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FRAGMENTATION IN THE CENTRAL REGION: POSSIBILITY OF A DISCONTINUITY AT $x = 0$*

Rudolph C. Hwa and Paul A. Zucker
Institute of Theoretical Science
University of Oregon, Eugene, Oregon

and

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

Abstract

The question of whether there may exist a discontinuity at $x = 0$ in the pion spectrum of an inclusive reaction initiated by two unequal particles is raised. While such an occurrence is impossible in a model with only short-range correlations, it is a distinct possibility in the diffractive excitation model when the energy is high enough. We relate the discontinuity to the properties of the spectra of single diffractive excitations. Arguments are given as to why the discontinuity, if it exists, is not likely to be substantial.

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I. INTRODUCTION

In the fragmentation picture,[1] or more specifically in the diffractive excitation model,[2] all produced particles found in the final state of a high-energy collision are identified as fragments, or the decay products of the excited states, of the initial particles. This interpretation is made even in the central region near $x = 0$. For this reason, the usual nomenclature of "pionization" and "fragmentation" regions is unsatisfactory in that it is biased toward a model which presupposes the existence of a region that does not involve fragmentation. We favor the use of the words "central" and "end" regions, the former referring to the region where the spectrum is flat in the rapidity plot.

If indeed all secondary particles are fragments of the initial particles, it is conceivable that different particles prefer to fragment differently. In the language of the diffractive excitation model, the probability of excitation to certain massive states may be different for different hadrons. Suppose that it is easier to excite a pion than a proton. Then the pion spectrum for the inclusive reaction of a πp collision should in the rapidity plot have a flat plateau on the pion side that is higher than the flat plateau on the proton side. A schematic representation of this is shown in Fig. 1. The overall distribution is the sum of the two parts so the transition across $y = 0$ is smooth, as indicated by the dashed line. However, in the x-plot in the infinite energy limit, the different altitudes of the two plateaus in the y plot imply a discontinuity at $x = 0$ as indicated in Fig. 2. Present data on πp collisions do not show any obvious indication of this discontinuity, but since they are at energies less than 30 GeV, the extensive overlap of the right and left regions can easily smooth out the distribution at $x = 0$.

It is <u>not</u> our intention here to suggest the inevitability of a discontinuity at $x = 0$. One result of this study is that if the discontinuity exists, it is not likely to be a glaring one. But, more importantly, we want to point out the possibility of its existence and to relate the behavior of the pion spectrum near $x = 0$ to the behavior of the leading-particle spectrum near $x = 1$.

A model based on short-range correlations in the longitudinal momentum, such as the multiperipheral model,[3] firmly predicts the absence of a discontinuity at $x = 0$. This is because a pion in the central region is far removed in rapidity from the ends and therefore carries no information about the initial particles. The assumption of double Pomeron dominance in the Mueller analysis[4] leads immediately to this result. Thus the experimental discovery of a discontinuity at $x = 0$ would create an insurmountable difficulty for the weakly-correlated models. The confirmation of no discontinuity there, however, is quite acceptable to the diffractive excitation model.

II. THE SPECTRUM NEAR $x = 0$

In the following, we shall consider hadron collisions in the infinite energy limit. In the diffractive excitation model, the two incident particles are excited into two massive states (fireballs), which then decay into two clusters of particles with invariant masses M_1 and M_2. The invariant distribution f(x) defined to be

$$f(x) = x_0 \frac{d\sigma}{dx\,dp_\perp^2} = p_0 \frac{d\sigma}{dp_\parallel\,dp_\perp^2} \tag{1}$$

where $x = 2p_\parallel/\sqrt{s}$ and $x_0 = 2p_0/\sqrt{s}$, can be calculated according to the relation[2]

$$f(x) = x_0 \int \frac{d\sigma}{dM_1}\, g(M_1, x)\, n_1\, dM_1, \qquad x > 0 \tag{2}$$

$$= x_0 \int \frac{d\sigma}{dM_2}\, g(M_2, x)\, n_2\, dM_2, \qquad x < 0 \tag{3}$$

where $d\sigma/dM_i$ is the cross section for the production of a cluster of mass M_i whatever the other cluster may be (including the possibility that it may be unexcited). $g(M_i, x)$ is the probability that a particle is observed at x among the cluster of particles of mass M_i; it is normalized such that its integral over all x is unity. Finally, n_i is the multiplicity of the particles in the i-th cluster that are identical in type to the observed one.

The question we address ourselves to is whether $f(0+)$ might be different from $f(0-)$ where we define for $\epsilon > 0$

$$f(0\pm) = \lim_{\epsilon \to 0} f(\pm\epsilon) \tag{4}$$

The probability function $g(M_i, x)$ is obtained by Lorentz transforming a Gaussian distribution (chosen for simplicity's sake) in the cluster rest frame to the center-of-mass frame. Using E to denote the average energy of a pion in the cluster rest frame, we have[2,5]

$$g(M_i, x) = \left(\frac{3}{2\pi}\right)^{\frac{1}{2}} e^{-\frac{3}{2}k^2/E^2} \frac{dk}{dx} \tag{5}$$

$$k = \frac{1}{4}\left[(x_0 - |x|)\frac{s}{M_i} - (x_0 + |x|)M_i\right] \tag{6}$$

We have assumed that the average energy E of a pion is the same in either cluster. The Gaussian peak occurs approximately at $x = E/M_1$ or $x = -E/M_2$. This is as expected from simple momentum conservation considerations since M_i/E is the average multiplicity of the cluster of mass M_i. Thus in the limit $x \to 0\pm$, the important values of M_i that contribute significantly to the integrals in (2) and (3) are infinitely large. Since we work in the $s \to \infty$ limit, this region of infinite M_i is to be taken while maintaining $M_i^2 \ll s$. Thus in the diffractive excitation model, the pions in the $x \simeq 0$ region are decay products of very massive clusters, each of which has nevertheless the (essentially) maximum momentum $K_i \simeq \sqrt{s}/2$.

From (6), we obtain

$$\frac{dk}{dx} \simeq - \frac{\mu_\perp^2}{2 x_0 x M_i} \quad , \qquad \mu_\perp^2 = \mu^2 + p_\perp^2 \tag{7}$$

where μ is the mass of the pion detected. For simplicity, we shall keep $p_\perp^2$ at a typical value of average $\langle p_\perp^2 \rangle$, i.e. $(350\,\mathrm{MeV/c})^2$. Recognizing that $n_i \approx \xi M_i/E$, where ξ is the fraction of the total multiplicity in either cluster that belongs to the particular type of particle observed, we have after collecting formulas

$$f(x) = \left(\frac{3}{8\pi}\right)^{\frac{1}{2}} \frac{\mu_\perp^2 \xi}{E|x|} \int \frac{d\sigma}{dM_i} \, e^{-3k^2/2E^2} \, dM_i \tag{8}$$

By taking ξ out of the integral, we have tacitly assumed that ξ is independent of M_i, particularly at large M_i. Since the large M_i behavior of the integrand determines the small x behavior of f(x), it is evident that scaling is attained if

$$\frac{d\sigma}{dM_i} \propto M_i^{-2} \quad , \quad \text{as } M_i \to \infty \tag{9}$$

In Ref. 2, this behavior is suggested and then derived in the Regge model.[6] Our main point here is that the value of f(x) at x = 0 depends on the normalization of $d\sigma/dM_i$ in such an essential way that we have

$$\frac{f(0+)}{f(0-)} = \frac{\lim\limits_{M_1 \to \infty} M_1^2 \, d\sigma/dM_1}{\lim\limits_{M_2 \to \infty} M_2^2 \, d\sigma/dM_2} \tag{10}$$

We reiterate that $d\sigma/dM_1$ is the cross section for the diffractive production of a cluster of mass M_1 with the mass of the other cluster being integrated over both the discrete state (no excitation) and the continuum.

III. SINGLE DIFFRACTIVE EXCITATION

The derivation of (10) is based on rather general ideas about diffractive dissociation, supplemented by the assumption that the pions in the clusters have an average energy E that is independent of the multiplicity or the hadronic origin of the clusters. However, the cross section $d\sigma/dM_i$ is not easily measurable. We now want to relate it to the single excitation processes which are experimentally accessible. This requires an additional assumption that the diffractive processes are factorizable, i.e.,

$$\frac{d\sigma_{12}}{dM_1\, dM_2\, dt}\ \frac{d\sigma_{e\ell}}{dt} = \frac{d\sigma_1}{dM_1\, dt}\ \frac{d\sigma_2}{dM_2\, dt} \tag{11}$$

where the four quantities are, respectively, the differential cross sections for double excitation, elastic scattering, single excitation of cluster 1 and of cluster 2. Equation (11) is assumed to be valid for all t, M_1, and M_2 in the limit of infinite s. An obviously valid special case is when M_1 (or M_2) remains in the unexcited state of the initial hadron. A pictorial representation of (11) in terms of the amplitudes is given in Fig. 3, where the wavy lines represent the factorizable Pomeron.

From (11) we obtain by integration

$$\frac{d\sigma}{dM_1} = \int dt \left\{ \frac{d\sigma_1}{dM_1 dt} \left[\int dM_2\ \frac{d\sigma_2}{dM_2 dt} \right] \left[\frac{d\sigma_{e\ell}}{dt} \right]^{-1} \right\} \tag{12}$$

Since we work in the infinite s limit, the limits of integration over t are independent of M_1 and M_2. The expression for $d\sigma/dM_2$ is identical to (12) except for the interchange of the labels 1 and 2. The integration over the cluster masses includes both the discrete and the continuum.

We now see from (10) and (12) that if the quantity

$$\int dM_1 \frac{d\sigma_1}{dM_1\, dt} \Bigg/ \lim_{M_1 \to \infty} M_1^2 \frac{d\sigma_1}{dM_1 dt} \tag{13}$$

is independent of the nature of particle 1, i.e., if (13) is equal to the same expression with 1 replaced by 2, then f(0+) = f(0−). The converse is not true in general because of the t integration. However, it would be very surprising if it were not true, on account of the characteristically monotonic behavior in the t dependence of the differential cross sections, elastic or single diffractive excitation.

The numerator of (13) is the sum of the elastic and all single excitation (of hadron 1) cross sections. The denominator is the normalization of the asymptotic cross section for producing massive M_1 clusters. Thus the quotient measures the area under the asymptotically <u>normalized</u> excitation spectrum. The high M_i portion of such a normalized curve is therefore not expected to depend on the nature of the particle being excited. The major diferences between normalized spectra for different hadrons, if any, should reside in the threshold effects and the excitation of low-lying resonances. In Fig. 4, we show a sketch of the excitation spectrum (the solid line) along with the M_i^{-2} asymptotic behavior extrapolated to lower values of M_i (the dashed line).

In general, one expects the spectra to show characteristic differences at low cluster masses. The minimal cluster for pion excitation consists of three pions, while a proton can be diffractively excited into a neutron and a π^+. The spin and isospin of the initial hadrons may also affect low-energy excitations. Of course, what is of interest in (13) is the area under the spectrum curve, which is not sensitively dependent on the precise shape of the curve itself. Nevertheless, the quantity expressed by (13) can quite possibly be different for different hadrons so that a discontinuity of f(x) at x = 0 cannot be ruled out.

Having stated the possibility of the discontinuity, we now argue for the inconspicuousness of such a discontinuity. The reasoning depends heavily on the notion of duality applied to particle-Pomeron scattering, which has never been tested empirically. Assuming its validity, one then expects the low-lying resonances to be interpolated on the average by the curve corresponding to the exchange of the leading Regge trajectory. In the present case, it is the f trajectory which dominates the asymptotic behavior, and gives rise to the M_i^{-2} fall-off. The dashed line in Fig. 4 exemplifies this interpolation. More explicitly, we have

$$\frac{d\sigma_1}{dM_1^2\,dt} \propto \frac{\beta^2(t)}{s^2}\left(\frac{s}{M_1^2}\right)^2 A\left(M_1^2, t\right) \tag{14}$$

where $A\left(M_1^2, t\right)$ is the forward absorptive part of the particle-Pomeron amplitude. Assuming

$$A\left(M_1^2, t\right) \propto \left(M_1^2\right)^{\alpha_f(0)}, \tag{15}$$

$\alpha_f(0) \approx 1/2$, not only for large M_1 but also for values of M_1 in the low-lying resonance region whatever the hadron being excited may be, we then get on the average a universal excitation spectrum. The area under such a curve with appropriate threshold factors built in should then be more or less independent of the nature of the particle. We thus conclude that the discontinuity of f(x) at x = 0 is not likely to be a substantial one.

Experimentally, $d\sigma_1/dM_1dt$ can be measured by analyzing the recoil against the cluster excitation. Defining $s' = M_1^2$, we have

$$\frac{d\sigma}{ds'\,dt} = \frac{x_0}{s}\,\frac{d\sigma}{dx\,dp_1^2} \tag{16}$$

where x, x_0, and $p_\perp$ now refer to the recoiling particle, and

$$s'/s = 1 - x_0 \tag{17}$$

$$t = -\left[m^2 (1-x_0)^2 + p_\perp^2\right]/x_0 , \tag{18}$$

m being the mass of the hadron before excitation. Using (14) and (15), we obtain for the recoil spectrum

$$x_0 \frac{d\sigma}{dx\,dp_\perp^2} \propto s^{-1/2} (1-x_0)^{-3/2} \tag{19}$$

Since $s' \ll s$, we have $x \approx x_0$. It thus follows from (19) that the leading particle spectrum recoiling against a cluster is not limiting as $s \to \infty$, but for every fixed s, there is a singularity at x = 1. A careful analysis of the kinematics shows that the physical boundary in x differs from x = 1 by terms of the order of 1/s. Thus if the cross section in (19) is integrated over x, the result is limiting as it should be, since it corresponds to $\int (d\sigma/ds'\,dt)\,ds'$. The excitation spectrum of low-lying resonances can be obtained by studying the recoil distribution $x_0\, d\sigma/dx\,dp_\perp^2$ near x = 1 even at the present machine energies.

IV. CONCLUSION

We have pointed out the possibility that the invariant pion distribution can have a discontinuity at x = 0. In πp collisions at very high energy the events that populate the x = 0+ region belong to the type where the incident pion is highly excited while the proton is lowly excited. In the x = 0− region, it is the other way around. There is no fundamental principle that requires that these two excitation modes occur with equal probability. However, by applying the notion of duality to the particle-Pomeron amplitude (a theoretical conjecture as yet untested by experiments), one does not expect the discontinuity to be very conspicuous.

Nevertheless, continuity should not be taken for granted. An experimental verification of the behavior at $x = 0$ is therefore of great importance.

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References

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4. A. Mueller, Phys. Rev. D 2, 2963 (1970).
5. R. C. Hwa and D. Robertson (to be published).
6. It was later also derived by M. Jacob and R. Slansky, Phys. Rev. D 5, 1847 (1972), following identical reasoning.

Figure Captions

Fig. 1. A sketch of a possible pion distribution in the rapidity plot.

Fig. 2. The same distribution as in Fig. 1 plotted in x.

Fig. 3. Factorization of diffractive amplitudes.

Fig. 4. A sketch of a diffractively induced excitation spectrum. The dashed line is an extrapolation of the M_i^{-2} behavior toward lower energies.

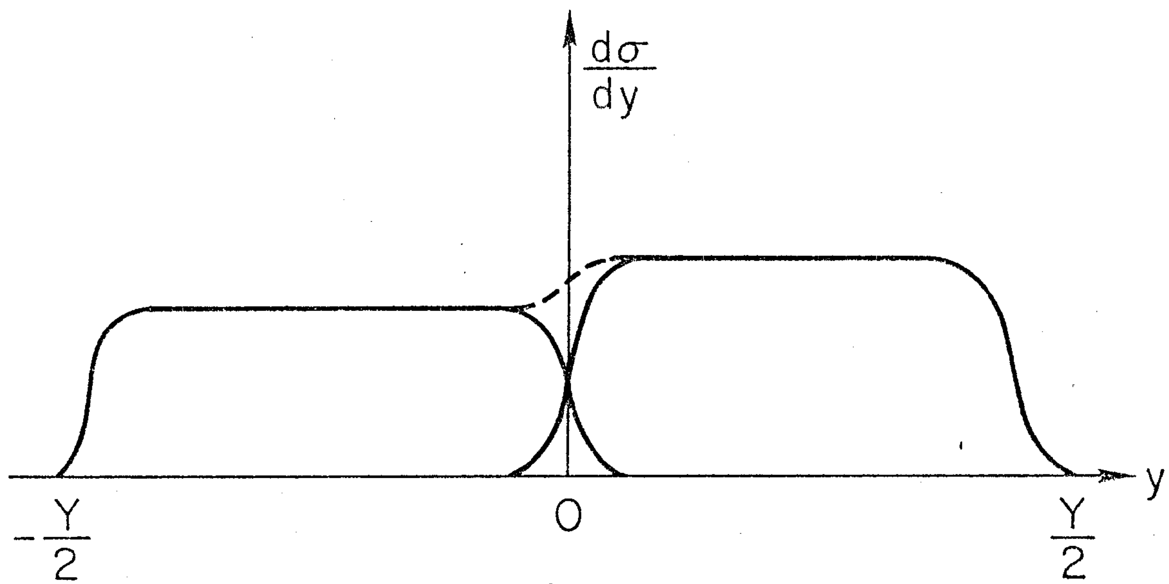

Fig 1

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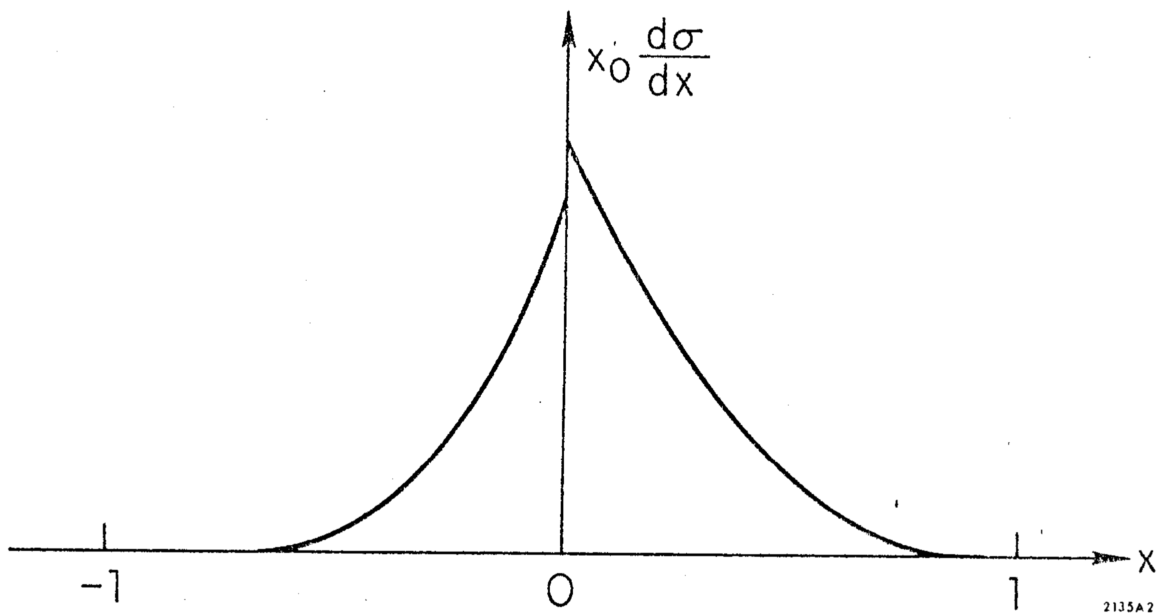

Fig. 2

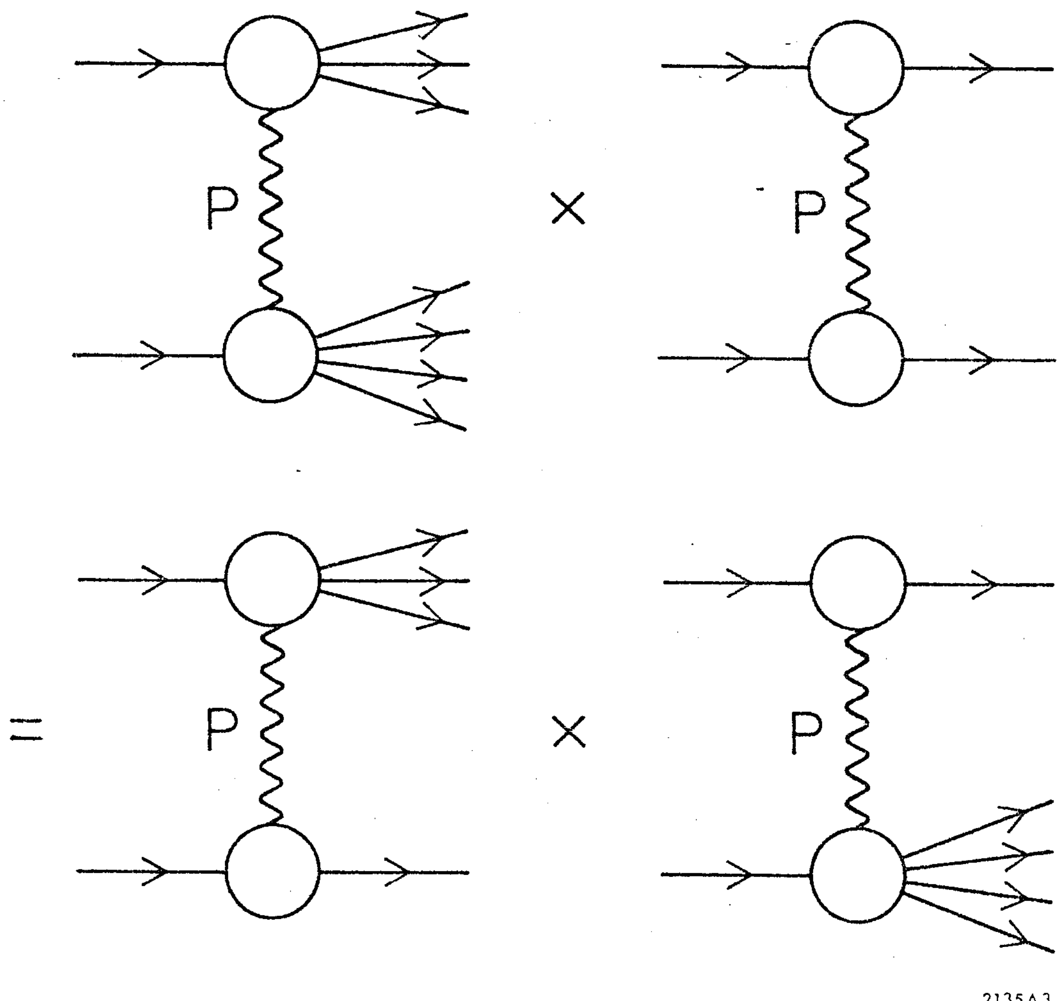

Fig. 3

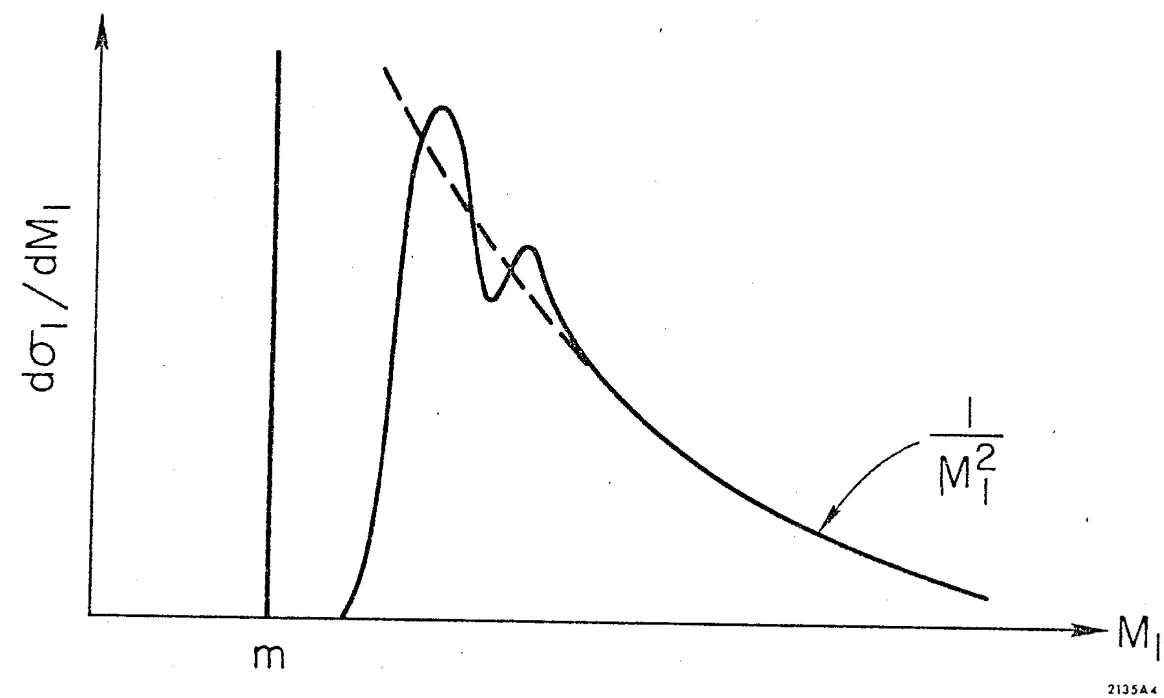

Fig. 4