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INCLUSIVE PHOTON SPECTRA AND HIGH ENERGY MULTIPLICITIES*

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ABSTRACT

The consequences of Feynman scaling for photon spectra at $x = 0$ arising from π^0 decays are examined. A consistency condition on the photon distribution is obtained. The low transverse momentum region is examined in detail and the contribution of bremsstrahlung computed to be $\frac{k}{\sigma_{tot}} \frac{d\sigma}{d^3k} = \frac{\alpha}{\pi^2 k_\perp^2} < (\Delta Q_R)^2 >$, where ΔQ_R is the final right-moving charge minus the initial-right moving charge. This relation, together with the consistency condition provides a new means of obtaining pertinent multiplicity information at very high energies.

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Data from the CERN Intersecting Storage Rings for photon spectra from p-p collisions at very high energies have been analyzed for photons with moderate transverse momenta.[1,2,3] In this paper we indicate the variety of information which can be obtained from photon spectra at very low transverse momenta. Throughout, we shall confine our discussion to the central rapidity region (where Feynman's variable, $x = p_{\parallel}^{cm}/(s^{\frac{1}{2}}/2)$, is zero).

There are two striking results of our analysis. The first relates the photon spectrum arising from π^o decays at zero transverse photon momentum to the integral of the same photon spectrum over the full transverse momentum range (Eq. 10). The second predicts the bremsstrahlung contribution to the photon spectrum. We find that the invariant cross section for bremsstrahlung photons is proportional to $<$(final right-moving charge - initial right-moving charge)$^2>/k_{\perp}^2$ (Eq. 13). If there are no correlations, this is proportional to $<n^{ch}>/k_{\perp}^2$ (Eq. 15). These results follow primarily from Feynman scaling and kinematic considerations.

Photon production in the central rapidity region is dominated by four sources: (a) π^o decays (2 photons per π^o), (b) η decays (3.2 photons per η), (c) K_S^o decays (1.25 photons per K_S^o), and (d) bremsstrahlung from charged particles. It is difficult to separate the η and K_S^o decays from the π^o decays, especially because of the $\eta \rightarrow 3\pi^o$ and $K_S^o \rightarrow 2\pi^o$ decay modes. For this reason, and because η and K_S^o production are significantly smaller than the π^o production, we shall ignore η and K_S^o as sources of photons.

We summarize first some straight-forward kinematic calculations which are needed for obtaining our results. Further details will be published elsewhere.[4]

The Lorentz invariant cross section for photon production from decays of π^0's is

$$k\frac{d\sigma}{d^3k}(\underset{\sim}{k}, s) = \int \frac{d^3p}{E}\left(E\frac{d\sigma}{d^3p}(\underset{\sim}{p}, s)\right)\frac{1}{\pi}\,\delta\left(p\cdot k - \frac{m^2}{2}\right) \tag{1}$$

where m is the π^0 mass, and where $E\frac{d\sigma}{d^3p}$ is the π^0 production cross section at momentum p and center-of-mass energy squared s. From (1) we see that the pions contributing to photon production at momentum k are constrained to lie on a paraboloid given by

$$p'_{\parallel} = p_0 + \frac{k}{m^2}p'^2_{\perp} \tag{2}$$

where $p'_{\parallel}$ and $p'_{\perp}$ are the components of π^0 momentum parallel to and perpendicular to the photon and where $p_0 = k - m^2/(4k)$. The delta function in (1) can be eliminated by integrating over θ, the angle between the π^0 and the photon, with the result,

$$k \frac{d\sigma}{d^3k}(\underset{\sim}{k}, s) = \frac{1}{\pi k} \int_{E_0}^{\infty} dE \int_0^{2\pi} d\phi \; E \frac{d\sigma}{d^3p}\left(E, p_\perp^2 = \left\{\left[\frac{m^2}{k}(E-E_0)\right]^{\frac{1}{2}} \frac{k_\parallel}{k} \cos\phi \right.\right.$$
$$\left.\left. + \left(E - \frac{m^2}{2k}\right)\frac{k_\perp}{k}\right\}^2 + \frac{m^2}{k}(E-E_0)\sin^2\phi, s\right) \tag{3}$$

where ϕ is the azimuthal angle between the photon and the plane containing the π^0 and the beam direction (see Figure 1). The components of photon momentum parallel to and perpendicular to the beam direction are $k_\parallel$ and $k_\perp$, and $E_0 = (p_0 + m^2)^{\frac{1}{2}} = k + m^2/(4k)$. We now introduce new coordinates, $Q_x = Q\cos\phi$, $Q_y = Q\sin\phi$, where $Q^2 = m^2\,(E - E_0)/k$. Then (3) becomes

$$k \frac{d\sigma}{d^3k}(\underset{\sim}{k}, s) = \frac{2}{\pi m^2} \int d^2Q \; E \frac{d\sigma}{d^3p}\left(E = \frac{kQ^2}{m^2} + E_0, \right.$$
$$\left. p_\perp^2 = \left[\left\{\frac{k_\parallel}{k} Q_x + \left(\frac{kQ^2}{m^2} + p_0\right)\frac{k_\perp}{k}\right\}^2 + Q_y^2\right], s\right) . \tag{4}$$

If we assume the validity of Feynman scaling, then as $s \to \infty$ the cross section becomes a function only of $p_\perp$ and $x = p_\parallel^{cm}/(s^{\frac{1}{2}}/2)$. From Eq. (4) we find that a scaled pion distribution yields a scaled photon distribution. Representing the scaled spectra by $f(x, p_\perp^2)$, we have:

$$f_\gamma(x, k_\perp^2) = \frac{2}{\pi m^2} \int d^2Q \, f_\pi\left(x_\pi = x\left(1 + \frac{Q^2}{m^2}\right),\right.$$
$$\left. p_\perp^2 = \left\{\left[Q_x + k_\perp\left(1 + \frac{Q^2}{m^2}\right)\right]^2 + Q_y^2\right\}\right). \tag{5}$$

For photons with fixed momentum in the center of mass, we have, assuming the pion distribution is limited in transverse momentum,

$$f_\gamma(0,k_\perp^2) = \frac{2}{\pi m^2}\int d^2Q\, f_\pi\left(0, p_\perp^2 = \left[Q_x + k_\perp\left(1+\frac{Q^2}{m^2}\right)\right]^2 + Q_y^2\right) . \tag{6}$$

The integration can be divided into two symmetrical regions, with the result

$$f_\gamma(0,k_\perp^2) = \frac{4}{\pi m^2}\int_{-\infty}^{\infty} dQ'_y \int_{-m^2/2k_\perp}^{\infty} dQ'_x\, f_\pi\left(0, p_\perp^2 = Q'^2 + 2Q'_x k_\perp\left(1+\frac{Q'^2}{m^2}\right)\right.$$
$$\left. + k_\perp^2\left(1+\frac{Q'^2}{m^2}\right)^2\right) \tag{7}$$

For $m^2/2k_\perp >> < p_\perp >$ we can consider the integral to extend over the entire Q' plane. To order $(k_\perp/m)^2$, we obtain

$$f_\gamma(0,k_\perp^2) = \frac{4}{\pi m^2}\int d^2Q\, f_\pi(0,Q^2)\left[1 + \frac{k_\perp^2}{m^2}\left(2+\frac{4Q^2}{m^2}\right)\right]. \tag{8}$$

Thus we are led to the somewhat surprising result that the photon spectrum arising from π^0 decays increases quadratically away from $k_\perp = 0$. For $m^2/2k_\perp \simeq < p_\perp >$, it is no longer permissible to extend the integration in (7) over the whole plane, and the value of $f_\gamma(0,k_\perp^2)$ begins to fall.

Since each π^0 produces two photons.

$$\int d^2Q\, f_\pi(0,Q^2) = \tfrac{1}{2}\int d^2Q\, f_\gamma(0,Q^2) . \tag{9}$$

Thus from (8) we have the interesting relation:

$$\lim_{k_\perp \to 0} f_\gamma(0,k_\perp^2) = \frac{2}{m^2}\int dk_\perp^2 f_\gamma(0,k_\perp^2) \tag{10}$$

Equation (10) is a consistency test on the photon spectrum arising from a scaled pion distribution. It should be satisfied at finite s insofar as the pion spectrum is flat in the central rapidity region. Of course $x = 0$ does not necessarily mean $k_\parallel = 0$; it suffices to consider fixed $k_\parallel$, as $s \to \infty$.

For low values of the transverse momentum, bremsstrahlung from charged particles becomes a source of photons comparable to π^o decays. The bremsstrahlung contribution can be estimated by the following argument. Treating all the particles as scalars we compute the matrix element for the exclusive process $a+b \to \gamma + c_1 \ldots + c_n$ in terms of the non-radiative matrix element. Assuming the extrapolations of the hadronic matrix element are negligible and that we need consider only externally attached photons, we have for a photon of momentum k and polarization of ϵ,

$$M_R = e\left(\sum_{i=i}^{n} Q_i \frac{q_i \cdot \epsilon}{q_i \cdot k} - Q_a \frac{p_a \epsilon}{p_a \cdot k} - Q_b \frac{p_b \cdot \epsilon}{p_b \cdot k}\right) M_{NR}, \tag{11}$$

where q_i and Q_i are the momentum and charge of c_i.[5] For high energy collisions we can assume each q_i is fully relativistic and either parallel (right-moving) or anti-parallel (left-moving) to the direction of p_a in the center of mass. From (11) we see that only photons polarized in the plane of the beam direction and the photon contribute. With the above assumptions,

$$M_R = e \frac{M_{NR}}{k_\perp} (\Delta Q_R - \Delta Q_L), \tag{12}$$

where $k_\perp$ is the component of photon momentum perpendicular to the beam direction and where $\Delta Q_R = -\Delta Q_L$ is the change in the right-moving charge between the final and initial states. Calculating the radiative cross-section for each exclusive process and summing yields

$$\frac{k\, d\sigma}{\sigma_{tot}\, d^3k} = \frac{\alpha}{\pi^2 k_\perp^2} \langle(\Delta Q_R)^2\rangle \tag{13}$$

Now while $<\Delta Q_R> = 0$, $<(\Delta Q_R)^2>$ can be expressed as

$$<(\Delta Q_R)^2> = <n_R^{+2}> + <n_R^{-2}> - 2 <n_R^+ n_R^-> - Q_a^2 \tag{14}$$

If there are no correlations, $< n_R^{+2}> = <n_R^+>^2 + <n_R^+>$ and $<n_R^+ n_R^-> = <n_R^+><n_R^->$ so that (13) becomes, in this crude approximation,

$$\frac{k d\sigma}{\sigma_{tot} d^3k} = \frac{\alpha}{2\pi^2 k_\perp^2} <n^{ch}> \tag{15}$$

To separate the bremsstrahlung from the photons due to π^0 decay would require careful measurement of the low transverse momentum region.

We can estimate the size of the bremsstrahlung spectrum from Eq. (15). If $<n^{ch}> \simeq 10$ at $s \simeq 2000$ GeV^2, the bremsstrahlung spectrum would be $\simeq 4 \times 10^{-3} k_\perp^{-2}$. Neuhofer et al.[1] have measured the photon spectrum over a large kinematic region (photon momenta from 100 MeV to 5 GeV) and find their data can be fitted with

$$\frac{k}{\sigma_{inel}} \frac{d\sigma}{d^3k} = \frac{A}{k_\perp} \exp\left(-\frac{k_\perp}{k_0} - \frac{x}{x_0}\right) \tag{16}$$

with $A = 1.48$ GeV^{-1}, $x_0 = 0.083$, and $k_0 = 0.162$ GeV. This indicates that the bremsstrahlung and π^0 contributions would be roughly equal for $k_\perp \simeq 3$ MeV.

In Figure 2 we show some of the fixed angle data of Neuhofer, together with the photon spectrum derived from a hypothetical π^0 spectrum with a transverse momentum dependence proportional to $\exp(-bp_\perp)$ with $b = 6$ GeV^{-1}. The normalization of the pion spectrum was determined by Eq. (10) using the experimental value determined from Eq. (16), 53 GeV^{-2}, for the right side. The general agreement is adequate. In the 10^0 data, the point for the lowest value of $k_\perp$ seems substantially higher than the curve and may be a reflection of the bremsstrahlung. A careful measurement of the low transverse momentum region should reveal a distribution like the curves in Figure 2, with a bremsstrahlung spectrum like Eq. (13) superimposed.

The asymptotic π^+, π^-, and π^0 multiplicities are expected to be of the form

$$n_i = A_i \log s + B_i \tag{17}$$

where $i = +, -,$ or 0, and where $A_+ = A_- = A_0$ and $B_0 = (B_+ + B_-)/2$.[6,7] The coefficient of log s is given by

$$A_i = \frac{1}{\sigma} \int d^2 p_\perp f_i(0, p_\perp^2) , \tag{18}$$

which is precisely the quantity appearing in Eqs. (9) and (10).

The experimental data on high energy multiplicities are contradictory. Results from the CERN ISR indicate higher asymptotic multiplicities than the cosmic ray data. In particular, the data of Breidenbach et al.[8] can be fitted in terms of the variable $\eta = \ell n(\cot\frac{\theta}{2})$ with the form

$$\left(\frac{1}{\sigma_{\mathrm{inel}}} \frac{d\sigma}{d\eta}\right)_{\eta = 0} \simeq A' + B' s^{-\frac{1}{4}}$$

with plausible fits ranging from $A' = 2.0$, $B' = -4.0$ to $A' = 2.6$, $B' = -7.0$. To convert this to multiplicity information, we note that if ξ is the center of mass rapidity

$$\left[\left(\frac{d\sigma}{d\eta}\right) \Big/ \left(\frac{d\sigma}{d\xi}\right)\right]_{\eta = \xi = 0} = \left< \frac{p_\perp}{m_\perp} \right>$$

where $m_\perp^2 = m^2 + p_\perp^2$ and the average is over transverse momentum squared. For a transverse momentum dependence $\exp(-bp_\perp)$ with $b = 6\ \mathrm{GeV}^{-1}$, $< p_\perp/m_\perp > \simeq 0.83$. Thus the coefficient of log s for charged multiplicity appears to be in the range 2.4 - 3.1. This is a surprisingly large number and further experimental tests are of great interest.

The low transverse momentum photon spectrum provides an independent means of obtaining pertinent information about high energy charged and neutral multiplicities. Because there are two significant contributions to the photon spectrum, careful measurements are required. The difficulty of separating the two components is lessened by their contrasting behaviors as $k_\perp$ decreases: the π^o contribution decreases slightly while the bremsstrahlung rises sharply. The factor $\langle \Delta Q_R^2 \rangle$ in Eq. (13) is directly accessible in bubble chamber experiments and this should aid in elucidating the very low transverse momentum spectrum.

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FOOTNOTES

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FIGURE CAPTIONS

Figure 1 The geometry for determining the photon spectrum from a given pion spectrum.

Figure 2 Some of the data of reference 1 for $(k/\sigma_{inel})(d\sigma/d^3k)$ at low transverse photon momenta and fixed angles of 10 and 24 degrees away from the beam direction. The curves are predictions for $x = 0$ photons based on an assumed pion spectrum proportional to $\exp(-bp_\perp)$ with $b = 6\ \mathrm{GeV}^{-1}$. The normalization for the curve is determined by Eq. (10). The data points have values of x between zero and 0.05.

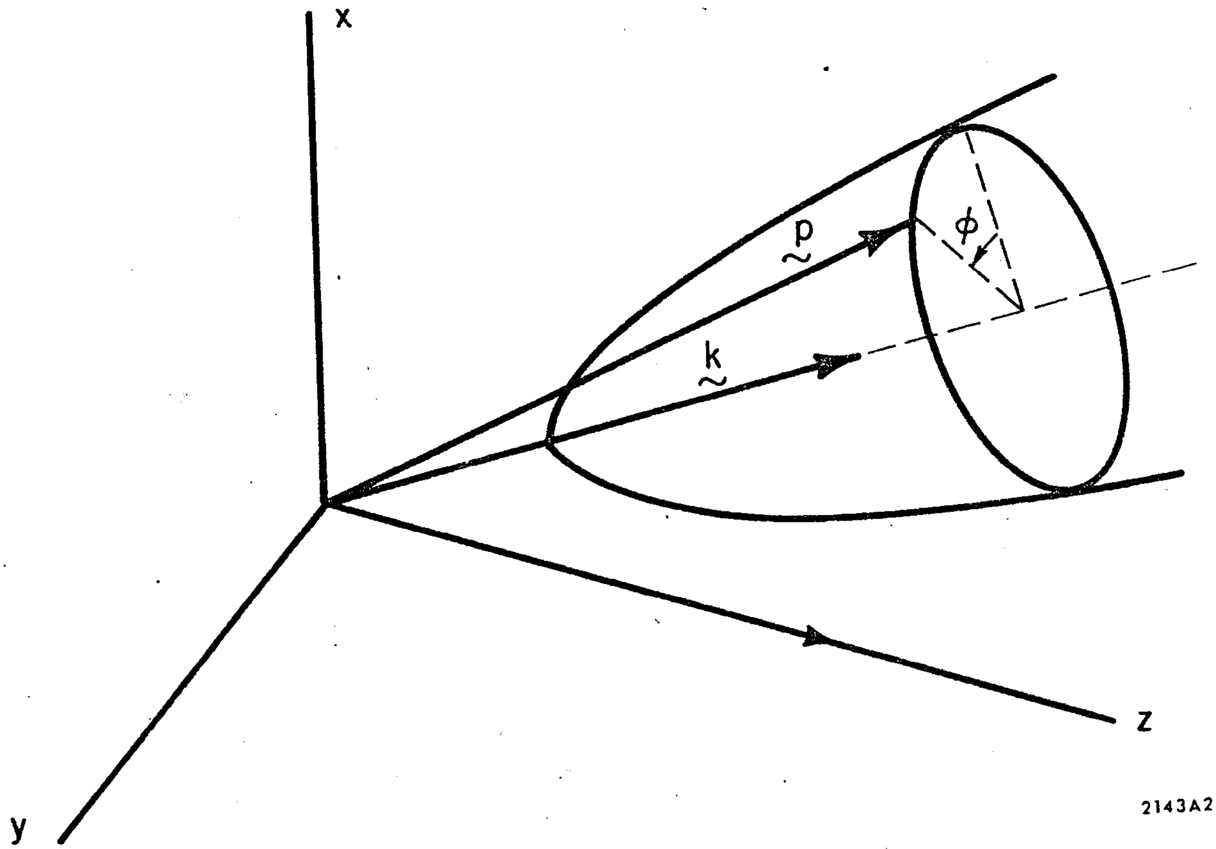

FIG. 1

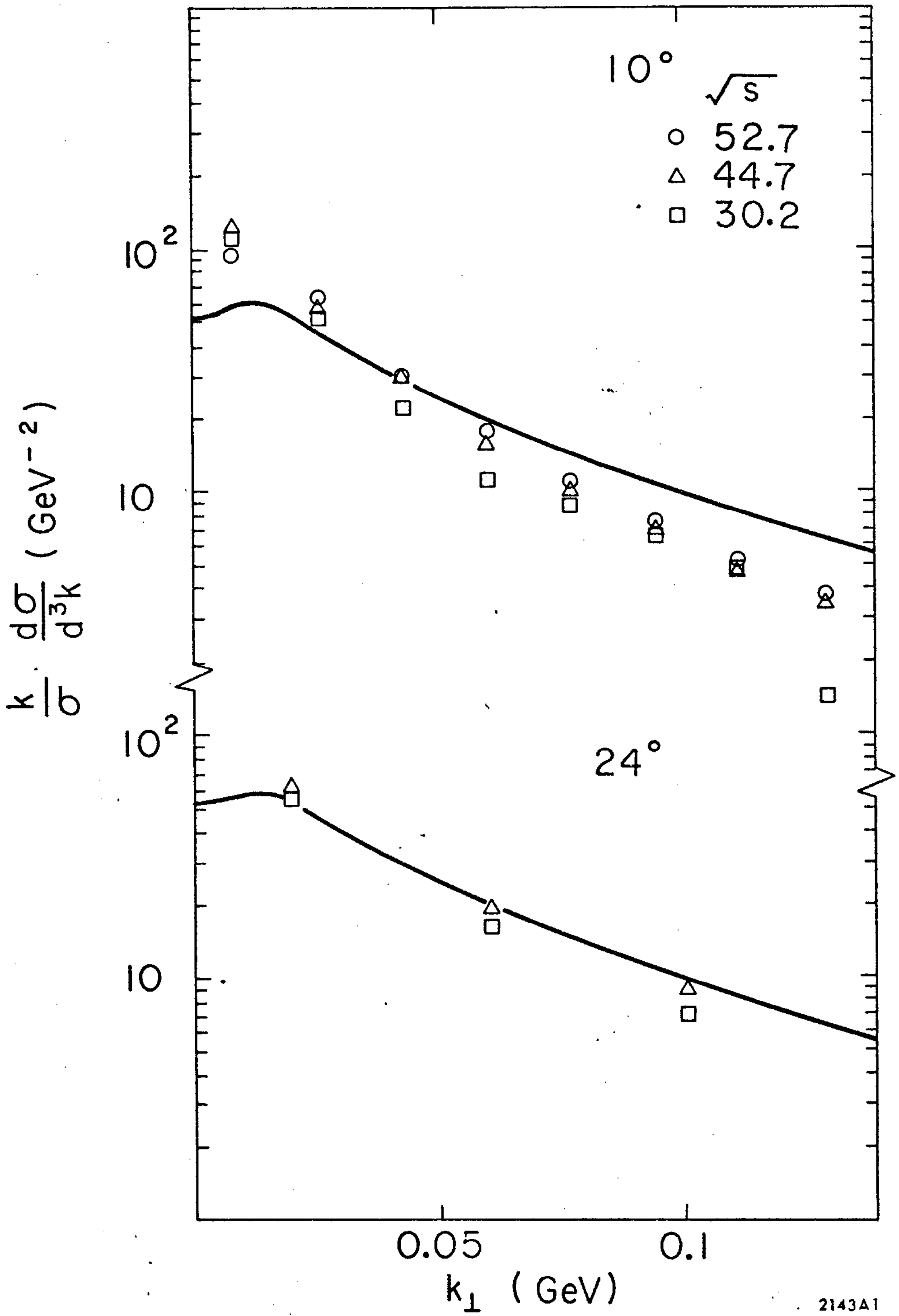
10°
$\sqrt{s}$
52.7
44.7
30.2
10^2
10
10^2
24°
10
$\frac{k}{\sigma}\frac{d\sigma}{d^3k}$ (GeV^{-2})
0.05
0.1
$k_\perp$ (GeV)
2143A1

FIG. 2