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CURRENT PROBLEMS AND FUTURE DIRECTIONS IN MESON SPECTROSCOPY: THEORY*

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ABSTRACT

Some current problems and open questions connected with the subject of meson spectroscopy are discussed. Emphasis is put on some of the experiments which might be done to clarify these topics.

INTRODUCTION

In looking back at the proceedings of the previous conferences in this series[1,2] and at previous theoretical reviews of the subject,[3] one cannot but be impressed by the lack of major experimental progress in meson spectroscopy over the last half a dozen years. Once important questions, like the A_2 splitting, that have occupied years of prime experimental effort and a considerable portion of previous conferences on the subject have been barely mentioned here, while our confusion over exactly what states exist at as low a mass as 0.9 to 1.0 GeV/c^2 is still manifest at this conference. Especially when compared to its brother subject of baryon spectroscopy, where phase shift analysis is pushing into the 2 GeV/c^2 mass region, meson spectroscopy is in a very primitive state.

There are of course good reasons why meson spectroscopy fares so badly in comparison to baryon spectroscopy. Many of the previous experiments were bubble chamber exposures with very limited statistics. Also, and very importantly, one did not have the equivalent of the polarization measurements which were so instrumental in restricting the possible baryon phase shift solutions. And, except for the nucleon-antinucleon channel, one was restricted to doing purely production experiments, rather than the formation experiments central to baryon spectroscopy.

At this conference, however, there are several developments which indicate some real hope for the future. First are the very high statistics, systematic spark chamber studies of specific channels, e.g., the study of the $\pi^+\pi^-$ system in $\pi^-p \to \pi^+\pi^-n$ presented to the conference by the CERN-Munich group.[4] Second are the multibody partial wave analyses, as pioneered by the Illinois group,[5] and extended now to the study of the A_3 region.[6] Third is the advent of e^+e^- colliding beams, and the opening up to experimentation of two new formation channels for studying meson resonances: e^+e^- and $\gamma\gamma$.[7] I shall return to the possible importance of this last development again at the end of the talk.

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In the remainder of this talk, with due apologies to the many authors whose work is arbitrarily omitted, I will discuss four topics which are of particular interest to me:

(1) Pomeron Exchange,
(2) The Quark Model and Meson Spectroscopy,
(3) Chiral $SU(2) \times SU(2)$, and
(4) Infinitely Rising Regge Trajectories and Narrow Resonances.

As this is a conference on experimental meson spectroscopy, I hope as I go along to try and emphasize some of the experiments which might be done to clarify these topics.

POMERON EXCHANGE

We define "Pomeron exchange" in an experimental manner, as that dynamical mechanism which produces energy independent cross sections (aside from possible factors of logarithms of the incident energy) in two-body $\rightarrow$ two-body processes. Whether this is due to the actual exchange of a factorizable (Regge?) pole we leave for the moment as an interesting theoretical question which we do not prejudice by our definition. However, whether a Regge pole or not, almost everyone would now agree that Pomeron exchange does have certain other properties: namely, that it carries only the quantum numbers of the vacuum, $I=0$, $C=+$, and therefore, $G=+$. Furthermore, what evidence we have, principally that from rho meson photoproduction, points to Pomeron exchange only having a natural spin-parity ($J^P=0^+, 1^-, 2^+, \ldots$) component.

There are a number of other properties of Pomeron exchange proposed through the years, among them Morrison's rule[8] that $P(-1)^J$ where P is the parity and J is the spin does not change across a hadron-Pomeron-hadron vertex. There have been a few recent experimental results which purport to challenge this "rule". Bubble chamber data for $K^-p \rightarrow K^*(890)^-p$ and $K^-p \rightarrow K^*(1420)^-p$ at 10 and 14.3 GeV/c suggest a flattening in the energy dependence of the cross sections due to natural spin-parity exchange.[9] However a similar effect is not seen in K^+ induced reactions in the same energy range. While perhaps suggestive, I do not find the present results compelling evidence of Pomeron exchange and therefore a violation of Morrison's rule both because of the absence of an effect in K^+p reactions and because the conclusion of a flattening of the energy dependence in the K^-p channel depends crucially on the highest energy experiments. A study of both the K^+p and K^-p reactions with high statistics, especially at high energy, is needed to clarify this situation.

Another possible piece of evidence for Pomeron exchange violating Morrison's rule lies in A_2 production by pions, i.e., $\pi^\pm p \rightarrow A_2^\pm p$. The CERN-IHEP collaboration,[10] from a fit to their missing mass spectra at 25 and 40 GeV/c, quote an energy dependence of $p_{lab}^{-0.7\pm0.3}$. A bubble chamber collaboration[11] with data on the $\pi^-\rho^0$ decay mode of the A_2^- from 5 to 25 GeV/c quote an energy dependence of $p_{lab}^{-0.79\pm0.08}$. However, a collection of bubble chamber data[12] on $\pi^-p \rightarrow A_2^-p$, $A_2^- \rightarrow K^0K^-$ around 5 GeV and a recent Brookhaven experiment[13] at 20 GeV indicate a cross section which falls faster than p_{lab}^{-1}.

Suppose the cross section does have a dependence on energy like p_{lab}^{-1}, does this imply Pomeron exchange? To decide this one might look at a well-defined and well-measured process like $\pi^- p \to \pi^0 n$, where almost no one would claim there is Pomeron exchange. This cross section has now been measured[14] up to 50 GeV/c, with a resulting energy dependence of $p_{lab}^{-1.09\pm0.03}$ above 6 GeV/c. By use of the optical theorem and the measured total cross section difference, $\sigma_T(\pi^- p) - \sigma_T(\pi^+ p)$, one can deduce an energy dependence of $p_{lab}^{-0.62\pm0.08}$ over the same energy region for the forward (t=0) differential cross section. Therefore, while the experimental situation for $\pi^- p \to A_2^- p$ is somewhat confusing, in my opinion there is no definite evidence for Pomeron exchange in A_2 production by pions. The present data are quite consistent with simply f and ρ exchange, with the isoscalar exchange being the dominant[11] one.

The absence of Pomeron exchange in this reaction is evidence against theories where the f (and f') and Pomeron couplings are proportional, such as that proposed by Freund and Rivers[15] and by Lovelace,[16] and investigated in great phenomenological detail by Carlitz, Green and Zee.[17] Indeed, if the ratio of f to Pomeron couplings is universal not only at t=0, as indicated by the analysis of Carlitz, Green, and Zee, but also for $t\neq0$, then $\pi p \to A_2 p$ and $\pi p \to \pi p$ (elastic scattering) should have essentially the same energy dependence.[18] This is already clearly not the case experimentally. Even without this additional assumption of f to Pomeron proportionality for $t\neq0$, one expects to see Pomeron exchange eventually dominate $\pi p \to A_2 p$ in such theories. A single experiment, studying $\pi p \to A_2 p$ in the clean decay channel $A_2 \to K\bar{K}$ (or $\eta\pi$) and which measures the energy dependence and density matrix (needed to separate natural from unnatural spin-parity exchange) over a range of energies to as high an energy as possible would be very welcome to settle this question.

Further evidence against the spirit, if not the substance, of the idea of a proportionality between f and Pomeron couplings, comes from recent work[19] on describing elastic pion-nucleon scattering and rho photoproduction in terms of the dual absorptive model.[20] There it is found that the imaginary part of the helicity nonflip f exchange amplitude is peripheral in impact parameter space, like other Regge exchanges (ω and ρ), with a corresponding zero at $t \simeq -0.2$ GeV/c^2, while the Pomeron exchange amplitude is central in impact parameter space and smoothly falls off with increasing $|t|$. Furthermore, the slope (in t) due to Pomeron exchange in pion-nucleon elastic scattering is not only increasing with s (i.e., shrinkage, as observed for Pomeron exchange at the same energies in K^+p and pp elastic scattering), but is the same as in K^+p elastic scattering at the same energy, in contradiction to the suggestion of Carlitz, Green and Zee.[17,18]

There is one further process which has been observed and may involve Pomeron exchange, thereby violating Morrison's rule. That is $\gamma p \to Bp$, where the B is the I=1, $J^P=1^+$ meson with a mass $\simeq 1235$ MeV/c^2. All the B quantum numbers are such as to allow diffraction production, but the vector (γ) to axial-vector (B) transition clearly violates conservation of $P(-1)^J$. This reaction has been observed in both bubble chamber experiments[21] and counter experiments[22] with compatible results for the cross section, which is observed to be roughly energy independent. Further work still is needed to establish the energy dependence more definitively, but this remains at present as possibly the most serious counter example to Morrison's rule.

Another proposed property of Pomeron exchange is the conservation of s-channel helicity across hadron vertices.[23] From the amplitude analysis[24] of $\pi N \to \pi N$ at 6 GeV/c it is known that s-channel helicity (and not t-channel) conservation at the nucleon vertex holds to the 10 to 20% level in the amplitude out to $-t=0.5$ GeV/c^2 (see Fig. 1). In rho production[25,26,28] there is s-channel helicity conservation at the γ-ρ vertex at the 10 to 20% level in amplitude up to 18 GeV/c. With lower statistical accuracy it is found to hold[28] in $\gamma p \to \omega p$ and $\gamma p \to \phi p$ at 9.3 GeV/c and at the photon vertex in Compton scattering[29] at 3.5 GeV/c. Using the additional hypothesis of two component duality, there is recent work[30] using low energy partial-wave analysis indicating consistency with s-channel helicity conservation for the Pomeron in $K^-p \to K^-p$. Finally, we have seen at this conference evidence[31] for possible s-channel helicity conservation for $\gamma p \to \rho' p$.

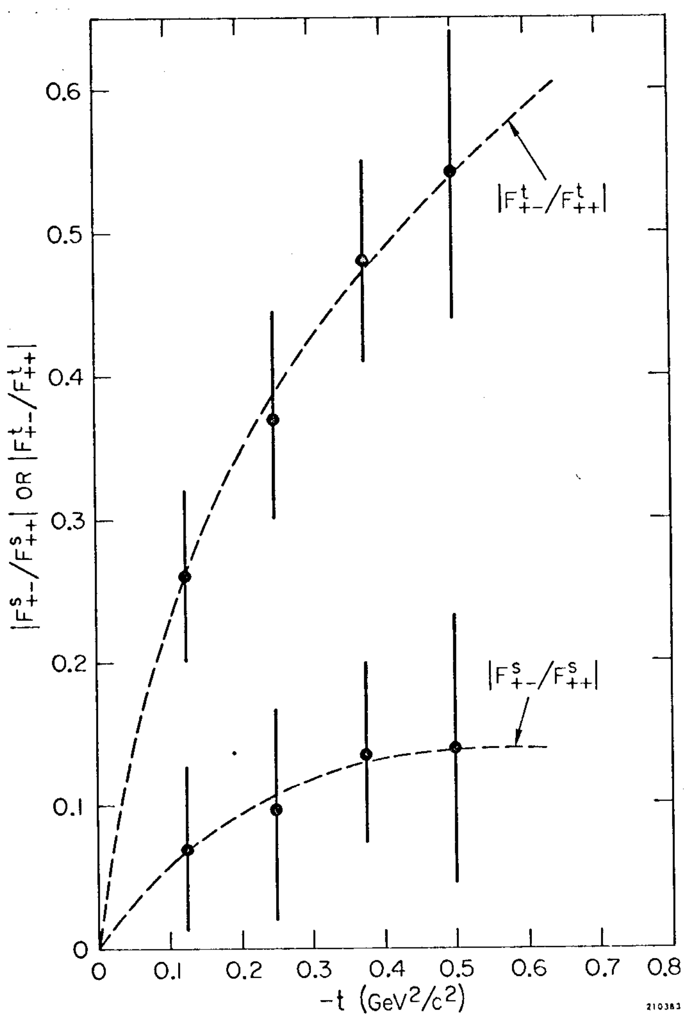

Fig. 1. Ratio of the isospin zero exchange helicity flip to nonflip amplitudes in the s-channel, $|F^s_{+-}/F^s_{++}|$, and t-channel, $|F^t_{+-}/F^t_{++}|$, from the analysis of pion-nucleon scattering at 6 GeV/c in Ref. 24.

However, s-channel helicity conservation is certainly not an exact property of $\pi N \to \pi N$ at 6 GeV/c or $\gamma p \to \rho p$ at 9.3 GeV/c. Is this due to Pomeron exchange or to some other trajectory? There seems to be only one definitive way to answer this question — measure the energy dependence of the helicity non-conserving amplitudes. Very accurate measurements at medium energies together with experiments at Serpukhov and NAL energies would be particularly desirable.

One may note that all the above processes for which s-channel helicity conservation is indicated are elastic or at least have the same spin in the initial and final state. What about inelastic processes like $\pi N \rightarrow (\pi\rho) N$, $KN \rightarrow (\pi K^*)N$, etc. where one sees broad, dominantly s-wave enhancements near threshold in the $\pi\rho$, πK^*, etc. systems?

As we have seen again at this conference, these bumps show no sign of being resonances, and generally appear to be threshold enhancements of the Deck variety.[32] As pointed out some time ago by Stodolsky,[33] and reiterated recently by Donohue[34] and others, the kinematics of the Deck model gives approximate t-channel helicity conservation for such threshold enhancements if they are treated as a single system, in agreement with experimental observation.[35] Until one shows (as does not appear to be the case) that resonances are the dominant part of the $\pi\rho$, πK^*, etc. enhancements, the situation for helicity conservation at inelastic hadron-Pomeron-hadron vertices is completely ambiguous on the basis of such experiments. If there is little or no resonant amplitude in these systems, the present measurements of these processes and their respective density matrices are irrelevant to deciding about s- or t-channel helicity conservation at inelastic hadron-Pomeron-hadron vertices.

At this point it has often become conventional to quote the paper of Chew and Pignotti[36] and conclude that by "duality" the Deck model threshold enhancements are made up of resonances (on the average at least). Not so often noted is that one is using duality (near threshold) for the pion exchange contribution to the process, $\pi +$ Pomeron $\rightarrow \rho + \pi$, in the case of the simplest Deck model for $\pi p \rightarrow (\pi\rho)p$. This is not a very conventional application of duality, first because one has an external Pomeron and second, and probably more importantly, because one is dealing with the predominantly real amplitude of pion exchange, whereas duality is conventionally applicable only to the imaginary part of an amplitude. That Deck enhancements in fact do occur in channels where no poles exist has been shown in a recent theoretical experiment carried out by Frampton and Tornqvist[37] using the dual B_5 model. An analogous real experiment, with a threshold enhancement in the (exotic) $\pi^-\rho^-$ system where there is no known resonance, has been carried out by Cohen et al.[38] in a paper submitted to this conference.

MESON SPECTROSCOPY AND THE QUARK MODEL

For purposes of classification of the meson states, the quark-antiquark bound state model with orbital angular momentum excitation continues to enjoy a general vitality. Unfortunately, the primary problem with the quark model classification scheme for mesons is the lack of observed candidates to fill out the various multiplets. However, the most striking evidence to me for such a scheme for the mesons lies at present not in the presence or absence of some state needed to fill a given slot in the model, but in the continued absence of any exotic states (exotic being defined as not a state of the quark model), while there is now good evidence for exotic exchanges (presumably double exchanges) with mesonic quantum numbers.[39]

For the quark model states with orbital angular momentum $L=0$, the pseudoscalar and vector mesons, there have been no basic changes in the past few years aside from the increasing confidence[40] that the $\eta'(958)$ has $J^{PC}=0^{-+}$. However, an interesting situation relevant to the mixing of the η and η' is developing. Defining $\eta = \eta_8 \cos\theta - \eta_1 \sin\theta$ and $\eta' = \eta_8 \sin\theta + \eta_1 \cos\theta$, in terms of the SU(3) octet and singlet states η_8 and η_1, there are a number[41] of ways of determining the mixing angle θ, all of which involve possibly dubious additional assumptions. Among them are:

(1) The Gell-Mann-Okubo mass formula for the masses squared gives $\theta \simeq \pm 10^{o}$. A linear mass formula on the other hand gives $\pm 24^{o}$. The quark model (with λ quarks heavier than n or p quarks) suggests taking a negative sign for θ so that the "λ quark content" of the η is reduced from what it is for a pure octet member.

(2) The reactions $\pi^{+}p \to \eta\Delta^{++}$ and $\pi^{+}p \to \eta'\Delta^{++}$ plus the quark model, or the assumption of A_2 exchange and the quark model ratio of the $A_2\pi\eta_1$ and $A_2\pi\eta_8$ couplings, permit a determination of θ. A recent determination by a Toronto group[42] yields a value for $\theta \simeq -30^{o}$.

(3) Martin and Michael[43] have carried out an analysis of the reactions $\pi^{-}p \to (\eta, \eta')n$ and $K^{-}p \to (\pi^{o}, \eta, \eta')\Lambda$ assuming A_2 and $K^*(890)$ plus $K^*(1420)$ exchange respectively, together with SU(3) for coupling constants at meson vertices (but no assumption on singlet to octet coupling ratios). They obtain a small mixing angle ($\theta \simeq -10^{o}$) from the combined analysis of relatively low energy data on all these reactions.

(4) The two photon decays of the π^{o}, η and η' are related through SU(3) by[44]

$$\frac{\Gamma(\eta' \to \gamma\gamma)}{M^3_{\eta'}} \sin^2\theta = \left[\left(\frac{\Gamma(\pi^{o} \to \gamma\gamma)}{3M^3_{\pi}}\right)^{1/2} \pm \cos\theta \left(\frac{\Gamma(\eta \to \gamma\gamma)}{M^3_{\eta}}\right)^{1/2}\right]^2 \qquad (1)$$

where the plus (minus) sign corresponds to positive (negative) values of θ. Inserting the measured widths[45] of the π^{o} and η leads to

$$\begin{aligned} \Gamma(\eta' \to \gamma\gamma) &= 55\,^{+23}_{-19} \text{ keV} \qquad (\theta = -10^{o}) \\ &= 9\,^{+3.9}_{-3.2} \text{ keV} \qquad (\theta = -24^{o}) \end{aligned} \qquad (2)$$

and therefore

$$\begin{aligned} \Gamma(\eta' \to \text{all}) &= 3.1\,^{+1.5}_{-1.3} \text{ MeV} \qquad (\theta = -10^{o}) \\ &= 0.51\,^{+0.25}_{-0.21} \text{ MeV} \qquad (\theta = -24^{o}) \end{aligned} \qquad (3)$$

using the measured branching ratio[45] $\Gamma(\eta' \to \gamma\gamma)/\Gamma(\eta' \to \text{all}) = 0.018 \pm 0.003$. The new limit[46] $\Gamma(\eta' \to \text{all}) < 1.9$ MeV would marginally favor a large mixing angle (and linear mass formula?).

The situation is particularly interesting in light of a recent suggestion of Odorico[47] that the mixing angle should be large and have the magnitude of the canonical "ideal" mixing angle $\theta_c \simeq 35^{o}$ such that $\tan\theta = \pm\tan\theta_c = \pm 1/\sqrt{2}$. With such a mixing angle one dismisses (as Odorico does) the mass formula determination(s) of θ as not relevant because the mass splitting of the pseudoscalar mesons is so large compared to the pion mass that a first order mass formula doesn't apply. In such a case, one cannot decide between a quadratic and linear mass formula from the size of θ, both formulae being inapplicable in principle. The positive sign for θ would lead to the canonical situation with the η being the analogue of the ϕ and purely $\lambda\bar{\lambda}$, something which we seem to want to avoid here.[48] Hence we want to choose the negative sign, have an ideal mixing angle, but the "wrong" sign so that both η and η' contain all three types of quarks and the $\pi A_2\eta'$ and $KK^*(1420)\eta$ couplings are arranged to vanish.[47] Whether this amusing possibility is realized by nature or not, a measurement of $\Gamma(\eta' \to \gamma\gamma)$ by observation of the Primakoff production of the η', and hence the determination of $\Gamma(\eta' \to \text{all})$, is both interesting theoretically and overdue experimentally.

For the case of the L=1 states, the experimental situation has not changed recently. Presumably the I=1 octet members corresponding to the spin triplet states with $J^{PC}=2^{++}$, 1^{++}, and 0^{++} are the $A_2(1310)$, $A_1(1070)$, and $\delta(970)$, respectively, while the singlet state with $J^{PC}=1^{+-}$ is to be identified with the B(1235). While the properties of the A_1 and δ can hardly be said to be well established, a more serious difficulty is the lack of candidates for the I=0 partners of the A_1 and B. Only the D(1285) has been definitely found and could be in the same octet (or mixed nonet) with the A_1. Another $J^{PC}=1^{++}$ state and two $J^{PC}=1^{+-}$ states are still to be found four years after Harari[3] noted our embarrassment at their absence at the Vienna conference. Where are they?

As for the L=2 states, a new situation has developed in the past few months in that it is quite possible that all the I=1 states have been observed. For the triplet $J^{PC}=3^{--}$ state there is the well established g(1660), the Regge recurrance of the ρ, whose ideally mixed nonet partners may also have been seen, as discussed by Samios[49] in his talk at this conference. The $F_1(1540)$ is a good candidate for the $J^{PC}=2^{--}$ triplet state, while the broad state in the 1500-1600 MeV region seen in photoproduction of pion pairs[50,51], in the reaction[52] $e^+e^- \to 2\pi^+2\pi^-$, and in photoproduction[53,54] of four charged pions, $\gamma p \to \rho^0\pi^+\pi^- p$, has the right quantum numbers to be the $J^{PC}=1^{--}$ state. The broadness of this state and the present experimental uncertainties make its exact mass difficult to determine. For our present discussion of the quark model we take advantage of this uncertainty and choose a value (like that quoted by the recent 9.3 GeV photoproduction experiment[54]) of 1500 MeV, keeping in mind that this could easily be 100 MeV too low (or even too high). Finally, the singlet state with $J^{PC}=2^{-+}$ could be the $A_3(1640)$. Note that if the ordering with increasing mass of the (spin) triplet L=2 states is really 1^{--}, 2^{--}, 3^{--}, as suggested here, then the spin orbit splitting has the same sign for the L=2 and L=1 states discussed above.

It is perhaps worth summarizing the L=0, 1, and 2 states with I=1 in terms of a simple mass formula. The naiveté of the usual quark model for spectroscopy is so great that I do not think it merits a sophisticated approach. We adopt the "bonehead" quark model and simply consider a nonrelativistic quark-antiquark system with harmonic forces plus perturbing spin-orbit and spin-spin interactions. We therefore assume a mass formula of the form

$$M^2 = a + bL + c(L)\,\vec{S}\cdot\vec{L} + d(L)\,\vec{S}\cdot\vec{S} \tag{4}$$

where $\vec{L}$ is the orbital angular momentum and $\vec{S}$ the total spin of the $q\bar{q}$ pair. We express all masses in simple multiples of what turn out to be convenient units of $m_\rho^2 - m_\pi^2 \simeq 0.56$ GeV^2 and demand that:

(1) The pion (L=0, S=0) state should have the correct mass. This forces $a = m_\pi^2$.

(2) The B meson (L=1, S=0, J=1) state should have approximately the correct mass. This gives $b = (5/2)(m_\rho^2 - m_\pi^2)$ if we demand a simple multiple of our mass units.

(3) The leading J-plane trajectory should pass through the ρ and have a slope $(\Delta J/\Delta M^2)$ of $1/2\,(m_\rho^2 - m_\pi^2)$.

This restricts Eq. (4) to be of the form

$$\frac{M^2 - m_\pi^2}{(m_\rho^2 - m_\pi^2)} = \frac{5}{2}L + \frac{1}{2}f(L)\vec{S}\cdot\vec{L} + \left[\frac{1}{2} - \frac{1}{4}L\,f(L) - \frac{L}{4}\right]\vec{S}\cdot\vec{S} \ , \tag{5}$$

where f(L) is an arbitrary function except that f(L) should go to zero like 1/L or faster as $L \to \infty$ if one is to avoid having particles above the leading trajectory (which is impossible by definition). While one could obtain better agreement with the mass values of some of the observed states in the discussion to follow by not forcing everything to be simple in units of $m_\rho^2 - m_\pi^2$ (e.g., forcing $b = 5/2\,(m_\rho^2 - m_\pi^2)$), the crudity of the model does not merit these refinements.

The masses of the various states with I=1 from Eq. (5) are then as follows (see Fig. 2): For L=0 we have:

$$\begin{aligned} M^2(0^{-+}) &= m_\pi^2 \\ M^2(1^{--}) &= m_\rho^2 \end{aligned} \tag{6}$$

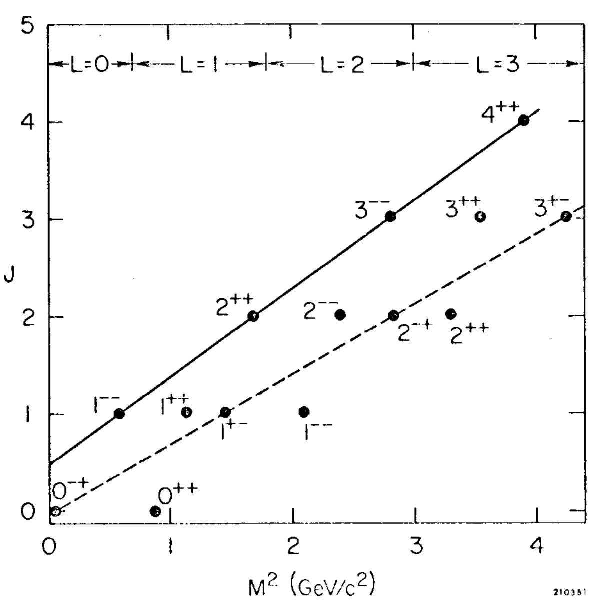

Fig. 2. Regge plot of the isospin one meson states with L=0, 1, 2, and 3 predicted by Eq. (5) with f(L)=1/L.

This was put in by assumption, of course. For L=1 with f(1)=1 we have

$$
\begin{aligned}
M^2(2^{++}) &= M^2(A_2) = (1.30\ \mathrm{GeV})^2 \\
M^2(1^{++}) &= M^2(A_1?) = (1.07\ \mathrm{GeV})^2 \\
M^2(0^{++}) &= M^2(\delta ?) = (0.93\ \mathrm{GeV})^2 \\
M^2(1^{+-}) &= M^2(B) = (1.20\ \mathrm{GeV})^2 \ .
\end{aligned}
\tag{7}
$$

Since the (approximate) B mass and (approximate) slope of the leading trajectory were put in, the A_2 and B masses are not "predictions". Furthermore, f(1) is arbitrary, and we have chosen it to give approximately the correct A_1 mass. Then the δ mass is fixed. It is probably 30 or 40 MeV too low, but within our rough model one shouldn't expect anything better.

For the L=2 states the situation is then quite restricted. Keeping in mind that (asymptotically) we want f(L) to decrease with increasing L like 1/L or faster, we choose f(2) = 1/2 and find

$$
\begin{aligned}
M^2(3^{--}) &= M^2(g) = (1.68\ \mathrm{GeV})^2 \\
M^2(2^{--}) &= M^2(F_1?) = (1.55\ \mathrm{GeV})^2 \\
M^2(1^{--}) &= M^2(\rho'?) = (1.45\ \mathrm{GeV})^2 \\
M^2(2^{-+}) &= M^2(A_3?) = (1.68\ \mathrm{GeV})^2 \ .
\end{aligned}
\tag{8}
$$

All the states have predicted masses fairly close to their observed ones, with the ρ' being perhaps a little low[55] (like the δ, which is also J=L-1) and the A_3 slightly high. More interesting is that the singlet state (the A_3 with $J^{PC}=2^{-+}$) is predicted to be degenerate in mass with the g meson which lies on the leading trajectory. This is to be contrasted with the situation for L=1 where the singlet state (B) was lower in mass then the corresponding state (A_2) on the leading trajectory. If we go to the L=3 states and take f(3) = 1/3, we have

$$
\begin{aligned}
M^2(4^{++}) &= (1.98\ \mathrm{GeV})^2 \\
M^2(3^{++}) &= (1.88\ \mathrm{GeV})^2 \\
M^2(2^{++}) &= (1.82\ \mathrm{GeV})^2 \\
M^2(3^{+-}) &= (2.06\ \mathrm{GeV})^2
\end{aligned}
\tag{9}
$$

Now the singlet state (3^{+-}) lies higher in mass than the relevant state (4^{++}) on the leading trajectory (see Fig. 2). One can phrase the question of the masses of the singlet states relative to those of the states on the leading trajectory in an alternate way which is independent of the quark model: Namely, is the slope of the Regge trajectory of the singlet states smaller than that of the leading trajectory? More generally, do all trajectories have the same slope? It would obviously be interesting to establish the mass of the 4^{++} and 3^{+-} states in this regard. In particular, a bump at or near the mass predicted for the 3^{+-} state by Eq. (9) has been seen[56] in backward $\pi^- p$ collisions. An attempt at determining its quantum numbers would seem worthwhile.

CHIRAL SU(2) × SU(2)

If one forms the vector and axial-vector charges

$$
\begin{aligned}
Q^i(t) &= -i \int V_4^i(\vec{x}, t)\, d^3x \\
Q_5^i(t) &= -i \int A_4^i(\vec{x}, t)\, d^3x
\end{aligned}
\tag{10}
$$

from the weak and electromagnetic vector and axial-vector currents, $V_\mu^i(x, t)$ and $A_\mu^i(x, t)$ respectively, then they obey a set of equal time commutation relations proposed by Gell-Mann[57]:

$$
\begin{aligned}
\left[Q^i, Q^j\right] &= i\,\epsilon_{ijk} Q^k \\
\left[Q_5^i, Q^j\right] &= i\,\epsilon_{ijk} Q_5^k \\
\left[Q_5^i, Q_5^j\right] &= i\,\epsilon_{ijk} Q^k \quad .
\end{aligned}
\tag{11}
$$

Here i, j, k are isospin (SU(2)) indices which run from 1 to 3. Forming $Q^i \pm Q_5^i$ shows that one has left-handed, $(Q^i - Q_5^i)$, and right-handed, $(Q^i + Q_5^i)$, SU(2) algebras which commute with each other, and therefore one has a chiral SU(2)×SU(2) algebra in Eq. (11). Since the vector current is conserved, $dQ^i(t)/dt = \dot{Q}^i = 0$; we define $\dot{Q}_5^i = D^i$, which is not zero since the axial-vector current is not conserved. Since D^i is an isotopic vector

$$
\left[D^i, Q^j\right] = i\,\epsilon_{ijk} D^k \quad , \tag{12}
$$

and we take

$$
\left[D^i, Q_5^j\right] = \delta_{ij}\, S \quad , \tag{13}
$$

where we have made the standard assumption that S is an isotopic scalar, i.e., there are no I=2 sigma terms on the right-hand side of Eq. (13). It then follows that

$$
\begin{aligned}
\left[S, Q^i\right] &= 0 \\
\left[S, Q_5^i\right] &= D^i \quad .
\end{aligned}
\tag{14}
$$

Equations (11) - (14) form an algebraic system of equations. If we sandwich, say, Eq. (11) between hadron states, and insert a complete set of intermediate states, then assuming the convergence of the resulting sum one has a sum rule on the vector or axial-vector transition amplitudes (squared) with a known right-hand side. Furthermore, PCAC relates the axial vector current to the pion field so that the sum rules involving the axial-vector charge Q_5 can be converted into sum rules involving pionic transition amplitudes. For example, the use of PCAC with the last of Eqs. (11) sandwiched between proton states leads directly to the Adler-Weisberger sum rule.[58] Thus one is lead to a set of sum rules[59,61] (one for each helicity λ) on the forward scattering amplitudes for the process π+hadron $\rightarrow$ π+hadron.

In general an infinite number of intermediate states contribute to each of these sum rules. If the infinite sum was truncated and the sum rules were to be saturated by a finite number of states, then this forces the finite set of states to be the basis vectors of a (generally reducible) representation of the $SU(2)\times SU(2)$ algebra of Eq. (11). This leads directly to relations between the pionic transition amplitudes connecting pairs of states from among that set of states which was assumed to be saturating the sum rules. There are also mass formulae since the mass squared values of all states in an irreducible representation of $SU(2)\times SU(2)$ are equal[60,61] (the mass formulae come from Eq. (13) and arise algebraically from the fact that Eqs. (12) - (14) show that D^i and S make up a chiral four-vector).

For example, sandwiching the commutators between ρ states and allowing only π and A_1 intermediate states for helicity λ=0 and only the ω for helicity λ=1 leads to[59]:

$$
\begin{aligned}
&M^2_{A_1} = 2m^2_\rho - m^2_\pi = (1.07\ \mathrm{GeV})^2 \quad ,\\
&\Gamma_{\lambda=0}(A_1 \rightarrow \rho\pi) \simeq 110\ \mathrm{MeV} \quad ,\\
&m_\omega = m_\rho \quad ,\\
&\Gamma_{\lambda=1}(A_1 \rightarrow \rho\pi) = 0,\\
&g_{\omega\rho\pi} = 21\ \mathrm{GeV}^{-1} \quad .
\end{aligned}
\tag{15}
$$

While these particular results, where they are capable of comparison, are perhaps not in such bad agreement with experiment as they stand, the real world is clearly more complicated than complete saturation of the $\pi\rho$ system of sum rules by only π, A_1, and ω states. Such simple schemes can only be a first approximation to reality.

There are two ways of extending this approach to make it correspond more closely with reality. One is to continue to consider a few simple systems of equations (like that for $\pi\rho$ above), but to add additional intermediate states. A recent example of this is the work of Rosner and Colglazier[62] who expand some of the saturation schemes of Ref. 61 in their consideration of a possible D' resonance at 953 MeV. A more ambitious approach is to consider an infinite number of intermediate and external hadron states, treating simultaneously all the possible sum rules which relate their pionic transition amplitudes.[63] Buccella et al.[64] have tried to use an infinite tower of $SU(6)\times O(3)$ (i.e., the quark model with orbital angular momentum excitation) states in a perturbative (in a certain mixing angle) attempt to satisfy the algebra. An interesting result of their work is that the coefficient of the spin orbit $(\vec{S}\cdot\vec{L})$ term in the mass squared formula, Eq. (5), should behave as $(-1)^{L+1}$, i.e., change sign on going through successive L values, in contradiction to what was conjectured in the last section for L=1 and 2. Whether or not the sign of the spin orbit term alternates in sign, experimentally what is needed is something that is at the heart of spectroscopy. Both to test existing saturation schemes and to extend them to more realistic and more ambitious schemes, we need a general knowledge of which states exist and the strength of their pionic transitions to other hadrons.

INFINITELY RISING REGGE TRAJECTORIES AND NARROW RESONANCES

Perhaps one's first reaction to the title of this section should be: are there in fact infinitely rising (and in particular, linear) Regge trajectories for mesons? While the I=1 states found by the missing mass technique with the CERN boson spectrometer[65] could fall on a straight line in a J vs M^2 plot, we in fact have no solid evidence for the spin-parity of states above J=3. Moreover, the next state above the g meson ($J^{PC}=3^{--}$) on the leading trajectory should have G=-1, but the Purdue group[66] sees a narrow state which corresponds fairly well in mass and width with the CERN boson spectrometer[64] object but decays into two pions. Thus this object can not be on the leading trajectory. Similarly the T meson, as observed in $p\bar{p}$ annihilations,[67] decays into $\rho\rho\pi$ and therefore has G=-1, so that it can not be the next state on the leading trajectory — which requires a state with a mass near that of the T but G=+1.

So we do not even seem to have solid candidates at this time for the J=4 and J=5 states on the leading trajectory. However, let's look at some of the possible leading baryon trajectories (Fig. 3) where the experimental

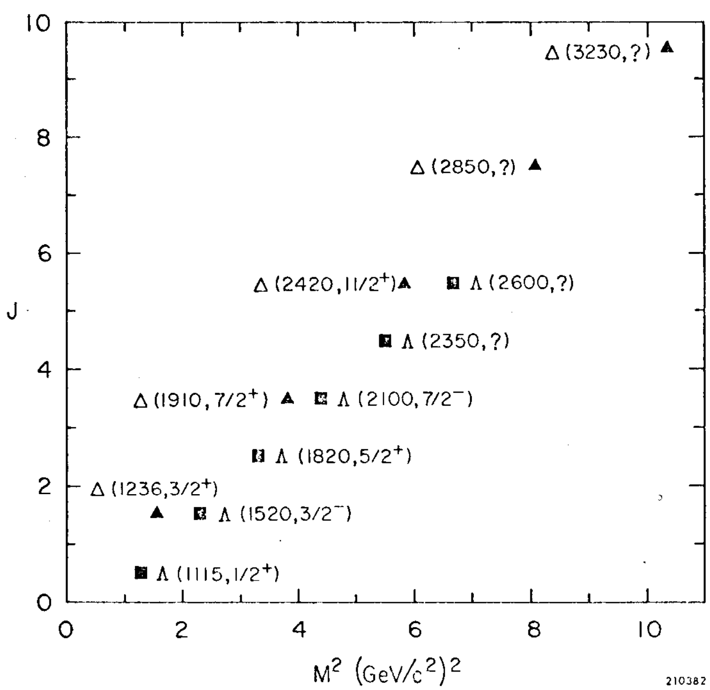

Fig. 3. Known states on some leading baryon Regge trajectories.

situation[45] is much better and spin-parities up to J=11/2 or so are established in some cases. There we find beautiful linear trajectories with masses extending well above $M=2\ GeV/c^2$. Given this situation for the baryons, there is every reason to expect that the leading meson trajectory (at least) continues rising above the g: the 4^+, 5^-, ... states should be there and an effort should be made to establish their existence and properties.

If the leading meson trajectories continue to rise linearly with M^2, should we expect to eventually have narrow states? A look at the high mass baryon states shows rather broad states with the total width slowly increasing with mass, if anything. But for masses greater than about 3 GeV, the evidence against possible narrow baryon states is not very strong, although there is no evidence for them either. In the case of mesons there is experimental disagreement on the width of the states seen in missing mass experiments[65,68] while some bubble chamber evidence from πp collisions[66] and $p\bar{p}$ annihilation[69] does indicate narrow states in the S(1930), U(2380), and X(2620) regions.

Theoretically, it was suggested by Goldberg[70] that the mesons on the leading trajectory should become more and more narrow as J increases. For $J \gtrsim 12$ he suggested that $\Gamma_{total} \sim (J \ell n J)^{-0.28\sqrt{J}}$ assuming linear trajectories and two body decays and using centrifugal barrier arguments. Recently von Hippel and Quigg[71] have completed an extensive analysis of centrifugal barrier effects in hadron decays. They find, in agreement with Goldberg, that high mass meson states on the leading linear (natural spin-parity) trajectory decay primarily by emitting the lowest mass particle possible (the pion) and jumping to lower mass states on the leading (unnatural spin-parity if available, otherwise, natural spin-parity) trajectory. In other words, one has dominantly cascade decays where one jumps down from state to state on the leading trajectories emitting a pion with each jump. Assuming a constant radius for the respective angular momentum barrier factors one again finds a decreasing width with increasing J. Daughter states turn out to be much broader than their parents.

The work of Goldberg and of von Hippel and Quigg is generally confirmed by that of Chan and Tsou,[72] who have calculated the widths of mesons up to J=15 in the n-point factorizable Veneziano model with the lowest mass states taken to be "pions".[73] Here one has the great advantage that one can directly check various approximations and assumptions made previously, such as the neglect of decays of states on the leading trajectory to daughter states, the constancy of the interaction radius, etc. Chan and Tsou find first of all that the total width of leading trajectory states decreases with increasing J, but rather slowly — like $1/\sqrt{J}$, which is a far cry from Goldberg's $(J \ell n J)^{-\sqrt{J}}$. Secondly, the first daughter states have widths ~5 times greater than their parents. Thirdly, the parent states (on the leading trajectory) mostly decay into another parent plus a pion, i.e., one has cascade decays successively from one parent state to the next highest mass parent state available and allowed by energy conservation. As a result of the cascade decay the net number of final "pions" increases very slowly with the mass (or J) of the initial state — perhaps only logarithmically. The daughter states also often decay into a "pion" plus a parent state, which then decays as described above.

Is the slow fall off of Γ_{total} in the work of Chan and Tsou consistent with the analysis of von Hippel and Quigg? The answer appears to be yes if one allows the interaction radius to increase linearly with the mass. In any case, it seems that it is no longer so clear that one must have very narrow states as J becomes large if one can go from the $(J\, \ell nJ)^{-\sqrt{J}}$ behavior of Γ_{total} proposed by Goldberg to the $\sim 1/\sqrt{J}$ behavior of Chan and Tsou using reasonable models. Chan and Tsou have $\Gamma \simeq 24$ MeV for the state with J=10 on the leading trajectory, normalizing to the rho width for J=1. At that point, whether for example the assumption of linear trajectories still holds in the real world is open to question. A further experimental investigation of the existence of narrow, high mass meson states is certainly warranted. If narrow states do exist, it is important to determine their quantum numbers and ascertain that they are not exotic.

In the above discussion we mentioned the likelihood that the daughter states will be much broader than their parents. This makes them difficult to detect by conventional means and therefore makes it difficult to determine which if any of them do exist. Recall in particular that the factorizable Veneziano n-point function demands that there be an enormous number of such states[74] — increasing with the mass M like $e^{5.1\ M/GeV}$ so that there should be literally thousands of such states below M=4 GeV.

At the moment there are rather few examples of such daughter meson states. The possible new vector meson with I=1 in the 1500-1600 MeV/c^2 mass region could well be a candidate for such a state, rather than (or in addition to?) being an L=2 state in the quark model, as discussed previously. The mass is presumably too high to be the long sought after daughter of the f, but one could easily contemplate it being a daughter of the g meson, in which case we might take advantage of the uncertainty in the mass to choose a larger value than previously and rename it $\rho''(1600)$.

Fortunately, we now have or soon will have at our disposal an ideal tool for studying a certain class of daughter states: e^+e^- colliding beams. For, assuming one photon annihilation, the intermediate state formed by e^+e^- annihilation can only have J=1, and by systematically increasing the energy of the beams one moves parallel to the M^2 axis in the usual Regge plot of J versus M^2, picking off the J=1 daughter states in a rather clean fashion. This is to be contrasted to most strong interaction experiments which see best the relatively narrow states along the leading trajectory with slope $\sim 1/GeV^2$ in the Regge plot, but have much difficulty (even with detailed partial wave analysis) in seeing broad, very inelastic daughter states in low partial waves.

Moreover, for very high masses centrifugal barrier effects will severely damp the decays into low mass (2 body) final states coming from states on the leading trajectory. Thus it will get more and more difficult to see even the states on the leading trajectory at high mass in strong interaction formation experiments, especially if they are not very narrow. If low mass exchanges dominate production experiments, as appears to be the case, then the same effect will eventually control the production of high mass states on the leading trajectory(s), making them very difficult to see there as well. But, we recall that Chan and Tsou found, albeit in a model calculation, that there is a fairly high probability for daughter states to decay into a "pion" plus a parent state. If this affinity of daughters for parents holds in the real world, then studying reactions like $e^+e^- \rightarrow \pi + X$, $K+X$, $p+X$ may well be a unique way not only to look for daughters, but to look for very high mass parents as well.

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