PRODUCTION OF K^O_L MESONS AND NEUTRONS FROM

ELECTRONS ON BERYLLIUM ABOVE 10 GeV*

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ABSTRACT

Measurements are presented of the yields of K^O_L mesons and neutrons produced by electrons of energies between 10 and 19 GeV incident on 0.70 and 1.75 radiation length beryllium targets for production angles between 1.6^O and 4°. Values for the K^O_L absorption cross section on lead are also found for momenta between 1.4 and 7.4 GeV/c. A successful interpretation of K^O_L production is made in terms of the process $\gamma N \rightarrow K^O_L X$, where X represents an inclusive sum over all final states. The invariant structure function for K^O_L photoproduction, extracted from the Be yields, is found to be simply related to the result for hydrogen by an overall multiplicative factor, A_{EFF}, the effective number of nucleons in the target nucleus. The results of the theoretical analysis are also compared to charged K photo-production and extrapolated to electron energies of 50 GeV.

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I. Introduction

We present measurements of the yield of K_L^O mesons and neutrons produced from electrons incident on a beryllium target at energies between 10 and 19 GeV. Table 1 summarizes the specific electron energies, production angles, and Be target thicknesses for which these measurements have been made. The data were acquired during three separate data runs at the Stanford Linear Accelerator Center (SLAC) in which the SLAC 40-inch hydrogen filled bubble chamber was exposed to a neutral beam. A description of the neutral beam is given in Section II. A preliminary report of the results for run I is given in Ref. 1. Measurements of K_L^O photoproduction below 10 GeV on complex nuclei have also been made in other experiments.[2-5]

The yields of K_L^O and neutrons are presented in Sections III and IV respectively. The yields are found to behave similarly above ~4 GeV/c, where both show a rapid fall off with increasing momentum of the neutral particle. However, for momenta below ~2 GeV/c the neutron yield has an intense low energy component which is not present in the K_L^O yield. This low energy component is probably due to the disintegration of the target nucleus.

The present measurements of K_L^O yields together with previous measurements[1-3] suggest that the photoproduction of K^O mesons is a complicated process with contributions not only from the obvious two-body and quasi-two-body reactions but also from multiparticle channels. A natural framework for dealing with such a complicated situation is provided by the inclusive single particle production process, $\gamma N \rightarrow K_L^O X$, where X represents the sum of all final state configurations. In Section V, we discuss the K_L^O yields in terms of inclusive K_L^O production and find a satisfactory description for the measurements. Extrapolations are then

made for K^O_L yields at higher energies and for various target thicknesses. Comparisons are also made to the available yield data for K^+ and K^- mesons.

II. Neutral Secondary Beam

A schematic illustration of the neutral beam is shown in Fig. 1. The primary electron beam first passed through a toroid charge monitor[6] which integrated the total charge per pulse for purposes of beam normalization. The production angle of the neutral beam with respect to the electron beam could be varied from 1.5^O to 5° by a dipole magnet placed immediately upstream of the Be target. The position, spot size, and angle of entry of the electron beam on the target was monitored visually by means of closed-circuit television displays of two ruled zinc sulfide screens. The first screen was attached to the upstream face of the target and the second was attached to the upstream face of a water cooled dump used to stop the electron beam. The neutral beam channel was defined by three carefully aligned lead collimators: a 2.2 meter tapered collimator centered 7 meters from the target, and two 0.5 meter untapered collimators located 10 meters and 22 meters from the target. Two sweeping magnets were used to remove charged particles. The halo of muons was absorbed by $\sim$ 15 meters of iron shielding surrounding the neutral beam channel. The photons in the beam were absorbed by placing suitable amounts of material in the beam, as summarized in Table 2. At the bubble chamber (55 meters from the target) the beam had a cross sectional area of 15 cm by 40 cm and subtended a solid angle of $\sim 2 \times 10^{-5}$ sr.

III. K^O_L Yields

A. Selection of Events

The film was scanned for the visible K^O_L decays: $K^O_L \to \pi^\pm e^\mp \nu$, $K^O_L \to \pi^\pm \mu^\mp \nu$, and $K^O_L \to \pi^+ \pi^- \pi^O$. These decays appear mainly as two-prong events which are not associated with any interaction in the chamber. However, a small fraction of the final sample of events (8%) were first classified by the scanners as associated

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with an interaction in the chamber but were properly reclassified as beam decays following measurement. The decay tracks were measured either on the SLAC spiral reader or on film plane digitizers, and spatially reconstructed with the program TVGP. The reconstructed tracks and the K_L^0 direction were then used to obtain kinematic solutions to the above decay modes. Events were included in the analysis of the K_L^0 momentum spectrum if any of the 5 decay modes was kinematically possible.[7] Events were excluded if any of the following were satisfied:

$$M(e^+e^-) < 35 \text{ MeV},$$

$$485 < M(\pi^+\pi^-) < 510 \text{ MeV},$$

or
$$1110 < M(p\pi^-) < 1120 \text{ MeV},$$

where the charged tracks were interpreted as the indicated particles. These cuts, which effectively remove all non-K_L^0 decays, also remove a small fraction ($\sim 5\%$) of the K_L^0 decays. However, these cuts introduce no bias since the same cuts are made on the Monte Carlo events used in the theoretical anlaysis (see below). A final selection required the K_L^0 decay to occur within a 55 cm long decay volume within the chamber.

B. Determination of the K_L^0 Momentum Spectrum at the Bubble Chamber.

The method of analysis described below determines both the shape and the absolute magnitude of the K_L^0 momentum spectrum at the bubble chamber from the observed distribution of the visible momentum, p_{VIS}, defined as

$$p_{VIS} = \hat{n} \cdot (\vec{p}_1 + \vec{p}_2),$$

where $\vec{p}_1$ and $\vec{p}_2$ are the three-momenta of the two charged tracks and $\hat{n}$ is a unit vector along the beam direction. On the average, the quantity p_{VIS} is roughly two thirds of the K_L^0 momentum, p_K, regardless of decay mode; therefore, the p_{VIS} distribution depends sensitively on the shape of the p_K distribution. The absolute

intensity of the K^o_L flux is fixed by the known K^o_L lifetime and the branching ratio $(K^o_L \rightarrow$ charged$)/(K^o_L \rightarrow$ all$)$.[8]

In Fig. 2, we illustrate the procedure used to determine the momentum distribution of the K^o_L particles which decay in the chamber. First the p_{VIS} data are binned such that for the $i\underline{th}$ bin there are N_i events between p_{VIS_i} and $p_{VIS_i} + \Delta p_{VIS_i}$. The momentum distribution of decays, denoted $Z(p_K)$, is then represented by a histogram, the $j\underline{th}$ bin of which covers the interval from p_{Kj} to $p_{Kj} + \Delta p_{Kj}$. The height of the $j\underline{th}$ bin, $Z(p_{Kj})$, is denoted by the parameter α_j. For each bin in p_K, a p_{VIS} distribution is then generated by Monte Carlo techniques;[9] for the $j\underline{th}$ bin in p_K the p_{VIS} distribution normalized to unity is denoted H_{ji}, where the index i runs over all p_{VIS} bins. Each p_{VIS} distribution is comprised of events from the five main K^o_L charged decay modes generated according to the standard decay matrix elements in proportion to their known decay rates.[8] The mass cuts used for the data selection (see Sect. III-A) are also applied to the Monte Carlo events. For an assumed set of α_j values, the expected number of events in the ith p_{VIS} bin is then given by:

$$T_i = \sum_j \alpha_j H_{ji}.$$

The best set of values for the α_j's is then determined by minimizing $\chi^2(\alpha_1, \alpha_2, \ldots)$, where

$$\chi^2(\alpha_1, \alpha_2, \ldots) \equiv \sum_i (T_i - N_i)^2 / T_i.$$

The value of α_j at the minimum of χ^2, α^*_j, is equal to $Z(p_{Kj})$.[10] In order to determine the statistical uncertainties in α^*_j, a number of successive reminimizations of χ^2 (typically 50) are done after independently changing each p_{VIS} bin from its original value, T_i, to $T_i \pm \delta T_i$ where δT_i is randomly chosen on successive minimizations according to a gaussian distribution having a standard deviation equal to $T_i^{\frac{1}{2}}$. The standard deviations of the values of α^*_j thus obtained are taken to be

the statistical uncertainties.

The procedure outlined above is illustrated in Fig. 2 for a K_L^o beam produced at 2° by 16 GeV electrons on a 1.75 radiation length (r.1.) Be target. The p_{VIS} distributions corresponding to the fitted components of the $Z(p_K)$ distribution are summed to provide the solid curve on the experimental p_{VIS} distribution. This curve is seen to reproduce the p_{VIS} spectrum very well.

The absolute flux of K_L^o at the chamber, denoted by $F(p_K)$, is related to $Z(p_K)$ as follows:

$$F(p_K) = \frac{1}{\epsilon r} \left[1 - e^{-L/\lambda(p_K)} \right]^{-1} Z(p_K)$$

where ϵ = efficiency factor for scanning and measuring of the film,

       r = branching ratio, $(K_L^o \rightarrow$ charged$)/(K_L^o \rightarrow$ all$)$,

       $\lambda(p_K)$ = $\dfrac{p_K c \tau}{m_K}$, the mean K_L^o decay length,

       L = length of decay volume in the chamber,

       m_K = K_L^o mass,

and τ = K_L^o lifetime.

An example of the K_L^o flux at the chamber is shown in Fig. 3. The uncertainties shown are the statistical uncertainties determined as described above.

As a check of the preceeding method we have also determined the K_L^o spectrum at the chamber by an alternate method.[11] First, by means of a transverse momentum selection, a sample of unique K_{e3} decays is isolated. This selection retains approximately 15% of the total number of K_L^o decays. Each of these decays has up to four kinematical solutions for p_K, and each of the solutions is assigned a weight which is normalized such that the total weight for a single decay is unity. The individual weights are proportional to a product of the Dalitz plot density, the Jacobian relating p_K to the measured variables, and the K_L^o spectrum. Thus the

spectrum is given by:

$$Z(p_{Kj}) = \sum_{i=1}^{N_j} W_{ij}(Z),$$

where the $W_{ij}(Z)$ are the weights of the N_j kinematical solutions in the jth p_K bin. Since the weights are functions of the K_L^0 spectrum (Z), this expression is a set of m non-linear equations, where m is the numer of p_K bins. These can be solved iteratively by substituting the spectrum Z of the previous iteration into $W_{ij}(Z)$ to obtain a new solution. Using an arbitrary spectrum as a starting value, this procedure converges after a few interations to a unique solution for $Z(p_K)$. The open circles in Fig. 3 show a K_L^0 spectrum obtained by this method, where $Z(p_K)$ has been converted to flux at the chamber. The agreement with the previously described spectrum determination is excellent, although the error bars are larger due to the smaller statistics.

C. Determination of K_L^0 Yields at the Target from K_L^0 Fluxes at the Chamber

If $F(p_K)$ is the flux of K_L^0 mesons per GeV/c at the chamber, D the distance between target at chamber, $\Delta\Omega$ the solid angle of the detector, and N_e the number of electrons incident on the target, then the yield is given by

$$Y_K(p_K) = F(p_K) \exp\left[D/\lambda\,(p_K)\right]\,\exp\left[\sum_i a_i \sigma_{Ki}(p_K)\right] / (N_e \Delta\Omega),$$

expressed in units of K_L^0/electron/sr/GeV/c. The first exponential factor corrects for the loss of K_L^0 particles due to decays in flight. The second exponential factor corrects for absorption of the K_L^0 beam in the photon absorbers (see Table 2), where for the ith absorber, $\sigma_{Ki}(p_K)$ is the total K_L^0 absorption cross section for momentum p_K, $a_i = \rho_i N_0/A_i$, ρ_i is the amount of absorber in g/cm^2, A_i is the atomic weight, and N_0 is Avogadro's number.

The solid angle factor, $\Delta\Omega$, has been determined from the geometry of the neutral beam; corrections for collimator edges have been made by a study of the

distribution of K_L^o decay positions within the bubble chamber. The number of incident electrons, N_e, has been measured for each accelerator pulse in the toroid charge monitor and summed over the pulses for each yield curve.

Values for the absorption cross sections for Li, W, and Pb have been interpolated from the K_L^o-nucleus measurements of Lakin et al,[12] and the $K^\pm$-nucleus measurements of Abrams et al.[13] In addition, values for the K_L^o-Pb absorption cross section have been measured in the present experiment and are presented in Table 3.[14] The available K-nucleus cross sections are shown in Fig. 4 together with interpolation curves which, for atomic weight A and momentum p_K, are given by the empirical parameterization[15]

$$\sigma(p_K, A) = \left[b_1(p_K + b_2 A)^{-n_1} + b_3\right] A^{n_2}. \tag{1}$$

Values for the $K_L^o p$ absorption cross section have been interpolated from $K^\pm n$ measurements,[16-18] also shown in Fig. 4, where the interpolation curve is of the form[15]

$$\sigma(p_K) = b_4 p_K^{-n_3} + b_5.$$

The K_L^o yield results for the 1.75 r.1. Be target are presented in Table 4 and Fig. 5; the results for the 0.70 r.1. Be target are presented in Table 5 and Fig. 6. The quoted uncertainties are the statistical uncertainties in $F(p_K)$, discussed in Sect. III-B. All the yield curves have a broad maximum for P_K equal to $\sim 15\%$ of the incident electron energy. The fall off in yield at larger p_K values becomes more rapid as the target thickness is increased and as the production angle is increased. The curves shown on Figs. 5 and 6 are discussed in detail in Section V.

D. Estimates of Systematic Uncertainties in the K_L^o Yields.

The relative systematic uncertainties between the various K_L^o yields are estimated to be 15%, with the following contributions taken in quadrature: film

analysis (5%), solid angle (5%), and targetting (focus and intensity) of the electron beam (10%).

The overall systematic uncertainty in the scale of the K_L^0 yields is estimated to be 20%, and is comprised of $\sim 15\%$ uncertainty in the absorption factor and $\sim 10\%$ uncertainty in the film analysis.

IV. Neutron Yields

A. Selection of Events

The neutron flux at the bubble chamber has been determined from a sample of events from the reaction

$$np \rightarrow pp\pi^- ,$$
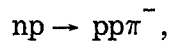
which appear as three-prong interactions in the chamber. The candidates have been measured on the SLAC spiral reader, and spatially reconstructed and kinematically fitted with the programs TVGP and SQUAW. These events have three kinematic constraints since the direction of the beam and the momenta of all outgoing tracks are measured. For momenta above ~ 5 GeV/c, a fraction of the events ($\sim 10 - 15\%$) are kinematically ambiguous with the reaction $K^0 p \rightarrow pK^+ \pi^-$. However, the majority of the ambiguities are resolved on the basis of the χ^2 probabilities of the kinematic fits, and we estimate that the remaining contamination from the $K^0 p$ events is less than 5% at all momenta. The same samples of film and the same active volume in the chamber were used for the $np \rightarrow pp\pi^-$ events as for the K_L^0 decays.

B. Determination of the Neutron Momentum Spectrum at the Bubble Chamber.

For a given neutron momentum, p_n, the neutron flux at the chamber per GeV/c is given by:

$$F(p_n) = C \ N(p_n)/\sigma(np \rightarrow pp\pi^-)$$

where $N(p_n)$ is the number of observed $np \rightarrow pp\pi^-$ events per GeV/c, and the

constant of proportionality, C, is determined from the density of the liquid hydrogen and the length of the interaction region.

Values for $\sigma(np \to pp\pi^-)$ have been interpolated from the pd experiments of Batson et al[19], Cohn et al,[20] Brunt et al,[21] Shapira et al, [22] and Rushbrooke et al[23] and from the np experiment of Gasparyan et al. [24] These cross section data are plotted in Fig. 7 together with a hand drawn interpolation curve (solid curve) used for the present analysis. The dashed curves represent our estimate of the uncertainty of the interpolation cruve.

An example of the neutron flux at the chamber is shown in Fig. 3 for comparison to the K_L^o flux. The uncertainties shown are dominated by the uncertainty in $\sigma(np \to pp\pi^-)$ rather than the statistical error on $N(p_n)$. The neutron flux peaks below 1 GeV/c and the n/K_L^o ratio is seen to decrease by an order of magnitude over the range from 2 to 6 GeV/c. For this example the flux of neutrons becomes equal to the flux of K_L^o mesons for $p \sim 3$ GeV/c. However, the n/K_L^o ratio at the chamber depends on the amount of absorber in the neutral beam and is diminished as the amount of absorber is increased since the nuclear absorption cross sections are larger for neutrons than for K_L^o mesons.

D. Determinations of Neutron Yields at the Target from Neutron Fluxes at the Chamber

If $F(p_n)$ is the number of neutrons per GeV/c at the chamber, $\Delta\Omega$ the solid angle of the detector, and N_e the number of electrons incident on the target, then the yield is given by

$$Y_n(p_n) = F(p_n)e^{\sum_i a_i \sigma_{ni}(p_n)}/(N_e \Delta\Omega),$$

expressed in units of neutrons/electron/sr/GeV/c. The same values for N_e, $\Delta\Omega$, and the a_i's apply to the neutron yields as applied to the K_L^o yields (see Sect. III.C).

Values for neutron absorption cross sections for Li, W, and Pb have been interpolated from the measurements of Refs. 12, 25 - 30. Values for np total cross

sections have been interpolated from the measurements of Refs. 31 - 33.

The results for the neutron yields are given in Table 6 and Fig. 8. The quoted uncertainties are the sum in quadrature of the statistical uncertainty in $N(p_n)$ and the estimated uncertainty of the interpolation curve for $\sigma(p_n)$ shown in Fig. 7. The neutron yields have an intense low energy component below ~ 2 GeV/c. Since this low energy component is not present in the K_L^o yields, it must arise from the disintegration of Be nuclei in the target. For momenta above ~ 4 GeV/c the yield of neturons and K_L^o mesons from the 1.75 r.l. Be target have similar dependences on momentum, with the ratio n/K_L^o being in the range ~ 1.2 to ~ 1.7.

D. Estimates of Systematic Uncertainties in the Neutron Yields.

The relative systematic uncertainties between the various neutron yields is estimated to be 15%, the same as for the relative systematic uncertainty between the various K_L^o yields (see Sect. III-D).

The overall systematic uncertainty in the scale of the neutron yields is estimated to be 20%, and is made up of $\sim 15\%$ uncertainty in the absorption factor, and $\sim 10\%$ uncertainty in the film analysis.

V. Interpretation of K_L^o Yields

A. Theoretical Analysis

The production of hadronic particles from high energy electrons incident on thick targets can be described qualitatively by the following sequence of steps: (a) real photons[34] are produced by bremsstrahlung throughout the volume of the target, (b) particles are photoproduced from the target nuclei, and (c) a fraction of the hadronic particles are absorbed before leaving the target. Although steps (a) and (c) are well understood and readily parameterized, step (b) is in general a complicated summation over many processes, and has not been systematically parameterized in previous studies.

The aim of this section is to obtain a simple description of the measured K_L^o

yields by considering only two production processes. The more important contri-

bution is incoherent photoproduction from the individual nucleons in the target

nucleus, $\gamma N \rightarrow K_L^o X$, where X represents the inclusive sum over all final state

configurations. The second contribution is from the reaction $\gamma Be \rightarrow \phi(1020) Be$,

$\phi(1020) \rightarrow K_L^o K_S^o$, which we assume to be the only important coherent process for

K_L^o photoproduction.

The quantitative description for steps (a)-(c) for the yield of hadronic particles

having laboratory momentum, p (GeV/c), and laboratory production angle, Θ,

from a target of thickness, T (in units of r.l.), for electrons incident with energy,

E_0 (GeV), is given by the following relation:[35]

$$Y(p, \Theta; T, E_0) = \left(\frac{N_0 X_0}{2\pi A}\right) \int_0^T dt \int_{k_{min}}^{E_0} dk\, e^{-\eta(p)(T-t)} I_\gamma(k, E_0, t) \frac{d^2 \sigma(k)}{dp\, d\cos\Theta} \quad (2)$$

in units of particles/electron/sr/GeV/c. In this expression, k is the photon energy,

$I_\gamma(k, E_0, t)$ is the distribution in energy of the photons for a single electron of energy

E_0 incident on a target of thickness t, $\frac{d^2\sigma}{dp\, d\cos\Theta}$ is the differential cross section

for photoproduction of the hadronic particles, k_{min} is the minimum energy kin-

ematically allowed, N_0 is Avagadro's number, X_0 is the unit r.l. of the target

material in gr/cm^2, and A is the atomic weight of the target nucleus. The factor

$\eta(p)$ accounts for the absorption of the hadronic particles leaving the target and is

given by

$$\eta(p) = \left(\frac{N_0 X_0}{A}\right) \sigma(p),$$

where $\sigma(p)$ is the absorption cross section for the target nucleus. For the

beryllium nucleus we have used the empirical parameterization of Eq. (1) for

$\sigma(p)$. The photon energy distribution is given by the thick-target approximation of

Tsai and Whitis:[35]

$$I_\gamma(k, E_o, t) = \frac{1}{k} \frac{(1 - k/E_0)^{(4/3)t} - e^{-(7/9)t}}{[(7/9) + (4/3)\ell n(1 - k/E_0)]} .$$

In order to describe K_L^o yields by electrons on thick targets, the single particle differential cross section for K_L^o photoproduction must be known. In the present analysis, we assume

$$\frac{d^2\sigma}{dpdcos\Theta} = \left(\frac{d^2\sigma}{dpdcos\Theta}\right)_{incoherent} + \left(\frac{d^2\sigma}{dpdcos\Theta}\right)_{coherent}$$

where the incoherent part is inclusive K_L^o photoproduction from the individual nucleons and the coherent part is $\phi(1020)$ photoproduction from the entire target nucleus.

For each measured K_L^o yield point, the value for $\left(\frac{d^2\sigma}{dpdcos\Theta}\right)_{coherent}$ is calculated by a Monte Carlo integration for the two-step process : $\gamma Be \rightarrow \phi(1020)Be$, $\phi(1020) \rightarrow K_L^o K_S^o$. The differential cross section is interpolated from the measurements of McClellan et al[36] of $\phi(1020)$ photoproduction from complex nuclei and is assumed to be independent of energy:

$$\frac{d\sigma}{dt} (\gamma Be \rightarrow \phi(1020) Be) = 125 \, e^{40t} \, \mu b/GeV^2 ,$$

where t is the square of invariant momtnum transfer to the Be nucleus. The branching ratio, $(\phi(1020) \rightarrow K_L^o K_S^o)/(\phi(1020) \rightarrow all)$, is taken to be 31%.[8] The decay angular distribution of the $\phi(1020)$ is required to be proportional to $sin^2\beta$, where β is the angle between the direction of the K_L^o in the $\phi(1020)$ rest system and the direction of the $\phi(1020)$ in the center of mass system.

The incoherent differential cross section is treated as an unknown to be determined in the present analysis. In terms of $p_\perp^2$ and the variable [37] $x = p_{\parallel}^*/p^*_{max}$, where $p_\perp$ and $p_{\parallel}^*$ are the transverse and longitudinal momenta of the K_L^o in the center of mass system and p^*_{max} is the kinematic maximum of the center of mass

K^0_L momentum, the differential cross section is:

$$\left(\frac{d^2\sigma(k)}{dp\,d\cos\Theta}\right)_{incoherent} = \frac{2p^2 E^*}{Ep^*_{max}} \frac{d^2\sigma(k)}{dx\,dp^2_\perp},$$

where $E(E^*)$ is the energy of the K^0_L in the laboratory (center of mass) system. In the notation of the invariant structure function, [37, 38] we then have

$$\frac{E^*}{\pi p^*_{max}} \frac{d^2\sigma(k)}{dx\,dp^2_\perp} = f(x, p^2_\perp, s),$$

where s is the square of the total energy in the center of mass. The integrated structure function, $\mathscr{F}(x, s)$, is defined by

$$\mathscr{F}(x, s) \equiv \int_0^\infty dp^2_\perp\, f(x, p^2_\perp, s).$$

If we assume that the hypotheses of scaling [37] and factorization [39, 40] hold for all s and x, and if we parameterize the $p^2_\perp$ dependence as a single exponential, we then have

$$f(x, p^2_\perp, s) = \mathscr{F}(x) B e^{-Bp^2_\perp},$$

a form which is approximately obeyed by the available data on $\pi^\pm$ and $K^\pm$ inclusive photoproduction from hydrogen. [41, 42] For the exponential parameter we have used $B = 4.5$ GeV^{-2}, a value consistent with the measurements of Boyarski et al. [42] With these approximations, the incoherent differential cross section becomes:

$$\left(\frac{d^2\sigma}{dp\,d\cos\Theta}\right)_{incoherent} = \left(\frac{2\pi p^2}{E}\right) B e^{-Bp^2_\perp} \mathscr{F}(x).$$

The object of the analysis is then to find the magnitude and shape of $\mathscr{F}(x)$ which best reproduces the K^0_L yield data. The procedure is possible since different values of the variable x contribute to distinctly different K^0_L momenta, as is illustrated in Fig. 9. In this Figure, we show calculated K^0_L yields as a function of p (for $\Theta = 2^0$, $E_0 = 16$ GeV, and T = 1.75 r.l.) corresponding to successive bins in $\mathscr{F}(x)$, where

for $i\frac{\text{th}}{}$ curve

$$\mathcal{F}(x) = 1\mu b \text{ for } x_i < x < x_{i+1} \tag{3}$$

$$= 0 \quad \text{for all other } x,$$

and where $x_i = (i-1)/10$. Note that the observed yields receive contributions from $\mathcal{F}(x)$ only in the interval $0 \lesssim x \lesssim 1$.

The shape and magnitude of $\mathcal{F}(x)$ has been determined by a χ^2 minimization procedure as follows. First, the function $\mathcal{F}(x)$ is represented by a histogram for which the $i\frac{\text{th}}{}$ bin covers the region x_i to x_{i+1} and has a height given by the parameter, $\alpha_i(\mu b)$. The incoherent yield for the $j\frac{\text{th}}{}$ data point (corresponding to $p = p_j$, $\Theta = \Theta_j$, $E_0 = E_{0j}$, and $T = T_j$) is then given by:

$$Y_{\text{incoherent}}(j) = \sum_i \alpha_i\, y_{ij}$$

where the y_{ij} values are found by integrating Eq. (2) using $\mathcal{F}(x)$ as in Eq. (3). Denoting the $j\frac{\text{th}}{}$ data point by D_j, χ^2 for the $j\frac{\text{th}}{}$ point is

$$\chi_j^2 = \left(\frac{D_j - S_j}{\Delta_j}\right)^2,$$

where
$$S_j = Y_{\text{coherent}}(j) + Y_{\text{incoherent}}(j).$$

The denominator, Δ_j, is taken to be

$$\Delta_j = \left[(\delta D_j)^2 + (\epsilon D_j)^2 \right]^{\frac{1}{2}}$$

where δD_j is the statistical uncertainty quoted in Tables 4-5 and $\epsilon = 0.10$ is our estimate of the systematic uncertainties expected between the various yield curves.

B. Discussion of Results

The results of the best fit are displayed in Figs. 5-6. The solid curves represent the sum of the coherent and incoherent yields whereas the dashed curves represent the incoherent yields alone. The yield measurements for all energies,

production angles, and target thicknesses are observed to be satisfactorily reproduced. As seen in Figs. 5-6, the major portion of yield arises from the incoherent contributions. The coherent $\phi(1020)$ contribution, which is $\sim 15\%$ of the total yield at 2°, is concentrated at low momenta and diminishes rapidly as the production angle is increased.

The fitted values for the integrated structure function, $\mathscr{F}(x)$, are given in Table 7 and shown in Fig. 10. For the integral over x, we find $\int_0^1 dx \ \mathscr{F}(x) = 6.0 \mu b$. The overall systematic uncertainty in $\mathscr{F}(x)$ may be as large as 25% arising from the overall systematic uncertainties in the data, in the thick target approximation for the photon energy spectrum, and in the theoretical assumptions of scaling and factorization.

It is interesting to compare the values of $\mathscr{F}(x)$ for Be to those for hydrogen. In Fig. 10 we also show the preliminary data of Boyarski et al[42] for the average of K^+ and K^- inclusive photoproduction from hydrogen at 18 GeV.[43] The charged K results agree well in shape with the K_L^0 results, differing only by a scale factor, A_{EFF}. A_{EFF} can be interpreted as the effective number of nucleons contributing to the incoherent particle production. For the present comparison we find $A_{EFF} = 6$, in agreement with the empirical relation, $A_{EFF} = A^{0.9}$, which applies over a wide range of photon energies in the measurement of γA total cross sections.[44-45] Thus the inclusive photoproduction of particles from light nuclei and from hydrogen appear to be related simply by the factor A_{EFF}. For nuclei much heavier than Be, the relation $A_{EFF} = A^{0.9}$ is likely to be modified because of the opposing effects of (a) nuclear absorption of the photoproduced kaons and (b) production of kaons in multistep processes such as $\gamma N \rightarrow \pi X$, $\pi N \rightarrow KX'$.

As a further test of the present analysis, we compare in Fig. 11 the average of K^+ and K^- photoproduction yields from electrons on Be targets[46-48] to the expected K_L^o yields.[49] Generally resonable agreement is found although there are large systematic differences between the experiments.

Extrapolations of K_L^o yields to higher energies for several different Be target thicknesses have also been made. The yields for 2° production from a 1.0 r.l. Be target at energies of 30, 40, and 50 GeV are given in Fig. 12. The yields for 2° production from 0.5, 1.0, and 2.0 radiation length Be targets at an energy of 40 GeV are given in Fig. 13.

In conclusion, the interpretation presented in this paper successfully describes the K_L^o photoproduction yields, together with the average of K^+ and K^- photoproduction yields, over a wide range of electron energies, production angles, and target thicknesses. In addition, we find that the inclusive photoproduction of particles from light nuclei and from hydrogen appear to be related simply by a multiplicative factor, A_{EFF}, the effective number of nucleons in the target nucleus. The success of the inclusive interpretation allows us with confidence to extrapolate K_L^o yields to 50 GeV.

REFERENCES

1. A.D. Brody, W.B. Johnson, D.W.G.S. Leith, G. Loew, J.S. Loos, G. Luste, R. Miller, K, Moriyasu, B.C. Shen, W.M. Smart, and R. Yamartino, Phys. Rev. Letters 22, 966 (1969).

2. D.G. Cassel, E. Engels, Jr., A.C. Entis, and A. Sadoff, Phys. Letters 34B, 223 (1971).

3. M.G. Albrow, D. Aston, D.P. Barber, L. Bird, R.J. Ellison, C. Halliwell, A.E. Harckham, F.K. Loebinger, P.G. Murphy, J.Walters, A.J. Wynroe, and R.F. Templeman, Nucl. Phys. B23, 509 (1970) .

4. J.F. Shivell, E. Engels, Jr., A. Entis, J.M. Paterson, L.N. Hand, and A. Sadoff, Phys. Rev. Letters 19, 1349 (1967).

5. C.D. Buchanan, C.Y. Chien, B. Cox, E. Dally, D.F. Drickey, L. Ettlinger, P. Innocenti, F.D. Rudnick, E. Seppi, P.F. Shepard, D.H. Stork, H.K. Ticho, R.A. Zdanis, D. Berk, A. Grigorian, and L. Resvanis, Phys. Lett. 32B, 396 (1970).

6. R.S. Larsen and D. Horelick, A Precision Toroidal Charge Monitor for SLAC, SLAC-PUB-398 (1968).

7. A small fraction of true K_{e3} decays failed to have a kinematic solution due to measurement resolution or small angle scatters on the electron track. Events in this category are readily identifiable and have been included in the analysis of the K_L^o spectrum. Only 7% of the total sample are in this category.

8. The lifetime and decay properties of K_L^o and $\phi(1020)$ mesons are taken from the compilation of the Particle Data Group, Phys. Lett. 39B, 1 (1972).

9. The Monte Carlo calculations have been done using the values $\xi(0) = -0.85$, $\lambda_+ = 0.045$ (parameters for the leptonic decay), and $A = -0.25$ (parameter for the $\pi^+ \pi^- \pi^o$ decay). However, the K_L^o momentum spectrum determined in

the present analysis is insensitive to the choice of matrix element parameters. A sample analysis using simple phase space for the Monte Carlo events produced a K_L^o momentum spectrum well within the statisical uncertainties.

10. In order to insure a smooth behavior of the α_j^* versus P_K, a contribution is added to χ^2 which for the j$\underline{th}$ bin is proportional to

$$\left[\left(\frac{\alpha_{j+1} - \alpha_j}{\Delta P_{Kj}} \right) - \left(\frac{\alpha_j - \alpha_{j-1}}{\Delta P_{Kj-1}} \right) \right]^2 .$$

This additional term forces a smooth behavior of $\frac{dZ(P_K)}{dP_K}$ without significantly changing the original χ^2 value.

11. A. Böhm, P. Darriulat, C. Grosso, V. Kaftanov, K. Kleinknecht, H. L. Lynch, C. Rubbia, H. Ticho and K. Tittel, Nucl. Phys. B9, 605 (1969).

12. W. L. Lakin, E. B. Hughes, L. H. O'Neill, J. N. Otis, and L. Madansky, Phys. Letters 31B, 677 (1970).

13. R. J. Abrams, R. L. Cool, G. Giacomelli, T. F. Kycia, B. A. Leontic, K. K. Li, A. Lundby, D. N. Michael, and J. Teiger, Phys. Rev. D4, 3235 (1971).

14. The K_L^o - Pb absorption cross sections have been measured by comparing the K_L^o flux at the chamber with and without an additional 130 g/cm^2 Pb in the neutral beam.

15. Expressing P_K in GeV/c and σ in mb, the parameters for the best fits to the cross section are: $b_1 = 14.76$, $b_2 = 0.00423$, $b_3 = 19.21$, $n_1 = 0.754$, $n_2 = 0.874$, $b_4 = 7.63$, $b_5 = 17.50$, and $n_3 = 0.804$.

16. R. L. Cool, G. Giacomelli, T. F. Kycia, B. A. Leontic, K. K. Li, A. Lunby, J. Tieger, and C. Wilkin, Phys. Rev. D1, 1887 (1970).

17. R. J. Abrams, R. L. Cool, G. Giacomelli, T. F. Kycia, B. A. Leontic, K. K. Li, and D. N. Michael, Phys. Rev. D1, 1917 (1970).

18. W. Galbraith, E. W. Jenkins, T. F. Kycia, B. A. Leontic, R. H. Phillips,

A. L. Read, and R. Rubinstein, Phys. Rev. 138, B913 (1965).

19. A. P. Batson, B. B. Culwick, H. B. Klepp, and L. Riddiford, Proc. of the Royal Soc. (London) 251, 233 (1959).

20. H. O. Cohn, R. D. McCulloch, W. M. Bugg, and G. T. Condo, Nucl. Phys. B21, 505 (1970).

21. D. C. Brunt, M. J. Clayton, and B. A. Westwood, Phys. Rev. 187, 1856 (1969).

22. A. Shapira, O. Benary, Y. Eisenberg, E. E. Ronat, D. Yaffe, and G. Yekutieli, Phys. Rev. Letters 21, 1835 (1968).

23. J. G. Rushbrooke, D. V. Bugg, A. J. Oxley, J. A. Zoll, M. Jobes, J. Kinson, L. Riddiford, and B. Tallini, Nuovo Cimento 33, 1509 (1964).

24. A. Gasparyan, A. V. Nikitin, and Yu. A. Troyan, Preprint-JINR-P1-5665, Dubna (1971).

25. W. Schimmerling, T. J. Devlin, W. W. Johnson, K. G. Vosburgh, and R. E. Mischke, Rutgers Univ. Preprint (Feb., 1972).

26. N. E. Booth, G. W. Hutchinson, and B. Ledley, Proc Phys. Soc. (London) 71, 293 (1958).

27. T. Coor, D. A. Hill, W. F. Hornyak, L. W. Smith, and G. Snow, Phys. Rev. 98, 1369 (1955).

28. V. P. Dzhelepov, V. I. Satarov, and B. M. Golovin, Soviet Phys. JETP 2, 293, (1959).

29. E. F. Parker, T. Dobrowolski, H. R. Gustafson, L. W. Jones, M. J. Longo, F. E. Ringia, and B. Cork, Phys. Letters 31B, 250 (1970).

30. J. Engler, K. Horn, F. Mönnig, P. Schludecker, W. Schmidt-Parzefall, H. Schopper, P. Sievers, H. Ullrich, R. Hartung, K. Runge, and Yu. Galaktionov, Phys. Letters 32B, 716 (1970).

31. R. E. Mischke, T. J. Devlin, W. Johnson, J. Norem, K. Vosburgh, and W. Schimmerling, Phys. Rev. Letters 25, 1724 (1970).

32. E. F. Parker, T. Dobrowolski, H. R. Gustafson, L. W. Jones, M. J. Longo, F. E. Ringia, and B. Cork, Phys. Letters 31B, 246 (1970).

33. J. Engler, K. Horn, J. König, F. Mönnig, P. Schludecker, H. Schopper, P. Sievers, H. Ullrich, and K. Runge, Phys. Letters 28B, 64 (1968).

34. The analagous production of particles by virtual photons (electroproduction) also occurs. However, for the present application we can ignore the electroproduction of particles since the ratio of the contribution of virtual/real photons is approximately 0.02/T, where T is the target thickness in units of radiation lengths. For a further discussion, see L. Hand and R. Wilson, SLAC Report 25, Part II (1963).

35. Y. S. Tsai and Van Whitis, Phys. Rev. 149, 1248 (1966).

36. G. McClellan, N. Mistry, P. Mostek, H. Ogren, A. Osborne, J. Swartz, R. Talman, and G. Diambrini-Palazzi, Phys. Rev. Letters 26, 1593 (1971).

37. R. P. Feynman, Phys. Rev. Letters 23, 1415 (1969).

38. For a review of inclusive reactions, see for example, W. R. Frazer, L. Ingber, C. H. Mehta, C. H. Poon, D. Silverman, K. Stowe, P. D. Ting, and H. J. Yesian, Rev. Mod. Phys. 44, 284 (1972).

39. N. F. Bali, L. S. Brown, R. D. Peccei, and A. Pignotti, Phys. Rev. Letters 25, 557 (1970).

40. J. L. Day, N. P. Johnson, A. D. Krisch, M. L. Marshak, J. K. Randolph, P. Schmueser, G. J. Marmer, and L. G. Ratner, Phys. Rev. Letters 23, 1055 (1969).

41. K. C. Moffeit, J. Ballam, G. B. Chadwick, M. Della-Negra, R. Gearhart, J. J. Murray, P. Seyboth, C. K. Sinclair, I. O. Skillicorn, H. Spitzer, G. Wolf, H. H. Bingham, W. B. Fretter, W. J. Podolsky, M. S. Rabin, A. H. Rosenfeld, R. Windmolders, G. P. Yost, and R. H. Milburn, Phys. Rev. D5, 1603 (1972).

42. A. M. Boyarski, D. Coward, S. Ecklund, B. Richter, D. Sherden, R. Siemann, and C. Sinclair, Inclusive Yields of π^+, π^-, K^+, and K^- from Hydrogen Photoproduced at 18 GeV at Forward Angles, paper submitted to the International Symposium of Electron and Photon Interactions at High Energies, Ithaca, N. Y. (1971); D. Sherden, private communication.

43. Values for $\mathscr{F}(x)$ for hydrogen have been evaluated from the data in Ref. 41 by assuming $E^* \dfrac{d^3\sigma}{d^3p} = B\exp(-Bp_\perp^2)\mathscr{F}(x)$, and using $B = 4.4$ GeV^{-2}. In making the comparison of K_L^o photoproduction from Be with $K^\pm$ photoproduction from hydrogen, we ignore possible systematic differences due to isospin and re-generation.

44. D. O. Caldwell, V. B. Elings, W. P. Hesse, G. E. Jahn, R. J. Morrison, F. V. Murphy, and D. E. Yount, Phys. Rev. Letters 23, 1256 (1969).

45. V. Heynen, H. Meyer, B. Naroska, and D. Notz, Phys. Letters 34B, 651 (1971).

46. A. Barna, J. Cox, F. Martin, M. L. Perl, T. H. Tan, W. T. Toner, T. F. Zipf, and E. H. Bellamy, Phys. Rev. Letters 18, 360 (1967).

47. S. M. Flatté, J. J. Murray, P. R. Klein, L. H. Johston, S. G. Wojcicki, Phys. Rev. 166, 1482 (1968).

48. A. M. Boyarski, F. Bulos, W. Busza, R. Diebold, S. D. Ecklund, G. E. Fischer, J. R. Rees, and B. Richter, SLAC Users Handbook, Section C.2 (Oct. 1971).

49. The calculated curves are identical to the expected K_L^o yields except that the branching ratio of $\phi(1020)$ into charged kaons has been used (see Ref. 8).

FIGURE CAPTIONS

1. Schematic illustration of the neutral secondary beam.

2. Distribution in p_{VIS} used to determine the K^o_L momentum spectrum from

    16 GeV electrons incident at 2° production angle on a 1.75 r.l. Be target.

    The variable p_{VIS} is the visible momentum from the three body K^o_L decays,

    as explained in the text. (a) - (s) Monte Carlo generated component histograms

    corresponding to the indicated narrow K^o_L momentum intervals. The intensities

    of the histograms are determined by a fit to the p_{VIS} spectrum. (t) Experimental

    p_{VIS} spectrum. The solid curve is the summation of the fitted component histo-

    grams.

3. Comparison of the K^o_L flux (circles) and neutron flux (squares) at the hydrogen

    bubble chamber. This comparison is a typical example of the relative K^o_L/n

    fluxes at the experimental apparatus, but this ratio does depend on the amount

    of photon absorber in the beam, the production angle, and the distance between

    the target and apparatus. The solid and open circles result from different

    methods of analysis of the K^o_L spectrum (see text).

4. Absorption cross sections for K mesons on various absorbers. The data are

    from the following sources: (■) K^o_L - Pb (this experiment); (△) K^o_L- Pb,

    K^o_L- Cu, and K^o_L- C (Ref. 12); (◆) average of K^+ and K^- on Cu, C, and n

    (Ref. 13, 16, 17); (◇) average of K^+ and K^- on n (Ref. 18). The curves

    shown are empirical interpolations described in the text.

5. Yields of K^o_L mesons from electrons incident on a 1.75 r.l. Be target. The

    various electron energies and producton angles are indicated. The curves

    represent an interpretation of the yields (discussed in detail in the text) in terms

    of the inclusive K^o_L photoproduction from the individual nucleons in the Be nucleus

    together with the coherent $\phi(1020)$ photoproduction from the entire nucleus. The

    sum of these two processes is shown by the solid curves, whereas the inclusive

production alone is shown by the dashed curves.

6. Yields of K_L^O mesons from electrons incident on an 0.70 r.l. Be target. The electron energies and production angles are indicated. The meaning of the curves is discussed in the caption to Figure 5.

7. Summary of cross section values for the reaction $np \to pp\pi^-$. The inset shows the low energy data on an expanded momentum scale. The data sources are: ($\triangle$) Ref. 19, ($\blacktriangledown$) Ref. 20, ($\square$) Ref. 21, ($\ast$) Ref. 22, ($\blacklozenge$) Ref. 23, and ($\lozenge$) Ref. 24. The solid curve is a hand drawn interpolation curve used to extract neutron flux at the chamber. The dashed curves represent our estimate of the uncertainty in the extrapolation curve.

8. Yields of neutrons from electrons incident on a 1.75 r.l. Be target. The electron energies and production angles are indicated.

9. Calculated yields for successive bins in $\mathscr{F}(x)$. The different curves result from distinct bins in $\mathscr{F}(x)$ which have widths of 0.1 in x and are centered at the indicated values of x (see text).

10. Integrated structure function, $\mathscr{F}(x)$, versus x. The solid circles are the values found in the present analysis for K_L^O photoproduction from Be. For comparison, the average of K^+ and K^- data from hydrogen (Ref. 42) are also shown. The hydrogen data are multiplied by a factor, $A_{EFF} = 6$, in order to account for the Be target (see text).

11. Comparison of expected yields for K_L^O from Be to the average of K^+ and K^- yields from Be. The data sources are: ($\blacklozenge$) Ref. 46, ($\blacklozenge$) Ref. 47, and ($\blacksquare$) Ref. 48. The electron energies, production angles, and target thicknesses are indicated. The curves are calculated from the fitted values for $\mathscr{F}(x)$ as explained in the text. The solid and dashed curves are as in the caption to Figure 5.

12. Predicted yields of K_L^O mesons for 2^O production from a 1.0 r.l. Be target

at electron energies of 30, 40, and 50 GeV.

13. Predicted yields of K_L^0 mesons for 2^0 production from 0.5, 1.0, and 2.0 r.l. Be targets at an electron energy of 40 GeV.

Table 1

Target thicknesses, electron energies, and production angles for

K_L^O and n yields in the present experiment

Experimental Run	Thickness of Be Target (r.l.)	e⁻ Energy (GeV)	Production Angle (degrees)	Particle Yields Reported
I	1.75	10	2	K_L^O
			4	K_L^O
		16	2	K_L^O and n
			3	K_L^O and n
			4	K_L^O and n
II	0.70	16	1.6	K_L^O
		18	1.6	K_L^O
III	1.75	19	1.6	K_L^O and n

Table 2

Photon absorbers used in secondary neutral beams

e⁻ Energy (GeV)	Production Angle	Amount of material in secondary beam (g/cm^2)			
		Hydrogen[a]	Lithium[a]	Tungsten	Lead
10	2°	6.5	44.6	147.1	173.0
10	4°	6.5	44.6	147.1	173.0
16	2°	6.5	44.6	147.1	173.0
16	3°	6.5	44.6	147.1	173.0
16	4°	6.5	44.6	147.1	173.0
16	1.6°	9.7	65.0	98.0	216.2
18	1.6°	9.7	65.0	98.0	259.5
19	1.6°	3.3	22.3	98.0	230.6

a. The hydrogen and lithium was in the form of compressed blocks of lithium hydride powder kept in an inert atmosphere.

Table 3 K^O_L-Pb Absorption Cross Sections

K^O_L Momentum (GeV/c)	σ^a (mb)
1.4 - 2.6	2730 ± 100
2.6 - 3.4	2660 ± 120
3.4 - 5.0	2540 ± 130
5.0 - 7.4	2370 ± 140

a. The uncertainties quoted are statistical only. The overall systematic error is estimated to be ± 5%.

Table 4

K^o_L yields for electrons incident on 1.75 rad. length Be target

Electron Energy (GeV)	10	10	16	16	16	19
Production Angle	2°	4°	2°	3°	4°	1.6°
No. Events in P_{VIS} Distribution	821	601	7112	2149	1524	2468
K^o_L Momentum Interval (GeV/c)	Yield at target $K^o_L/(10^4$ electrons)/sr/GeV/c					
1.0 - 1.5	1.10 ± 0.13	1.12 ± 0.13	2.40 ± 0.12	1.92 ± 0.15	1.70 ± 0.20	2.40 ± 0.20
1.5 - 2.0	1.18 ± 0.10	1.19 ± 0.11	2.59 ± 0.12	2.10 ± 0.13	1.89 ± 0.17	2.76 ± 0.15
2.0 - 2.5	1.21 ± 0.10	1.12 ± 0.10	2.70 ± 0.12	2.15 ± 0.13	2.00 ± 0.16	3.00 ± 0.15
2.5 - 3.0	1.19 ± 0.10	0.93 ± 0.09	2.62 ± 0.11	2.06 ± 0.12	1.99 ± 0.14	3.17 ± 0.15
3.0 - 3.5	1.08 ± 0.11	0.65 ± 0.09	2.44 ± 0.11	1.89 ± 0.12	1.78 ± 0.12	3.13 ± 0.15
3.5 - 4.0	0.86 ± 0.13	0.42 ± 0.08	2.14 ± 0.10	1.67 ± 0.11	1.53 ± 0.10	2.94 ± 0.15
4.0 - 4.5	0.61 ± 0.11	0.27 ± 0.07	1.85 ± 0.10	1.46 ± 0.10	1.24 ± 0.09	2.70 ± 0.14
4.5 - 5.0	0.38 ± 0.10	0.18 ± 0.06	1.58 ± 0.10	1.25 ± 0.09	1.01 ± 0.08	2.45 ± 0.14
5.0 - 6.0	0.16 ± 0.06	0.12 ± 0.05	1.21 ± 0.09	0.96 ± 0.08	0.74 ± 0.07	2.10 ± 0.12
6.0 - 7.0	0.06 ± 0.03		0.84 ± 0.07	0.62 ± 0.07	0.48 ± 0.06	1.66 ± 0.11
7.0 - 8.0			0.54 ± 0.05	0.38 ± 0.05	0.32 ± 0.05	1.34 ± 0.10
8.0 - 9.0			0.35 ± 0.04	0.23 ± 0.04	0.19 ± 0.05	1.04 ± 0.09
9.0 - 10.0			0.21 ± 0.03	0.14 ± 0.04	0.01 ± 0.04	0.76 ± 0.08
10.0 - 11.0			0.09 ± 0.03	0.07 ± 0.03		0.50 ± 0.07
11.0 - 12.0			0.05 ± 0.02			0.31 ± 0.06
12.0 - 13.0						0.17 ± 0.06

Table 5

K_L^0 yields for electrons incident on 0.70 rad. length Be target

Electron Energy (GeV)	16	18
Production Angle	1.6°	1.6°
No. Events in P_{VIS} Distribution	4195	2117
K_L^0 momentum interval (GeV/c)	Yield at target $K_L^0/(10^4$ electrons)/sr/GeV/c	
1.0 - 1.5	0.83 ± 0.09	0.92 ± 0.09
1.5 - 2.0	1.00 ± 0.08	1.09 ± 0.09
2.0 - 2.5	1.13 ± 0.08	1.21 ± 0.09
2.5 - 3.0	1.17 ± 0.07	1.27 ± 0.08
3.0 - 3.5	1.14 ± 0.07	1.24 ± 0.08
3.5 - 4.0	1.10 ± 0.07	1.20 ± 0.08
4.0 - 4.5	1.03 ± 0.07	1.11 ± 0.07
4.5 - 5.0	0.96 ± 0.07	1.04 ± 0.07
5.0 - 6.0	0.82 ± 0.07	0.89 ± 0.07
6.0 - 7.0	0.60 ± 0.06	0.68 ± 0.06
7.0 - 8.0	0.40 ± 0.05	0.50 ± 0.06
8.0 - 9.0	0.25 ± 0.04	0.36 ± 0.05
9.0 - 10.0	0.13 ± 0.03	0.26 ± 0.04
10.0 - 11.0	0.06 ± 0.02	0.16 ± 0.04
11.0 - 12.0		0.09 ± 0.03

Table 6

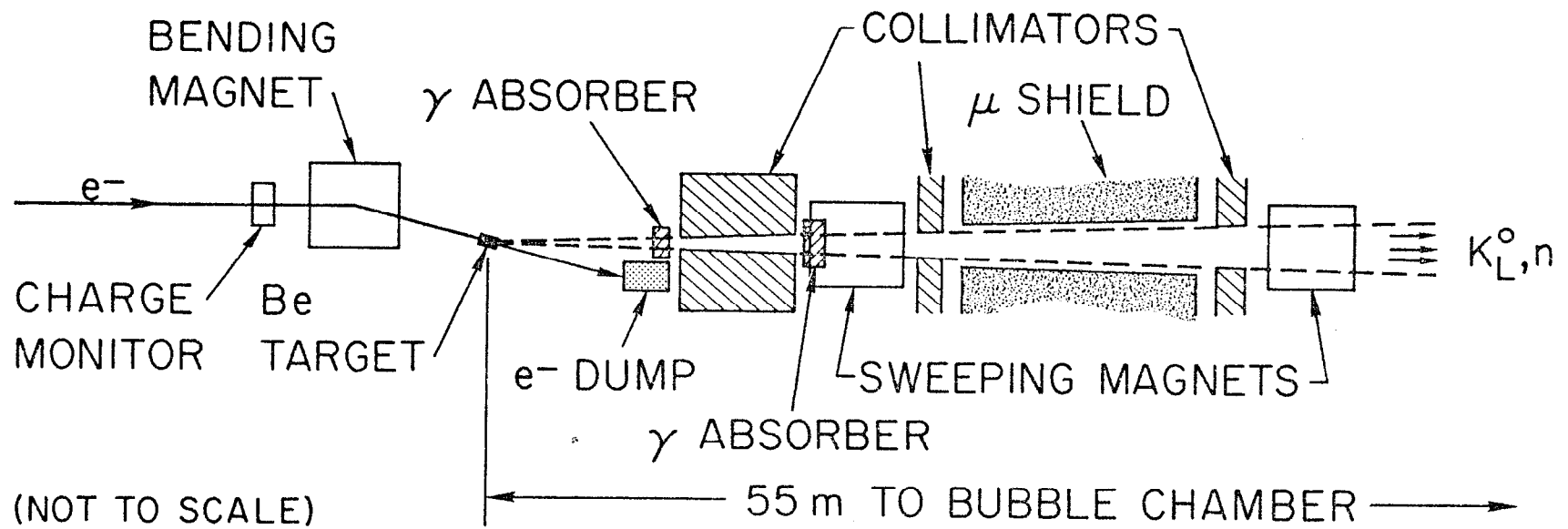Neutron yields for electrons incident on 1.75 rad. length Be target

Electron Energy (GeV)	16	16	16	19
Production Angle	$2°$	$3°$	$4°$	$1.6°$
No. Events np → ppπ⁻	10596	3809	2722	3297
n Momentum Interval (GeV/c)	Yield at target n/(10^4 electrons)/sr/GeV/c			
1.1 - 1.2	12.27 ± 3.00	11.22 ± 2.87	10.85 ± 2.81	10.34 ± 2.71
1.2 - 1.4	9.99 ± 1.31	9.52 ± 1.30	9.04 ± 1.26	9.13 ± 1.28
1.4 - 1.6	9.74 ± 1.10	8.15 ± 0.98	8.54 ± 1.04	8.41 ± 1.03
1.6 - 1.8	9.03 ± 1.04	8.99 ± 1.09	7.35 ± 0.93	8.83 ± 1.10
1.8 - 2.0	8.68 ± 0.86	7.35 ± 0.81	6.98 ± 0.80	7.60 ± 0.86
2.0 - 2.5	7.31 ± 0.75	6.52 ± 0.70	5.46 ± 0.61	6.84 ± 0.75
2.5 - 3.0	5.77 ± 0.66	5.47 ± 0.66	5.24 ± 0.65	5.40 ± 0.67
3.0 - 3.5	5.20 ± 0.67	4.33 ± 0.60	4.04 ± 0.58	5.00 ± 0.70
3.5 - 4.0	3.88 ± 0.56	3.91 ± 0.59	3.12 ± 0.50	3.87 ± 0.60
4.0 - 4.5	3.14 ± 0.50	3.45 ± 0.58	2.30 ± 0.42	4.09 ± 0.69
4.5 - 5.0	2.49 ± 0.44	2.20 ± 0.42	2.00 ± 0.40	3.30 ± 0.62
5.0 - 6.0	1.80 ± 0.28	1.56 ± 0.27	1.35 ± 0.24	2.57 ± 0.42
6.0 - 7.0	1.34 ± 0.24	1.24 ± 0.24	0.86 ± 0.19	1.78 ± 0.34
7.0 - 9.0	0.52 ± 0.10	0.53 ± 0.12	0.32 ± 0.08	1.03 ± 0.21

TABLE 7

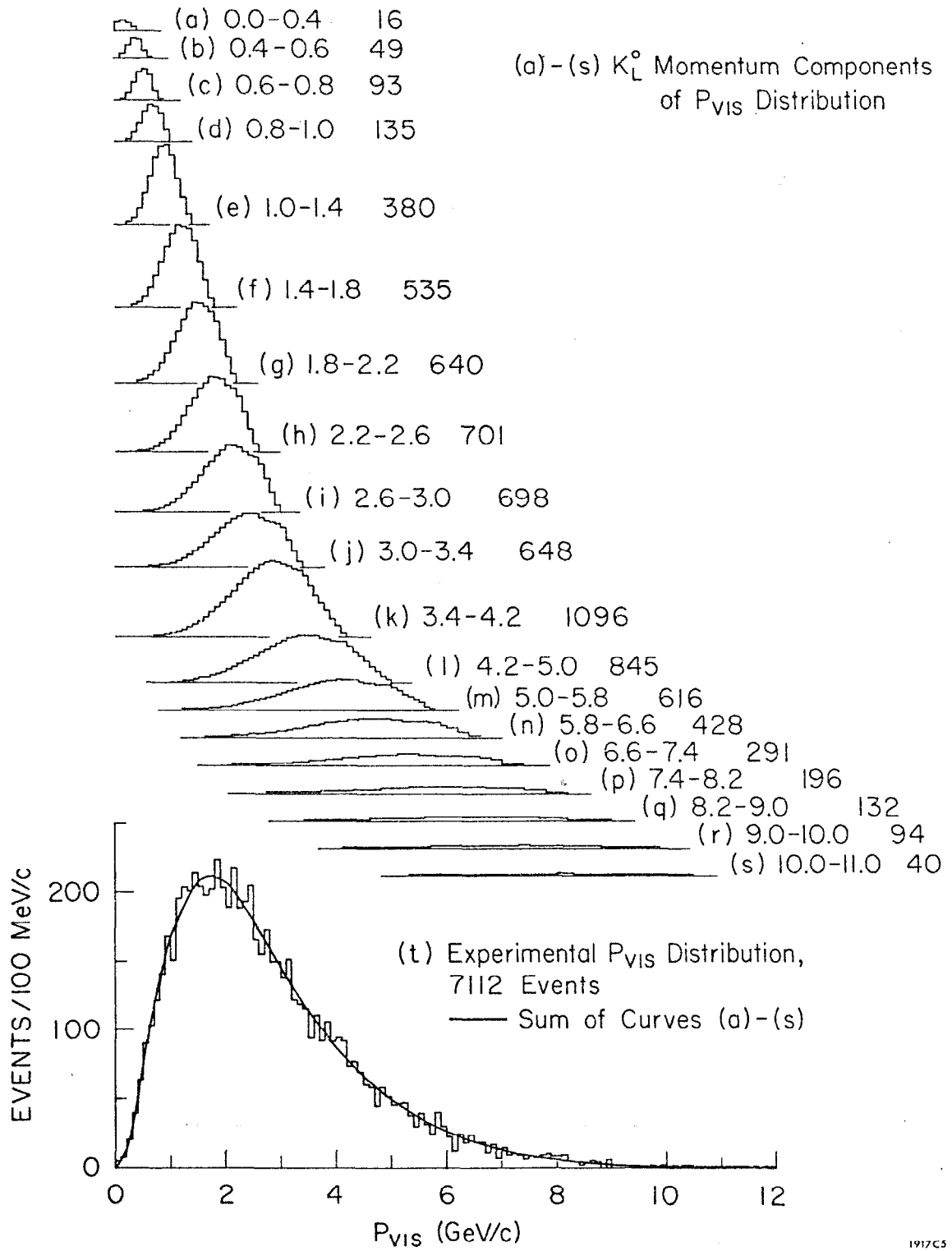Integrated Structure Function for $\gamma Be \rightarrow K_L^O X$

x	$\mathscr{F}(x)$ (μb)
0.0 - 0.2	11.4 ± 0.8
0.2 - 0.3	9.5 ± 1.6
0.3 - 0.4	8.6 ± 1.6
0.4 - 0.5	6.0 ± 1.4
0.5 - 0.6	6.7 ± 1.1
0.6 - 0.7	4.2 ± 0.9
0.7 - 0.9	1.0 ± 0.3

BENDING MAGNET

γ ABSORBER

COLLIMATORS

μ SHIELD

e⁻

CHARGE MONITOR

Be TARGET

e⁻ DUMP

γ ABSORBER

SWEEPING MAGNETS

K^o_L, n

(NOT TO SCALE)

55 m TO BUBBLE CHAMBER

191783

FIG. 1

(a) 0.0-0.4   16
(b) 0.4-0.6   49
(c) 0.6-0.8   93
(d) 0.8-1.0   135

(a)-(s) K_L^0 Momentum Components
        of P_{VIS} Distribution

(e) 1.0-1.4   380

(f) 1.4-1.8   535

(g) 1.8-2.2   640

(h) 2.2-2.6   701

(i) 2.6-3.0   698

(j) 3.0-3.4   648

(k) 3.4-4.2   1096

(l) 4.2-5.0   845
(m) 5.0-5.8   616
(n) 5.8-6.6   428
(o) 6.6-7.4   291
(p) 7.4-8.2   196
(q) 8.2-9.0   132
(r) 9.0-10.0   94
(s) 10.0-11.0   40

(t) Experimental P_{VIS} Distribution,
    7112 Events
    —— Sum of Curves (a)-(s)

EVENTS/100 MeV/c

200

100

0

0   2   4   6   8   10   12

P_{VIS} (GeV/c)

1917C5

FIG. 2

FIG. 3

FIG. 4

K$_L^0$ YIELD FROM 1.75 R.L. Be TARGET

YIELD (K$_L^0$/e$^-$/sr/GeV/c)

19 GeV e$^-$
1.6°

16 GeV e$^-$
2°

16 GeV e$^-$
3°

16 GeV e$^-$
4°

10 GeV e$^-$
2°

10 GeV e$^-$
4°

K$_L^0$ MOMENTUM (GeV/c)

2126C4

FIG. 5

FIG. 6

$\sigma\,(pn \longrightarrow pp\pi^-)$

- ● Rushbrooke et al.
- ○ Gasparyan et al.
- △ Batson et al.
- □ Brunt et al.
- ▼ Cohn et al.
- ✕ Shapira et al.

FIG. 7

2126A6

NEUTRON YIELD FROM
1.75 R.L. Be TARGET

19 GeV e⁻
1.6°

16 GeV e⁻
2°

16 GeV e⁻
3°

16 GeV e⁻
4°

YIELD (n/e⁻/sr/GeV/c)

NEUTRON MOMENTUM (GeV/c)

212687

FIG. 8

FIG. 9

Figure axes: K$_L^0$ MOMENTUM (GeV/c) (horizontal axis, 0 to 16); YIELD FOR BINS IN $\mathscr{G}(x)$ (K$_L^0$/e$^-$/sr/GeV/c) (vertical axis, 10^{-4} to 10^{-6}). Curves labeled x = .05, .15, .25, .35, .45, .55, .65, .75, .85, .95.

2126A8

FIG. 10

2126A10

AVERAGE K$^{\pm}$ YIELD

- ■ Boyarski et al. (E_0=18 GeV, T=0.3 R.L.)
- ● Flatte' et al. (E_0=18 GeV, T=0.3 R.L.)
- ◆ Barna et al. (E_0=16 GeV, T=1.8 R.L.)

$\theta = 1°$

$\theta = 2°$

$\theta = 3°$

$\theta = 5°$

$\theta = 0°$

YIELD (K/e$^-$/sr/GeV/c)

K MOMENTUM (GeV/c)

2126C13

FIG. 11

FIG. 12

FIG. 13