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DEEP INELASTIC SCATTERING AND CHIRAL SYMMETRY BREAKING IN THE GLUON MODEL*

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ABSTRACT

The formal light-cone properties of commutators involving current divergences are studied in the gluon model. Relations are derived which make it possible (in principle) to distinguish the vector from the (pseudo) scalar gluon model. In the vector gluon model these relations provide an experimental determination of the bare quark masses. The additional assumption that the residues of any $\alpha = 0$ fixed poles in current scattering amplitudes are polynomials in q^2 makes it possible to relate fixed pole residues in γp, γn, νp and νn scattering (which are related to deep inelastic data), the sigma term in pion-nucleon scattering, baryon mass differences and the bare quark masses. Approximate values are obtained for the bare quark masses and the parameter μ_0. The pattern of chiral symmetry breaking and a striking implication for the behavior of $F_2^{\nu p + \nu n}(x)$ are discussed.

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I. Introduction

In this paper we study properties of light-cone commutators involving current divergences in the vector-gluon model. All our considerations are formal; that is to say they are untrue in perturbation theory. It is frequently argued that the scaling observed in the SLAC-MIT inelastic electron scattering experiments implies that formal field theory might be relevant to the real world. This argument is not totally compelling (especially since the data do not exclude log Q^2 terms) but at least we are in good company in pursuing the formal approach.

Despite the dubious validity of formal field theory, we find it interesting that relations can be obtained which probe the structure of the Hamiltonian. Briefly, we shall derive relations which (although hard to test experimentally) make it possible in principle to distinguish between vector and (pseudo) scalar gluon models and determine the bare quark masses in the vector gluon model. With the further assumption that the residues of any $\alpha = 0$ fixed poles in current amplitudes are polynomials in Q^2, we are able to relate fixed pole residues in γp and γn scattering, the sigma terms in meson nucleon scattering, deep inelastic data, baryon mass differences and the bare quark masses. Existing data already allow us to make a preliminary numerical investigation of the bare quark masses.

Since this paper is rather discursive we have summarized our results explicitly in the last section, to which the reader may wish to turn first. In the rest of this section we shall review some previous work on this subject and explain the plan of our exposition.

In one of their first papers on the subject, Brandt and Preparata[1] pointed out that in the vector-gluon model the most singular terms in operator product expansions on the light cone would be chirally symmetric. This implies that[2]

$$\lim_{\nu, q^2 \to \infty} \nu W_{4,5} = 0 ,$$

$$x = \frac{2\nu}{-q^2} \text{ fixed}$$

where ν and q^2 are defined in the usual way (see Section II) and W_4 and W_5 are structure functions, defined in Eq. (2.5) below[3], which can (in principle) be measured in neutrino reactions. Subsequently, several authors[4-7] have discussed the scaling properties of W_4 and W_5, which probe the nature of chiral symmetry breaking.

Mandula et al. showed[4] that the dimension (d) of the operator in the part of the Hamiltonian which breaks chiral symmetry can be bounded in terms of the scaling properties of $W_{4,5}$. However, Ng and Vinciarelli have recently pointed out[7] that the hope that these scaling properties might therefore be used to measure d is thwarted in the vector-gluon model in which the bound is not saturated (the bound is clearly not saturated in the free quark model since it allows $\nu^{3/2} W_{4,5}$ to scale; but free field theory cannot yield fractional powers of ν in the scaling laws).

Ng and Vinciarelli argue that $\nu^2 W_{4,5}$ scale and derive a sum rule for the sigma term in pion nucleon scattering (σ_π) in terms of the scaling functions. We observe that their sum rule (like the σ-term sum rule in Ref. 6) diverges linearly if conventional Regge behavior is assumed. This led us to reexamine the derivation.

In Section II we present a light-cone analysis of commutators involving divergences which determine $W_{4,5}$. We find that $\lim_{bj} \nu^2 W_5 = 0$ (which implies that the σ-term sum rules of Refs. 6 and 7 are the same). We also find (among other things) that $\lim_{bj}\nu^2 W_4$ and F_2 (= $\lim_{bj}\nu W_2$) are related in the vector-gluon model in such a way that this σ-term sum rule diverges unless

$$\int_0^1 \frac{F_2(x)dx}{x^2}$$

exists. This seems unlikely even without appeal to Regge theory. We find that the divergence of the sum rule vitiates the derivation.

In Section III we turn to a momentum space calculation which takes account of asymptotic behavior from the start and cannot yield divergent sum rules. We find that σ_π is related to the asymptotic behavior of a subtraction constant. In Section IV we show that the latter quantity can be calculated if we accept the assertion that the residues of any $\alpha = 0$ fixed poles in current nucleon scattering amplitudes are polynomials in Q^2. Our results can then be combined to relate a variety of diverse quantities, as stated above.

The reader should be forewarned that the experimental measurement of $W_{4,5}$ to the accuracy required to test the results in Section II is almost impossible.[8] However, the very existence of results such as Eq. (2.14) (which can in principle serve to determine the interaction) is interesting. It will be observed that $W_{4,5}$ have been eliminated from our results in Section IV which relate more easily measurable quantities.

II. Light-Cone Calculations

In this section we shall derive scaling laws for the structure functions W_4 and W_5 in the vector-gluon model and investigate the origins of the possibly divergent sum rule for σ_π referred to in the introduction. We use the quark model light-cone algebra developed by Fritzsch and Gell-Mann[9] as extended to the vector-gluon model by Gross and Treiman.[10] Identical results may be obtained by studying almost equal time commutators in the BJL limit.[11]

By means of formal manipulations, based on the canonical equations of motion, Gross and Treiman found that to leading order on the light cone, the vector gluon can be treated as a massless, external, C-number field. That is, to leading order on the light cone the equations of motion:

$$(i\not\partial - g\not B(y) - m)\psi(y) = 0 \tag{2.1}$$

and

$$(\square + \mu^2)B_\nu(y) = g\bar\psi(y)\,\gamma_\nu\psi(y)$$

may be replaced by

$$(i\not\partial - g\not B(y))\psi(y) = 0$$

and

$$\square B_\nu(y) = 0$$

with

$$(x-y)_\mu(x-y)_\nu\left[B^\mu(x), B^\nu(x)\right] = 0\,, \quad \text{for } (x-y)^2 = 0\,.$$

Consequently, the free-field results of Fritzsch and Gell-Mann are modified only by the inclusion of a phase in the bilocal operator which modulates the leading free-field, light-cone singularity.

For our purposes it will be necessary to know the second most singular term in the light-cone expansion of the commutator of a current with a divergence. The particularly simple mechanism of chiral symmetry breaking in the vector-gluon model allows us to calculate this term in the same manner that Gross and Treiman computed the leading term. We find that the first corrections to the results of Gross and Treiman are given by the quark mass term in Eq. (2.1): the gluon may still be regarded as a massless, external, C-number field and modifies the free-field results only by the phase mentioned above.[12] The effective equations of motions are:

$$\begin{aligned} &(i\not\partial - g\not B(y) - m)\psi(y) = 0 \\ &\Box B_\nu(y) = 0 \end{aligned} \tag{2.2}$$

with

$$(x-y)_\mu (x-y)_\nu \left[B^\mu(x), B^\nu(y)\right] = 0 \text{ for } (x-y)^2 = 0 \ .$$

The quark field anti-commutator, correct to leading and next to leading order on the light cone is therefore given by:

$$\left\{\psi(x), \bar\psi(y)\right\} \approx e^{-ig\int_y^x B_\mu(\xi)\, d\xi^\mu} (-\not\partial + im)\, D(x-y) \tag{2.3}$$

where $D(x-y) \equiv \frac{1}{2\pi}\,\delta\left((x-y)^2\right)\epsilon\,(x_0 - y_0)$ and the integral is taken along a light-like path from y to x.

This result is derived as follows. Formal manipulations involving the equations of motion can introduce only positive integral powers of the terms $m\psi(y)$, $\mu^2 B_\nu(y)$ and $g\bar\psi(y)\gamma_\nu\psi(y)$ into the light-cone commutator of two currents. If the leading light-cone singularity results in a scaling law for some structure function, e.g.:

$$W \to \frac{1}{\nu^n} f(x) \qquad x \equiv Q^2/2\nu$$

(where ν and Q^2 are defined below), then a term involving the quark mass can scale at most like $\frac{m}{\nu^{n+\frac{1}{2}}}\, g(x)$ while terms involving μ^2 can scale at most like $\frac{\mu^2}{\nu^{n+1}}\, h(x)$. A similar argument may be given for $\bar\psi(y)\gamma_\nu\psi(y)$. A more careful (but essentially identical) derivation of these results follows from "twist" arguments similar to those of Gross and Treiman or from power counting in the BJL expansion.[11]

To proceed, we define the structure functions for neutrino or antineutrino scattering as follows:

$$W_{\mu\nu}^{\overline{\nu}/\nu} \equiv \frac{1}{4\pi} \int e^{iq\cdot y} d^4y \langle P | [J_\mu^{\pm}(y), J_\nu^{\mp}(0)] | P \rangle \tag{2.4}$$

$$\begin{aligned} &\equiv -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1^{\overline{\nu}/\nu}(q^2, \nu) \\ &+ \frac{1}{M^2}\left(P_\mu - \frac{\nu}{q^2} q_\mu\right)\left(P_\nu - \frac{\nu}{q^2} q_\nu\right) W_2^{\overline{\nu}/\nu}(q^2, \nu) \\ &- \frac{i\epsilon_{\mu\nu\alpha\beta}}{2M^2} p^\alpha q^\beta W_3^{\overline{\nu}/\nu}(q^2, \nu) \\ &+ \frac{q_\mu q_\nu}{M^2} W_4^{\overline{\nu}/\nu}(q^2, \nu) + \frac{P_\mu q_\nu + P_\nu q_\mu}{2M^2} W_5^{\overline{\nu}/\nu}(q^2, \nu) \end{aligned} \tag{2.5}$$

where $\nu \equiv P \cdot q$, $q^2 \equiv -Q^2 < 0$, the matrix element is understood to be averaged over the proton's spin,[13] and we have assumed T-conservation and set $W_6 = 0$. Throughout this section, we consider only the strangeness-conserving weak current.

$$J_\mu^{\pm}(y) \equiv \overline{\psi}(y)\gamma_\mu(1-\gamma_5)\lambda^{\pm}\psi(y).$$

Generalization to the $\Delta S = 1$ current is straightforward and the results are stated in Section V.

Taking the double divergence of Eq.(2.4) and using

$$\partial^\mu J_\mu^{\pm}(y) = -2\, i\, m_p \overline{\psi}(y)\gamma_5 \lambda^{\pm}\psi(y),$$

where m_p is the mass of the proton quark (assumed equal to the mass of the neutron quark), we find:

$$\begin{aligned} q^\mu q^\nu W_{\mu\nu}^{\overline{\nu}/\nu} = \frac{m_p^2}{\pi} \int d^4y\, e^{iq\cdot y} \langle P | [\overline{\psi}(y)\gamma_5\lambda^{\pm}\psi(y), \\ \overline{\psi}(0)\gamma_5\lambda^{\mp}\psi(0)] | P \rangle . \end{aligned} \tag{2.6}$$

In the Bjorken limit (ν, $Q^2 \equiv -q^2 \to \infty$; $x \equiv Q^2/2\nu$ finite) only the leading light-cone singularity of this commutator is required:

$$
\begin{aligned}
&<P|[\bar{\psi}(y)\gamma_5\lambda^{\pm}\psi(y),\ \bar{\psi}(0)\gamma_5\lambda^{\mp}\psi(0)]|P> \\
&\quad = \partial^{\rho}D(y) < P|\bar{\psi}(y)\gamma_{\rho}\lambda^{\pm}\lambda^{\mp} I(y,0)\psi(0) \\
&\quad - \bar{\psi}(\ )\gamma_{\rho}\lambda^{\mp}\lambda^{\pm} I(0,y)\psi(y)|P> \\
&\quad + \text{ terms less singular on the light cone}
\end{aligned}
\tag{2.7}
$$

where $I(y,0) \equiv e^{-ig\int_0^y B_\mu(\xi)\, d\xi^\mu}$. We now define $\mathscr{F}_{\rho}^{\pm}(y\cdot p)$ to be the light-cone restriction of the matrix element of Eq. (2.7):

$$
\begin{aligned}
\mathscr{F}_{\rho}^{\pm}(y\cdot p) &\equiv <P|\bar{\psi}(y)\gamma_{\rho}\lambda^{\pm}\lambda^{\mp} I(y,0)\psi(0)|P>\big|_{y^2=0} \\
&\equiv P_{\rho} F^{\pm}(y\cdot p) + y_{\rho} G^{\pm}(y\cdot p)
\end{aligned}
\tag{2.8}
$$

Combining Eqs. 2.6-2.8 and using the Fourier transform representation of $\partial^{\rho}D(y) \equiv \frac{1}{(2\pi)^3}\int d^4k\, k^{\rho} e^{-ik\cdot y}\delta(k^2)\,\epsilon(k_0)$ we obtain:

$$
\mathrm{Lim}_{bj}\, q^{\mu}q^{\nu}\ W_{\mu\nu}^{\bar{\nu}/\nu} = m_p^2\,(\hat{F}^{\pm}(x) - \hat{F}^{\mp}(-x))
$$

where $\hat{F}^{\pm}(x)$ is the Fourier transform of $F^{\pm}(y\cdot p)$:

$$
F^{\pm}(y\cdot p) \equiv \int e^{ixy\cdot p}\,\hat{F}^{\pm}(x)\,dx.
$$

The bilocal operator of Eq.(2.8) is identical to that which occurs in the light-cone analysis of the structure function $W_2^{\bar{\nu}/\nu}(q^2,\nu)$, specifically:

$$
x\left(\hat{F}^{\pm}(x) - \hat{F}^{\mp}(-x)\right) = F_2^{\bar{\nu}/\nu}(x)
$$

Expressing the double divergence in terms of $W_4^{\bar{\nu}/\nu}$ and $W_5^{\bar{\nu}/\nu}$, we obtain finally:

$$\lim_{bj}\left(\frac{\nu^2}{M^4}\,W_4^{\bar{\nu}/\nu}(q^2,\ \nu) - \frac{1}{2x}\,\frac{\nu^2}{M^4}\,W_5^{\bar{\nu}/\nu}(q^2,\ \nu)\right) = \frac{1}{4x^3}\left(\frac{m_p^2}{M^2}\right) F_2^{\bar{\nu}/\nu}(x) \tag{2.9}$$

We shall find that $\nu^2 W_5$ actually vanishes in the Bjorken limit so that Eq. (2.9) relates $\nu^2 W_4$ to the more easily measured structure function $F_2(x)$. Proportionality between $\nu^2 W_4(q^2,\ \nu)$ and $\frac{1}{x^3}\nu W_2(q^2,\ \nu)$ in the Bjorken limit obtains only in the vector gluon model and represents, in principle, a test of the interaction. If proportionality is observed the constant measures the bare quark mass.

We now turn to the single divergence of $W_{\mu\nu}^{\bar{\nu}/\nu}$:

$$q^\mu W_{\mu\nu}^{\bar{\nu}/\nu} = \frac{m_p}{2\pi}\int d^4y\, e^{iq\cdot y} <P|[\bar{\psi}(y)\gamma_5\lambda^\pm\psi(y),\ \bar{\psi}(0)\gamma_\nu(1-\gamma_5)$$

$$\times\ \lambda^\mp\psi(0)]\,|P> \tag{2.10}$$

The leading contribution to this commutator is computed in the Appendix. The result is:

$$<P|[\bar{\psi}(y)\gamma_5\lambda^\pm\psi(y),\ \bar{\psi}(0)\gamma_\nu(1-\gamma_5)\lambda^\mp\psi(0)]\,|P>$$

$$= \partial_\nu\left\{D(y) <P|\bar{\psi}(y)\lambda^\pm\lambda^\mp I(y,0)\psi(0) + \bar{\psi}(0)\lambda^\mp\lambda^\pm I(0,y)\psi(y)|P>\right\} \tag{2.11}$$

+ terms less singular on the light cone.

It is here that it was necessary to keep the term proportional to the quark mass in Eq. (2.3). The reason lies in the spin structure of the bilocal operator which multiplies the leading singularity: the term proportional to $\not{\partial}D(y)$ in Eq. (2.3) contributes a term to Eq. (2.11) of the form

$$\partial^\rho D(y) <P|\bar{\psi}(y)\,g_{\nu\rho}\lambda^\pm\lambda^\mp I(y,0)\psi(0)|P>$$

This twist three bilocal is normalized to some mass. The contribution of the mass term in Eq. (2.3) is of the form

$$m\, D(y) \langle P | \bar{\psi}(y)\, \gamma_\nu\, \lambda^{\pm}\lambda^{\mp}\, I(y,0)\, \psi(0) | P \rangle$$

Although less singular on the light cone, this term contains a twist two bilocal normalized to P_ν, and is equally important in the Bjorken limit. In the Appendix the two terms are combined to yield Eq. (2.11).

Combining Eq. (2.10-2.11) and performing the y-integral we obtain:

$$\lim_{bj} q^\mu W_{\mu\nu}^{\bar{\nu}/\nu} = \frac{-m_p q_\nu}{2\nu} \left(\hat{H}^{\pm}(x) + \hat{H}^{\mp}(-x) \right) \tag{2.12}$$

where

$$H(y \cdot p) \equiv \langle P | \bar{\psi}(y) \lambda^{\pm}\lambda^{\mp} I(y,0)\, \psi(0) | P \rangle \Big|_{y^2=0} \tag{2.13}$$

and $\hat{H}(x)$ is its Fourier transform.[14] That Eq. (2.12) should contain no term proportional to P_ν may be seen by partially integrating ∂_ν in Eq. (2.11) back onto $e^{iq \cdot y}$. Since

$$q^\mu W_{\mu\nu} = \frac{q^2 q_\nu W_4}{M^2} + \frac{(\nu q_\nu + q^2 p_\nu)}{2M^2} W_5$$

and since Eq. (2.12) would contain any term of order P_ν/ν were it present, we conclude that

$$\lim_{bj} \nu^2 W_5^{\bar{\nu}/\nu}(q^2, \nu) = 0 \tag{2.13a}$$

$$\lim_{bj} \frac{\nu^2}{M^4} W_4^{\bar{\nu}/\nu}(q^2, \nu) \equiv F_4^{\bar{\nu}/\nu}(x) = \frac{m_p}{M^2} \frac{1}{4x} \left(H^{\pm}(x) + H^{\mp}(-x) \right), \tag{2.13b}$$

while Eq. (2.9) reduces to

$$\begin{aligned} F_4^{\bar{\nu}/\nu}(x) &= \frac{1}{4x^2} \left(\frac{m_p^2}{M^2} \right) \left(\hat{F}^{\pm}(x) - \hat{F}^{\mp}(-x) \right) \\ &= \frac{1}{4x^3} \left(\frac{m_p^2}{M^2} \right) F_2^{\bar{\nu}/\nu}(x) \end{aligned} \tag{2.14}$$

The import of this result was already noted above following Eq. (2.9) (the $1/x^3$ factor is to be expected if both F_4 and F_2 are Regge behaved).

At this point it is useful to compare our results with those of Ref. 4 and 7. According to Mandula et al., $\nu^{3/2}W_4$ and $\nu^{3/2}W_5$ may scale in the Bjorken limit in the gluon model. This result is based on the assumption that the bilocal operator which occurs in the expansion of the current-divergence commutator will in general contain a piece with twist as low as is allowed by the observed scaling of W_2, i.e., twist 2. In the vector gluon model such a piece is absent: the bilocal operator $\bar{\psi}(y)\psi(0)$ has twist 3 from which Ng and Vinciarelli's result that $\nu^2 W_4$ scales follows. The further result that $\nu^2 W_5 \to 0$ rests on the careful calculation of seemingly lower order terms in the light-cone expansion (cf. the Appendix).

Returning to Eq's. (2.13) and (2.14) we find the relation:

$$\hat{H}^{\pm}(x) = m_p \hat{F}^{\pm}(x)/x \tag{2.15}$$

If the Fourier transform of $\hat{F}^{\pm}(x)/x$ exists, we obtain

$$<P|\bar{\psi}(y)\lambda^{\pm}\lambda^{\mp} I(y,0)\psi(0)|P> = \int_{-1}^{1} dx\, e^{ixy\cdot p} \frac{m_p \hat{F}^{\pm}(x)}{x} \tag{2.16}$$

or at $y \cdot p = 0$:

$$<P|\bar{\psi}(0)\lambda^{\pm}\lambda^{\mp}\psi(0)|P> = \frac{4M^2}{m_p} \int_0^1 x\,dx\, F_4^{\bar{\nu}/\nu}(x) \tag{2.17a}$$

$$= m_p \int_0^1 \frac{dx}{x^2} F_2^{\bar{\nu}/\nu}(x) \tag{2.17b}$$

Eq. (2.17a) leads immediately to the σ-term sum rule derived by Lee and Mandula[6] and Ng and Vinciarelli[7] (with $\nu^2 W_5 \to 0$ in the case of Ref. 7).

Unless $F_2(x) \to x^{1+\epsilon}$ as $x \to 0$, Eq's. (2.17) diverge and the inversion of the Fourier transform leading from Eq. (2.15) to Eq. (2.16) is not allowed. Since we expect $F_2(x) \to$ const as $x \to 0$ if Regge behavior obtains, it is likely that Eqs. (2.17) are incorrect. It is nevertheless possible to extract a meaningful result from Eq. (2.15): $xH^{\pm}(x)$ is the Fourier transform of $\frac{d}{dy\cdot p} H^{\pm}(y\cdot p)$ so that Eq. (2.15) may be inverted to yield

$$\frac{d}{dy\cdot p} \langle P|\bar{\psi}(y)\lambda^{\pm}\lambda^{\mp} I(y,0)\psi(0)|P\rangle\Big|_{y^2=0} = im_p \int_{-1}^{1} dx\, e^{ixy\cdot p}\, \hat{F}^{\pm}(x) \qquad (2.18)$$

In contrast to Eq. (2.16), the integral in the above equation does converge when $F_2(x)$ and $F_4(x)$ have Regge behavior. We cannot integrate this relation with respect to $y\cdot p$ and interchange the $y\cdot p$ and x integrals because the result, Eq. (2.16), is divergent.

There is another, particularly simple, way to derive Eq. (2.18). Consider the definition of $F^{\pm}(y\cdot p)$, from Eq. (2.8):

$$F^{\pm}(y\cdot p) = \frac{y^{\rho}\langle P|\bar{\psi}(y)\gamma_{\rho}\lambda^{\pm}\lambda^{\mp} I(y,0)\psi(0)|P\rangle}{y\cdot p}\Bigg|_{y^2=0}$$

Using the equation of motion for $\psi(y)$ (cf. Eq. 2.2):

$$\bar{\psi}(y) = \bar{\psi}(y)\left(\frac{-i\overleftarrow{\not\partial} - g\not{B}(y)}{m_p}\right)$$

together with the identity $\gamma_{\rho}\gamma_{\lambda} = g_{\rho\lambda} - i\sigma_{\rho\lambda}$ and the translational invariance of the matrix element we find

$$\frac{y^\rho < P|\bar{\psi}(y)\gamma_\rho \lambda^{\pm}\lambda^{\mp} I(y,0)\psi(0)|P>}{y\cdot p}\Bigg|_{y^2=0}$$

$$= \frac{1}{im_p} \frac{d}{dy\cdot p} < P|\bar{\psi}(y)\lambda^{\pm}\lambda^{\mp} I(y,0)\bar{\psi}(0)|P>\Big|_{y^2=0} \qquad (2.19)$$

from which Eq. (2.18) follows directly.

In conclusion, we have found that the consistency of the single and double divergence calculations of $F_4^{\bar{\nu}/\nu}(x)$ leads us to the identity Eq. (2.19) between the matrix elements of bilocal operators. Formal integration of Eq. (2.19) yields:

$$< P|\bar{\psi}(y)\lambda^{\pm}\lambda^{\mp} I(y,0)\psi(0)|P> = \int_{-1}^{1} dx\, e^{ixy\cdot p} \frac{F_2^{\bar{\nu}/\nu}(x)}{x^2} + C \qquad (2.20)$$

In the absence of all Regge contributions to $F_2(x)$ with $\alpha > 0$, $C = 0$ and the sum rules of Eq. (2.17) are obtained. When $\alpha > 0$ terms are present C cannot be zero (it is in fact infinite), and vitiates any attempt to derive a sum rule for $<P|\bar{\psi}(0)\lambda^{\pm}\lambda^{\mp}\psi(0)|P>$ in this approach. In order to proceed we turn to the BJL expansion where subtraction constants such as C may be handled in a more transparent manner.

III. Momentum Space Calculation

In the previous section we found that, unless $F_2(x)$ has quite unexpected behavior as $x \to 0$, we could not derive an expression for σ_π using coordinate space methods. We now turn to a momentum space calculation which employs the Bjorken-Johnson-Low (BJL) limit, in which prejudices about asymptotic behavior are incorporated from the start and divergent expressions are therefore never encountered.

We define:

$$T_{\mu\nu} = i\int dx^4 e^{iq\cdot x}\theta(x_0) < P|[J_\mu^-(x),\ J_\nu^+(0)]|P> \tag{3.1}$$

$$+ \text{ seagulls,}$$

where the "seagulls" are polynomials in q which may possibly be present. $T_{\mu\nu}$ can be expanded in terms of structure functions T_i in analogy with Eq. (2.5). The T_i satisfy:

$$\mathrm{Im}\, T_i = 2\pi W_i \tag{3.2}$$

Assuming conventional Regge behavior, $W_4 \sim \nu$ and $W_5 \sim$ const. as $\nu \to \infty$ with q^2 fixed, we can therefore write the following dispersion relations:

$$T_4(q^2,\nu) = T_4(q^2,0) + 2\nu\int_0^\infty \frac{W_4^-(q^2,\nu')d\nu'}{\nu'^2 - \nu^2} + 2\nu^2\int_0^\infty \frac{W_4^+(q^2,\nu')d\nu'}{\nu'(\nu'^2 - \nu^2)}$$

$$T_5(q^2,\nu) = 2\nu\int_0^\infty \frac{W_5^+(q^2,\nu')d\nu'}{\nu'^2 - \nu^2} + 2\int_0^\infty \frac{\nu' W_5^-(q^2,\nu')d\nu'}{\nu'^2 - \nu^2} \tag{3.3}$$

where:

$$W_{4,5}^{\pm} = W_{4,5}^{\nu} \pm W_{4,5}^{\bar{\nu}}.$$

We consider

$$q^\mu T_{\mu\nu} = \frac{q^2 q_\nu}{M^2} T_4 + \frac{q^2 P_\nu + \nu q_\nu}{2M^2} T_5$$

$$= -\int d^4x e^{iq\cdot x}\ \theta(x^0) < P|[\partial^\mu J_\mu^-(x),\ J_\nu^+(0)]|P> \tag{3.4}$$

$$+ \text{ seagulls}$$

(where the "equal time" contribution from $\partial_\mu \theta(x_o)$ has been absorbed in the seagulls). We now let $q_o \to i\infty$ in Eq. (3.4) and identify the coefficient of $1/q_o$ in the retarded commutator with the equal time commutator, according to the BJL theorem.[15] Taking the time component and using Eq. (3.3) and the scaling laws derived above we obtain:

$$\lim_{q^2 \to -\infty} \frac{q^4 \overline{T}_4(q^2,0)}{M^2}$$

$$\equiv \lim_{q_o \to i\infty} \left(\frac{q^2 q_o^2 T_4(q^2,0)}{M^2} - q_o \times \text{seagulls} \right) \tag{3.5}$$

$$= -i \int d\vec{x}\, e^{-i\vec{q}\cdot\vec{x}} \langle P | [\partial^\mu J_\mu^-(\vec{x},0), J_o^+(0)] | P \rangle .$$

The right hand side of this equation is proportional to the σ term. Explicitly: σ_π is defined by:

$$\sigma_\pi = \frac{1}{4iM} \int \langle P | [\partial^\mu J_\mu^-(\vec{x},0), J_o^+(0)] | P \rangle \, d\vec{x}. \tag{3.6}$$

This is the conventional σ-term ($\sim$ 10 - 100 Mev ?); the factor $(2M)^{-1}$ occurs because our states are normalized covariantly. In the vector-gluon model:

$$\sigma_\pi = \frac{m_p}{2M} \langle P | \overline{\psi}(0) \begin{pmatrix} 1 & \\ & 1_0 \end{pmatrix} \psi(0) | P \rangle \tag{3.7}$$

Since the seagull is a polynomial in q Eq. (3.5) relates σ_π to the piece of the subtraction constant $T_4(q^2,0)$ which behaves as $1/q^4$ as $q^2 \to -\infty$ which appears theoretically (and experimentally) intractable. However, we shall see in the next section that (given an additional assumption) it can be calculated.

We note that using the BJL limit to all orders (as in Ref. 11) we can rederive all the results of Section II.[16] (We have checked this explicitly.)

IV. Fixed Pole Residues, the σ-Term and Baryon Mass Differences

Several authors have argued[17,18] that the residues of any fixed poles with $\alpha = 0$ in kinematic singularity free current-nucleon scattering amplitudes are polynomials in q^2; the arguments advanced are most compelling in the context of field theories[18] (constituent models) such as those considered here (we adopt the language of the Regge model although our specific assumption and results may be valid more generally). In this section, we shall pursue the consequences of assuming polynomial residues. We note that this assumption can be tested since it leads to the Cornwall Corrigan Norton, Rajaraman Rajesakaran[19] sum rule:

$$\int_0^\infty \tilde{F}_2^{ep,en} \frac{dx}{x^2} = C^{p,n} \tag{4.1}$$

where:

$$C^p = 1 + \frac{1}{2\pi^2\alpha} \int_0^\infty \tilde{\sigma}_{\gamma p}(\nu)\, d\nu$$

$$C^n = \frac{1}{2\pi^2\alpha} \int_0^\infty \tilde{\sigma}_{\gamma n}(\nu)\, d\nu \tag{4.2}$$

$$\tilde{F}_2(x) = F_2(x) - F_2^R(x)$$

$$\tilde{\sigma}(\nu) = \sigma(\nu) - \sigma^R(\nu)$$

and $F_2^R(x)\left(\sigma^R(\nu)\right)$ are Regge fits to the small x (large ν) behavior of $F_2(x)\left(\sigma(\nu)\right)$ including all contributions (cuts and poles) with $\alpha > 0$.[20]

Phenomenological analyses[21,22] suggest that $C_p \approx 1$. The sum rule of Eq. (4.1) therefore predicts behavior of the type illustrated in Fig. 1. This striking behavior is not ruled out by existing data. If it fails, the ensuing considerations are invalid.

We consider a function $W_4^R\left(q^2,\nu\right)$ which fits the asymptotic behavior of $W_4\left(q^2,\nu\right)$ and includes all Regge contributions with $\alpha > 0$. An analytic function $T_4^R\left(q^2,\nu\right)$ whose discontinuity equals $2\pi W_4^R$ can be constructed from a dispersion relation. Then the residue of any $\alpha = 0$ fixed pole in T_4 is defined as follows:

$$C_4(q^2) = \lim_{\nu\to\infty}\left(T_4(q^2,\nu) - T_4^R(q^2,\nu)\right)$$

However, the assumption that the $\alpha = 0$ fixed poles in the kinematic singularity free amplitudes, $\hat{T}_i$ (defined in Eq. (2.5) of Ref. 8), are polynomials, together with the fact that $\frac{\nu^2}{M^4}\, T_4(q^2,\nu)$ scales and $\nu^2\, T_5(q^2,\nu)$ vanishes in the Bjorken limit, are sufficient to prove $C_4(q^2) = 0$.[23] Combining the dispersion relations for T_4 (Eq. (3.3)) and T_4^R (and noting that with a Regge expansion in ν^α subtraction terms at $\nu = 0$ in T_4^R vanish) and using Eq. (4.3), we obtain:

$$C_4(q^2) = T_4\,(q^2,0) - 2\int \frac{\widetilde{W}_4^+(q^2,\nu')\,d\nu'}{\nu'} = 0$$

when

$$\widetilde{W}_4^+ = W_4^+ - W_4^{+R}\quad .$$

Comparing Eqs. (4.4) and (3.5), we find the σ term sum rule:

$$\frac{1}{2i}\int d\vec{x}\, e^{-i\vec{q}\cdot\vec{x}}\langle P\left|\left[\partial^\mu J_\mu^-(\vec{x},0), J_0^+(0)\right]\right|P\rangle = 4M^2\int \widetilde{F}_4(x)\, x\, dx \tag{4.4}$$

where $\widetilde{F}_4$ is defined in the same way as $\widetilde{F}_2$ (Eq. (4.2)).

It is satisfying to observe that if F_4 is well enough behaved so that $F_4 \equiv \tilde{F}_4$, Eq. (4.4) becomes the σ term sum rule of Refs. 6 and 7 (cf. Eqs. (2.17) and (3.7)). If this is not the case, we see that the assumption of a polynomial residue has provided us with a prescription for calculating the infinite constant in Eq. (2.20). (The probably divergent result of Eq. (2.17b) can be derived immediately in the parton model[24]; the subtracted form of this equation is probably implicit in the parton calculations of Brodsky, Close and Gunion.[18])

Combining Eqs. (2.14), (3.6), (3.7) and (4.4) and generalizing to all SU(3) currents we obtain the following equations for the three independent scalar densities:

$$\begin{aligned}
\langle P|\bar{\psi}(0)\begin{pmatrix}1 & \\ & 1 \\ & & 0\end{pmatrix}\psi(0)|P\rangle &= m_p \int \tilde{F}_2^{\nu p+\nu n}\frac{dx}{x^2} \\
\langle P|\bar{\psi}(0)\begin{pmatrix}1 & \\ & -1 \\ & & 0\end{pmatrix}\psi(0)|P\rangle &= 6m_p \int \tilde{F}_2^{ep-en}\frac{dx}{x^2} \qquad (4.5)\\
\langle P|\bar{\psi}(0)\begin{pmatrix}0 & \\ & 0 \\ & & 1\end{pmatrix}\psi(0)|P\rangle &= m_\lambda \int \left(9\tilde{F}_2^{ep+en} - \frac{5}{2}\tilde{F}_2^{\nu p+\nu n}\right)\frac{dx}{x^2} .
\end{aligned}$$

These equations are exact in this model.

We can now make further progress by noting that $\bar{\psi}(0)\lambda_i\psi(0)$ is in the same octet as the operator $\bar{\psi}(0)\lambda_8\psi(0)$ which is responsible for baryon mass differences in this model. Hence we can give an approximate expression for $\langle P|\bar{\psi}(0)\lambda_i\psi(0)|P\rangle$ (as a ratio of baryon mass differences to quark mass differences) which would become exact in the SU(3) symmetry limit. To next order, the error in using this expression is the same as the error in neglecting second order corrections to the Gell-Mann Okubo mass formula—i.e., empirically it is expected to be $\lesssim$ 10%. It would seem at first sight that in order to be strictly mathematically consistent, we should also assume exact SU(3) symmetry on the right-hand side of Eq. (4.5) and set $m_p = m_\lambda$. However, we shall see that if we persist with

$m_p \neq m_\lambda$, existing data suggest $\dfrac{m_\lambda - m_p}{m_p} \sim 1$, which indicates that the breaking of SU(3) × SU(3) and of SU(3) are comparable. Furthermore, our results suggest that SU(3) × SU(3) is spontaneously broken in which case the error in our approximation for $< P \,|\, \overline{\psi}(0)\lambda_i \psi(0) \,|\, P >$ is of order $\dfrac{m_\lambda - m_p}{M_P}$ and is not to be compared with $\dfrac{m_\lambda - m_p}{m_p}$. Therefore we proceed with $m_\lambda \neq m_p$ and note that if (as we assume) the matrix elements $\langle P|\overline{\psi}(0)\lambda_i\psi(0)|P\rangle$ are approximately SU(3) symmetric, then the fixed pole residues—$\widetilde{F}_2$ integrals—are not (according to the results of phenomenological analysis of existing data).

Expressing the left-hand sides of Eq. (4.5) in terms of baryon masses (M_i) and quark masses (m_i), we obtain

$$m_\lambda \approx m_p + \frac{M_P\left[2(M_\Xi - M_P) + 3(M_\Lambda - M_\Sigma)\right]}{12\,B\,m_p} \tag{4.6a}$$

$$\sigma_\pi = \frac{m_p^2}{2M_P}\int F_2^{\nu p+\nu n}\,\frac{dx}{x^2} \tag{4.6b}$$

$$\approx \frac{18\,m_p^2}{2M_P\left(m_p + 5m_\lambda\right)}\left[m_\lambda D + \frac{B\left[2(M_\Xi - M_P) - (M_\Lambda - M_\Sigma)\right]}{2(M_\Xi - M_P) + 3(M_\Lambda - M_\Sigma)}\right] \tag{4.6c}$$

where

$$B = \int \widetilde{F}_2^{ep-en}\,\frac{dx}{x^2}$$

$$D = \int \widetilde{F}_2^{ep+en}\,\frac{dx}{x^2} \quad .$$

If all the fixed pole residues were known experimentally, it would be possible to solve Eq. (4.6) for m_p, m_λ, and σ_π. From these results, we could obtain other quantities (such as σ_K) which are of interest for low-energy theorems. These quantities can all be expressed in terms of the fundamental parameters

C $\left(= -\sqrt{2}\right.$ in the SU(2) × SU(2) limit$\left.\right)$ and μ_0 which, we recall, are given by

$$c = \frac{-\sqrt{2}\,(m_\lambda - m_p)}{m_\lambda + 2\,m_p} \tag{4.7}$$

$$\mu_0 = \frac{\sum_i m_i}{2\,M_P \times 3} \left\langle P \right| \bar{\psi}(0) \left(^1{}_1{}_1\right) \psi(0) \left| P \right\rangle$$

$$= \frac{3\,\sigma_\pi}{\sqrt{2}\left(\sqrt{2} + c\right)} - \frac{\left(2m_p + m_\lambda\right)}{2\sqrt{3}} \; \frac{\left\langle P \right| \bar{\psi}(0)\lambda_8\psi(0)\left| P \right\rangle}{2\,M_P}$$

$$\simeq \frac{3\sigma_\pi}{\sqrt{2}\left(\sqrt{2} + c\right)} + \frac{1}{4\sqrt{2}\;c}\left(2\,M_{\bar{\Xi}} + M_\Sigma - 2\,M_P - M_\Lambda\right)$$

in this model. (Note that these expressions for μ_0 are not specific to our model but are true in any model in which the part of H_{strong} which breaks SU(3) × SU(3) belongs to the (3, 3*) ⊕ (3*, 3) representation and in which there is no nonelectromagnetic breaking of isospin symmetry.)

At present, estimates of the σ-term and fixed pole residues (from γp and γd data using Eq. (4.2))[22] range as follows:

$$10\ \text{MeV} \gtrsim \sigma_\pi \gtrsim 100\ \text{MeV},$$

$$0.7 \gtrsim \int \tilde{F}_2^{ep}\,\frac{dx}{x^2} \gtrsim 1.3\ ,$$

$$-0.3 \gtrsim \int \tilde{F}_2^{en}\,\frac{dx}{x^2} \gtrsim 0.5\ .$$

In Figs. 2-4, we have plotted σ_π and m_λ against m_p for values of the fixed pole residues and σ_π within these ranges. In Fig. 5, we have plotted the parameter C (Eq. (4.7)) against σ_π for the "best" values of the fixed pole residues: $C^P = 1.0$ and $C^n = 0.1$. Note the expected behavior (which is of course built into our equation) $c \to -\sqrt{2}$, $\sigma_\pi \to 0$ as $m_p \to 0$ (SU(2) × SU(2) limit).

In all cases we find that the fixed pole in neutrino production (cf. Eq. (4.6b)) has a substantial residue. Typically (e.g., $C^p = 1.0$, $C^n = 0.1$, $\sigma_\pi = 40$ MeV)

$$\int \tilde{F}_2^{\nu p + \nu n} \frac{dx}{x^2} = \sim 5 \; ,$$

which has the striking consequence that $F_2^{\nu p+\nu n}$ should have an exaggerated form of the behavior illustrated in Fig. 1 for F_2^{ep} – with(Area A – Area B) ~ 5 in this case.

The results in Figs. 2-4 seem reasonable. In addition to determining the parameters of the model in terms of data on C^p, C^n, and σ_π as they become available, they may be used as a theoretical laboratory for studying various properties of chiral symmetry breaking. Of course, for the latter purpose it must be remembered that C^p, C^n and the baryon masses are implicit functions of m_p and m_λ. (They would presumably be insensitive functions in models in which SU(3) × SU(3) is spontaneously broken and the quark masses may be treated as perturbations.)

Let us, for example, impose as a boundary condition the value $c \approx -1.25$, derived by Gell-Mann, Oakes and Renner[25] from the observed pseudoscalar meson masses in the framework of approximate SU(3) × SU(3) symmetry. Unfortunately, the results are extremely sensitive to the exact values of c and of the fixed pole residues. Taking the "best" values $C^p = 1.0$ and $C^n = 0.1$ with $c = -1.25$ gives

$$m_p \approx 45 \text{ MeV}$$

$$m_\lambda \approx 1{,}100 \text{ MeV}$$

$$\sigma_\pi \approx 4 \text{ MeV}$$

while with $c = -1.0$ we find

$$m_p \approx 80 \text{ MeV}$$

$$m_\lambda \approx 640 \text{ MeV}$$

$$\sigma_\pi \approx 15 \text{ MeV} \; .$$

In any case, as can be seen from Fig. 5, if c is at all close to $-\sqrt{2}$, then σ_π must be very small. (Thus we see explicitly that, at least in our model, the results of Cheng and Dashen[26] and of Gell-Mann, Oakes and Renner are incompatible.)

Turning to an evaluation of μ_0, we find that it is negative in the cases of interest (typically $\mu_0 \sim -100$ MeV). There is no fundamental problem here since the matrix elements which define μ_0 (Eq. (4.7)) are understood to be connected (i.e., when we write $\langle P|X|P\rangle$ we understand $\langle P|X|P\rangle - \langle 0|X|0\rangle \langle P|P\rangle$), so μ_0 is not a positive definite quantity. This result means that when the quark masses are "switched on" the baryon masses decrease slightly; we know of no reason why this rather bizarre situation should not obtain.

Its origin is manifest in Eq. (4.7); although the quantity $(2M_\Xi + M_\Sigma - 2M_P - M_\Lambda)$ in the last line is an approximation, presumably the fact that it is positive and of order 800 MeV is true of the exact expression which it replaces. Hence

$$\mu_0 \approx \frac{2.12\,\sigma_\pi}{\left(\sqrt{2} + C\right)} + \frac{145 \text{ MeV}}{C} \tag{4.8}$$

To see that this is, in fact, negative, consider first the SU(2) × SU(2) limit. In this limit μ_0 is, in general, rather sensitive to the manner in which $C \to -\sqrt{2}$ and $\sigma_\pi \to 0$. (For example, taking $C = -1.25$ and $\sigma_\pi = 10$ MeV, $\mu_0 = +13$ MeV, while for $C = -1.25$ and $\sigma_\pi = 4$ MeV, $\mu_0 = -66$ MeV.) However, Eqs. (4.6b) and (4.7) provide us with an analytic relation between σ_π and $C + \sqrt{2}$, which gives near $m_p = 0$: $C + \sqrt{2} \sim m_p$ and $\sigma_\pi \sim m_p^2$. (Note that although the neutrino fixed pole residue in Eq. (4.6b) must be regarded as an implicit function of m_p, nevertheless it cannot become singular as $m_p \to 0$ and therefore σ_π vanishes at least as fast as m_p^2.) Hence the second term in (4.8) dominates and $\mu_0 \xrightarrow{m_p \to 0} -100$ MeV. As m_p increases, this dominance continues and μ_0 remains negative.

V. Conclusions and Analysis

In order for our analysis of current divergences in the gluon model to be valid, it is necessary (a) that the real world is described by (results abstracted from) "formal" field theory, (b) that the relevant field theory is the quark model.

Therefore, our results will be irrelevant unless the well-known quark model sum rules derived previously (which are much more easily tested) prove to be correct. However, if prerequisite (a) holds but (b) fails, similar results can doubtless be obtained in other models.

Our main results are:

1. In the vector-gluon model

$$\lim_{bj} \frac{\nu^2 W_4}{M^4} = F_4(x) ,$$

a result previously obtained by Ng and Vinciarelli,[7] and

$$\lim_{bj} \nu^2 W_5 = 0 .$$

2. The following sum rule for the σ term can be obtained[6,7]

$$\sigma_\pi = 2 M_P \int F_4^{\nu+\bar{\nu}}(x)\, x\, dx .$$

However, it diverges linearly if Regge behavior is assumed. If it diverges, the derivation is invalid and we find instead

$$\sigma_\pi = \lim_{q^2 \to \infty} \frac{q^4 \bar{T}_4(q^2, 0)}{4 M_P^3}$$

($\bar{T}_4(q^2, 0)$ is equal to $T_4(q^2, 0)$ with pieces which fall less rapidly than $1/q^4$ subtracted.)

3. If $\dfrac{x^3 F_4(x)}{F_2(x)}$ = const. (independent of x), this would imply that the interaction

between the quarks is vector in nature. In this case

$$\left.\frac{4x^3 F_4(x)}{F_2(x)}\right|_{\Delta S=0} = \left(\frac{m_p}{M_P}\right)^2$$

$$\left.\frac{4x^3 F_4(x)}{F_2(x)}\right|_{\Delta S=1} = \frac{m_p m_\lambda}{M_P^2}$$

so that it is possible (in principle) to measure the bare quark masses in the vector-gluon model.

4. If the residue of any fixed $\alpha = 0$ Regge pole is a polynomial in q^2:

$$\sigma_\pi = \lim_{q^2 \to -\infty} \frac{q^4 \bar{T}_4(q^2, 0)}{4 M_P^3}$$

$$= 2 M_P \int_0^\infty \tilde{F}_4(x)\, x\, dx$$

where $\tilde{F}_i = F_i - F_i^R$ and F_i^R is a Regge fit to the small x behavior of F_i including all contributions (poles or cuts) with $\alpha > 0$. (This result is also true in models with a scalar or pseudoscalar interaction.) As discussed in Section IV, the assumption of polynomial residues can be tested experimentally.

5. If the interaction is vector in nature and fixed poles have polynomial residues, results 3 and 4 can be combined to yield the exact expressions for the scalar bilocals in Eq. (4.5); in addition,

6. The octet scalar bilocals can be expressed approximately in terms of baryon mass differences and $m_\lambda - m_p$. Hence we obtain a relation between fixed pole residues in γp, γn, and $\nu p + \nu n$ scattering (which can be expressed in terms of

deep inelastic $[F_2]$ data), baryon mass differences, σ_π and bare quark masses (cf. Eq. 4.6). The results of a preliminary analysis of these relations were discussed in Section IV (see also Figs. 2-5). Two important results are that $\mu_0 \sim -100$ MeV (a negative value of μ_0, while somewhat bizarre, is not impossible), and

$$\int \tilde{F}_2^{\nu p + \nu n} \frac{dx}{x^2} \sim 5 \quad .$$

which would have the rather striking implications discussed in Section IV. In addition, we find that if c is near the SU(2)$\times$ SU(2) limit $(-\sqrt{2})$, σ_π must be extremely small (see Fig. 5).

Independent of the particular model considered, and despite the dubious nature of the formal manipulations in which we have freely indulged, we think it an interesting point of principle that relations which probe the structure of the Hamiltonian can be devised without solving the field theory. It is very hard to measure $W_{4,5}$, so results 1-4 are difficult to test. However, these results illustrate the fact that these structure functions may contain a wealth of information; it will be worth trying to measure them in future generations of experiments. Once accurate measurements of

$$\int \tilde{F}_2^{ep,\, en,\, \nu p + \nu n} \frac{dx}{x^2}$$

are available, our results make it possible to calculate σ_π , m_p, and m_λ (as noted above, it will be a nontrivial result if this yields the right order of magnitude for σ_π since this seems to require a large $\tilde{F}_2^{\nu p + \nu n}$ integral).

Appendix

Here we calculate the leading light-cone contribution to the commutator of Eq. (2.9). Following the techniques of Fritzsch and Gell-Mann, but keeping both the leading and next to leading contributions to $\{\psi(y), \bar{\psi}(z)\}$ we obtain:

$$\frac{1}{4\pi} \langle P|[\partial_\mu J^{\mu\pm}(y), J_\nu^{\mp}(z)]|P\rangle$$

$$= -\frac{m_p}{2\pi} \langle P|\bar{\psi}(y)\gamma_5 I(y,z)(-\not\partial + im_p) D(y-z)\gamma_\nu\gamma_5\lambda^{\pm}\lambda^{\mp}\psi(z)$$

$$- \bar{\psi}(z)\gamma_\nu\gamma_5(-\not\partial + im_p) D(z-y) I(z,y)\gamma_5\lambda^{\mp}\lambda^{\pm}\psi(y)|P\rangle \qquad \text{(A1)}$$

where the derivative is with respect to y and I(y, z) is defined as after Eq. (2.7). Naively the $\not\partial$ term would seem to dominate the term proportional to the quark mass; but as discussed in Section II this is not the case.

Separating the contribution of the two terms we find:

$$\frac{1}{4\pi} \langle P|[\partial^\mu J_\mu^{\pm}(y), J^{\mp}(z)]|P\rangle = \frac{m_p \partial^\rho D(y-z)}{2\pi} \langle P|\bar{\psi}(y) I(y,z)\gamma_\rho\gamma_\nu\lambda^{\pm}\lambda^{\mp}\psi(z) +$$

$$+ \bar{\psi}(z) I(z,y)\gamma_\nu\gamma_\rho\lambda^{\mp}\lambda^{\pm}\psi(y)|P\rangle$$

$$+ \frac{im_p^2 D(y-z)}{2\pi} \langle P|\bar{\psi}(y)\gamma_\nu I(y,z)\lambda^{\pm}\lambda^{\mp}\psi(z)$$

$$- \bar{\psi}(z)\gamma_\nu I(z,y)\lambda^{\mp}\lambda^{\pm}\psi(y)|P\rangle \qquad \text{(A2)}$$

$\gamma_\rho\gamma_\nu$ contains a $g_{\mu\nu}$ and a $\sigma_{\mu\nu}$ piece. Using the crossing properties of the currents, i.e.,

$$\int e^{iq\cdot y} \langle P|[\partial^\mu J_\mu^{\pm}(y), J_\nu^{\mp}(0)]|P\rangle = \int e^{iq\cdot y} \langle P|[\partial^\mu J_\mu^{\mp}(0), J_\nu^{\pm}(y)]|P\rangle$$

the $\sigma_{\mu\nu}$ term may be shown to vanish. The first term in Eq. A2 reduces to

$$\frac{m_p \partial_\nu D(y-z)}{2\pi} \quad <P|\overline{\psi}(y)\, I(y,z)\, \lambda^{\pm}\lambda^{\mp}\psi(z) + \overline{\psi}(z)\, I(z,y)\, \lambda^{\mp}\lambda^{\pm}\psi(y)|P> \qquad (A3)$$

This matrix element is normalized to some mass, M.

The vector bilocal in the term explicitly proportional to the quark mass may be reduced to a derivative of the scalar bilocal using the equation of motion:

$$<P|\overline{\psi}(y)\gamma_\nu\, I(y,z)\,\psi(z)|P>$$

$$= \frac{1}{2} <P|\overline{\psi}(y) \left(\frac{-i\overleftarrow{\not\partial}_y - g\not B(y)}{m_p}\right) \gamma_\nu\, I(y,z)\,\psi(z)|P>$$

$$+ \frac{1}{2} <P|\overline{\psi}(y)\gamma_\nu\, I(y,z) \left(\frac{i\not\partial_z - g\not B(z)}{m_p}\right) \psi(z)|P>$$

After some algebra, using especially the translational invariance of forward matrix elements, we obtain

$$<P|\overline{\psi}(y)\gamma_\nu\, I(y,z)\,\psi(z)|P> = \frac{-i\partial_\nu}{m_p} <P|\overline{\psi}(y)\, I(y,z)\,\psi(z)|P> \qquad (A4)$$

where the derivative is with respect to y. Combining Eqs. A2-A4 we obtain the result:

$$\frac{1}{4\pi} <P|[\partial^\mu J^{\pm}_\mu(y),\, J^{\mp}_\nu(z)]|P> = \frac{m_p}{2\pi}\,\partial_\nu \Big\{ D(y-z) <P|\overline{\psi}(y)\, I(y,z)\lambda^{\pm}\lambda^{\mp}\,\psi(z) +$$

$$+ \overline{\psi}(z)\, I(z,y)\,\lambda^{\mp}\lambda^{\pm}\psi(y)|P> \Big\}$$

quoted in Eq. (2. 11).

Footnotes and References

1. R. A. Brandt and G. Preparata, Nucl. Phys. B27, 541 (1971).

2. With our definition W_5 is the interference term between the scalar (divergence) and longitudinal components of the current in the forward current-nucleon scattering amplitude. It follows from the Schwartz inequality that νW_5 must vanish in the Bjorken limit in models (such as the quark model) in which $\frac{\sigma_L}{\sigma_T} \to 0$. (The inequalities are given explicitly by T. D. Lee and C. N. Yang, Phys. Rev. 126, 2239 (1962), and M. G. Doncel and E. de Rafael, Nuovo Cimento, 4A, 363 (1971) (see also Ref. 8).)

3. Different authors use different definitions for $W_{4,5}$. The definition used in this paper (Eq. 2.5) is useful for discussions of chiral symmetry breaking but it is inconvenient in other cases since it implies that the W_i are not completely independent as $q^2 \to 0$.

4. J. E. Mandula, A. Schwimmer, J. Weyers and G. Zweig, Phys. Letters 37B, 109 (1971).

5. S. Brown, Phys. Rev. Letters 27, 347 (1971); Phys. Letters 38B, 399 (1972).

6. B. W. Lee and J. E. Mandula, Phys. Rev. D4, 3475 (1971).

7. W. C. Ng and P. Vinciarelli, Phys. Letters 38B, 219 (1972).

8. The formula which relate the W_i to observables may be found, e.g., in C. H. Llewellyn Smith "Neutrino Reactions at Accelerator Energies" SLAC-PUB-958 (to be published in Physics Reports) (note that the definition of $W_{4,5}$ in this reference is different from that used here). There are two difficulties in measuring $W_{4,5}$: (1) They are only determined if both the muon polarization and the double differential cross section are observed in inelastic neutrino reactions. (2) Their contribution vanishes

in the approximation $m_\mu = 0$ (they would be more amenable if heavy leptons existed coupled to ν_μ).

9. H. Fritzsch and M. Gell-Mann in: "Center for Theoretical Studies University of Miami Tracts in Mathematics and Natural Science, Vol. 2" (Gordon and Breach, 1971).

10. D. J. Gross and S. B. Treiman, Phys. Rev. D4, 1059 (1971).

11. C. H. Llewellyn Smith, Phys. Rev. D4, 2392 (1971).

12. Once we are satisfied that the interaction only introduces a phase factor (which can in fact be eliminated by a suitable choice of gauge) into the free field results we are assured that any scaling laws or sum rules we obtain with light-cone techniques may also be obtained (with considerably less effort) in the parton model. However, it has become popular to do free field theory in coordinate space in order to circumvent the obvious and embarassing appearance of free field quanta, a tradition which we reluctantly follow.

13. States are normalized covariantly: $\langle P | P' \rangle = (2\pi)^3 \, 2E \, \delta^3(\vec{P} - \vec{P}')$. Our metric and other conventions follow those of J. D. Bjorken and S. D. Drell Relativistic Quantum Mechanics (McGraw Hill, New York, 1964).

14. With our normalization $H^{\pm}(y \cdot p)$ has dimensions of mass, accounting for the superficial dimensional discrepancy in Eq. (2.12).

15. J. D. Bjorken, Phys. Rev. 148, 1467 (1966). K. Johnson and F. E. Low, Prog. Th. Phys. Supp. 37-38, 74 (1966).

16. This involves first equating the coefficients of $1/q_0^n$ as $q_0 \to i\infty$ and then identifying and equating parts which have the same behavior when P_0 varies. This is most easily done by taking $P_0 \to \infty$ which is here simply a trick (in contrast to the use of $P_0 \to \infty$ in attempting to derive fixed mass sum rules). In fact the methods of Refs. 10 and 11 are essentially equivalent,

except that the former is more elegant while with the latter (as we have seen) any desired asymptotic behavior can automatically be taken into account.

17. T. P. Cheng and Wu-Ki Tung, Phys. Rev. Letters 24, 851 (1970).
J. M. Cornwall, D. Corrigan and R. Norton, Phys. Rev. 24, 1141 (1970).

18. S. J. Brodsky, F. E. Close and J. F. Gunion, Phys. Rev. D5, 1384 (1972).
P. V. Landshoff and J. C. Polkinghorne, Phys. Rev. D5, 2050 (1972).
M. Bander, "Fixed Poles in Compton Scattering; Light Cone Approach" U.C. Irvine preprint (1972).

19. J. M. Cornwall, D. Corrigan and R. Norton (loc. cit.). R. Rajaraman and G. Rajesakaran, Phys. Rev. D3, 266 (1971) and E:D4 2940 (1971).

20. Note that we have assumed in Eqs. (4.1) and (4.2) that the Regge fits are in terms of $\nu^{\alpha}(x^{-\alpha})$. If instead they were in terms of $(\nu - \nu_0)^{\alpha}$, $[(x-x_0)^{-\alpha}]$ there would be extra terms in these equations and the integrals would not have to run from 0 [to ∞].

21. M. Damashek and F. J. Gilman, Phys. Rev. D1, 1319 (1970). C. A. Dominquez, C. Ferro-Fontan and R. Suaya, Phys. Letters 31B, 365 (1970).

22. C. A. Dominquez, J. F. Gunion and R. Suaya, SLAC-PUB-1042.

23. The proof is as follows: in terms of the $\hat{T}_i$:

$$T_4 = \hat{T}_4 - \frac{M^2}{q^2}\hat{T}_1 - \frac{\nu^2}{q^4}\hat{T}_2$$

$$T_5 = \hat{T}_5 + \frac{2\nu}{q^2}\hat{T}_2$$

If the residues of fixed poles in $\hat{T}_i$ are polynomials $(P_i(q^2))$, then the fixed pole contributions to T_4 and T_5 are:

$$T_4^{F.P.}(q^2,\nu) = P_4(q^2) - \frac{M^2}{q^2} P_1(q^2) - \frac{M^4}{q^4} P_2(q^2)$$

$$T_5^{F.P.}(q^2,\nu) = \frac{M^2}{\nu} P_5(q^2) + \frac{2M^4}{\nu q^2} P_2(q^2)$$

Requiring $\frac{\nu^2}{M^4} T_4(q^2,\nu)$ to scale sets

$$T_4^{F.P.}(q^2) = -\frac{M^4}{q^4} P_2(0) ,$$

while requiring $\nu^2 T_5$ to vanish demands $P_2(0) = 0$. Note that $P_2(0) = 0$ does not exclude the usual fixed pole in T_2 since it appears as the term in $P_2(q^2)$ linear in q^2.

24. W. I. Weisberger, "Partons, Electromagnetic Mass Shifts and the Approach to Scaling," Stonybrook preprint (1972).

25. M. Gell-Mann, R. J. Oakes and B. Renner, Phys. Rev. 175, 2195 (1968).

26. T. P. Cheng and R. Dashen, Phys. Rev. Letters 26, 598 (1971).

Figure Captions

1. Illustration of the behavior of F_2^{ep} implied by the assumption that fixed pole residues are polynomials in Q^2 ($\omega \equiv 1/x$). As discussed in the text (cf. Eq. (4.1), this assumption yields: Area A — Area B $\simeq$ 1.

2. σ_π and m_λ as functions of m_p with (cf. Eq. 4.1) $C^p = 0.8$ and different values of C^n, as marked on the corresponding curves.

3. As for Fig. 2 except $C^p = 1.0$ here.

4. As for Fig. 2 except $C^p = 1.2$ here.

5. c (defined in Eq. 4.7) as a function of σ_π assuming $C^p = 1.0$ and $C^n = 0.1$.

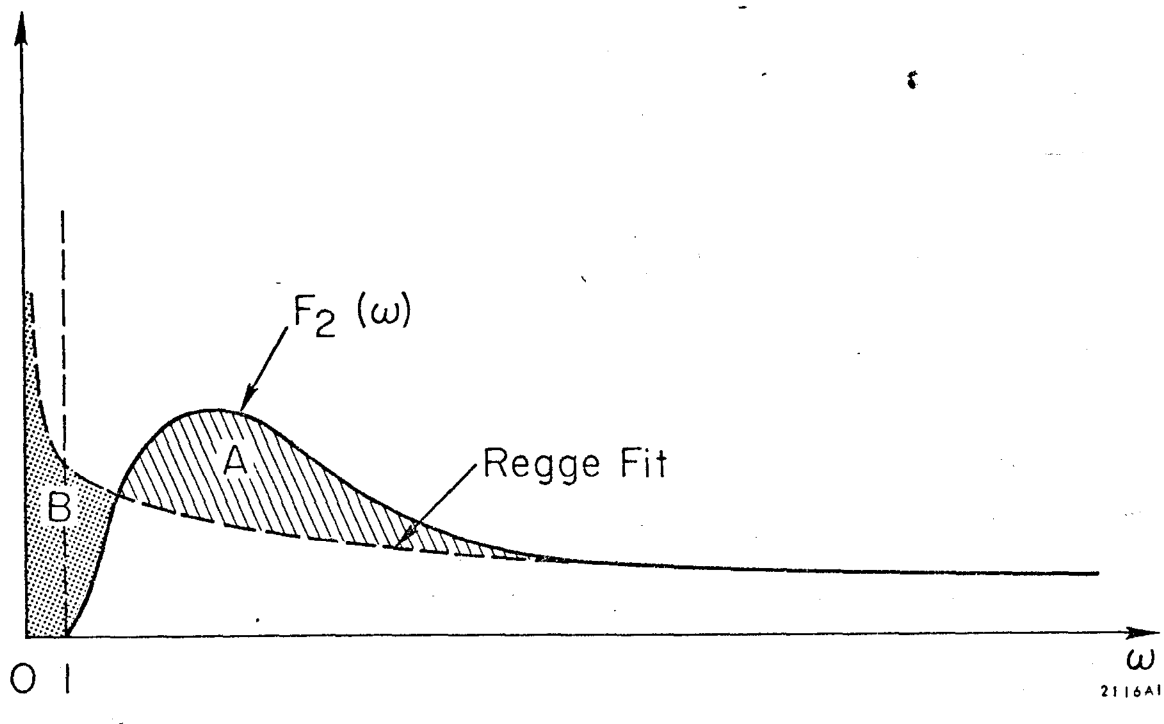
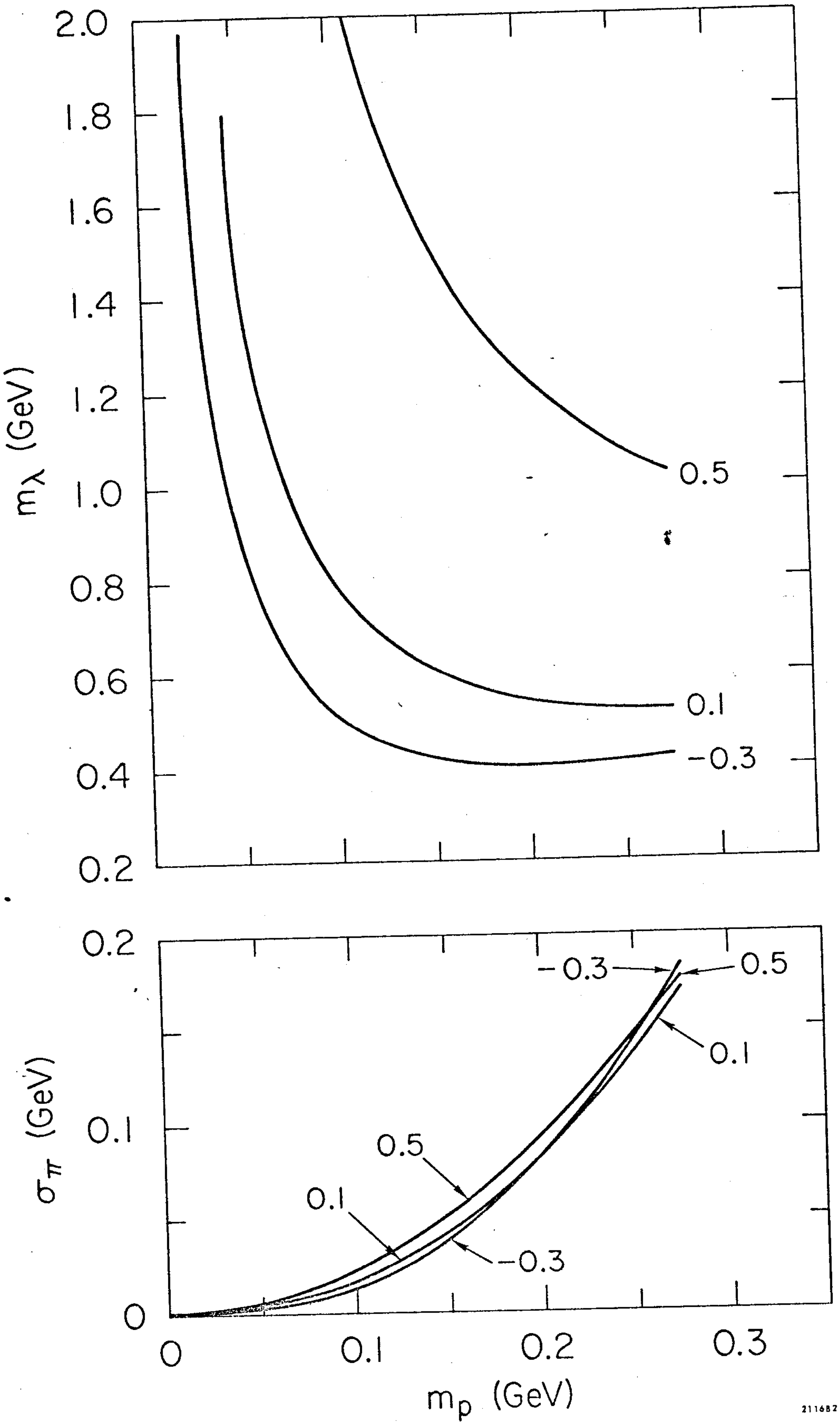


Fig. 1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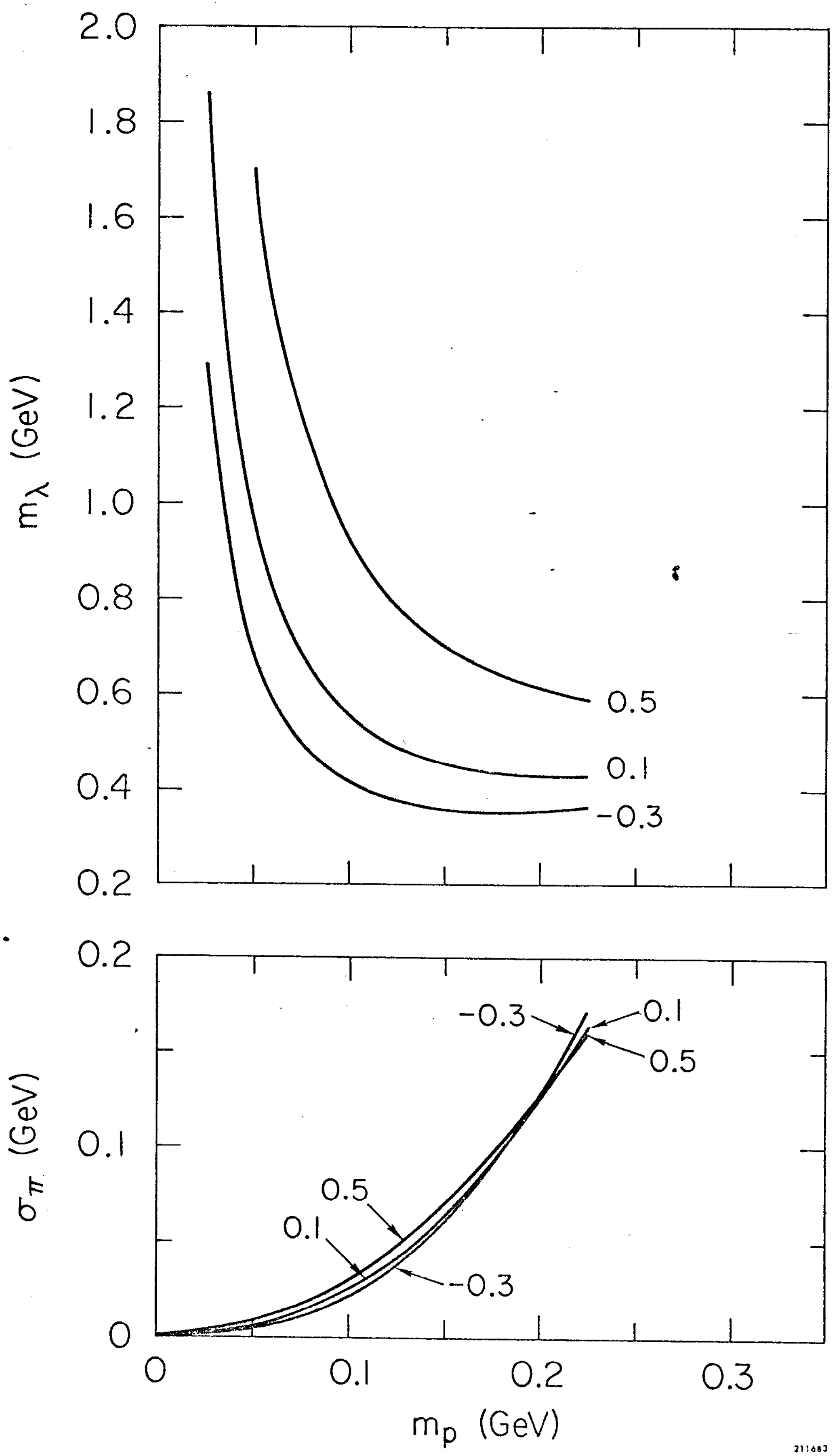
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
mλ (GeV)
0.5
0.1
−0.3
0.2
0.1
0
σπ (GeV)
−0.3
0.5
0.1
0.5
0.1
−0.3
0
0.1
0.2
0.3
mp (GeV)
211682

Fig. 2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1.0
0.8
0.6
0.4
0.2
m_λ (GeV)
0.5
0.1
−0.3
0.2
0.1
0
σ_π (GeV)
−0.3
0.1
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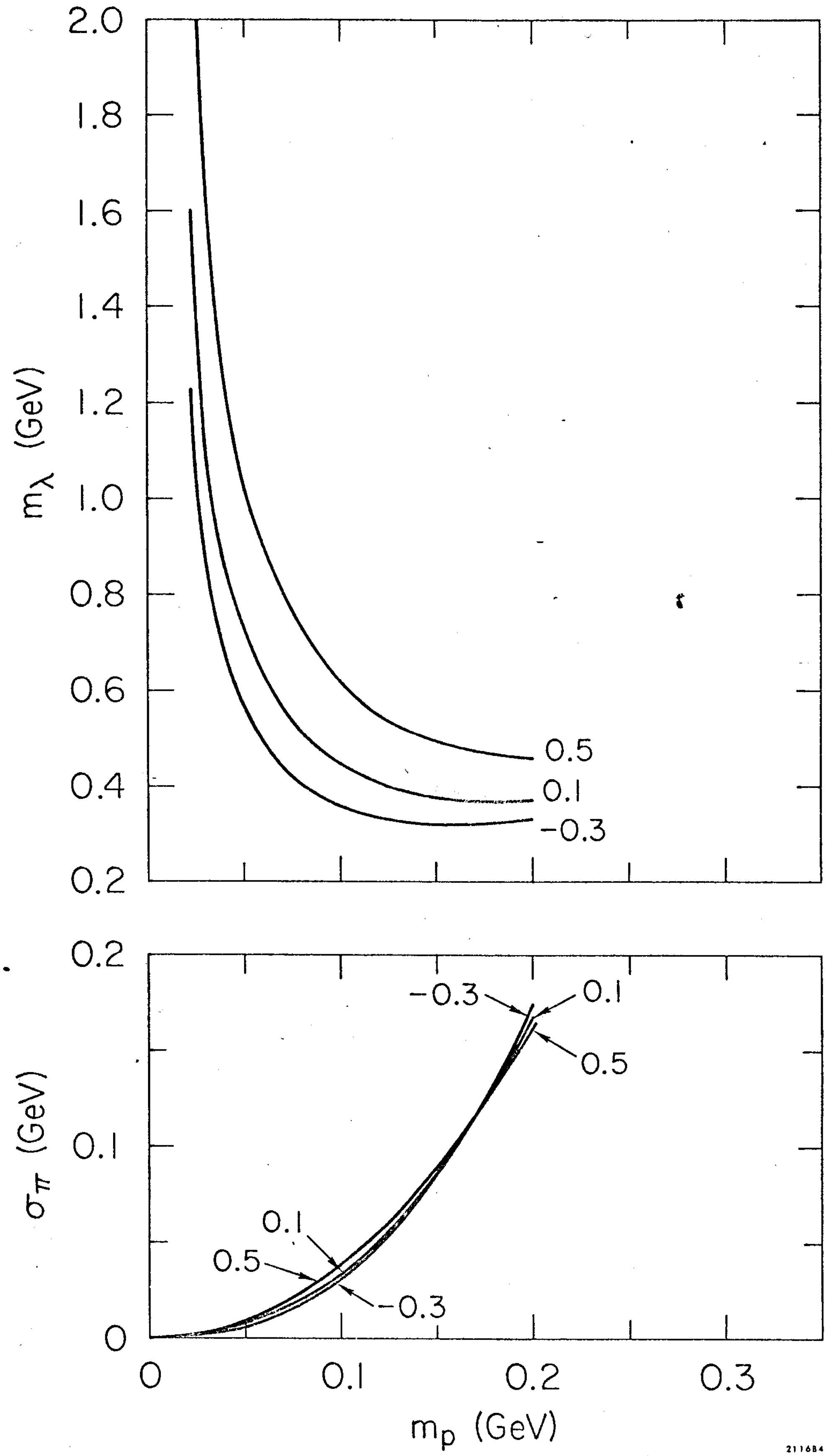
0.5
0.1
−0.3
0
0.1
0.2
0.3
m_p (GeV)
211683

Fig. 3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0.2
m_λ (GeV)
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0.1
−0.3
0.2
0.1
0
σ_π (GeV)
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−0.3
0
0.1
0.2
0.3
m_p (GeV)
2116B4

Fig. 4

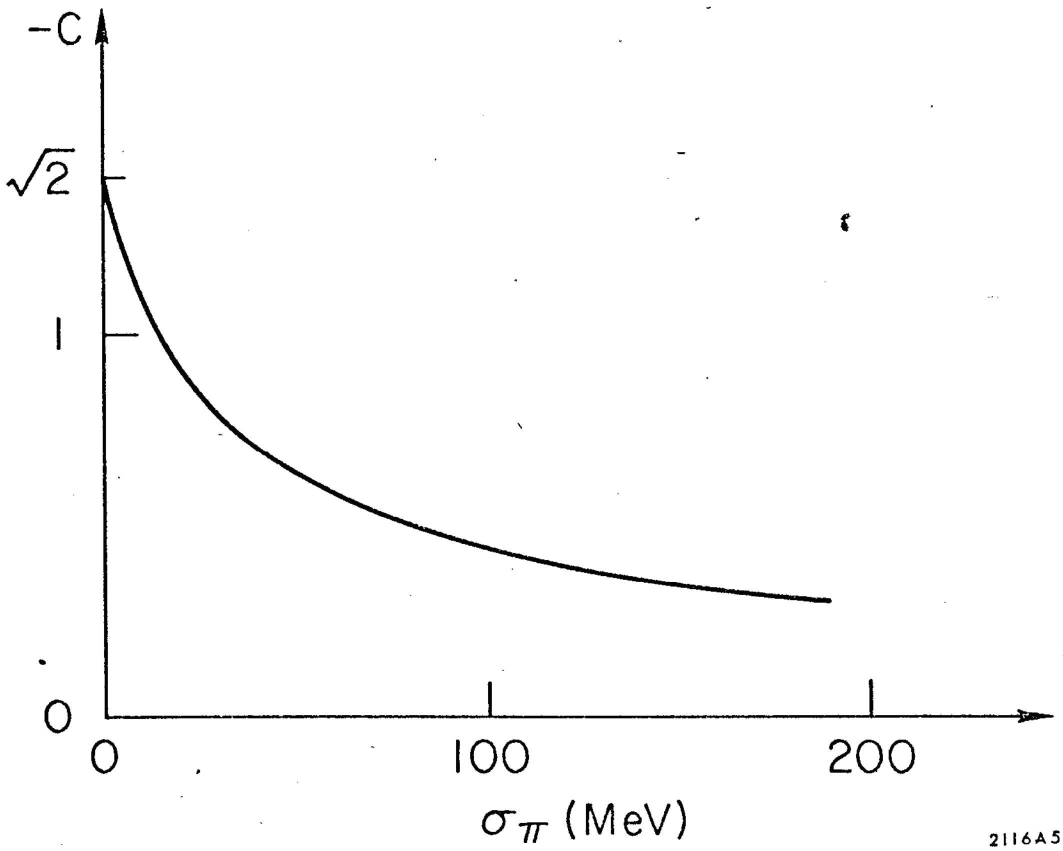

Fig. 5