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THE (6, 6*) ⊕ (6*, 6) REPRESENTATION OF SU(3) ⊗ SU(3) AND THE BREAKING OF CHIRAL SYMMETRY*†

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ABSTRACT

The (6, 6*) ⊕ (6*, 6) representation of SU(3) ⊗ SU(3) is presented and its use in breaking chiral symmetry is discussed in terms of its contribution to meson masses, pion-pion scattering lengths, baryon masses, and the nucleon sigma term. We include singlet, octet, and 27 plet SU(3) pieces in the symmetry breaking Hamiltonian and also discuss the possible SU(2) ⊗ SU(2) classifications of the Hamiltonian.

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I. Introduction

Recent experimental evidence on the s-wave pion-pion scattering lengths[1,2] seems to indicate the need for a chiral symmetry breaking Hamiltonian which transforms other than $(3, 3^*) \oplus (3^*, 3)$. In order to produce a large isospin zero s-wave scattering length, the original Weinberg analysis[3] must be modified to include isospin two contributions to the sigma commutator. This in turn requires the symmetry breaking Hamiltonian to contain pieces which belong to an $SU(3) \otimes (SU(3)$ representation which has isospin two components in its reduction to SU(3) and hence to SU(2). It is also possible that a large value of the nucleon sigma term would require these other terms but this conclusion is not definitely confirmed. Indirectly, a recent analysis of the hard-pion Ward identity approach to the pion-pion scattering problem[4] which inforces unitarity within certain smoothness approximations, also requires isospin two sigma terms for the optimal solution. This result is, however, also rather uncertain because of the many assumptions involved.

Assuming that such additional pieces are necessary in the Hamiltonian, it is natural to investigate the consequences of the simplest possible choices. In order to have isospin two we require at least the 27 dimensional representation of SU(3). The two smallest $SU(3) \otimes SU(3)$ representations containing this are $(8, 8)$ and $(6, 6^*) \oplus (6^*, 6)$ which reduce under parity and SU(3) as $1^+ \oplus 8^+ \oplus 27^+ \oplus 8^- \oplus 10^- \oplus \overline{10}^-$ and $1^+ \oplus 8^+ \oplus 27^+ \oplus 1^- \oplus 8^- \oplus 27^-$, respectively. The consequences of using the former have been discussed by several authors.[5,6,7] In this paper we shall explore the later possibility.

Although both of the above symmetry breaking mechanisms have been suggested on the basis of simplicity, no dynamical model has been proposed. If we use the quark model where the triplet belongs to $(1, 3) \oplus (3^*, 1) \oplus (1, 3^*) \oplus (3, 1)$, then a Fermi like coupling could induce either of the above breaking mechanisms. For (8, 8) one could also have a three point coupling to an octet of vector gluons. However, neither of these mechanisms is attractive from a theoretical standpoint.

We shall develop the $(6, 6^*) \oplus (6^*, 6)$ representation in analogy to the $(3, 3^*) \oplus (3^*, 3)$ case.[8] In section II we review the $(3, 3^*) \oplus (3^*, 3)$ development and then in section III we present the $(6, 6^*) \oplus (6^*, 6)$ representation. Section IV is a discussion of the possible forms of the symmetry breaking Hamiltonian in terms of its SU(3) and SU(2) $\otimes$ SU(2) properties. In section V we apply this Hamiltonian to the calculation of the symmetry breaking contribution to meson masses, pion-pion scattering lengths, baryon masses, and the nucleon sigma term. We discuss these results in section VI.

II. Review of $(3, 3^*) \oplus (3^*, 3)^{\dagger}$

The 3 and 3* representations of SU(3) are defined by the commutation relations

$$[F_\alpha, T_i] = \frac{1}{2} T_j \lambda^\alpha_{ji} \qquad \text{for } 3 \tag{1}$$

and

$$[F_\alpha, W_i] = -\frac{1}{2} \lambda^\alpha_{ij} W_j \qquad \text{for } 3^* \tag{2}$$

where the λ^α_{ij} are the eight 3×3 matrices of the three dimensional representation of SU(3). They satisfy

$$\lambda^{\alpha *}_{ij} = \lambda^\alpha_{ji},$$

$$[\lambda^\alpha, \lambda^\beta] = 2i f_{\alpha\beta\gamma} \lambda^\gamma,$$

and

$$\{\lambda^\alpha, \lambda^\beta\} = 2 d_{\alpha\beta\gamma} \lambda^\gamma + \frac{2}{3} \delta_{\alpha\beta} I .$$

† In this section Greek indices run from 1, . . 8 and Latin from 1, . . 3.

From the above we write for $(3, 3^*)$ in $SU(3) \otimes SU(3)$

$$[F^+_\alpha, T_{ij}] = \frac{1}{2} \lambda^{\alpha *}_{ik} T_{kj}$$

$$[F^-_\alpha, T_{ij}] = -\frac{1}{2} \lambda^{\alpha}_{jk} T_{ik} \tag{3}$$

and for $(3^*, 3)$

$$[F^+_\alpha, W_{ij}] = -\frac{1}{2} \lambda^{\alpha}_{ik} W_{kj}$$

$$[F^-_\alpha, W_{ij}] = \frac{1}{2} \lambda^{\alpha *}_{jk} W_{ik}$$

where $F^+_\alpha = \frac{1}{2}(F_\alpha + F^5_\alpha)$ and $F^-_\alpha = \frac{1}{2}(F_\alpha - F^5_\alpha)$. Since $T^\dagger_{ij}$ transforms like $(3^*, 3)$ we can parity double our decomposition by requiring

$$P\,T_{ij} P^{-1} = T^\dagger_{ji}$$

so that T_{ij} is now said to transform under $(3, 3^*) \oplus (3^*, 3)$.

In order to reduce this representation under parity, we define

$$P_{ij} = T_{ij} + T^\dagger_{ji} \tag{4}$$

and

$$M_{ij} = i(T_{ij} - T^\dagger_{ji})$$

so that

$$P\,P_{ij} P^{-1} = P_{ij}, \quad P^\dagger_{ij} = P_{ji}$$

$$P\,M_{ij} P^{-1} = -M_{ij}, \quad M^\dagger_{ij} = M_{ji} .$$

The SU(3) content can be made manifest by writing

$$P_{ij} = \frac{1}{\sqrt{3}} U_0 \delta_{ji} + \frac{1}{\sqrt{2}} \lambda^\alpha_{ji} U_\alpha$$

and

$$M_{ij} = \frac{1}{\sqrt{3}} V_0 \delta_{ij} + \frac{1}{\sqrt{2}} \lambda^\alpha_{ji} V_\alpha .$$

We can invert these relations as

$$U_0 = \frac{1}{\sqrt{3}} P_{ii}$$

(5)

$$U_i = \frac{1}{\sqrt{2}} \lambda^{\alpha}_{ij} P_{ji}$$

(5)

$$V_0 = \frac{1}{\sqrt{3}} M_{ii}$$

$$V_{\alpha} = \frac{1}{\sqrt{2}} \lambda^{\alpha}_{ij} M_{ji}$$

where the U's and V's are Hermitian scalar and pseudoscalar fields, respectively. They satisfy the well known commutation relations[8] (from Eq. (3))

$$[F_{\alpha}, U_0] = [F_i, V_0] = 0$$

$$[F_{\alpha}, U_{\beta}] = i f_{\alpha\beta\gamma} U_{\gamma}, \quad [F_{\alpha}, V_{\beta}] = i f_{\alpha\beta\gamma} V_{\gamma}$$

which identify U_0 and V_0 as SU(3) singlets and $\{U_{\alpha}\}$ and $\{V_{\alpha}\}$ as SU(3) octets. Also,

$$[F^5_{\alpha}, U_0] = -i\sqrt{\tfrac{2}{3}}\, V_{\alpha}, \quad [F^5_{\alpha}, V_0] = i\sqrt{\tfrac{2}{3}}\, U_{\alpha}$$

$$[F^5_{\alpha}, U_{\beta}] = -i\, d_{\alpha\beta\gamma} V_{\gamma} - i\sqrt{\tfrac{2}{3}}\, \delta_{\alpha\beta} V_0$$

$$[F^5_{\alpha}, V_{\beta}] = i\, d_{\alpha\beta\gamma} U_{\gamma} + i\sqrt{\tfrac{2}{3}}\, \delta_{\alpha\beta} U_0$$

For calculations involving $(3, 3^*) \oplus (3^*, 3)$ it is customary to work directly with the U's and V's since their commutation relations are simple and the properties of $f_{\alpha\beta\gamma}$ and $d_{\alpha\beta\gamma}$ are well tabulated.[9] However, as we shall see, for more complicated representations, it proves simpler to work directly in terms of the analogues of the T_{ij}. Thus, for example, instead of writing the perturbing Hamiltonian as[11]

$$H^1 = C_{30}U_0 + C_{38}U_8$$

we could as well use

$$H^1 = \frac{1}{\sqrt{3}} C_{30}P_{ii} + \frac{1}{\sqrt{2}} C_{38}\lambda^8_{ij}P_{ji} .$$

This later approach reduces calculations such as those in section V to trace calculations with the $\{\lambda^\alpha\}$ matrices.

III. The (6, 6*) ⊕ (6*, 6) Representation

We develop this representation in analogy to section II by writing the commutation relations

$$[F_\alpha, T_i] = \frac{1}{2} T_j S^\alpha_{ji} \qquad \text{for } 6$$

and

$$[F_\alpha, W_i] = -\frac{1}{2} S^\alpha_{ij} W_j \qquad \text{for } 6^*$$

where the Latin indices now run from 1-6 rather than the 1-3 in section II. The eight 6 × 6 matrices, $\{S^\alpha\}$, are the representation of the SU(3) generators in the 6 representation of SU(3). In the Appendix these matrices are explicitly presented using the phase conventions of R. Behrends et al.[10] They satisfy

$$S^{\alpha *}_{ij} = S^\alpha_{ji}$$

and

$$[S^\alpha, S^\beta] = 2i f_{\alpha\beta\gamma} S^\gamma$$

For (6, 6*) we write

$$[F^+_\alpha, T_{ij}] = \frac{1}{2} S^{\alpha *}_{ik} T_{kj}$$

$$[F^-_\alpha, T_{ij}] = -\frac{1}{2} S^\alpha_{jk} T_{ik} \qquad (6)$$

and extend this to $(6,6^*) \oplus (6^*,6)$ by introducing parity via $P\,T_{ij}P^{-1} = T_{ij}^{\dagger}$ as before. The parity content of this representation is reduced by

$$P_{ij} = T_{ij} + T_{ji}^{\dagger}$$

$$M_{ij} = i(T_{ij} - T_{ji}^{\dagger})$$

as in Eq. (4).

Now, however, the SU(3) decomposition is slightly more complicated due to the presence of the 27 dimensional representation. We write

$$\begin{aligned} P_{ij} &= \frac{1}{\sqrt{6}}\,\delta_{ji}U_0 + \frac{1}{\sqrt{10}}\,S_{ji}^{\alpha}\,U_{\alpha} + T_{ji}^{\ominus}\,U_{\ominus} \\ M_{ij} &= \frac{1}{\sqrt{6}}\,\delta_{ji}\,V_0 + \frac{1}{\sqrt{10}}\,S_{ji}^{\alpha}V_{\alpha} + T_{ji}^{\ominus}\,V_{\ominus} \end{aligned} \tag{7}$$

where $-$ is summed from 1 to 27. The matrices δ, $\{S^{\alpha}\}$, and $\{T^{\ominus}\}$ satisfy

$$T_R(\delta\,\delta) = 6, \quad T_R(\delta\,S^{\alpha}) = T_R(\delta\,T^{-}) = 0$$

$$T_R(S^{\alpha}S^{\beta}) = 10\,\delta_{\alpha\beta}, \quad T_R(S^{\alpha}T^{\ominus}) = 0$$

$$T_R(T^{\ominus}\,T^{\ominus'}) = \delta_{\ominus\ominus'}\,.$$

We can explicitly construct the $\{T^{\ominus}\}$ by writing the Clebsch-Gordon series for $6 \otimes 6^* = 1 \oplus 8 \oplus 27$. We do not present the general result since we shall need only T_{ij}^{27} (corresponding to the $I = Y = 0$ member of 27) in our subsequent calculations. It is given by

$$T_{ij}^{27} = \frac{1}{\sqrt{30}} \begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & 0 \\ & & & -3 & & \\ 0 & & & & -3 & \\ & & & & & 3 \end{pmatrix}$$

Using the trace relations, we invert Eqs. (7) as

$$U_0 = \frac{1}{\sqrt{6}} P_{ii}, \quad V_0 = \frac{1}{\sqrt{6}} M_{ii}$$
$$U_\alpha = \frac{1}{\sqrt{10}} S^\alpha_{ij} P_{ji}, \quad V_\alpha = \frac{1}{\sqrt{10}} S^\alpha_{ij} M_{ji} \tag{8}$$
$$U_\ominus = T^\ominus_{ij} P_{ji}, \quad V_\ominus = T^\ominus_{ij} M_{ji}$$

The U's and V's are scalar and pseudoscalar fields, respectively and $\{U_0\}$ and $\{V_0\}$, $\{U_\alpha\}$ and $\{V_\alpha\}$, and $\{U_\ominus\}$ and $\{V_\ominus\}$ transform as singlet, octet, and 27-plet representations of SU(3) respectively.

The commutation relations of the U's and V's are easily written down from Eq. (8) in analogy to Eq. (5). However, it is more convenient to use the relations for P_{ij} and M_{ij} directly. These are found from Eqs. (6) to be

$$[F_\alpha, P_{ij}] = \frac{1}{2} S^{\alpha*}_{ik} P_{kj} - \frac{1}{2} S^\alpha_{jk} P_{ik}$$
$$[F_\alpha, M_{ij}] = \frac{1}{2} S^{\alpha*}_{ik} M_{kj} - \frac{1}{2} S^\alpha_{jk} M_{ik}$$
$$[F^5_\alpha, P_{ij}] = -\frac{i}{2} S^{\alpha*}_{ik} M_{kj} - \frac{i}{2} S^\alpha_{jk} M_{ik} \tag{9}$$
$$[F^5_\alpha, M_{ij}] = \frac{i}{2} S^{\alpha*}_{ik} P_{kj} + \frac{i}{2} S^\alpha_{jk} P_{ik}$$

IV. Hamiltonian Forms

To construct a suitable symmetry breaking Hamiltonian for the strong interactions, we wish to include terms which conserve parity, isospin, and hypercharge. Thus, we can include components proportional to U_0(singlet), U_8 (from $\{U_\alpha\}$) and U_{27} (from $\{U_\ominus\}$), i.e., we write

$$H^1 = \frac{1}{\sqrt{6}} C_{60} P_{ii} + \frac{1}{\sqrt{10}} C_{68} S^8_{ij} P_{ji} + C_{627} T^{27}_{ij} P_{ji} \tag{10}$$

Two special cases of this general form may be of interest. If we set $C_{627} = 0$, then H^1 contains only singlet and octet pieces and thus represents an octet dominance type breaking which will, for example, lead automatically to the Gell-Mann-Okubo mass formula for the meson and baryon states.

It is also interesting to ask how H^1 transforms under the subgroup $SU(2) \otimes SU(2)^\dagger$ since the $(3, 3^*) \oplus (3^*, 3)$ symmetry breaking scheme seems to indicate that the breaking term may be approximately in a $(0, 0)$ representation.[11] It is easy to see that if we are to induce isospin two components in the sigma commutator, then we have to include an $SU(2) \otimes SU(2)$ piece from the $(1, 1)$ representation (the highest representation contained in $(6, 6^*) \oplus (6^*, 6)$). Thus, the two most interesting cases are the $(0, 0)$ and $(1, 1)$ representations contained in $(6, 6^*) \oplus (6^*, 6)$.

Because we have parity doubling, we have both a $(0, 0)^+$ and a $(0, 0)^-$. Also, for $(1, 1)$ we have two cases which reduce under SU(2) as $0^+ \oplus \frac{1}{2}^- \oplus 1^+$ and $0^- + \frac{1}{2}^+ \oplus 1^-$, respectively. From these, clearly the $(0, 0)^+$ state and the 0^+ member of a $(1, 1)$ are the suitable candidates for forming H^1. By using a group theoretic reduction or simply by examining the commutation relations directly, it is easy to see that

P_{66}	tramsforms like $(0, 0)$ with respect to $SU(2) \times SU(2)$ and has positive parity,
$\sum_{i=1}^{3} P_{ii}$	transforms like the 0^+ $(SU(2))$ member of a $(1, 1)$ representation of $SU(2) \otimes SU(2)$.

Examining the general form, Eq. (10), for H^1, we see that the choice

$$C_{60} : C_{68} : C_{627} \sim 1 : \frac{-4}{\sqrt{5}} : \frac{3}{\sqrt{5}} \tag{11}$$

† We use the conventional notation of labeling SU(3) representations by their dimension but SU(2) representations by their spin content.

implies $H^1 \sim (0,0)^{\dagger}$ in $SU(2) \otimes SU(2)$ and

$$C_{60} : C_{68} : C_{627} \sim 1 : \frac{2}{\sqrt{5}} : \frac{1}{\sqrt{5}} \tag{12}$$

implies $H^1 \sim (1,1)$ in $SU(2) \otimes SU(2)$ (the $I = 0^+$ member). Note that each of these separately requires a 27 plet piece in H^1 but the mixture, $C_{60} : C_{68} : C_{627} \sim 1 : \sqrt{5} : 0$, removes this dependence while maintaining a mixture of pure $(0,0) \oplus (1,1)$.

V. Calculations

In this section we shall employ the most general form of H^1 (Eq. (10)) and use it to find the $(6,6^*) \oplus (6^*,6)$ contributions to A) meson masses, B) pion-pion scattering lengths, C) baryon masses, and D) nucleon sigma terms. In (A) and (B) we shall use the soft meson approximation, but this is not needed for (C) and (D). We shall also neglect the possible effects on the breaking of scale invariance by a scalar meson,[12,13] effectively assuming our symmetry breaking H^1 to have dimention, $\ell = 3$. Additional factors due to such effects[14] can easily be included in our results. We shall use the simple assumption that H is given by $H_0 + H^1$ where H_0 is invariant under $SU(3) \otimes SU(3)$, does not contribute to meson masses, and gives a uniform mass, M_0, to the baryon octet.

(A) Meson Masses

Using the usual soft meson reduction, the meson mass is given by

$$<\mu_\alpha | H^1 | \mu_\alpha> \;=\; -\frac{1}{F^2} <0|[F_5^\alpha, [F_5^\alpha, H^1]]|0>$$

where to this order we assume that the PCAC constants are all equal,[15] $F_\pi = F_k = F_\eta = F$, i.e.,

$$\partial_\mu A_\mu^\alpha = m_\alpha^2 \, F \phi^\alpha .$$

We also write

$$<0|P_{ij}|0> \;=\; \frac{1}{\sqrt{8}} \quad \delta_{ij} <0|U_0|0>$$

neglecting any possible contribution from $<0|U_8|0>$ and $<0|U_{27}|0>$. With these assumptions

$$<\mu_\alpha|H^1|\mu_\alpha> = -\frac{1}{F^2}\frac{<0|U_0|0>}{\sqrt{6}}\left[\frac{C_{60}}{\sqrt{6}}T_R(S^\alpha S^\alpha) + \frac{C_{68}}{\sqrt{10}}T_R(S^8S^\alpha S^\alpha) + C_{627}\,T_R(T^{27}S^\alpha S^\alpha)\right]$$

which yields

$$m_\alpha^2 = \frac{-5}{3F^2}<0|U_0|0>\left[C_{60} + \frac{7\sqrt{3}}{5\sqrt{5}}C_{68}d_{8\alpha\alpha} + \frac{2\sqrt{6}}{5}C_{627}\,\xi^{27}_{\alpha\alpha}\right] \tag{13}$$

where

$$d_{8\alpha\beta} = \frac{1}{2\sqrt{3}}\begin{pmatrix} 2 & & & & & & & 0 \\ & 2 & & & & & & \\ & & 2 & & & & & \\ & & & -1 & & & & \\ & & & & -1 & & & \\ & & & & & -1 & & \\ & & & & & & -1 & \\ 0 & & & & & & & -2 \end{pmatrix}$$

and

$$\xi^{27}_{\alpha\beta} = \frac{1}{2\sqrt{30}}\begin{pmatrix} 1 & & & & & & & 0 \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & -3 & & & & \\ & & & & -3 & & & \\ & & & & & -3 & & \\ & & & & & & -3 & \\ 0 & & & & & & & 9 \end{pmatrix}$$

Note that although the traces involved in finding Eq. (13) can be derived by using commutator identities, for the calculations involved here it is simpler to compute them by hand using the explicitly form for $\{S^\alpha\}$ given in the Appendix. We also remark that $d_{8\alpha\beta}$ and $\xi^{27}_{\alpha\beta}$ are the standard matrices obtained from coupling $8\otimes 8$ to 8_{sym} $(I = Y = 0)$ and 8×8 to 27 $(I = Y = 0)$, respectively.[6]

(B) Pion-Pion Scattering Lengths

On the soft meson limit, the s-wave, isospin zero scattering length is given by[16]

$$a_0^{(0)} = \frac{1}{(96\pi)} \left(5A + \frac{16\, m_\pi^2}{F^2}\right) \Big/ m_\pi$$

and

$$a_0^{(2)} = \frac{2}{5}\, a_0^{(0)} - \frac{3}{20} \frac{m_\pi^2}{F^2}$$

where $a_0^{(2)}$ is the s-wave, isospin two scattering length. In the case where the sigma commutator has no isospin two piece, $A = \frac{m_\pi^2}{F^2}$. In general it is given by

$$A = -\frac{1}{F^4} <0|\,[F_5^i,[F_5^i,[F_5^i,[F_5^i, H^1]]]]\,|0>$$

where i = 1 or 2 or 3 (no sum). A more general isospin decomposition of the four-fold commutator involves L_0 and L_2 which measure the relative isospin zero and two components of the sigma commutator. In terms of these

$$A = \frac{1}{3} L_0 + \frac{2}{3} L_2$$

but L_0 and L_2 are also constrained by a Jacobi identity relation[17] which yields

$$2L_0 - 5L_2 = 6 \frac{m_\pi^2}{F^2}.$$

Thus, $L_2 = 0$ implies $L_0 = 3 \frac{m_\pi^2}{F^2}$ which yields $A = \frac{m_\pi^2}{F^2}$ for the pure isospin zero case.

Using our form for H^1, we find

$$A = -\frac{1}{F^4} \frac{<0|U_0|0>}{\sqrt{6}} \left\{ \frac{C_{60}}{\sqrt{6}} T_R[(S^1)^4] + \frac{C_{68}}{\sqrt{10}} T_R[S^8 (S^1)^4] + C_{627} T_R[T^{27}(S^1)^4] \right\}$$

$$= -\frac{17}{3F^4} <0|U_0|0> \left[C_{60} + \frac{31}{17\sqrt{5}} C_{68} + \frac{13}{17\sqrt{5}} C_{627} \right] \qquad (14)$$

(C) Baryon Masses

With our neglect of scale symmetry breaking considerations, we write simply

$$M_\alpha = M_0 + \langle B_\alpha | H^1 | B_\alpha \rangle$$

Now we must clearly keep $\langle N|U_0|N\rangle$, $\langle N|U_8|N\rangle$, and $\langle N|U_{27}|N\rangle$ all non-zero. We denote these by N_0, N_8, and N_{27}, respectively and we let $D\,(F+D=1)$ be the mixing parameter in $\langle B_\alpha|U_8|B_\alpha\rangle$. In this notation

$$\langle N|U_0|N\rangle = \langle \Lambda|U_0|\Lambda\rangle = \langle \Sigma|U_0|\Sigma\rangle = \langle \Xi|U_0|\Xi\rangle = N_0$$

$$\langle \Sigma|U_8|\Sigma\rangle = \frac{-2/3\,D}{(1-2/3\,D)}\,N_8$$

$$\langle \Lambda|U_8|\Lambda\rangle = \frac{+2/3\,D}{(1-2/3\,D)}\,N_8$$

$$\langle \Xi|U_8|\Xi\rangle = \frac{-(1-4/3\,D)}{(1-2/3\,D)}\,N_8$$

$$\langle \Sigma|U_{27}|\Sigma\rangle = -\frac{1}{3}N_{27}$$

$$\langle \Lambda|U_{27}|\Lambda\rangle = -3\,N_{27}$$

$$\langle \Xi|U_{27}|\Xi\rangle = N_{27}$$

Thus,

$$\begin{aligned} M_N &= M_0 + C_{60}N_0 + C_{68}N_8 + C_{627}N_{27} \\ M_\Sigma &= M_0 + C_{60}N_0 - \frac{2/3\,D}{(1-2/3\,D)}\,C_{68}N_8 - \frac{1}{3}\,C_{627}N_{27} \end{aligned} \tag{15}$$

etc., with the obvious replacements for Λ and Ξ.

(D) Nucleon Sigma Term

The nucleon sigma term is given by

$$\sigma_N = \langle N|\,[F_5^i,[F_5^i,\ H^1]]\,|N\rangle$$

where i = 1 or 2 or 3 (no sum). To evaluate this we again need N_0, N_8, and N_{27}. To perform the projection we write symbolically

$$\langle N | P_{ij} | N \rangle \to \frac{1}{\sqrt{6}} \; \delta_{ij} \, N_0 + \frac{1}{\sqrt{10}} \, S^8_{ji} \, N_8 + T^{27}_{ji} \, N_{27}$$

which yields

$$\begin{aligned}
\sigma_N = {} & \frac{C_{60}}{\sqrt{6}} \left[\frac{N_0}{\sqrt{6}} \; T_R(S^1 S^1) + \frac{N_8}{\sqrt{10}} \; T_R(S^8 S^1 S^1) + N_{27} \; T_R(T^{27} S^1 S^1) \right] \\
& + \frac{C_{68}}{\sqrt{10}} \left\{ \frac{N_0}{\sqrt{6}} \; T_R(S^8 S^1 S^1) + \frac{1}{2} \frac{N_8}{\sqrt{10}} \left[T_R(S^8 S^8 S^1 S^1) + T_R(S^8 S^1 S^8 S^1) \right] \right. \\
& \left. + \frac{1}{2} \, N_{27} \left[T_R(S^8 T^{27} S^1 S^1) + T_R(S^8 S^1 S^{27} S^1) \right] \right\} \\
& + C_{627} \left\{ \frac{N_0}{\sqrt{6}} \; T_R(T^{27} S^1 S^1) + \frac{1}{2} \frac{N_8}{\sqrt{10}} \left[T_R(S^8 T^{27} S^1 S^1) + T_R(S^8 S^1 S^{27} S^1) \right] \right. \\
& \left. + \frac{1}{2} \, N_{27} \left[T_R(T^{27} T^{27} S^1 S^1) + T_R(T^{27} S^1 T^{27} S^1) \right] \right\} \\
= {} & \frac{5}{3} \; N_0 \left[C_{60} + \frac{7}{5\sqrt{5}} \; C_{68} + \frac{1}{5\sqrt{5}} \; C_{627} \right] \\
& + \frac{7}{3\sqrt{5}} \; N_8 \left[C_{60} + \frac{17}{7\sqrt{5}} \; C_{68} + \frac{11}{7\sqrt{5}} \; C_{627} \right] \qquad (16) \\
& + \frac{1}{3\sqrt{5}} \; N_{27} \left[C_{60} + \frac{11}{\sqrt{5}} \; C_{68} + \frac{13}{\sqrt{5}} \; C_{627} \right]
\end{aligned}$$

Note that even if H^1 has no 27plet component ($C_{627} = 0$), Eq. (16) still will include contributions from N_{27}.

VI. Discussion

Evidence from the $K_{\ell 4}$ decay[1, 2] seems to favor a value of $a_0^{(0)}$ which is larger than the original Weinberg prediction[3] (A = 1). For A = 1, $a_0^{(0)} \simeq .16/m_\pi$

but increase S rapidly with A. For example if A = 10, $a_0^{(0)} \simeq .5/m_\pi$. The experimental data indicate a value of $a_0^{(0)}$ of order $.5/m_\pi$. However, one must also realize that there are theoretical corrections to the soft pion predictions. A recent theoretical calculation which included unitarity corrections,[4] a value of $A \simeq \frac{1}{4}$ was found. However, in the same calculation, an effective value of $A \simeq \frac{4}{5}$ would be needed, in the simple soft pion formula for $a_0^{(0)}$, to produce the calculated scattering length. Although this calculation may not be quantitatively reliable, it does indicate that unitarity corrections can enhance the effective value of A. In view of this we may conclude that even though the experimental evidence favors $A > 1$, it may not have to be as large as $A \sim 10$ which the simple soft pion formula would indicate.

However, a simple estimate using the meson mass formula (Eq. (13)) and our calculation of A (Eq. (14)) indicates that the use of $(6,6^*) + (6^*,6)$ alone for H^1 is not reasonable. To make this estimate, we set $C_{627} = 0$ although a similar result holds in any case. Let $N \equiv \frac{<0|U_0|0>}{F^2}$ and $\alpha \equiv \frac{C_{68}}{C_{60}}$. Then from Eq. (13),

$$m_\pi^2 = -\frac{5}{3} N \left(1 + \frac{7\alpha}{5\sqrt{5}}\right)$$

$$m_K^2 = -\frac{5}{3} N \left(1 - \frac{7\alpha}{10\sqrt{5}}\right)$$

from which we find

$$\alpha = -\frac{10\sqrt{5}}{7}\frac{\left(1 - m_\pi^2/m_K^2\right)}{\left(2 + m_\pi^2/m_K^2\right)}$$

and

$$N = -\frac{1}{5}\left(m_\pi^2 + 2m_K^2\right) .$$

Using these we can calculate A from Eq. (14),

$$A = -\frac{\left(24\,m_K^2 - 143\,m_\pi^2\right)}{35\ F^2} \sim -5\ \frac{m_\pi^2}{F^2}$$

Such a large negative value is clearly ruled our by the experimental data. We might note that pure (8, 8) breaking would also produce a negative value for A.

It is clear from the preceeding remarks that $(6, 6^*) \oplus (6^*, 6)$ symmetry breaking cannot be the only contribution to H^1. On the other hand, since all of our results in section V are linear in H^1, these calculations can be used to discuss more general schemes[7, 18] which involve using $(6, 6^*) \oplus (6^*, 6)$ breaking in addition to some other contribution to H^1. Classification of H^1 pieces under the subgroup $(SU(2) \otimes SU(2)$[18] as discussed in section IV may provide a tractable approach to this problem. We shall present several alternatives of combining $(3, 3^*) \oplus (3^*, 3)$, $(6, 6^*) \oplus (6^*, 6)$ and (8, 8) in a subsequent article.

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Appendix

The $\underline{6}$ Dimensional Representation of SU(3)

We use the phase conventions and notation of Behrends, et al.,[10] to construct the representation of the eight generators of SU(3) on the six dimensional representation. In terms of a spherical basis set, these are

$$H_1 = \frac{1}{\sqrt{3}} \left(|1\rangle\langle 1| - |3\rangle\langle 3|\right) + \frac{1}{2\sqrt{3}} \left(|4\rangle\langle 4| - |5\rangle\langle 5|\right)$$

$$H_2 = \frac{1}{3} \left(|1\rangle\langle 1| + |2\rangle\langle 2| + |3\rangle\langle 3|\right) - \frac{1}{6} \left(|4\rangle\langle 4| + |5\rangle\langle 5|\right) - \frac{2}{3} \left(|6\rangle\langle 6|\right)$$

$$E_1 = \frac{1}{\sqrt{3}} \left(|2\rangle\langle 3| + |1\rangle\langle 2|\right) + \frac{1}{\sqrt{6}} \left(|4\rangle\langle 5|\right)$$

$$E_2 = \frac{1}{\sqrt{3}} \left(|4\rangle\langle 6| + |1\rangle\langle 4)\right) + \frac{1}{\sqrt{6}} \left(|2\rangle\langle 5|\right)$$

$$E_3 = \frac{1}{\sqrt{3}} \left(|5\rangle\langle 6| + |3\rangle\langle 5|\right) + \frac{1}{\sqrt{6}} \left(|2\rangle\langle 4|\right)$$

and $E_{-i} = E_i^\dagger$. The states $\{|1\rangle, |2\rangle, |3\rangle\}$ form an isotriplet, $\{|4\rangle, |5\rangle\}$ an isodoublet, and $|6\rangle\}$ is an isosinglet. We transform to a cartesian basis by writing

$$\begin{aligned}
S_1 &= 6\,(E_1 + E_{-1}) \\
S_2 &= -i\sqrt{6}\,(E_1 - E_{-1}) \\
S_3 &= 2\sqrt{3}\,H_1 \\
S_4 &= \sqrt{6}\,(E_2 + E_{-2}) \\
S_5 &= -i\sqrt{6}\,(E_2 - E_{-2}) \\
S_6 &= \sqrt{6}\,(E_3 + E_{-3}) \\
S_7 &= -i\sqrt{6}\,(E_3 - E_{-3}) \\
S_8 &= 2\sqrt{3}\,H_2
\end{aligned}$$

such that $[S_i, S_j] = 2if_{ijk} S_k$. These matrices form the $\underline{6}$ dimensional representation of SU(3).

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