A LARGE-APERTURE CERENKOV HODOSCOPE FOR IDENTIFICATION OF MULTI-PARTICLE STATES*

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ABSTRACT

A large-aperture Čerenkov chamber has been designed and constructed at SLAC for use in identifying particles in 1,2, and 3-body final states. It is basically a threshold type counter, but the optics is designed such that each of the eight 62.5×62.5 cm spherical mirrors reflects the Čerenkov light into a corresponding light horn and phototube; the counter may therefore be used as a hodoscope of eight optically independent Čerenkov cells. In addition, the pulse-height information from each phototube is recorded so that particle identification may be made in some cases on the basis of how much light was emitted. The performance of the counter for single and multi-particle identification is discussed.

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A. Introduction

Although the phenomenon of Čerenkov radiation has been widely utilized by a variety of counters and detectors in particle physics, its application to the study of particle interactions at high energies has been limited primarily to two kinds of devices: differential counters and threshold counters. Both counters aid in particle identification by virtue of being velocity selectors but their modes of operation are rather different.

In differential counters, particles of a particular velocity, β , are selected by requiring that light be radiated at the corresponding Cerenkov angle, $\cos\theta = \frac{1}{\beta n}$, where n is the index of refraction of the radiator. This is usually accomplished by focusing the radiated light onto a thin annular slit such that the light is detected only if it is emitted at the predetermined angle. Extremely good velocity separation and hence particle identification, may be obtained in this manner but the technique requires that the angular dispersion and momentum spread of the incident particles be rather small. Hence, the usage of differential counters has been largely confined to identifying incident beam particles, or secondary particles when the momentum and angular phase space is limited.

Threshold counters, on the other hand, identify particles of different masses solely on the basis of whether or not Čerenkov light was emitted. Since no use is made of the angular information, such counters can be, and have been, built with large solid-angle acceptances. By the nature of their operation, however, they have been used primarily in relatively simple applications in which they determine either that no particles were above threshold or that at least one was above threshold. In addition, particle identification is limited to the momentum range extending from the threshold momentum of the lighter particle to that of the heavier one $(p_{THR ESH} = m/(n^2 - 1)^{\frac{1}{2}}$ where m is the mass of the particle). For larger or

smaller momenta no information is obtained since either both of the particles radiate or neither of them do.

As there is now considerable interest in studying multibody states at reasonably high energies, e.g., >10 GeV, there is a need for large-aperture detectors which can identify each of the particles in many-body final states. States such as $K\overline{K}$ or $p\overline{p}$ may be identified simply by threshold counter techniques, but a clean separation of more complicated states such as $K\pi$, $\pi\pi K$, $K\overline{K\pi}$ or $p\overline{p\pi}$ requires separate identification of several, if not all of the particles. In general, it is necessary not only to correctly distinguish the final states, but in addition to identify each of the individual particles; otherwise the kinematic correlations that are characteristic of a particular final state may be obscured by particle misidentification.

Although the Čerenkov chamber that is described here is basically a threshold-type counter, the optical system is designed such that each of the eight mirrors reflects the Čerenkov light into a corresponding light horn and phototube (see Fig. 1). Hence the counter may be used as a hodoscope of eight optically independent Čerenkov cells. The different optical units are not physically separated, ¹ but if the Čerenkov cone of a particle intersects only one mirror, it will be detected only by the opposing phototube. For multibody states in which several particles are incident upon different areas of the counter, independent information is therefore available on the identity of each particle.

The chamber may also be used as a single, large, uniformly efficient counter by summing the outputs of each of the phototubes.

The other feature, of operation more than design, that distinguishes it from simple "Yes-No" threshold counters is that the individual pulse heights from the phototubes are recorded. Thus, identification may be made, in some cases, on the basis of how much light was emitted. This extends the useful range of the counter to momenta above the threshold of the heavier particle, and may aid in identification when more than one particle enters a single mirror. The detailed design of the counter, and the extent to which it is successful in obtaining these objectives, is discussed below.

B. Construction and Optical Design

The main frame of the counter was a large, rectangular, steel box designed to withstand pressures between 0 and 3 atm (Fig. 1); its size was determined primarily by the desire to match its solid-angle acceptance to that of a momentum analyzing magnet and spectrometer system.² The entrance window, which is 2 mm thick Al, is $2.5 \text{ m} \times 1 \text{ m}$ and the minimum path length of radiator is 1.75 m. At the back of the counter there is a plane of mirrors covering a total area of $2.5 \text{ m} \times 1.25 \text{ m}$ and consisting of eight square sections of spherical mirrors set edge to edge. Each of the upper four mirrors is inclined at an angle of 10° with respect to the horizontal so that it reflects light into one of the upper four light horns; similarly, each of the lower mirrors is inclined to reflect light into one of the lower light horns. The horns are designed to accept all light rays which have an angle with respect to the axis of the horn that is less than a maximum cutoff angle of 25° .³ The light from each horn is then detected by a single photomultiplier tube (Amperex 58 UVP) having a photocathode diameter of 110 mm.

The mirrors and light horns were machined from acrylic and handpolished. A thin UV-reflecting aluminum coating was deposited by evaporation. The size of the horns was essentially fixed by the diameter of the phototubes, while the optimal radius of curvature for the mirrors was determined by conventional ray tracing and by detailed Monte Carlo studies. The latter consisted of simulating photon emission (at the proper Čerenkov angle) from each of the possible particle trajectories, calculating the trajectory of these photons through the optical system and thereby determining the final number of photons to be detected by the phototube

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(including loss due to imperfect reflectance of the mirrors and light horns). The radius of curvature, r = 1.75 m, was chosen so as to optimize the uniformity of the optical efficiency for the various particle trajectories.

A particularly important feature of the optics was that it was designed to detect ultraviolet Čerenkov radiation for wavelengths as low as 2300 Å. Since the photon spectrum from Čerenkov radiation is proportional to $1/\lambda^2$, the amount of light emitted between 2300 and 5000 Å is almost three times that emitted between 3500-5000 Å, the visible region normally detected when no effort is made to prevent absorption of the ultraviolet. For this reason phototubes with quartz windows were used, and the ports between the light horns and the phototubes were also constructed of quartz. The reflectance of the mirrors and horns was measured as a function of wavelength and was > 80% for wavelengths down to 2300 Å. This comprised the primary quality control on the mirror production. Mirrors which did not achieve this reflectance were polished further and re-coated. The radiator was chosen to be Freon-12 (CCl₂F₂) because of its relatively high index of refraction (n-1=1.15×10⁻³ at STP) and good transmission of ultraviolet frequencies; the transmission is $\approx 100\%$ for $\lambda > 2300$ Å (although it falls rapidly to zero for shorter wavelengths).⁴

C. Čerenkov Counter Electronics

The electronics for the Čerenkov counter was designed not only to record which of the eight phototubes had fired, but to record the individual pulse heights as well. Each of the anode signals from the phototubes , which were $\approx .05 - 1$ volts and 10 ns wide, were stretched and amplified so that the output pulse was $\approx .1-2$ volts and 1-2 μ s in width. The peak of the ouput pulse was proportional to the total charge of the phototube pulse, and hence to the total amount of light detected by the phototube. The stretcher-amplifier modules were gated so that only that part of the input signal which was coincident with the gate was integrated. During the test runs, the gate was generated by a three-fold coincidence of two scintillation counters upstream, and one downstream, which served to identify that a beam particle had passed through the chamber; a gas Čerenkov counter in the beam line was also included in the trigger when it was desired that only pions be selected.

The peak of the stretched pulse was determined by modules which charged up to the maximum voltage of the input and held that voltage until reset to zero. The outputs of these "peak-hold" units then passed through a multiplexer so that each of the channels could be successively selected by the analog-to-digital converter (A/D).

In addition, a signal proportional to the sum of the anode signals was obtained with a high-input-impedance adder. This signal was split, one half being digitized by the pulse-height electronics and the other half being discriminated so that direct coincidences with the trigger could be obtained if desired.

The A/D converter and the multiplexer were interfaced to an IBM model 1800 computer which recorded and analyzed the data. During both the testing of the counter and data-taking, on-line histograms were compiled of the pulse-height spectra for each of the mirrors separately, and for the sum of all eight mirrors. These pulse height spectra were used for particle identification and for determining the efficiency of the counter as is discussed below.

D. Testing and Performance

1. Uniformity

The counter was tested using a pion beam at 8 and 5 GeV/c which, when undeflected, passed through the center of the Cerenkov chamber. Two bending magnets 10 m and 6.35 m upstream from the counter were used to deflect the beam vertically and horizontally towards various points on the mirror plane. In this way, it was verified that the efficiency of the counter to detect a particle was uniform for various particle trajectories. It was particularly important to check the regions at the edges of the mirrors where the Čerenkov light may be split two or even four ways and detected by separate phototubes. The uniformity was found to be very good. For example, Fig. 2 shows the pulse-height spectrum obtained for two different incident trajectories of the pion beam (5GeV/c) with a gas pressure of 1.5 atm Freon - 12 in the counter. Note that although the Čerenkov light divides equally between two mirrors for the trajectory shown in parts (a) and (b), the pulse-height spectrum obtained by summing the outputs of the two phototubes, Fig. 2c, is essentially undegraded. The pulse-height spectrum for the beam incident on the center of a mirror is shown in part (d) for comparison.

Additional information on the uniformity of the counter detection efficiency was provided by offline analysis of 400,000 particle trajectories collected during a study of π^-p interactions at 15 GeV.² The area covered by the mirror plane of the counter was divided into 12.5 × 12.5 cm² intervals and the pulse height spec trum obtained for all particles incident on each of these intervals with momenta between 5 and 10 GeV/c. The momentum cut selected particles significantly above π threshold (2.7 GeV/c) but with a momentum low enough that kaons did not count significantly (K-threshold was 9.2 GeV/c). Each of these pulse height spectra was similar to those shown in Fig. 2c and d. A cutoff was chosen (for the entire set of spectra) to separate the particles in the zero-pulse -height spike from those which emitted light (mostly pions). The fraction of the pion events which fall below this cutoff, due to the low-pulse-height tail of the pion distribution is then just the inefficiency of the counter. This inefficiency was estimated for each of the spectra, and in this way the pion detection efficiency of the counter obtained as a function of position. The results are shown in Fig. 3. The errors quoted are estimated

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uncertainties in the efficiency (inefficiency) for each spectra as determined by the above procedure. In addition, there could be an overall systematic error in the above procedure of .3 - .5%. The uniformity of the detection efficiency of the counter as a function of position is, however, clearly excellent.

2. Efficiency

An 8 GeV/c pion beam was used to measure the efficiency of the counter as a function of the gas pressure for a typical trajectory, as shown in Fig. 4. The trigger was a four-fold coincidence between three scintillation counters (two upstream and one downstream of the chamber) and a threshold gas Čerenkov counter in the beam line which was set to count on π 's. The efficiency may be obtained directly (i.e., in a hardware manner) as the ratio of trigger-Cerenkov chamber coincidences to the total number of triggers. Alternatively it may be calculated in a manner analogous to that discussed in the preceding section. The pulse-height spectrum of the chamber, gated by the trigger as discussed in Section C, is compiled and a cut introduced in the software to separate those events in which light was detected by the chamber from those in which a null signal was observed. Since each particle producing a trigger is positively identified to be a pion, the fraction of events falling in the null-signal pedestal is precisely the inefficiency of the counter. This latter method is the more accurate one and was therefore used for each of the measurements presented here. The inefficiency of the counter, as a function of pressure, is shown in Fig. 4.

In addition, a series of measurements were made without the beam Čerenkov counter. These measurements, also shown in Fig. 4, illustrate that the chamber is capable of detecting a .1 - .2% contamination of K⁻ mesons in the beam.

Although the maximum efficiency of the counter is indicated by Fig. 4 to be 99.87 \pm .1%, a subsequent pressure curve at 7.2 GeV/c with higher statistics and a better

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trigger resulted in a maximum efficiency of 99.986 \pm .01%. The primary objective of this second set of measurements was to obtain an efficiency vs. pressure curve for K mesons, and to compare the pulse-height spectra for π and K mesons when both were above threshold. Consequently, an rf-separator was used and the beam consisted largely of K⁻ (\approx 65%). Since the remaining 35% of the beam particles were pions, it was possible to repeat the π^- efficiency measurements by including the beam Cerenkov counter in the trigger. The counter is thus capable of discriminating between π and K mesons (or K and p at higher momentum) with a rejection ratio in excess of 5000:1.

3. Identification of Multibody Final States

To illustrate the usefulness of the counter in identifying multibody final states, we now discuss briefly the separation of the two-body states detected in the experiment on π p interactions mentioned above.² The generalization of the results to final states with three or more particles is relatively straightforward.

The total pulse-height spectrum from the Čerenkov counter for those events in which two oppositely charged particles were detected by the spectrometer, and at least one particle entered the counter with a momentum between 4 and 11 GeV/c, is shown in Fig. 5. The momentum cut extends from the point at which pions are > 99% efficient to ≈ 2 GeV/c above K-meson threshold. This sample consists largely of $\pi^+\pi^-$, K^+K^- , $p\bar{p}$, $K\pi$ and $p\pi$ events, and it is clear that a rather clean separation is obtained between the $\pi^+\pi^-$ events, which emit light in the counter, and the K^+K^- (and $p\bar{p}$) events which occupy the zero-pulse-height pedestal (or the pulse height region up to ≈ 125 for K mesons with a momentum above 9.2 GeV/c).†

[†]The reason the events which produced no Cerenkov light fall at channel number 50, rather than at 0, is that the stretcher amplifier for the signal from each phototube was set to have a slightly positive pedestal. This pedestal varied from unit to unit but the average value corresponded to channel number 6 or 7, thus resulting in a pedestal of \approx 50 for the sum of all eight mirrors. The slight instability of the individual pedestals (variations in channel number of 2-3units) produced a small but finite width of the zero-pulse-height spike.

Most of the $\pi K(\pi p)$ events also occupy the non-zero pulse-height region, although those events in which the kaon (proton) was the only particle entering the counter produce a null signal. An even cleaner separation is obtained if it is required that both particles enter the counter. In this case, selection of K^+K^- (or $p\bar{p}$) events may evidently be made simply by imposing a cut on the total pulse height. Introduction of a cut at ≈ 125 retains the K^- mesons up to 11 GeV/c without resulting in significant pion background.

As noted in the Introduction, however, selection of πK , and more generally $\pi \pi K$, $\pi K^+ K^-$, and $\pi p \bar{p}$, final states requires identification of each of the particles. This was accomplished using the hodoscopic properties of the counter.

Events were selected in which both particles entered the counter, subject to the momentum cut above, and the separate pulse heights for the positive and negative particles were determined in the following manner: Each track was projected to the mirror plane of the Čerenkov counter and it was determined which mirrors would have been intersected by the Čerenkov cone of the corresponding particle. The pulse heights for those mirrors were then summed. In the event that the light cones from the two particles intersect the same mirrors, it may be necessary to resolve the identification on the basis of how much light is detected in each mirror. The extent to which this is possible is discussed in Section 5 but is ignored here since it occurred for less than 10% of the events.

A convenient display of the results obtained is shown in Fig. 6 which is a two-dimensional scatter plot of the pulse height for the positive particle (CERSMP) versus that for the negative one (CERSMN). It is evident that there is quite a clean separation between the events in the horizontal and vertical bands along the edges, the events at the intersection of these two bands, and the events occupying the rest

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of the plot; these correspond, respectively, to $K^-\pi^+$, $K^+\pi^-$, K^+K^- and $\pi^+\pi^-$ final states. † Note in particular that there is almost no feedthrough from the $\pi^+\pi^-$ to the K^+K^- events; out of a total of $\approx 25,000$ events there are at most only a few $\pi^+\pi^-$ events that could be construed as K^+K^- .

As a further illustration of the effectiveness of the counter in identifying the positive and negative particles separately, Fig. 7 shows the mass spectrum for $K^+\pi^-$ events selected from Fig. 6. The selection criterion was CERSMP<50 and CERSMN > 34. A strong K^* (890) signal is observed with relatively little background; the events in this spectrum were also subjected to a missing mass cut of 1.02 - 1.28 GeV and therefore were produced in the reactions $\pi^-p \rightarrow K^*$ (Λ, Σ).

4. Pulse Height Information

It is evident from the preceding paragraph that multi-particle final states may be readily identified when the particles intersect different regions of the Čerenkov hodoscope. For the particular two-body case discussed above, this occurred $\approx 90\%$ of the time. The usefulness of such a hodoscope, however, is greatly increased if, in addition, it is possible to obtain information on the basis of how much light is emitted: First, in the event that two particles enter the same mirror, it may be possible to determine whether these two particles were $\pi\pi$, πK or $K\overline{K}$; and second, the useful momentum range of the counter may be extended beyond the threshold momentum of the heavier particle: for example, it is then possible to separate π and K mesons at momenta above the threshold for K mesons.

†Since the pressure of the counter (1.25 atm) was set to separate π and K mesons, no information was available to distinguish kaons from protons (or muons and electrons from pions). Background from these particles is not explicitly referred to in the text, but the reader should bear in mind that they may exist. Pulse-height information was used in the experiment mentioned above to extend the momentum range of the counter. However, the low pressure of Freon-12 in the counter during this run $(1\frac{1}{4} \text{ atm})$ did not produce sufficient light to enable the fuller use of pulse-height information that is possible at higher pressures. As an illustration of the kind of separation that can be obtained, we present in Fig. 8 the pulse-height spectrum for a 7.2 GeV/c beam of 65% K⁻ and 35% π^{-} as a function of pressure. Figures 8a-f correspond to Freon pressures of 2.22, 2.36, 2.50, 2.62, 2.71 and 2.85 atm, respectively. A clean separation is maintained between the π and K event samples even as the kaons move from just above threshold to a counting efficiency of 100%.

It should be noted that the spectra presented correspond approximately to those that would be observed for π and K mesons as a function of momentum for a particular pressure of the counter. For example, the pulse-height distributions of kaons in Figs. 8a-f correspond exactly to the spectra that would be observed at momenta of 7.2, 7.45, 7.75, 8.05 and 8.65 GeV/c for a counter pressure of 2.22 atm (K-threshold equal to 6.9 GeV/c). For this same momentum range, the pulse-height distributions of the pions could be approximately the same as in Fig. 8, coinciding at 7.2 GeV/c and differing at the higher momentum by 20%. One is therefore able to separate pions and kaons on the basis of pulseheight for a substantial range above K-threshold.

In a similar vein, it is evident from Fig. 8 that the pulse-height resolution for the pions is sufficiently good that one may correctly distinguish between one and two pions a large fraction of the time. Thus it is possible, when two particles enter a single mirror, to distinguish whether they were $\pi\pi$, π K, or K \overline{K} .

E. Conclusion

In conclusion, we have described a large, wide-aperture $\stackrel{\vee}{\operatorname{Cerenkov}}$ detector that is capable of extremely good single-particle separation when used as a

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threshold counter; for example, when operated at a pressure to select K mesons in the approximate momentum range 5-10 GeV/c, the π rejection ratio is in excess of 5000: 1. In addition, the hodoscopic properties and pulse-height information of the counter have been proven to be useful in identifying multi-particle reactions (e.g., $K^{+}\pi^{-}$ final states).

We would like to express our appreciation to Bill Walsh, who was invaluable in all stages of the construction of the counter, and to Bob Friday for his help with the electronics. We also thank Ron Stickley for his patience and care during the rather tedious job of polishing the mirros, and Werner Schulz for supervising the aluminization. Harry Saal and Jack Steinberger were kind enough to lend us drawings of a Cerenkov counter they had constructed which aided in the initial design. Finally, we thank Walter Zawojski for his artistic drawing of the counter.

REFERENCES

- A similar counter, but one which has optical cells that are physically separated, has been reported. See J.J. Aubert <u>et al.</u>, Nucl. Instr. and Meth. <u>87</u>, 79 (1970).
- For a description of the spectrometer, see F. Bulos et al., Phys. Rev. Letters <u>26</u>, 1453 (1971); H.H. Williams, Stanford Doctoral Dissertation (SLAC Report No. 142, 1972); B. Ratcliff, Stanford Doctoral Dissertation (SLAC Report No. 141, 1972); and G.T. Armstrong <u>et al.</u>, SLAC-PUB-801, 1970 (unpublished).
- 3. H. Hintenberger and R. Winston, Rev. Sci. Instr. <u>37</u>, 1094 (1966).
- 4. E. L. Garwin and A. Roder, Nucl. Instr. and Meth. <u>93</u>, 593 (1971).

FIGURE CAPTIONS

- (a) An artist's conception of the chamber showing particularly the optical arrangements.
 - (b) A side view of the Cerenkov chamber.
- 2. The pulse height spectrum is shown for two different trajectories of a 5 GeV/c pion beam, as indicated by the inserts. The shaded region indicates for which mirror (phototube) the spectrum is plotted. Figures (a), (b), and (c) show for a trajectory which intersects the crack between two mirrors, the spectra of the two corresponding phototubes, both separately and summed together. The solid line in (d) is the best fit to a Poisson distribution. The small spike near the origin indicates the zero-pulse-height pedestal and consists largely of mistriggers.
- 3. A map showing the detection efficiency as a function of position for four mirrors comprising one half of the counter (x=0, y=0 is the center point): results for the other half are identical within errors. Each square is 12.5 $\times 12.5 \text{ cm}^2$. Measurements for the vertical extremes of the counter are not shown since the angular acceptance of the spectrometer prevented detection of events in those regions.
- 4. Detection inefficiency of the Čerenkov counter for 8 GeV/c π⁻. For the solid points, a beam Čerenkov counter requiring the incident particle to be a π⁻ was included in the trigger. For the open points, this requirement was omitted. The difference between the two measurements indicates that the counter is capable of detecting a .2% contamination of K⁻ in the beam.
- 5. The distribution of total pulse-height from the Čerenkov counter showing the clear separation of $K^{+}K^{-}$ and $\pi^{+}\pi^{-}$ events. See discussion in text

- 6. Scatter plot of the pulse heights for the positive and negative particles. The horizontal and vertical bands, the intersection of the two bands, and the rest of the plot correspond to $K^-\pi^+$, $K^+\pi^-$, K^+K^- , and $\pi^+\pi^-$ events respectively. There are $\approx 20,000 \pi^+\pi^-$ events (including over-flow) and $\approx 2500 K^+K^-$ events in the plot.
- 7. $K^{+}\pi^{-}$ mass spectrum. The events are selected from the vertical band in Fig. 6, subject to the missing mass cut 1.02 1.28 GeV.
- 8. Pulse height distribution from the counter at pressures of 2.22, 2.36, 2.50, 2.62, 2.71 and 2.85 atm Freon-12, for a 7.2 GeV/c beam of ≈35% π⁻ and 65% K⁻. The spectra correspond closely to those that would be observed at a pressure of 2.22 atm and particle momenta of 7.2, 7.45, 7.75, 8.05, 8.28, and 8.65 GeV/c (see text).



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Fig. 1





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99.7	7 99.6	99.4	99.6	98.6	99.3	99.5	99.7	99.6	97.8	y=0
±.2	2 ±.2	±.2	±.2	±.4	±.3	±.3	±.2	±.1	±.6	
99.6	3 99.7	99.4	99.5	99.6	99.7	99.8	99.7	99.7	99.8	
±.2	2 ±.2	±.2	±.2	±.2	±.2	±.1	±.1	±.1	±.3	
99.7	7 99.7	99.7	99.7	99.6	99.7	99.8	99.8	99.7	99.6	
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Fig. 4

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Fig. 6





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