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A TEST FOR FRACTIONALLY CHARGED PARTONS FROM DEEP INELASTIC BREMSSTRAHLUNG IN THE SCALING REGION

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Equation (6)	$Q^2 = -(p-p'-k)^2$
Equations (7), (A1)	change s to $2p\cdot P$
Equation (A7)	change $(sQ^2\pi)$ to $(2p\cdot P Q^2\pi^2)$
Equation (A2)	interchange L_2 and L_1

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A TEST FOR FRACTIONALLY CHARGED PARTONS FROM DEEP INELASTIC BREMSSTRAHLUNG IN THE SCALING REGION*

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ABSTRACT

We show that measurements of deep inelastic bremsstrahlung

$$e^{\pm} + p \to e^{\pm} + \gamma + \text{anything}$$

in the appropriate scaling region will provide a definitive test for fractionally charged constituents in the proton, provided the parton model is valid. More precisely, measurement of the difference between the scaling inclusive bremsstrahlung cross sections of the positron and electron will allow the determination of a proton structure function V(x) which, unlike the deep inelastic e-p structure functions, obeys an exact sum rule based on conserved quantum numbers. In particular, we show that

$$\int_0^1 dx\, V(x) = \frac{1}{3} Q + \frac{2}{9} B$$

(= 5/9 for a proton target) in the quark model, whereas

$$\int_0^1 dx\, V(x) = Q$$

in the case of integrally-charged constituents. Since the result is independent of the momentum distribution of the partons, the sum rule holds for nuclear targets as well. Since V(x), which involves the cube of the parton charge, is related to odd charge conjugation exchange in the t-channel, Pomeron and other C-even contributions are not present, so that V(x) should have a readily integrable quasi-elastic peak. This, combined with the fact that there exists a simple kinematic region in which the difference is of the same order as the inclusive bremsstrahlung cross sections themselves, and the fact that there is no hadronic decay background, should make this a feasible experiment on proton and nuclear targets.

INTRODUCTION

The observation of scaling in the highly inelastic limit of electron-proton scattering has excited considerable interest in constituent models of hadrons. The existence of charged, structureless "partons" in the nucleon, together with an assumption limiting the partons' momentum distribution, is sufficient to derive scaling.[1] It is also well known that to account for scaling it is not necessary to postulate the full apparatus of a parton model but instead only to abstract from such a theory the singular behavior of current commutators in the vicinity of the light cone.[2]

Since they are more specific, however, parton models make concrete predictions which cannot be obtained from more general light-cone considerations. An example is the prediction of scaling in the process $p + p \rightarrow \mu^+ + \mu^- +$ anything[3] at high energy and large $(\mu^+\mu^-)$ invariant mass. A test of this prediction will be central in establishing the parton model independently of the light-cone approach.[4] More recently the parton model has been found to provide a particularly simple explanation of large angle exclusive scattering.[5] Although the parton model may be only an abstraction of a more complete theory, it is important to obtain and test all of its predictions, particularly in cases where the number of new assumptions is minimal.[6]

If partons are taken seriously it is important to find ways of determining their quantum numbers. Although the electroproduction structure functions $\nu W_2^{eP}(x)$ and $\nu W_2^{eN}(x)$ are sensitive to the squared charges of the partons, it is impossible to extract from them values of the charges without making additional, strong assumptions regarding the distribution of partons within the nucleon.[7] Our object is to describe an experiment which admits a parton model description and which provides a definitive probe of the partons' charges. The experiment involves the process:

$$e^{\pm} + P \rightarrow e^{\pm} + \gamma + \text{anything}$$

in an appropriate "scaling" region. More precisely, measurement of the difference between the scaling inclusive bremsstrahlung cross sections of the positron and electron will allow determination of a structure function dependent upon the charge cubed of the various partons.[8] As we shall see, this provides a definitive test for fractionally charged partons. This process avoids the complications of Pomeron subtractions and hadronic decay backgrounds. The assumption of a particular longitudinal momentum distribution for the partons is not necessary in the derivation of sum rules.

If the parton model is correct and scaling is observed, then the corresponding structure function depends only on the odd charge conjugation piece of the parton distribution functions:

$$V(x) \equiv \sum_a \lambda_a^3 U_a(x)$$

$$= \sum_a \lambda_a^3 U_a^{odd}(x) \qquad U_a^{odd}(x) = \frac{1}{2}\left[U_a(x) - U_{\bar{a}}(x)\right]$$

where $U_a(x)$ is the probability to find a parton of type "a" with charge λ_a and fraction x of the proton's momentum in an infinite momentum reference frame. Unlike $\nu W_2^{eP}(x)$, which obtains contributions from even charge conjugation (e.g., Pomeron) t-channel exchange terms, the new structure function should show a quasi-elastic peak (vanish as $x \rightarrow 0$); sum rules involving the integral of V(x) can be expected to converge in a finite experimentally accessible region. Moreover, integrals over V(x) are determined by the normalization of various odd charge conjugation form factors (e.g., charge, baryon number, hypercharge) and thus provide a definitive test for fractional charge. We also note that since the $U_a^{odd}(x)$ are related in parton models to the structure functions for highly inelastic neutrino scattering, V(x) should be completely determined by the results of neutrino experiments.

THE BREMSSTRAHLUNG CROSS SECTION

The diagrams relevant to the inclusive bremsstrahlung process $e^{\pm} + P \to e^{\pm} + \gamma +$ anything are shown in Fig. 1. In general there are contributions from both the standard Bethe-Heitler bremsstrahlung amplitude and the virtual inelastic Compton amplitude. The difference of the inclusive cross sections:

$$d\sigma(e^+ + P \to e^+ + \gamma + X) - d\sigma(e^- + P \to e^- + \gamma + X)$$

is due, in order α^3, to the interference of these two amplitudes (see Fig. 2), which is a particular discontinuity of the 3-photon "double" Compton amplitude[9]:

$$V_{\mu\nu\lambda} = \frac{4\pi^2 E_P}{M} \int d^4x\, d^4y\, e^{iq\cdot y + ik\cdot x} \langle P | J_\nu(y)\, T^*(J_\lambda(0) J_\mu(x)) | P\rangle \tag{1}$$

We shall work in the Bjorken kinematic region[10]:

$$\begin{aligned} 2P\cdot q = 2P\cdot(\tilde{q} - k) &\gg M^2 \\ Q^2 \equiv -q^2 = -(\tilde{q} - k)^2 &\gg M^2 \end{aligned} \tag{2a}$$

with $x \equiv Q^2/2P\cdot q$ fixed. In addition we require that

$$\begin{aligned} \tilde{Q}^2 \equiv -\tilde{q}^2 &\gg M^2 \\ 2P\cdot\tilde{q} &\gg M^2 \\ Q^2 - \tilde{Q}^2 = 2k\cdot q &\gg M^2 \quad . \end{aligned} \tag{2b}$$

In the parton model the leading contribution to $V_{\mu\nu\lambda}$ in this kinematic region arises when all three photons scatter on an individual parton (see Fig. 3) and is given by kinematical factors multiplying the scale invariant function V(x).

This result is derived from the following considerations:

(a) As long as both space like photons, q and $\tilde{q}$, are massive (i.e., have large transverse momenta in an infinite momentum frame) and are such that $Q^2 - \tilde{Q}^2 = 2k \cdot q >> M^2$ (which constrains k also to have large transverse momentum in an infinite momentum frame), then all diagrams in which photons interact with more than one parton line are strongly suppressed. [This assumption is, in general, not satisfied for inelastic Compton processes. In the case of small transverse momentum transfer, $P_T^2 \equiv \frac{ut}{s}$, where

$$t \equiv (\tilde{q} - k)^2, \qquad s \equiv (P + \tilde{q})^2, \qquad u \equiv (\bar{P} - k)^2$$

multiple parton processes can be important even in the scaling region.[11] This has been shown explicitly for the case of[12]

$$\gamma + P \to "\gamma"(Q^2) + \text{anything} \qquad Q^2 >> M^2$$

On the other hand, for large P_T^2 the elementary parton process calculated by Bjorken and Paschos[7] can be shown to dominate.[13]]

Since the interference contribution requires that both the Bethe-Heitler and Compton amplitudes have the same final state, we see that our kinematic restriction requires large transverse momentum in the hadronic wavefunction unless q, k and $\tilde{q}$ all interact with the same parton, as in Fig. 3.[14] Of course, if the photon were taken to be in the soft, infrared region ($k \cdot P \ll M^2$) then it can bremsstrahlung off of any of the constituent partons. This generates the usual target bremsstrahlung term in the soft photon radiative correction formulae.

(b) As in the usual application of the parton model, the requirements of large q^2, $\tilde{q}^2$, $P \cdot q$ and $P \cdot \tilde{q}$ are assumed to justify the neglect of interparton interactions during the time period of the photon processes (the impulse approximation) and final state interactions (incoherence approximation).

Thus the standard assumptions of parton models imply that the difference of positron and electron inclusive bremsstrahlung cross sections scales and is weighted by the cube of the partons' charges. Denoting, as usual, the fraction of the proton's momentum in an infinite momentum frame carried by parton i as η_i we find that $\left(|P\rangle = \sum_n \mathscr{A}_n \; |n\rangle\right)$:

$$V_{\mu\nu\lambda} = \frac{1}{2P\cdot q}\;\frac{1}{x}\;\sum_{n,i} |\mathscr{A}_n|^2 \langle n|\delta(\eta_i - x)\lambda_i^3|n\rangle \, M^i_{\mu\nu\lambda} \tag{3}$$

with

$$M^i_{\mu\nu\lambda} = \frac{1}{2}\,\mathrm{Tr}\, \not{p}_i\gamma_\nu(\not{p}_i + \not{q})\,[\gamma_\mu(\not{p}_i + \not{q} + \not{k})^{-1}\,\gamma_\lambda + \gamma_\lambda(\not{p}_i - \not{k})^{-1}\,\gamma_\mu] \tag{4}$$

where $x \equiv Q^2/2P\cdot q$. We have written $M^i_{\mu\nu\lambda}$ for the case of spin 1/2 partons; the spin zero case is analogous. From $V_{\mu\nu\lambda}$ we may extract the structure function V(x):

$$\begin{aligned} V(x) &\equiv \sum_{n,i} |\mathscr{A}_n|^2 \langle n|\delta(\eta_i - x)\,\lambda_i^3|n\rangle \\ &\equiv \sum_a U_a(x)\,\lambda_a^3 \quad , \end{aligned} \tag{5}$$

the sum being over parton and anti-parton of different types, a.

The cross section is a function of six independent variables

$$\begin{aligned} P\cdot p &\equiv ME \equiv \frac{Q^2}{2}\alpha \\ P\cdot p' &\equiv ME' \equiv \frac{Q^2}{2}\alpha' \\ P\cdot k &\equiv Mk_0 \equiv \frac{Q^2}{2}\gamma \\ p\cdot k &\equiv \frac{Q^2}{2}\beta \\ p'\cdot k &\equiv \frac{Q^2}{2}\beta' \end{aligned} \tag{6}$$

and $$Q^2 \equiv -(p' - p - k)^2 \;.$$

The difference of cross sections may be written as

$$\frac{d^6\sigma_+}{\frac{d^3p'}{p_0'}\frac{d^3k}{k_0}} - \frac{d^6\sigma_-}{\frac{d^3p'}{p_0'}\frac{d^3k}{k_0}} = \frac{\left(\frac{e^2}{4\pi}\right)^3}{\pi^2 s Q^2} \sum_a |T^a_{int}|^2 \, U_a(x) \, \lambda_a^3 \tag{7}$$

with

$$|T^a_{int}|^2 = \left. \frac{-M^a_{\mu\nu\lambda} L^{\mu\nu\lambda}}{q^2 \tilde{q}^2} \right|_{\eta_a = x} \tag{8}$$

and

$$L_{\mu\nu\lambda} = \frac{1}{2} \mathrm{Tr}\, \not{p}' \gamma_\lambda \not{p} \left[\gamma_\nu (\not{p} - \not{q})^{-1} \gamma_\mu + \gamma_\mu (\not{p} - \not{k})^{-1} \gamma_\nu\right] \tag{9}$$

The expression for $M^a_{\mu\nu\lambda}$ changes depending on whether partons of type a have spin 0 or 1/2. The product $M^a_{\mu\nu\lambda} L^{\mu\nu\lambda}$ was computed using A. C. Hearn's program REDUCE.[15] The complete expressions are given in the Appendix for both spin 0 and spin 1/2 partons.

Here we will concentrate on a particularly simple region, namely:

$$\frac{1}{\beta - \beta'} \gg \alpha, \alpha', \gamma, \beta, \beta' \gg 1 \qquad \text{with} \qquad \alpha - \alpha' - \gamma \equiv \frac{1}{x}$$

of order 1, in which the formulae of the Appendix simplify considerably. To preserve the condition $Q^2 - \tilde{Q}^2 = Q^2(\beta - \beta') \gg M^2$, we are required to take Q^2 quite large. We choose this region for illustrative purposes only — in general Q^2 need not be larger than the onset of scaling in inelastic electroproduction (e.g., 1 GeV^2), and the full formulae of the Appendix must be used. In this region we find:

$$\frac{d^6\sigma_+}{\frac{d^3p'}{p_0'}\frac{d^3k}{k_0}} - \frac{d^6\sigma_-}{\frac{d^3p'}{p_0'}\frac{d^3k}{k_0}} \cong \frac{-8\left(\frac{e^2}{4\pi}\right)^3}{\pi^2\alpha\beta\gamma x Q^6}\Bigg\{[\alpha^2x^2-\alpha\beta x-\alpha x+\beta^2-\beta\alpha'x+x^2\alpha'^2+x\alpha'+1]\sum_{\text{spin }\frac{1}{2}}\lambda_a^3 U_a(x)$$

$$+[\alpha\beta x-2\alpha\alpha' x^2-\alpha x+\alpha'\beta x+\alpha' x+1-x^2\gamma^2]\sum_{\text{spin }0}\lambda_a^3 U_a(x)\Bigg\}\ . \tag{10}$$

Clearly the different dependences of spin 0 and spin-1/2 terms on the invariants allows one in principle to distinguish the parton's spin.

Besides simplifying our formulae this kinematic region satisfies the important experimental requirement that the interference be a substantial fraction of the signal. To estimate the individual electron and positron cross section we have calculated the squared amplitudes for the Bethe-Heitler and inelastic Compton processes off of a single parton. We find (e.g., for spin 1/2 partons) for the squared Compton amplitude:

$$|T_C|^2 \cong \frac{8}{x^2\gamma^2Q^2}\left(\alpha^2x^2-\alpha\beta x-\alpha x+\beta^2-\alpha'\beta x+\alpha'^2x^2+\alpha' x+1\right) \tag{11}$$

and for the squared Bethe-Heitler amplitude:

$$|T_{BH}|^2 \cong \frac{8}{\beta^2Q^2}\left(\alpha^2x^2-\alpha\beta x-\alpha x+\beta^2-\alpha'\beta x+\alpha'^2x^2+\alpha' x+1\right) \tag{12}$$

From these we can construct the interference to signal ratio:

$$\frac{d\sigma_+ - d\sigma_-}{d\sigma_+ + d\sigma_-} \cong -2\,\frac{\sum\lambda_a^3 U_a(x)}{\frac{x\gamma}{\beta}\sum_a \lambda_a^2 U_a(x) + \frac{\beta}{x\gamma}\sum_a \lambda_a^4 U_a(x)} \tag{13}$$

which is clearly of order unity. The choice of $\beta \cong \beta'$ yields this result because with that restriction the denominators which enter the inelastic Compton as well as the Bethe-Heitler amplitudes are approximately equal. It is therefore experimentally quite feasible to measure the quantity $\sum_a U_a(x)\lambda_a^3$ from the e^+-e^- cross section difference.

SUM RULES

To realize the particular utility of the interference measurement, one must recall that the usual sum rules for sums over the squares of the parton's charges involving $F_2(x) = \nu W_2(x)$ depend on a variety of questionable assumptions.[7] Rigorous sum rules must derive from quantum number conservation. Specifically we have

$$
\begin{aligned}
Q &= \int_0^1 dx \sum_a \lambda_a U_a(x) \\
Y &= \int_0^1 dx \sum_a y_a U_a(x) \qquad\qquad (14) \\
B &= \int_0^1 dx \sum_a b_a U_a(x)
\end{aligned}
$$

where Q, Y, and B (λ_a, y_a, and b_a) are the charge, hypercharge, and baryon numbers of the target hadron (parton) of interest. All of these sum rules depend only on the odd charge conjugation part of $U_a(x)$: $U_a^{odd}(x) \equiv \frac{1}{2}(U_a(x) - U_{\bar{a}}(x))$. In general it is possible to reduce λ_a^3 (which is odd under charge conjugation) to a linear combination of λ_a, y_a, and b_a so that the integral

$$\int_0^1 V(x)\, dx = \int_0^1 dx \sum_a \lambda_a^3 U_a(x) \qquad (15)$$

is determined by quantum number conservation.

This is in striking contrast to the sum rules involving the electroproduction structure functions $\nu W_2^{ep}(x)$ and $\nu W_2^{en}(x)$, defined by:

$$\begin{aligned} \nu W_2^{ep}(x) &= x \sum_a \lambda_a^2 \, U_a(x) \\ \nu W_2^{en}(x) &= x \sum_a \lambda_a^2 \, U_{\hat{a}}(x) \end{aligned} \tag{16}$$

where $\hat{a}$ is the isospin reflection of the parton a. νW_2^{ep} and νW_2^{en} depend only on the combination $U_a^{even} = 1/2\ (U_a(x) + U_{\bar{a}}(x))$ and are therefore unrelated to the conserved quantum numbers. The following sum rules are easily constructed:

$$\int_0^1 \frac{dx}{x}\ \nu W_2(x) = \sum_a \lambda_a^2\ N_a \tag{17a}$$

$$\int_0^1 dx\ \nu W_2(x) = \sum_a \lambda_a^2\ \bar{x}_a N_a \tag{17b}$$

where $N_a \equiv \int_0^1 dx\, U_a(x)$ is the mean multiplicity for a parton of type a, and $\bar{x}_a N_a \equiv \int_0^1 x dx\, U_a(x)$ is the momentum fraction for partons of type a. The right-hand side of (17a) is completely unknown without strong assumptions. If it is possible to define a distribution function for the momenta of partons in each constituent state $|n\rangle$ ($|P\rangle \equiv \sum_n \mathcal{A}_n |n\rangle$), and, if one assumes the distribution function to be symmetric in all its variables, then the right-hand side of (17b) reduces to the mean square charge of the partons.[16] The usefulness of (17b) is further diminished in the presence of neutral gluons for which case the mean square charge defined by the sum rule will be anomalously low. Similar remarks apply to sum rules for $\nu W_2^{ep} - \nu W_2^{en}$: They are valid only with specific assumptions about the distribution of partons in the nucleons.

Sum rules for $V(x) \equiv \sum_a \lambda_a^3 U_a(x)$ suffer from none of these difficulties:

(a) In all models with partons of charge 0 or ±1 (e.g., Drell, Levy, Yan; Han, Nambu; σ-model; etc.), $\lambda_a^3 \equiv \lambda_a$ so that

$$\int_0^1 dx\, V(x) = Q = \begin{cases} 1 \text{ for protons} \\ 0 \text{ for neutrons} \end{cases} \tag{18}$$

(b) In the standard quark parton model: $\lambda_a^3 = \frac{1}{3}\lambda_a + \frac{2}{9} b_a$ so that

$$\int_0^1 dx\, V(x) = \frac{1}{3} Q + \frac{2}{9} B = \begin{cases} 5/9 \text{ for protons} \\ 2/9 \text{ for neutrons} \end{cases} \tag{19}$$

The sum rule provides a striking test for fractionally charged partons. Since the sum rule is independent of the parton distribution, similar results hold for nuclear targets as well:

$$\int_0^1 dx\, V(x) = \frac{3Z + 2A}{9} \quad \text{(Quark model)}$$

For nuclei $A=2Z$, the quark model sum rule gives 7/9 of the corresponding result for integrally-charged constituents. Thus tests of the sum rule and the parton model can be performed on nuclear targets with the additional benefit of large cross sections.

Lastly, in various models it is possible to extract most of the functions $U_a(x)$ from deep inelastic neutrino, and electron scattering off of protons and neutrons,[17] and thereby relate V(x) back to these processes. In the quark model, for example, one obtains

$$V(x) = \frac{1}{108} \left\{ \frac{9}{x} \left(F_2^{\nu n}(x) - F_2^{\nu p}(x) \right) - 7 \left(F_3^{\nu n}(x) + F_3^{\nu p}(x) \right) \right\} - \frac{1}{27} \left(U_\lambda(x) - U_{\bar{\lambda}}(x) \right) .$$

If one assumes $U_\lambda(x) = U_{\bar\lambda}(x)$ in nonstrange baryons the last term drops out; if not it may be expressed in terms of $\Delta S=1$ deep inelastic neutrino scattering.[17] Of course the integral $\int_0^1 dx\,(U_\lambda(x) - U_{\bar\lambda}(x)) \equiv S$ vanishes for nonstrange baryons. Similar analyses may be performed in other models.

CONCLUSION

In conclusion, we have shown that the parton model predicts a very specific scaling form for deep inelastic bremsstrahlung. The prediction that the right-hand side of Eq. (A7) depends in the scaling region only on the variable x and not on any of the four other dimensionless ratios of invariants provides a strong test of the validity of the parton model. Second, since the structure function V(x) depends on the cube of the parton charge, it is possible to obtain exact sum rules, Eqs. (18) and (19), which provide a definitive test of whether the constituents of the proton have fractional versus integral charge.

Since V(x) does not receive contributions from diffractive, Pomeron, or other C-even exchange components, it should have a readily-integrable quasi-elastic peak. This, combined with the fact that there exists a simple kinematic region in which the Bethe-Heitler/Compton interference signal is maximal, and with the absence of a hadronic decay background, should make deep inelastic bremsstrahlung a feasible experiment for proton and nuclear targets.

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APPENDIX

In this appendix we give the complete parton model prediction for deep inelastic bremsstrahlung in the scaling region. The inclusive cross section assuming spin 1/2 partons is ($e^2/4\pi = \alpha = 1/137.036$, $Q^2 = -q^2$, $M\nu = P\cdot q$)

$$\frac{d\sigma(e^{\pm}p \to e^{\pm}\gamma X)}{\frac{d^3p'}{p'_0}\,\frac{d^3k}{k_0}} = \frac{s}{2M\pi}\;\frac{d\sigma}{dQ^2 d\nu\,\frac{d^3k}{k_0}}$$

$$= \left(\frac{e^2}{4\pi}\right)^3 \frac{1}{4\pi^2 sQ^2}\left\{\sum_a \lambda_a^2\, U_a(x)\, |T_{BH}|^2 \pm 2\sum_a \lambda_a^3\, U_a(x)\, |T_{int}|^2 + \sum_a \lambda_a^4\, U_a(x)\, |T_C|^2\right\} \qquad \text{(A1)}$$

where

$$|T_{BH}|^2 = \frac{-4}{Q^2}\Big[-2x^2 L_1^2 \alpha\beta'(\alpha-\alpha') - 2x^2 L_1 L_2(\alpha^2\beta - \alpha^2 - \alpha\beta\alpha' + \alpha\alpha'\beta' - \alpha'^2\beta' - \alpha'^2)$$
$$-2x^2 L_2^2 \beta\alpha'(\alpha-\alpha') - x L_1^2 \beta'(\alpha\beta - 2\alpha\beta' - 3\alpha + \alpha'\beta' + \alpha')$$
$$-x L_1 L_2(\alpha\beta^2 - 3\alpha\beta\beta' - 3\alpha\beta + 3\alpha\beta' + 2\alpha + 3\beta\alpha'\beta' + 3\beta\alpha' - \alpha'\beta'^2 - 3\alpha'\beta' - 2\alpha')$$
$$+x L_2^2 \beta(\alpha\beta - \alpha - 2\beta\alpha' + \alpha'\beta' + 3\alpha') - L_1^2\beta'\,(\beta'+1)$$
$$+L_1 L_2(\beta^2 - 2\beta\beta' - 3\beta + \beta'^2 + 3\beta' + 2) - L_2^2\beta(\beta-1)\Big] \qquad \text{(A2)}$$

$$
\begin{aligned}
|T\ \ |^2 = \frac{-4Q^2}{\tilde{Q}^4} \Big[& -x^2 D_1^2(\alpha^2\beta' + \alpha\beta\alpha' - \alpha\alpha'\beta' - \beta\alpha'^2) \\
& -2x^2 D_1 D_2(\alpha^2\beta - \alpha^2 + \alpha\beta\alpha' - \alpha\alpha'\beta' - \alpha'^2\beta' - \alpha'^2) \\
& -x^2 D_2^2(\alpha^2\beta' + \alpha\beta\alpha' - \alpha\alpha'\beta' - \beta\alpha'^2) \\
& -x D_1^2(\alpha\beta\beta' - \alpha\beta'^2 - \alpha\beta' + \beta^2\alpha' - \beta\alpha'\beta' - \beta\alpha') \\
& +x D_1 D_2(\alpha\beta^2 + \alpha\beta\beta' + \alpha\beta - \alpha\beta' - 2\alpha - \beta\alpha'\beta' - \beta\alpha' - \alpha'\beta'^2 + \alpha'\beta' + 2\alpha') \\
& +x D_2^2(\alpha\beta^2 - \alpha\beta + \alpha\beta'^2 + 2\alpha\beta' - \beta^2\alpha' + 2\beta\alpha' - \alpha'\beta'^2 - \alpha'\beta') \\
& +D_1 D_2(\beta^2 - 2\beta\beta' - 3\beta + \beta'^2 + 3\beta' + 2) - D_2^2(\beta^2 - \beta + \beta'^2 + \beta') \Big] \qquad \text{(A3)}
\end{aligned}
$$

$$
\begin{aligned}
|T_{int}|^2 = \frac{-2}{\tilde{Q}^2} \Big[& 4x^3 P_{22}\,\alpha^2\alpha' + 4x^3 P_{21}\,\alpha^2\alpha' + 4x^3 P_{12}\,\alpha\alpha'^2 + 4x^3 P_{11}\,\alpha\alpha'^2 \\
& -x^2 P_{22}(\alpha^2\beta + 2\alpha^2\beta' - \alpha^2 + \alpha\beta\alpha' + 3\alpha\alpha'\beta' + 4\alpha\alpha' + \alpha'^2\beta' + \alpha'^2) \\
& -x^2 P_{21}(3\alpha^2\beta - 2\alpha^2\beta' - 3\alpha^2 - \alpha\beta\alpha' + 5\alpha\alpha'\beta' + 4\alpha\alpha' - \alpha'^2\beta' - \alpha'^2) \\
& -x^2 P_{12}(\alpha^2\beta - \alpha^2 + 3\alpha\beta\alpha' + \alpha\alpha'\beta' - 4\alpha\alpha' + 2\beta\alpha'^2 + \alpha'^2\beta' + \alpha'^2) \\
& +x^2 P_{11}(\alpha^2\beta - \alpha^2 - 5\alpha\beta\alpha' + \alpha\alpha'\beta' + 4\alpha\alpha' + 2\beta\alpha'^2 - 3\alpha'^2\beta' - 3\alpha'^2) \\
& +x P_{22}(\alpha\beta^2 - \alpha\beta + 2\alpha\beta'^2 + 3\alpha\beta' + \beta\alpha'\beta' + 2\beta\alpha') \\
& +x P_{21}(3\alpha\beta\beta' + 2\alpha\beta - 2\alpha\beta'^2 - 4\alpha\beta' - 2\alpha - 2\beta\alpha'\beta' - \beta\alpha' + \alpha'\beta'^2 + 3\alpha'\beta' + 2\alpha') \\
& +x P_{12}(\alpha\beta\beta' - 2\alpha\beta' + 2\beta^2\alpha' - 3\beta\alpha' + \alpha'\beta'^2 + \alpha'\beta') \\
& +x P_{11}(\alpha\beta^2 - 2\alpha\beta\beta' - 3\alpha\beta + \alpha\beta' + 2\alpha - 2\beta^2\alpha' + 3\beta\alpha'\beta' + 4\beta\alpha' - 2\alpha'\beta' - 2\alpha') \\
& -P_{22}(\beta^2 - \beta\beta' - 2\beta + 2\beta'^2 + 3\beta' + 1) - P_{21}(\beta\beta' + \beta - \beta'^2 - 2\beta' - 1) \\
& +P_{12}(2\beta^2 - \beta\beta' - 3\beta + \beta'^2 + 2\beta' + 1) - P_{11}(\beta^2 - \beta\beta' - 2\beta + \beta' + 1) \Big] \qquad \text{(A4)}
\end{aligned}
$$

For the case of spin 0 partons

$$
\begin{aligned}
|T_{int}|^2 = \frac{-1}{Q^2} \Big[& 8x^3 P_{22}\alpha^2\alpha' + 8x^3 P_{21}\alpha^2\alpha' + 8x^3 P_{12}\alpha\alpha'^2 + 8x^3 P_{11}\alpha\alpha'^2 \\
& -4x^2 P_{22}\alpha(\alpha\beta-\alpha+3\alpha'\beta'+3\alpha') - 4x^2 P_{21}\alpha(\alpha\beta-\alpha+\alpha'\beta'+\alpha') \\
& -4x^2 P_{12}\alpha'(3\alpha\beta-3\alpha+\alpha'\beta'+\alpha') - 4x^2 P_{11}\alpha'(\alpha\beta-\alpha+\alpha'\beta'+\alpha') \\
& +4x L_1(\alpha\beta-\alpha-2\beta\alpha'+\alpha'\beta'+\alpha') - 4x L_2(\alpha\beta-2\alpha\beta'-\alpha+\alpha'\beta'+\alpha') \\
& +x P_{22}(5\alpha\beta\beta'+8\alpha\beta-9\alpha\beta'-8\alpha+5\alpha'\beta'^2+9\alpha'\beta'+4\alpha') \\
& +x P_{21}(\alpha\beta\beta'+4\alpha\beta-5\alpha\beta'-4\alpha+\alpha'\beta'^2+\alpha'\beta') \\
& +x P_{12}(5\alpha\beta^2-9\alpha\beta+4\alpha+5\beta\alpha'\beta'+9\beta\alpha'-8\alpha'\beta'-8\alpha') \\
& +x P_{11}(\alpha\beta^2-\alpha\beta+\beta\alpha'\beta'+5\beta\alpha'-4\alpha'\beta'-4\alpha') \\
& +4 L_1(\beta^2-\beta\beta'-2\beta+\beta'+1) + 4 L_2(\beta\beta'+\beta-\beta'^2-2\beta'-1) \\
& -P_{22}(2\beta\beta'^2+5\beta\beta'+4\beta-5\beta'^2-9\beta'-4) - P_{21}\beta'(\beta-\beta'-1) \\
& -P_{12}(2\beta^2\beta'+5\beta^2-5\beta\beta'-9\beta+4\beta'+4) - P_{11}\beta(\beta-\beta'-1) \Big]
\end{aligned} \tag{A5}
$$

The quantities $\alpha, \beta, \alpha', \beta'$ and kinematics are defined in Eq. (6). The propagators are (mass terms are neglected)

$$L_1^{-1} = \frac{(p-k)^2}{Q^2} = -\beta$$

$$L_2^{-1} = \frac{(p'+k)^2}{Q^2} = \beta'$$

$$D_1^{-1} = \frac{(xP+q+k)^2}{Q^2} = x(\alpha-\alpha') - 1 + \beta-\beta'$$

$$D_2^{-1} = \frac{(xP-k)^2}{Q^2} = -x\gamma \tag{A6}$$

and $P_{ij} = L_i \cdot D_j$, i, j=1, 2. Also note the relation $x = (\alpha-\alpha'-\gamma)^{-1}$.

The odd conjugation structure function $V(x) = \sum_a \lambda_a^3 U_a(x)$ is thus obtained from experiment by the relation

$$V(x) = \frac{\left\{\frac{d\sigma(e^+p \to e^+\gamma X)}{d^3p'/p_0 \;\; d^3k/k_0} - \frac{d\sigma(e^-p \to e^-\gamma X)}{d^3p'/p_0 \;\; d^3k/k_0}\right\}}{(e^2/4\pi)^3 \; |T_{int}|^2/(sQ^2\pi)} \tag{A7}$$

A severe test of the parton model is obtained from the requirement that the right-hand side of (A7) is in fact a function of x alone. Note that hadronic decay processes, e.g., $ep \to e\pi^0 X \to e\gamma\gamma X$ contributes to the sum but not to the difference of $e^\pm$ cross sections. The nominal order of the total scaling inelastic bremsstrahlung e-p cross section is $[\alpha/\pi]$ times the total scaling inelastic e-p cross section.

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8. We also note that measurements of the Bethe-Heitler/Compton interference contribution from the difference of positron and electron elastic bremsstrahlung determines the real part of the Compton amplitude and a check of the Kramers-Kronig dispersion relation (for photon mass $\tilde{Q}^2 \to 0$, $t = -Q^2 \to 0$). Furthermore, the parton model predicts the existence of

an energy-independent ("seagull") contribution to the real part of the amplitude, which is independent of the photon mass $\tilde{Q}^2$ at fixed t. See Brodsky, Close, and Gunion, Ref. 6.

9. The proton state is normalized to $\langle P | P' \rangle = (2\pi)^3 \delta^3(P-P')$. We use the metric $(a \cdot b = a^0 b^0 - \vec{a} \cdot \vec{b})$ and other conventions of J. D. Bjorken and S. D. Drell, Relativistic Quantum Mechanics (McGraw Hill, New York, 1964).

10. The early scaling observed in inelastic e-p scattering indicates that $Q^2 \geq 1\ \mathrm{GeV}^2$ should be sufficient here.

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14. These results may be equally well obtained in the covariant approach of Landshoff, Polkinghorne, and Short (Ref. 1).

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16. This is proved as follows: For any constituent state of the proton, $|n\rangle$, containing N_n particles we define a probability function $f^n(x_1 \ldots x_{N_n})$ (integrated over transverse momenta), normalized by $\int_0^1 dx_1 \int_0^1 dx_2 \ldots \int_0^1 dx_{N_n}\, f^n(x_1 \ldots x_{N_n}) = 1$. Then νW_2 is given by:

$$\nu W_2(x) = \sum_n P_n \sum_i \lambda_i^2 \int dx_1 \ldots dx_{N_n}\, x_i \delta(x_i - x)\, f^n(x_1 \ldots x_{N_n})$$

where P_n is the probability for finding the state $|n\rangle$ in the proton. Integrating from 0 to 1:

$$\int_0^1 \nu W_2(x)\, dx = \sum_n P_n \sum_i \lambda_i^2 \int dx_1 \ldots dx_{N_n}\, x_i f^n(x_1 \ldots x_{N_n})$$

$$= \sum_n P_n \sum_i \lambda_i^2 (\bar{x}_i)_n$$

where $(\bar{x}_i)_n$ is the average momentum of particle i in the state $|n\rangle$. Since for any values of $x_1 \ldots x_{N_n}$ momentum conservation requires $\sum_{i=1}^{N_n} x_i = 1$ we find $\sum_{i=1}^{N_n} (\bar{x}_i)_n = 1$. If $f(x_1 \ldots x_{N_n})$ is symmetric under interchange of any indices i and j, $(\bar{x}_i)_n$ is independent of i. This implies $(\bar{x}_i)_n = (\bar{x})_n = 1/N_n$ which proves the contention that $\int_0^1 \nu W_2(x) dx$ is the mean square charge.

17. C. H. Llewellyn Smith, Ref. 6.

LIST OF FIGURES

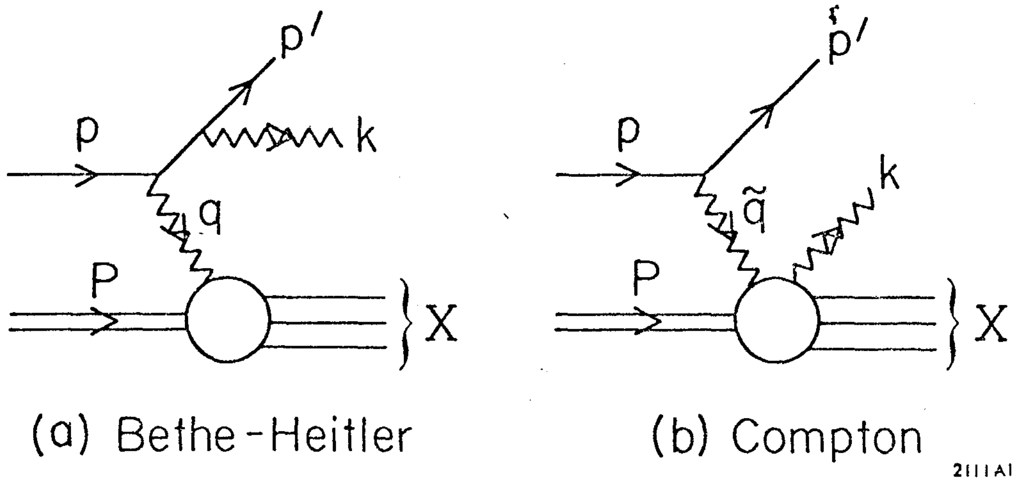

Fig. 1

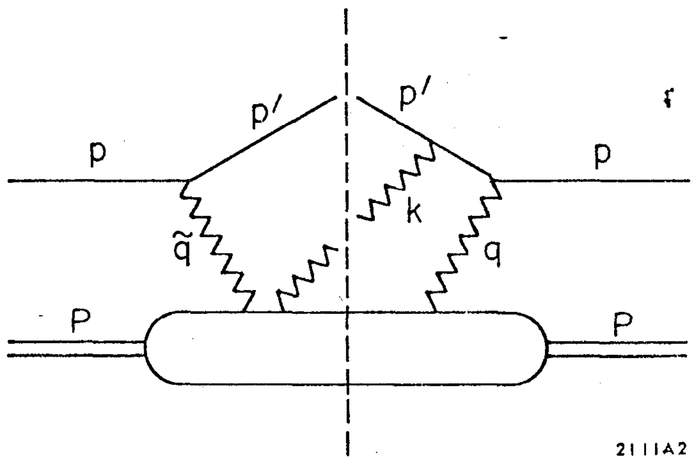

Fig. 2

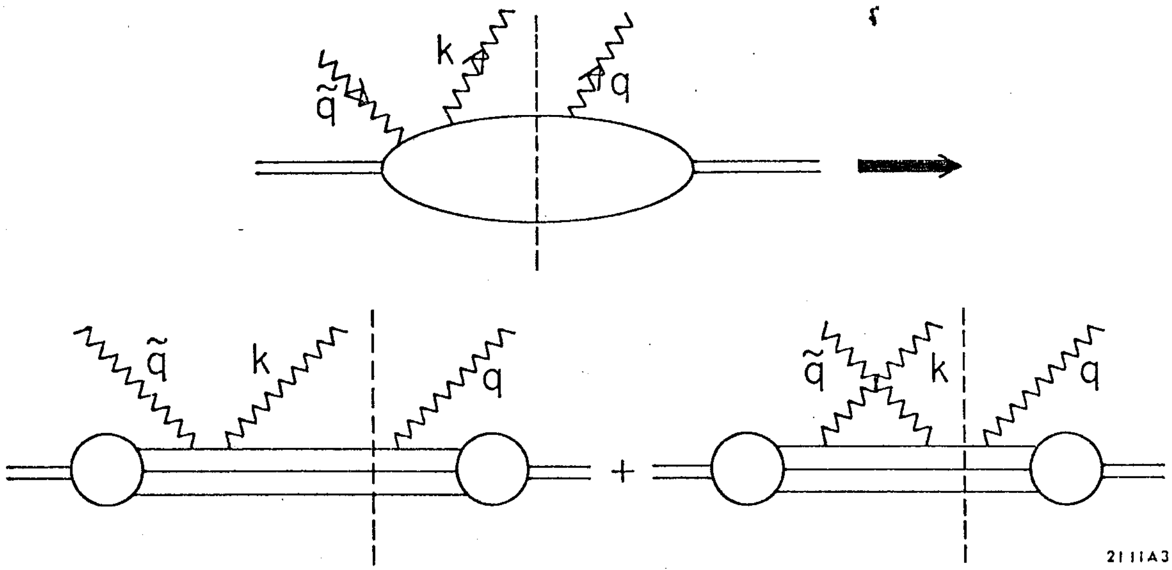

Fig. 3