSTRACT

A dynamical quark model of hadrons is constructed in terms of the usual triplet of fractionally charged spin 1/2 quarks (q). They have a light mass (≈ 300 MeV) and obey Fermi-Dirac statistics. In approximate nonrelativistic terms, the quark interactions are assumed to be described by a long range effective single particle Hartree potential with infinitely rising walls and by a strong short range Yukawa-type one between quark pairs. These combine to prevent single quark emission from a hadron and to give the observed early scaling and the asymptotic electromagnetic form factors $\sim 1/t^2$. The energy-momentum propagating in the field between the interacting quarks is idealized as a virtual particle (the "soul" of the baryon) so that the center-of-mass of the three quarks in the baryon and of the $q\bar{q}$ in the meson is not constrained. Implications of this model are discussed — in particular, the baryon wave functions and energy spectrum, the form factor behavior, the Adler anomaly for $\pi^0 \rightarrow 2\gamma$ decay, and the hadronic interactions.

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Composite models of the physical hadrons have been constructed recently in order to explain two basic properties of their observed structure and interactions:

I. The scaling properties of the structure functions, viz. $W_1(Q^2, 2M\nu) = F_1(x \equiv Q^2/2M\nu)$ in the Bjorken limit of deep inelastic scattering, can be derived from a bound state model that incorporates the essential feature that the amplitude of the relativistic wave function is finite at the origin.[1] Furthermore, if the binding potential has the form of a superposition of Yukawa potentials at small distances, i.e.,

$$V(q^2) = \int_{\sigma^2_{min}}^{\infty} \frac{\rho(\sigma^2)\, d\sigma^2}{q^2 - \sigma^2} \quad \text{with} \quad 0 < \int_{\sigma^2_{min}}^{\infty} \rho(\sigma^2)\, d\sigma^2 < \infty \tag{1}$$

the electromagnetic form factors for elastic scattering decrease as $G_M(t) \sim 1/t^2$ up to logarithmic factors for large t in accord with experiment.[2] If the range of the "short range" Yukawa potential is $\approx 1\ \text{GeV}^{-1}$ the onset of scaling will occur as observed at momenta of ≈ 1 GeV. These results which emphasize the small distance or high momentum components of the bound state can be derived rigorously from a model of the proton constructed of two point-like particles with spin 0 and 1/2, respectively, obeying the ladder approximation to the Bethe-Salpeter equation.

II. The fact that no point-like constituents of the proton (quarks or partons?) have been observed in nature has been explained in a relativistic model that generates a deep potential well (with infinitely rising walls) and at the same time, self-consistently, rising hadron trajectories.[3] A specific realization of this model in terms of a covariant harmonic oscillator leads to approximately linear trajectories; and just as there are no continuum scattering states in the eigenspectrum of the nonrelativistic simple harmonic oscillator, in this case there is also a total absence of free constituents in asymptotic scattering states. These results emphasize the long range or low momentum components of the binding potential. In particular, singular behavior of the potential at low or zero momenta, corresponding to the rising walls of the potential at large distances, is the mechanism killing the free constituent states. The typical range parameter for the well is taken as (few hundred MeV)$^{-1}$ $\approx$ 1f. It should be emphasized that in the model, the long range effective single quark potential is a consequence of a distortion of the interaction between quarks within a given hadron and is analogous to a Hartree potential. It does not act between the quarks in different hadrons. Thus, the size of the hadron is not characterized by the growing width of the effective r^2 potential, but rather is determined by the size of the wave function for the quarks moving in the potential within a hadron.

We are now proposing a dynamical quark model of the hadrons that incorporates these two nonoverlapping features into one picture. As a consequence of these observations we should like to formulate a quark model as follows in nonrelativistic terms, where $\vec{r}_i$ denotes the spatial coordinate of the ith quark:

1. We suppose the quarks to be a triplet of fractionally charged spin 1/2 particles obeying Fermi-Dirac statistics in accord with the ordinary connection between spin and statistics. They are taken to be light with rest masses $m_Q \sim 300$ MeV.

2. In order to keep the quarks bound to hadrons and thereby guarantee that only states of zero triality appear in the world, we suppose there is a long range component to an effective single particle potential of the (approximate) form $\omega(r_i/r_0)^2$ leading to linearly rising trajectories. The magnitude of r_0 will determine the size of the hadron wave function and the parameter ω gives the

characteristic splittings, $\omega \approx 500$ MeV, of the mass spectrum. In a nonrelativistic simple harmonic oscillator model r_0 can be determined from the quark mass m_Q and the level spacing ω and is given by $r_0 = \ 2/m_Q\omega \sim 0.7$f. We take this long range potential to a first approximation to be spin independent and an SU3 singlet in order to assure a common slope and a near degeneracy among the baryon trajectories, and also among the meson trajectories.

3. For the baryon (B) we do not put any constraint on the center of mass coordinate of the three quarks for the following reason. We imagine that the origin of the long range effective single particle potential which acts on each quark in the baryon is a consequence of its interaction with the other two quarks, plus their remaining large field mass or interaction energy. There will be momentum and energy in the field propagating between the three quarks and binding them together. We might think of this "field" as a virtual particle[4] (called the "soul" of the baryon). Its static analogue in nuclear physics is the shell model potential. However in contrast to the usual nuclear case the "soul" of the proton is relatively light as we discuss below. The average momentum and energy in the field propagating between the three quarks will in general depend on the particular quark states, and therefore so will the mass of the soul. The added three degrees of freedom that we associate with the "soul" are the essential residue in a nonrelativistic description of all the infinite degrees of freedom associated with the "field" energy. Therefore there will be no overall center-of-mass constraint on the 3 quark coordinates; that is, we have B=qqqs, where s is, in effect, a spinless, neutral boson, which together with the three quarks fixes the center-of-mass of the baryon. The long range effective single particle potential is assumed to bind the three quarks with coordinates $\vec{r}_1, \vec{r}_2, \vec{r}_3$

to the "soul" at $\vec{r}_s$; viz.

$$V = \frac{\omega}{r_0^2}\left\{(\vec{r}_1-\vec{r}_s)^2 + (\vec{r}_2-\vec{r}_s)^2 + (\vec{r}_3-\vec{r}_s)^2\right\} = \omega\left(\rho^2+\lambda^2+4\sigma^2\right)/r_0^2 \qquad (2)$$

where

$$\rho \equiv (\vec{r}_1-\vec{r}_2)/\sqrt{2}\ ; \qquad \lambda \equiv (\vec{r}_1+\vec{r}_2-2\vec{r}_3)/\sqrt{6}\ ; \qquad \sigma \equiv (\vec{r}_1+\vec{r}_2+\vec{r}_3-3\vec{r}_s)/\sqrt{12}$$

are the usual relative coordinates for a four body system. Similarly in the case of the meson there will be an effective long range single quark (antiquark) potential acting on each of the two constituents and binding them together.

4. We introduce a strong, short range interaction between pairs of quarks (qq) and also between each quark and the soul (qs). If we assume that this short distance interaction is mediated by neutral vector gluons we are led to a repulsion between all pairs (qq) and (qs). If the range of this repulsion is taken to be of order of 1 $\mathrm{GeV}^{-1} \approx 0.2$ f as discussed above, then it will have no appreciable effect on the spectrum or wave functions for those states which inhibit the probability of all pairs (qq) and (qs) approaching within 0.2 f of one another. We assume then that this short range interaction is repulsive and strong in order to push the 3-quark S state, which is an SU(6) $\underline{20}$ (antisymmetric), up in energy by $\gtrsim$ several GeV. In terms of a short range quark-quark repulsion of form $be^{-Mr_{ij}}/r_{ij}$ this requires $b \approx 10$. The lowest state will then be one in which the orbital angular momentum for each quark is the smallest, consistent with the short range repulsion being absent for all the quarks. It is clear that if we construct a totally antisymmetric spatial wave function of the form ($\vec{r}_{is} \equiv \vec{r}_i-\vec{r}_s$)

$$\phi(\vec{r}_1,\vec{r}_2,\vec{r}_3) \propto (\vec{r}_{1s}\cdot\vec{r}_{2s}\times\vec{r}_{3s})\ u\left(r_{1s}^2, r_{2s}^2, r_{3s}^2\right)$$

$$\propto (\vec{\rho}\cdot\vec{\lambda}\times\vec{\sigma})\,u \qquad (3)$$

where u is symmetric and nodeless, the quarks will maintain a maximum relative separation, consistent with the smallest ℓ value for each quark and will form a totally symmetric $\underline{56}$ representation of SU(6). It is the freeing of the center-of-mass constraint on the three quark coordinates that makes the form of the wave function in (3), i.e., a (p^3) L=0 state, possible.

Due to the short range (qs) repulsion the totally antisymmetric state formed from an (sp^2) configuration of the three quarks, viz.

$$\overrightarrow{(r_1 \times r_2)} + \overrightarrow{(r_2 \times r_3)} + \overrightarrow{(r_3 \times r_1)}$$

will also be pushed up in energy. This (unwanted) L=1 $\underline{56}$ will then appear in the excited spectrum.[5] If the (qs) short distance potential like the long range one is also in the nature of a Hartree-effective potential it will presumably be state dependent and, for example, it could be stronger when the quarks overlap.

Equation (3) is, of course, to be understood as no more than a nonrelativistic ansatz to a proper relativistic wave function. Familiar theorems requiring that ground states of a Schroedinger equation must have no nodes cannot be applied against (3) due to the relativistic effects that are neglected, and which will be particularly important for wave functions which overlap the strong short-range repulsions.[6]

Since we have assumed that the short-range strong potential between pairs of quarks is mediated by neutral vector gluons it becomes an attraction between the $q\bar{q}$ pair in a meson which will therefore contain no factor as in (3) in its wave function to keep the $q\bar{q}$ apart. In contrast the attraction will enhance the $q\bar{q}$ wave function at the origin relative to the standard quark model; we shall discuss the qualitative effects of such an enhancement in predictions of meson decays based on quark model calculations later on.

At this point we should like to stress that the existence of the short range, $\approx$ (1 GeV)$^{-1}$, interaction is not just an ad hoc assumption designed to remove the SU(6) $\underline{20}$ from the lowest position in the baryon spectrum. It is a consequence of the short distance dynamics needed in order to obtain the $1/t^2$ falloff of the elastic electromagnetic form factor of the baryon in contrast to the exponential behavior $e^{-|t|}$ which would be characteristic of a pure harmonic force. It also gives the correct threshold behavior for the inelastic form factors and is compatible with the observed early onset of scaling at $q^2 \gtrsim 1$ GeV^2. If we assume that this short distance interaction is mediated by neutral vector gluons we are led to a repulsion between pairs of quarks.

Although we cannot make specific predictions on the basis of our crude model without entering into dynamical details, we have presented here the basic ideas that are required in order to overcome the usual arguments against a simple picture of hadrons as being composed of three light quarks obeying the ordinary spin-statistics connection. The ultimate success or failure of such a model must lie with its ability to make specific predictions of transition matrix elements and hadron spectra to be confronted by data. These we are not here providing. In particular we offer no direct test of the existence of the "soul" s of the hadron that was necessarily introduced in order to free the center-of-mass constraint on the three quarks and thereby write (3). It is clear however that we cannot totally ignore all the dynamical degrees of freedom in the interaction field energy and momentum and construct states only in terms of the three valence quarks of the Gell-Mann-Zweig scheme that give the symmetry classification. There is a correspondence to this soul in the familiar parton models. In calculations of the structure functions of deep inelastic scattering that are based on quark-parton models it has been found necessary to introduce a sea of quark-antiquark pairs plus neutral gluons, all with vacuum quantum numbers, in addition to the three valence quarks.[7] Moreover some configuration mixing[8] is strongly suggested by recent results on the difference of neutron and proton structure. This presence of a "sea plus glue" corresponds in parton models, in the infinite momentum frame, to the soul introduced here.

Baryons

What we are proposing then for the model of a baryon is a system of 3 quarks moving relative to a core, or "soul". The lowest state will have the antisymmetric spatial wave function (3) and the spin-unitary spin part will be the standard totally symmetric SU(6) $\underline{56}$. In this way we shall obtain a quark picture which will be completely consistent with the ordinary relation between spin and statistics for the quarks. Because of the cross product factor in (3) the quarks are inhibited from approaching close to one another. Whereas the short range repulsion that we have introduced: $be^{-Mr_{ij}}/r_{ij}$ with $b \approx 10$, shifts the L=0 $\underline{20}$ and L=1 $\underline{20}$ configurations[5] up by several GeV in energy, enhancing thereby also the kinetic energy of the individual quarks, its effect on the $\underline{56}$ described by (3) is reduced by a factor $\approx (1/Mr_0)^2 \sim 2/25$. We thus retain a picture of nonrelativistic free quark motion in this ground state as well as in those excitations which retain the factor $\vec{r}_1 \cdot \vec{r}_2 \times \vec{r}_3$ and are, thereby, also low lying. Furthermore in relating our model to current algebra we expect the free quark aspect as well as the triplet structure of the current operators to apply for all matrix elements dominated by the low lying states.

Since the factor $\vec{r}_1 \cdot \vec{r}_2 \times \vec{r}_3$ is a (pseudo) scalar, it will have no influence on the results of the conventional quark model which depend only on the SU(6) part of the wave function.

That is, for example, the baryon magnetic moments and the famous proton-neutron ratio $\mu_p/\mu_n = -3/2$ will be maintained. In particular, we can assign normal g-values to our light quarks[10] with masses $m_Q \approx 300$ MeV.

Turning to the excited baryon spectrum, experiment indicates that the first excited level should be an L=1 $\underline{70}$ of negative parity relative to the ground state as given by the ordinary orbital excitations of single quarks in the standard quark model. We form such an excited state here, retaining the additional factor $\vec{r}_1 \cdot \vec{r}_2 \times \vec{r}_3$ as in (3) in order to avoid the short-range repulsion, with the radial wavefunction,

$$\vec{r}_{1s}(\vec{r}_1 \cdot \vec{r}_2 \times \vec{r}_3)\, u\left(r_{1s}^2, r_{2s}^2, r_{3s}^2\right) \tag{4}$$

This is the state that we want for the first excited baryon level. However there also appear in our model new families of states due to the added degrees of freedom introduced with the coordinates of the soul and these create a problem. Some of these states are usually dismissed as spurious in shell models,[11] since they correspond to a rigid translation of the 3 quarks relative to the soul at $\vec{r}_s$. There is an L=1 $\underline{56}^-$ excitation of this type of the form

$$(\vec{r}_{1s}+\vec{r}_{2s}+\vec{r}_{3s})(\vec{r}_1 \cdot \vec{r}_2 \times \vec{r}_3)u \propto \vec{r}_s(\vec{r}_1 \cdot \vec{r}_2 \times \vec{r}_3)u \tag{5}$$

Since there are no strong indications that such levels are excited[12] — for example an I=3/2, J=5/2$^-$ (D_{35}) level appearing in the decouplet is not strongly excited in π^+p scattering — we must either suppress such matrix elements or else raise these levels in energy above the relatively well identified region of the baryon spectrum. Lacking a reason for the former we choose the latter course of raising the excitation energies for levels of type (5). This is most simply accomplished in the oscillator model (2) by specifying the soul to be of lighter mass than the quarks since the excitation intervals for states of type (5) relative to the L=1 70 in (4) scale as $\sqrt{(3m_Q + m_S)/m_S}$. We cannot of course make the soul arbitrarily light relative to the quarks since the energy associated with the core excitations, i.e., $\Omega_{core} = \omega \sqrt{(3m_Q + m_S)/m_S}$ must be less than reduction in energy associated with the quarks staying out of the short-range repulsion — i.e.,

$$\omega \sqrt{(3m_Q + m_S)/m_S} < \text{several GeV} \quad . \tag{6}$$

If for example we specify $m_S \approx 200$ MeV, we satisfy (6) for choices of $\omega \approx 500$ MeV and $m_Q \simeq 300$ MeV as discussed earlier and, since $\Omega_{core} \approx 2.3\,\omega > 1$ GeV the 56 1^- multiplet will lie above the first excited 70 1^- levels.

In addition there are excited 56 multiplets with totally antisymmetric space wave functions formed from (sp^2) and (p^2d) configurations with total L=1 and 3. *A priori*, and without appealing to spin and unitary spin corrections to a predominantly singlet potential, these should be equal candidates with the L=1 70^- in (4) for the first excited hadron states. The mechanism suppressing these states or pushing their energies up (perhaps spin-orbit effects) will have to be introduced ad hoc and we have no general arguments against their (unwanted) appearance.

Specific predictions of our model that depend on the actual form of the spatial wave function may differ from those of the conventional quark model. Examples of such modification will be in calculations of the variation of matrix elements with the magnitude of the momentum transfers, such as a comparison of electroproduction with photoproduction amplitudes.[13] A specific question of importance is how does the node that we have introduced in our spatial wave function (3) whenever one of the quarks is at the origin affect the behavior of the electromagnetic form factor at large momentum transfer t. This question is raised because of the familiar result from the nonrelativistic Schrodinger equation according to which the form factors generally decrease more rapidly with increasing three-momentum transfer $\vec{q}$ when such nodes are present than when they are not.[14] In particular, (3) leads to a form factor decreasing as $1/|q|^6$ whereas the rate of decrease would be as $1/|q|^4$ for a nodeless state.

There is no directly analogous result for the relativistic problem because of retardation effects that become important when we are probing the small distance or high momentum components of the bound state wave function as we do in a study of $G_M(t)$ for large t, or of $W_1(x)$ as $x \to 1$. Therefore a non-relativistic wave function as in (3) cannot be used to study such asymptotic behavior. To illustrate this explicitly we can turn to the specific model[1] of a composite proton built of two constituents of spin 0 and spin 1/2, respectively, that are bound to another by scalar gluons in a ladder approximation to the Bethe-Salpeter equation. In such a model it is easy to show[15] that the asymptotic behavior of the relativistic wave function which controls the large t behavior $G_M(t) \sim (1/t^2)$ is not altered by a requirement that its static approximation vanish at $\vec{r}=0$.

Mesons

Turning next to the case of mesons, the strong short range interaction which we have assumed to be mediated by neutral vector gluons is one of strong attraction between quark-antiquark pairs. Therefore the wave function will not vanish when the $q\bar{q}$ are close together but rather will be large. Furthermore since the quarks have a sizable probability to be within the strong potential we would not expect a nonrelativistic description to be possible for quark motion in a meson, in contrast to the baryon. Therefore we would not expect the formulas for the meson mass spectrum to be expressed simply in terms of m, as in the Gell-Mann-Okubo formula for baryons. Furthermore the freeing of the center-of-mass constraint on the $q\bar{q}$ pair will lead to the existence of "spurious" excited states corresponding to rigid translation of the $q\bar{q}$ relative to the soul. Thus an additional $L=1^-$ 35 multiplet will occur and lie above the standard one for a suitably light soul mass.

The strong short-range attraction will also have an important quantitative effect on the calculated lifetime for $\pi^0 \to 2\gamma$ decay. The analysis of Adler[16] seems to require quarks with integer charge in contrast to the common fractionally charged triplet that leads to a decay rate that is too small by a factor 1/9. However the Adler calculation is likely to be strongly altered, and the decay rate increased, if the π^0 is a bound state[17] of a $q\bar{q}$ with a strong short-range attraction. Formally this is because the calculation requires an extrapolation of the form factor at the $\pi^0 \to 2\gamma$ vertex over the time-like interval from 0 to m_π^2. Such an extrapolation is assumed to be smooth in a perturbation theory calculation of gluon radiative corrections and, in particular, the corrections are assumed to be characterized by a small mass ratio m_π/M where $M \sim 1$ GeV is the gluon mass. However such an order-by-order perturbation analysis fails to generate the bound state that we are assuming in our dynamical pion model. For a bound system of a $q\bar{q}$ pair forming the π^0 there is a lower mass scale against which to measure how great an extrapolation is being made from 0 to m_π^2. This scale can be expressed in terms of the size of the bound state wave function. If in analogy to the case for baryons we take $1/r_0 \sim 300$ MeV, the extrapolation is over a large range, $m_\pi r_0 \sim 1/2$, and large effects may occur in the vertex over this interval.

Physically what is happening is that the wave function of the $q\bar{q}$ system will be enhanced at the origin by the attraction and thus we expect the decay rate to be increased above the perturbation result. As an exceedingly crude estimate of how big this enhancement might be we take the ratio of the density at the origin for a $q\bar{q}$ pair bound together by a strong attraction with a wave function

$$\psi_{BS}(r) \sim \sqrt{\frac{b^3}{\pi}}\ e^{-br}\ , \qquad \text{with}\ \ b = \sqrt{m_Q(2m_Q - m_\pi)}$$

to the density for a free quark in a sphere of volume $\frac{4\pi}{3}\ r_0^3$. This ratio is $\psi^2_{BS}(0)/\psi^2_{Free}(0) \sim 10$ and suggests very crudely that the Adler anomaly might be made compatible with fractional quark charges in a bound state model. No such factor enters into the standard PCAC analyses, such as the Goldberger Treiman relation, at nucleon vertices or for hadron scattering, and therefore the smoothness assumption required for their success in the extrapolation over the interval from 0 to m_π^2 will not be similarly disturbed.

The enhancement of the $q\bar{q}$ wave function at small separations resulting from the strong short range attraction also provides a possible mechanism for explaining the variation of the boson decay rates to lepton pairs.[18]

A further quantitative elaboration of these conjectures and their impact on the Gell-Mann-Okubo mass formula is beyond the scope of the present speculations. We comment only that they are not incompatible with the structure of the first order mass formulae.

Hadron Processes

We would now like to discuss the qualitative features that our quark model will have in the case of hadron-hadron processes. In the case of high energy forward scattering we expect that our model will coincide with the ordinary Regge picture. This is because the potential wells of the two hadrons do not overlap for large impact parameter collisions and therefore the individual quarks within the two wells do not interact directly. Rather it is the long range exchanges that are playing the dominant role and these are via the hadron trajectories which in the model are linearly rising. Further, the residue functions $\beta(t)$ which express the coupling of the hadrons to a trajectory will fall with t on a scale given by the hadron size, or several hundred MeV.

For high momentum transfer, on the other hand, the characteristic quark processes will manifest themselves. In this case the incident hadron wave functions will overlap and the interaction will be characterized by the two processes of quark-quark scattering between the constituents of the overlapping hadrons, and by quark-antiquark exchange. While the former will transfer units of angular momentum and charge conjugation, the latter mechanism of quark-antiquark exchange will also transfer SU3 quantum numbers when the potential wells of the respective hadrons overlap. Thus the effective matrix element will be a superposition of the quark-quark gluon interaction at short range, and of the $q\bar{q}$ particle exchange. Gluon exchange will lead to an amplitude of the form of the product of two currents multiplied by a propagator $1/t-M^2$ for invariant squared momentum transfer t carried by a gluon of mass $M \sim 1$ GeV. It is natural to expect the structure functions for the gluon emission and absorption to be similar to that for the electromagnetic currents, leading to the Wu-Yang result of the product of electromagnetic form factors for elastic scattering or resonance excitations[19] (exclusive reactions), or to scaling functions for inclusive ones as proposed by Berman and Jacob.[20] In contrast to the original proposals the t dependence also contains an additional factor of $(1/t)^2$, for $|t| >> 1$ GeV^2, arising from the gluon propagator. Concerning the $q\bar{q}$ particle exchange, the qualitative behavior will coincide with the one described by quark-dual diagrams,[21] and in particular when quantum numbers are exchanged the absence of "exotic" reactions is understood in terms of the dynamical model with the baryon built of three quarks and the meson of one $q\bar{q}$ pair.[3] We cannot make an a priori prediction as to the relative strengths of these two interactions.

The physics picture that emerges is similar to the one which occurs in molecular scattering. Thus, we will have mutual quark excitation in the respective wells through the quark-quark interaction and $q\bar{q}$ exchange processes. Subsequently the excited quarks in the outgoing hadrons will make radiative decays, transforming their potential energy into $q\bar{q}$ pairs which emerge as ordinary mesons, just as radiation from the excited vibrational rotational levels of molecular wells leads the molecules back to their ground states in the absence of collisional de-excitation.

Finally, within the context of this model we have nothing to say about the possible radiation of the neutral vector (or perhaps axial-vector) gluons that mediate the strong, short range interaction. If such radiation is not suppressed it would appear as a vector (or an axial-vector), SU(3) singlet of mass ≈ 1 GeV perhaps mixed with the octet. This question of gluon radiation is an important one for all composite bound state models of hadrons.

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4. The notion of a baryon "core" in addition to the three quarks has occasionally appeared in the literature. In particular in F. Gursey and L. Radicati, Phys. Rev. Letters 13, 173 (1964) and as reported by R. Dalitz, High Energy Physics (Les Houches 1965), p. 286, the notion of a quark-core attraction and of a strong quark-quark repulsion is introduced. See also F. Gursey, T. D. Lee and M. Nauenberg, Phys. Rev. 135, B467 (1964). L. Kisslinger and H. Feshbach, Ann. Phys. (N.Y.) 66, 651 (1971). The "soul" also appears in H. Fritzsch and M. Gell-Mann (Proceedings of Tel Aviv Conference on Duality, 1971, p. 317) as a spinless fermion to fix up the statistics for baryons with boson quarks.

5. A $\underset{\sim}{20}$ state with L=1 formed with a wave function containing the factor $[\vec{\underset{\sim}{r}}_1(\vec{\underset{\sim}{r}}_2\cdot\vec{\underset{\sim}{r}}_3) + \vec{\underset{\sim}{r}}_2(\vec{\underset{\sim}{r}}_3\cdot\vec{\underset{\sim}{r}}_1) + \vec{\underset{\sim}{r}}_3(\vec{\underset{\sim}{r}}_1\cdot\vec{\underset{\sim}{r}}_2)]$ is also pushed up in energy with the L=0 $\underset{\sim}{20}$ state because the dot product does not keep pairs of quarks separated as does the cross product in (3).

6. Simple models of the inapplicability of the no-node theorem are provided by the Dirac equation in both one and three dimensions with a static potential that is the fourth component of a vector, and which is strong enough to illustrate the "Klein paradox". In contrast with the Schrodinger equation the kinetic energy increases linearly not quadratically with momentum and if the potential energy also increases with momentum the overall energy

balance can favor the presence of nodes. We mention the three dimensional case explicitly. The one dimensional example is exactly analogous and is, of course, even simpler.

The Dirac equation is

$$E\psi = (\vec{\alpha}\cdot\vec{p} + m\beta + v)\psi$$

and can be separated using the Dirac operator

$$k = \beta(\vec{\sigma}\cdot\vec{L} + 1) \quad .$$

The lowest states are expected to be the $k=\pm 1$ states. $k=1$ corresponds to the nonrelativistic S states and $k=-1$ to the nonrelativistic P states. We define

$$\psi = \frac{1}{r}\left(u_s + i\frac{\vec{\alpha}\cdot\vec{r}}{r}u_p\right)$$

u_s obeys, in the case where v is a sum of square wells,

$$-\left(\frac{\partial}{\partial r}\right)^2 u_s = \left((E-v)^2 - m^2\right)u_s \quad . \tag{A}$$

With the boundary condition that u_s and u_p are continuous at the discontinuities, we obtain

$$\frac{1}{E\pm m-v_>}\left(-\left(\frac{1}{u_s}\frac{\partial u_s}{\partial r}\right)_> + \frac{1}{r_>}\right) = \frac{1}{E\pm m-v_<}\left(-\left(\frac{1}{u_s}\frac{\partial u_s}{\partial r}\right)_< + \frac{1}{r_<}\right) \tag{B}$$

at the jumps. $\pm m$ corresponds to the cases $k=\pm 1$ respectively. We study (A) and (B) for the potential

$$v(r) = \begin{cases} v_0 & r<a \\ -v_1 & a<r<b \\ 0 & r>b \end{cases}$$

where v_0 is strong enough that (A) manifests the Klein paradox, i.e., $(E-v_0)^2 - m^2 > 0$.

For bound states $E<m$, in the limit where $v_0 \gg m$, and where for the sake of illustration only, we take

$$\begin{aligned} a(E\pm m - v_1) &\gg 1 \\ b(E\pm m - v_1) &\gg 1 \quad , \end{aligned} \tag{C}$$

the conditions (B) at r=a and r=b become,

$$\begin{aligned} \frac{\sqrt{m^2-E^2}}{\pm m + E} &= \sqrt{\frac{E+v_1\mp m}{E+v_1\pm m}}\ \frac{\tan\left(b\sqrt{(E+v_1)^2-m^2}\right)-R}{1+R\tan b\sqrt{(E+v_1)^2-m^2}} \\ \operatorname{ctn}(v_0-E)a &= \sqrt{\frac{E+v_1\mp m}{E+v_1\pm m}}\ \frac{\tan\left(a\sqrt{(E+v_1)^2-m^2}\right)-R}{1+R\tan a\sqrt{(E+v_1)^2-m^2}} \end{aligned} \tag{D}$$

The S state wave function in both cases in the 3 regions has the form, which also defines R,

$$u_s = A_1\, e^{-r\sqrt{m^2-E^2}} \qquad r>b$$

$$u_s = A_2\left(\cos r\sqrt{(E+v_1)^2-m^2} + R\sin r\sqrt{(E+v_1)^2-m^2}\right) \qquad b>r>a$$

$$u_s = A_3 \sin r(v_0-E) \qquad r<a$$

We see that this wave function will have nodes when v_0 is large enough in the domain $r<a$. We see that when $m\to -m$, so the "S" and "P" cases are interchanged, (D) is restored by letting $av_0 \to av_0+\frac{\pi}{2}$ (by letting the dummy $R\to -\frac{1}{R}$). That is, if $E_s<E_p$ for the potential av_0, then $E_s>E_p$ for the potential $av_0+\frac{\pi}{2}$, or vice-versa. Therefore there is no preferred ordering of the ground state levels, and the ground state wave function will have nodes. We remark that the condition (C) and $v_0 \gg m$, are sufficient for this. They may not be necessary. Continuum states exhibiting oscillations and the Klein phenomena for $r>b$ and with $E<-m$ are ruled out in our model.

7. J. Bjorken and E. Paschos, Phys. Rev. 185, 1975 (1969).

J. Kuti and V. Weisskopf, Phys. Rev. D4, 3418 (1971).

8. C. H. Llewellyn Smith, Nucl. Phys. B17, 277 (1970).

9. R. Dalitz, op. cit.; also Oxford High Energy Conference (1965) and Proceedings of the XIIIth International Conference on High Energy Physics, Berkeley, Aug. 31 - Sep. 7, 1966 (University of California Press, Berkeley, 1967). G. Morpurgo, Proceedings of the XIVth International Conference on High Energy Physics, Vienna, Aug. 28 - Sep. 5, 1968 (CERN, Geneva, 1968). R. van Royen and V. Weisskopf, Nuovo Cimento 50, 617 (1967). B. T. Feld, Models of Elementary Particles (Blaisdell, Waltham, Mass., 1969).

10. There will be radiative corrections to the g values due to emission and absorption of virtual gluons by the quarks. The magnitude of such self effects may be in fact relatively small as a result of the large gluon to quark mass ratio $(M/m_Q) \sim 3$.

11. It is of course also true that our candidate ground state (3) is "spurious" since it vanishes if the center-of-mass of the three quark system is fixed — i.e., if $m_s=0$. We are here using the term "spurious" with reference to higher levels in which the three quarks are excited to rigid modes of motion relative to the soul.

12. Alan Rittenberg et al., Rev. Mod. Phys. 43, No. 2 (April 1971).

13. Such an effect has been discussed recently by F. Close and F. Gilman, to be published (SLAC-PUB-1007).

14. cf. S. D. Drell, A. Finn and M. H. Goldhaber, Phys. Rev. 157, 1402 (1967). See also the discussion in Ref. 9 of oscillations due to the presence of nodel planes.

15. Introduce p, with $p^2 = m_p^2$, for the proton's on-shell momentum and $\frac{1}{2}p \pm q$ with $s_{1,2} = (\frac{1}{2}p \pm q)^2$ for the off-shell momenta of its two constituents of mass M and μ, respectively, and with f and λ representing the coupling strength and mass of the exchanged gluons, then

$$(\tfrac{1}{2}\not{p} + \not{q} - M) \left[(\tfrac{1}{2}p - q)^2 - \mu^2\right] \psi_p(q) = \frac{f^2}{(2\pi)^4} \int \frac{d^4k}{(k-q)^2 - \lambda^2} \psi_p(k). \qquad \text{(A)}$$

In order to make the nonrelativistic reduction of this Bethe-Salpeter equation for an (almost) instantaneous interaction, the well known procedure is to construct $\int dk_0 \psi_p(\vec{k}, k_0) \equiv \phi_p(\vec{k})$ for the momentum space wave function. Fourier transforming to coordinate space we have

$$\phi_p(\vec{x}) = \int e^{i\vec{k}\cdot\vec{x}} \phi_p(k)\, d^3k = \int d^4k\, e^{i\vec{k}\cdot\vec{x}} \psi_p(k)$$

or

$$\phi_p(0) \equiv \int d^4k\, \psi_p(k) \equiv \tilde{\psi}_p(0) \qquad \text{(B)}$$

Therefore in this almost static limit it is the vanishing of the right-hand side of (B) that corresponds to a node at the origin as in (3). According to (A), however, the asymptotic behavior of $\psi_p(q)$, which controls the rate of falloff of the electromagnetic form factor, is unaltered by a requirement such as the vanishing of (B). Explicitly writing the propagator under the integral in (A) for large q but bounded k, since the r.h.s. converges, we have

$$\lim_{q \to \infty} (k-q)^2 - \lambda^2 \to \frac{s_1 + s_2}{2} \left[1 - \frac{k \cdot n_q}{m_p} \left(\frac{s_1 - s_2}{s_1 + s_2}\right)\right]$$

where n_q is a unit null vector with space components along q. As was shown earlier[1, 2] the asymptotic behavior of the form factor is controlled by the behavior of $\psi_p(q)$ when the mass of one of the two constituents

(s_1 or s_2) is very large ($\sim q^2 \equiv t$) and the mass of the second one is bounded, in which case we have

$$\text{r.h.s. (7)} \rightarrow \frac{1}{s} \int \frac{\psi_p(k)\, d^4k}{1 - \frac{k \cdot n_q}{m_p} + i\epsilon} \propto \frac{1}{s} \int_0^\infty du\, e^{iu}\, \psi_p\left(\frac{u n_q^\mu}{m_p}\right)$$

which will not vanish in general independent of the vanishing of (B). Although this does not exhibit the complete t dependence, and is not a realistic hadron model with 3 q's plus the s, it shows the failure of the nonrelativistic argument against nodes. Also in this demonstration we make no apology for neglecting the long range (low t) parts of the interaction since we are only interested in the short distance part of the wave function for this analysis.

16. S. Adler, Phys. Rev. 177, 2426 (1969).

17. See comments by S. L. Adler, Brandeis University Summer Institute in Theoretical Physics (Massachusetts Institute of Technology Press, Cambridge, Mass., 1970), p. 88.

18. As first emphasized by van Royen and Weisskopf[9] the ratio of $K \rightarrow \mu\nu$ to $\pi \rightarrow \mu\nu$ decay rates exceeds naive expectations based on a nonrelativistic quark model calculation (and using the Cabibbo angle) by approximately m_K/m_π which they interpret as the ratio of the density of the $q\bar{q}$ systems at zero separation within the K and the π; i.e.,

$$\left| \frac{\psi_K(0)}{\psi_\pi(0)} \right|^2 \approx \frac{m_K}{m_\pi} \quad . \tag{C}$$

Equation (C) may simply reflect the failure of the nonrelativistic quark model in the presence of a strong $q\bar{q}$ attraction. An additional possibility for generating such a factor is by introducing an SU(3) breaking component to the strength of the gluon-quark interaction. If the breaking at the vertex for gluon emission or absorption by the quark is proportional to λ_8 and is in the direction of increasing the coupling to the strange quark then the strong short range attraction between a strange and nonstrange $q\bar{q}$ pair will be greater than that between two normal ones. This could be responsible for the factor (C). A similar mass factor is also needed to explain the ratio of decay rates for vector mesons to lepton pairs (ρ^o, $\omega^o \rightarrow e\bar{e}$, $\mu\bar{\mu}$) to the pion decay rate. In this case a spin dependence in the $q\bar{q}$ attraction is required. Such an interaction which breaks SU(6) symmetry could arise from the exchange of massive axial-vector gluons in addition to the vector gluons described thus far.

In contrast to the Bell-Jackiw-Adler anomaly for $\pi^o \rightarrow 2\gamma$ which is independent of any structure within the quark triangle graph and therefore lends itself to numerical calculation, the lepton decay rates for $\pi^\pm$, $K^\pm$, ρ^o, ω^o, ϕ^o are sensitive to all the detailed structure within the quark loop. Therefore we attempt no calculation of absolute rates for these processes based on our simple pictures. A quantitative and realistic study of the detailed properties of the bound state is called for!

19. T. T. Wu and C. N. Yang, Phys. Rev. 137, B708 (1965). H.D.I. Abarbanel, S. D. Drell, and F. J. Gilman, Phys. Rev. 177, 2458 (1969).

20. S. Berman and M. Jacob, Phys. Rev. Letters 25, 1683 (1970).

21. J. Rosner, Phys. Rev. Letters 22, 689 (1969). H. Harari, Phys. Rev. Letters 22, 562 (1969).