SLAC-PUB-1049
(TH) and (EXP)
May 1972

IS THE ADLER SUM RULE FOR INELASTIC LEPTON-HADRON PROCESSES CORRECT??*

J. D. Bjorken

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

and

S. F. Tuan

Department of Physics and Astronomy
University of Hawaii, Honolulu, Hawaii 96822

(Submitted to Comments on Nuclear and Particle Physics)

---

* Supported by the U. S. Atomic Energy Commission.

The beautiful series of SLAC-MIT experiments[1] on inelastic electron scattering from protons and neutrons has generated excitement, confusion and hysteria, especially in the community of high energy particle theorists[2]. One of the reasons for this interest is the regularity seen in the data; there is a simple scaling property inviting a simple explanation. Another reason is the relatively large cross section for production of secondary electrons of high transverse momentum. It is reminiscent of the Rutherford experiments and suggestive of a similar interpretation in terms of incoherent scattering of the electron probe by pointlike constitutents of the nucleon, the seeds in the raspberry jam, named partons by Feynman. Such an interpretation, however, is far from justified at present.

It is the purpose of this comment first to review the original motivation for the scaling property from the sum rule of Adler for neutrino processes[3], based on the local current algebra proposed by Gell-Mann.[4] Then looking at the data, we find it is not at all evident that the sum rule will in fact be true! It will be tested in the next generation of neutrino experiments, especially at NAL. If it fails, the foundations of Gell-Mann's current algebra will not be affected, because the derivation of the Adler sum rule requires a technical assumption of an unsubtracted dispersion relation for a certain amplitude. However, a great deal of the existing theoretical superstructure would collapse, including the parton ideas.

Finally we try to look at the alternatives if the Adler sum rule were to fail. One obvious one is that the sum rule works, but only when very massive hadron states (quarks?) are included in the sum. Another is simply that the derivation of the Adler sum rule is not reliable.

I. The Motivation for Scaling

To understand the issues involved we must simultaneously consider the closely related processes

$$\nu_\mu + \begin{pmatrix} p \\ n \end{pmatrix} \rightarrow \mu^- + \text{hadrons} \tag{1}$$

$$e^- + \begin{pmatrix} p \\ n \end{pmatrix} \rightarrow e^- + \text{hadrons} \tag{2}$$

in the limit of high incident energy and large transferred momentum from lepton to hadron. We consider the case of the final hadron system not observed, other than determination of its mass W. It is also convenient to think about the collision in the overall center-of-mass frame, in the limit of infinite incident lepton energy. Then process (2) is just Coulomb excitation, and the cross section may be written

$$\lim_{E\to\infty} \frac{d\sigma}{dQ^2 dW^2} \sim \left(\frac{4\pi\alpha^2}{Q^4}\right) \frac{F(W^2, Q^2)^{ep}}{[W^2 + Q^2 - m_p^2]}. \tag{3}$$

where $Q^2 = |\underset{\sim}{Q}|^2$ is the square of the transverse momentum transferred from lepton to hadron. The factor in parentheses is just the Rutherford cross section and the denominator $\left[W^2 + Q^2 - m_p^2\right]^{-1}$ is inserted as a matter of convention. It makes the structure function F dimensionless. For the neutrino process, there is a similar formula

$$\lim_{E\to\infty} \frac{d\sigma}{dQ^2 dW^2} = \frac{G^2}{2\pi} \frac{F(W^2, Q^2)^{\nu p}}{[W^2 + Q^2 - m_p^2]} \tag{4}$$

with $G \approx 10^{-5}/m_p^2$ the Fermi constant of weak interactions. The structure function F is proportional to the square of the matrix element of the weak or electromagnetic current operator from the initial nucleon state to the states excited by the scattering process. The equal time commutation relation of the currents for the weak processes

$$< P \mid \left[J(x), J^{\dagger}(0) \right] \mid P > = < P \mid J^3(0) \mid P > \delta^3(\underset{\sim}{x}) \tag{5}$$

taken between nucleon states of high momentum leads then to Adler's sum rule for neutrino processes

$$\int_0^{\infty} \frac{dW^2}{W^2 + Q^2 - m_p^2} \left[F(W^2, Q^2)^{\bar{\nu}p} - F(W^2, Q^2)^{\nu p} \right] = 2(\cos^2\theta_c + 2\sin^2\theta_c) \approx 2 \tag{6}$$

where θ_c is the Cabibbo angle in the weak current. It is derived in a manner similar to that used by Heisenberg and all his followers in deriving such sum rules down through history.

A crucial feature of the sum rule[3] is the independence of the right-hand side on Q^2, suggesting that the functions $F^{\nu p}$ and $F^{\bar{\nu}p}$ are large at large Q^2. The connection between neutrino processes (1) and electroproduction (2) following from the conserved vector current hypothesis allows the same conclusion to be made there as well, a conclusion borne out by the experiments. Furthermore, it is possible to estimate the value of W^2 for which the sum rule converges. This is done by estimating the longitudinal distance over which the process is incoherent. As W^2 increases (at fixed Q^2) this distance increases, and for $W^2 \gtrsim (5\text{-}10)Q^2$ exceeds the thickness of the nucleon.[5] Diffractive processes described by a generalized vector dominance model are then expected to be applicable and the distinction between ν and $\bar{\nu}$ as incident probe should disappear.

If the Adler sum rule converges when $W^2 \sim c Q^2$, with c a pure number $\sim$ 5-10, then, rewriting F in terms of a scale variable $\omega = (W^2 + Q^2 - m^2)/Q^2$

$$\int_1^{\sim(c+1)} \frac{d\omega}{\omega} \left[F(\omega, Q^2)^{\bar{\nu}p} - F^{\nu p}(\omega, Q^2) \right] \sim 2 \; . \qquad Q^2 >> m^2 \tag{7}$$

Viewed in this form, it becomes eminently reasonable that F becomes independent of Q^2 at large Q^2 and only a function of ω. This is the observed scaling property. The conserved vector-current hypothesis then implies similar behavior for the associated electroproduction processes. Many other ways of motivating the scaling behavior are possible, but we believe this choice rests on a minimum number of ad hoc assumptions; namely, two: validity of Adler's sum rule, and validity of the estimate for its convergence.

II. The Shape of the Data

F^{ep} and F^{en} have been measured for a large range of W($\lesssim$ 4 GeV) and Q^2($\lesssim$ 15 GeV^2) and the scaling property works remarkably well. $F^{ep}(\omega)$ is shown in Figure 1, and $F^{ep} - F^{en}$ shown in Figure 2. We see from both these measurements that the convergence estimate, i.e., the value of W^2 (or ω) where $F^{en} \to F^{ep}$ and where F^{ep} attains its asymptotic behavior appears to be in line with the theoretical estimate based on the longitudinal coherence length; namely, $\omega \sim$ 5-10.

This might suggest at first sight that the Adler sum rule is sure to work, but that just is not the case. The numerical magnitude of the electroproduction F^{ep} is uncomfortably small. To see this we use the conserved vector current hypothesis to estimate the vector, $\Delta S = 0$, part of the neutrino structure functions F, which satisfy Eqs. (6)-(7) with 1 instead of 2 on the right-hand side (the other half is contributed by axial vector currents). The conserved vector current relation is

$$\begin{aligned} F^{ep} + F^{en} &= (F^{ep} + F^{en})_{\text{isovector}\ \gamma} + (F^{ep} + F^{en})_{\text{isoscalar}\ \gamma} \\ &\geq (F^{ep} + F^{en})_{\text{isovector}\ \gamma} = \tfrac{1}{2}(F^{\nu p} + F^{\bar{\nu} p})_{\text{vector}\ \Delta S=0} \end{aligned} \tag{8}$$

Thus, using the electroproduction data in Figure 1, the average of $F^{\nu p}_{V,\Delta S=0}$ and $F^{\bar{\nu} p}_{V,\Delta S=0}$ never exceeds $\sim 0.6 - 0.7$ in magnitude, but the difference of areas weighted by a factor ω^{-1} must come out to be unity! It is hard to draw curves of $F^{\nu p}$ and $F^{\bar{\nu} p}$ which do this and which have a rapid rate of convergence. The ratio $F^{\nu p}/F^{\bar{\nu} p}$ must be taken $<< 1$ for $\omega \lesssim 5$ in order to do this. Model calculations which incorporate the sum rule do not have this feature but instead require very slow convergence. In Figure 3 we also plot two examples of such model predictions[6] for these structure functions. Note that even at $\omega \sim 100$ the differences are quite large.

III. What If the Sum Rule Fails?

Suppose that the Adler sum rule actually fails at present energies; i. e., for large Q^2

$$\int \frac{d\omega}{\omega} \left[F^{\bar{\nu} p}(\omega) - F^{\nu p}(\omega) \right] \cong 1, \quad \text{not } 2 \ . \tag{9}$$

One immediate speculation is that an additional contribution from production of a new class of hadrons X associated with a higher mass scale M becomes relevant to the sum rule for $W^2 \gtrsim M^2$, so that Eq. (3) is satisfied. Schematically the relation between F and ω is depicted in Figure 4 for $Q^2 << M^2$, $Q^2 \gtrsim M^2$, and $Q^2 >> M^2$.

For very small Q^2, the Adler sum rule reduces to well-verified sum rules, so that the conjectured new contribution need not be large in the limit of photoproduction or pion scattering. However, it is necessary for the new contribution to be large and contribute the remainder of the sum already at $Q^2 \gtrsim 1\ GeV^2$. This might pose some dynamical problems, about which we have nothing to say. As to what composes the new class of hadron states, we also have little to say. However, some must have isospin in order to contribute to the Adler sum rule. We invite the reader to have fun making his own speculations, such as massive quarks, integrally charged SU(3) triplets, chimerons, and also speculating on the magnitude of the new mass scale.

A less spectacular alternative than new states is that the Adler sum rule is wrong but scaling is correct. The derivations of the sum rule have well-analyzed loopholes, which would allow this to happen. But the attitude of the theoretical community on this point has been well put by Sam Treiman at last year's Cornell Conference[7]: "No test of any idea... is utterly pure, but compared to others on the subject the Adler sum rule stands out remarkably."

It would be ironic indeed were it to happen that scale invariance turns out to be true and the sum rule which motivated it to be false. But such a situation in physics is not without precedent.

IV. Conclusions

The cornerstone of many theoretical edifices used to describe inelastic lepton-hadron scattering lies in the Adler sum rule for neutrino processes. At present there is some empirical reason to be skeptical of its validity; if true the sum rule probably converges quite slowly or is saturated only when production of massive exotic hadrons are included in the sum. Accurate high energy neutrino experiments will be needed to resolve this very fundamental issue.

References

1. See for instance H. Kendall, Proceedings of the 1971 Symposium on Electron and Photon Interactions at High Energies, p. 247.
2. Discussion of these experiments can be found in this journal, see e.g. R. Wilson, 2, 169 (1967), V. Weisskopf, 3, 1 (1969), and S. Drell, 4, 147 (1971).
3. S. Adler, Phys. Rev. 143, B1144 (1966).
4. This subject is reviewed in this journal by F. Low, 1, 164 (1967); 2, 15 (1968).
5. For example, see H. Nieh, Phys. Rev. D1, 3161 (1970).
6. J. Kuti and V. F. Weisskopf, Phys. Rev. D4, 3418 (1971); J. Iizuka, M. Kobayashi, and H. Nitto, Prog. Theor. Phys. 45, 482 (1971).
7. S. B. Treiman, Proceedings of the 1971 International Symposium on Electron and Photon Interactions at High Energies, p. 8.

Figure Captions

Figure 1 The structure function $F(\omega)^{ep}$, conventionally called νW_2, as measured[1] for $1 \lesssim Q^2 \lesssim 15$ GeV^2 and 2 GeV $\lesssim W \lesssim 5$ GeV.

Figure 2 The measured difference $F^{ep} - F^{en}$; the curve is the theoretical model of Kuti and Weisskopf, reference 6.

Figure 3 Two typical models[6] of the neutrino structure functions $F_V^{\bar{\nu}p}$ and $F_V^{\nu p}$, illustrating the slow convergence of their difference.

Figure 4 Conjectured behavior of $F(\omega)$ were a new class of hadrons X of mass $\gtrsim$ M needed to saturate the Adler sum rule.

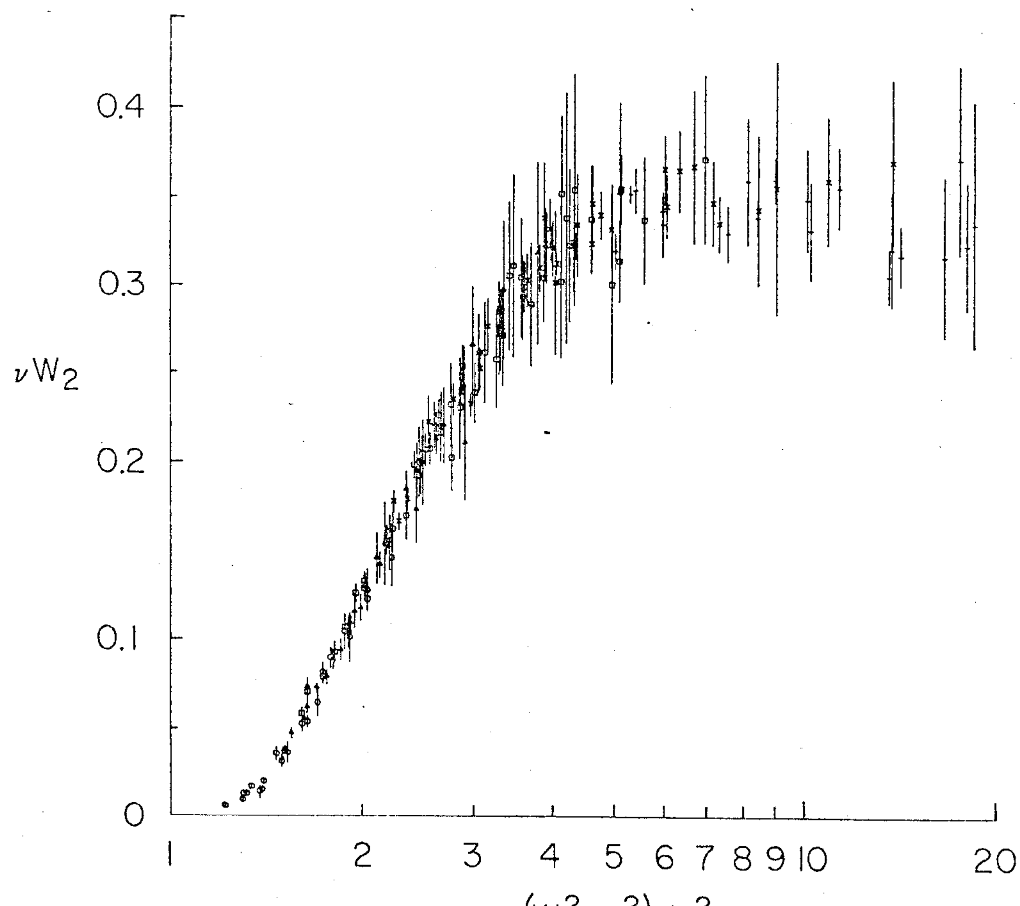


Fig. 1

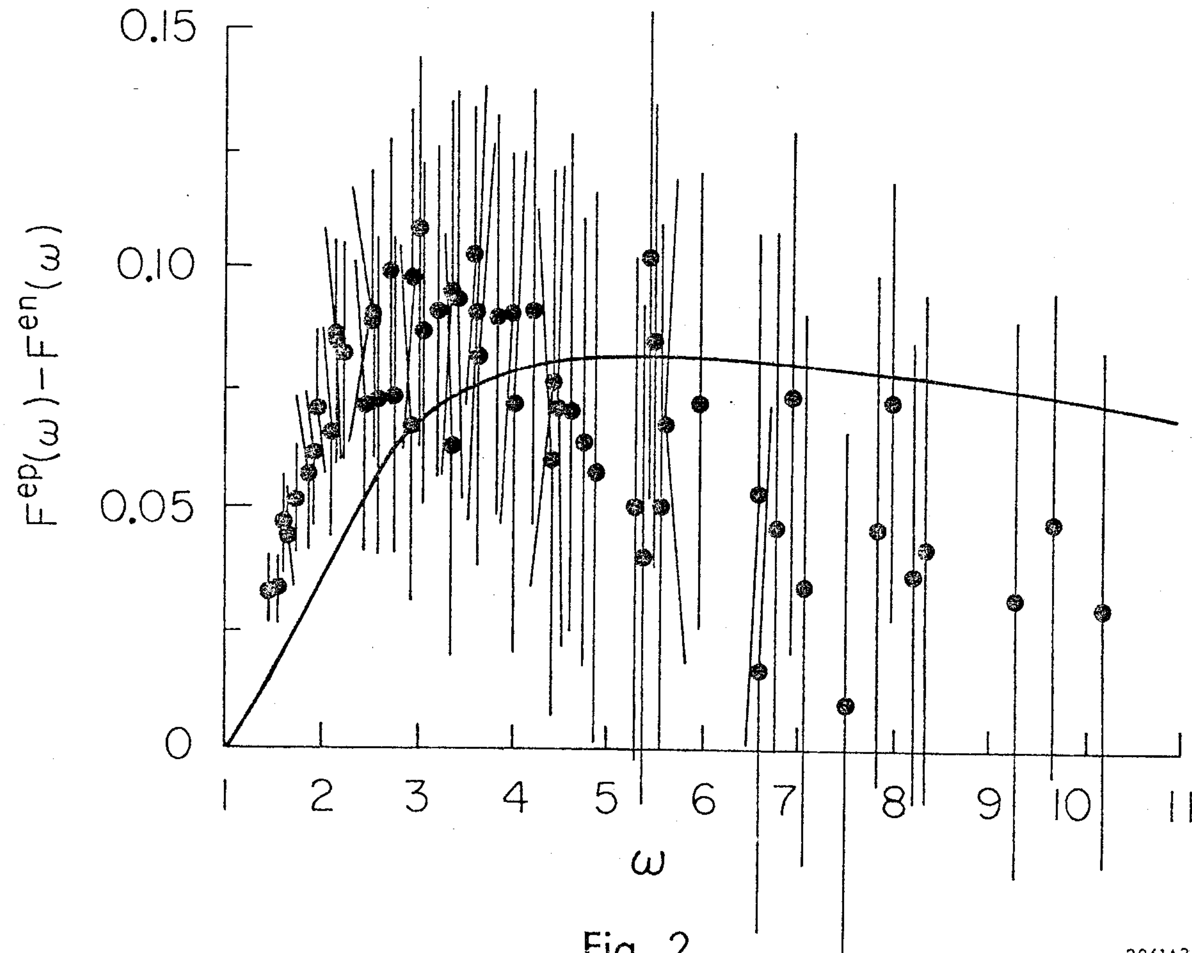
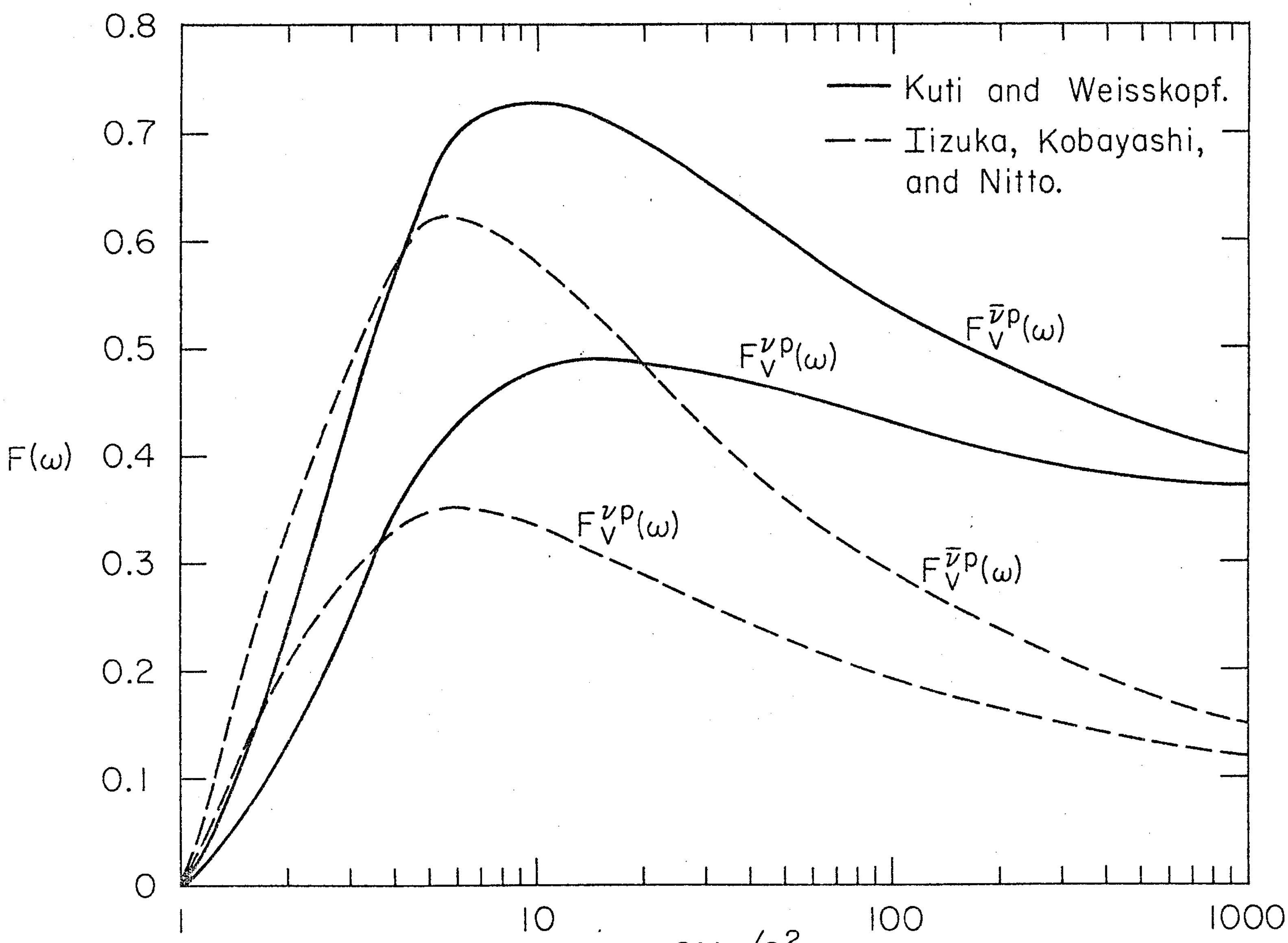


Fig. 2

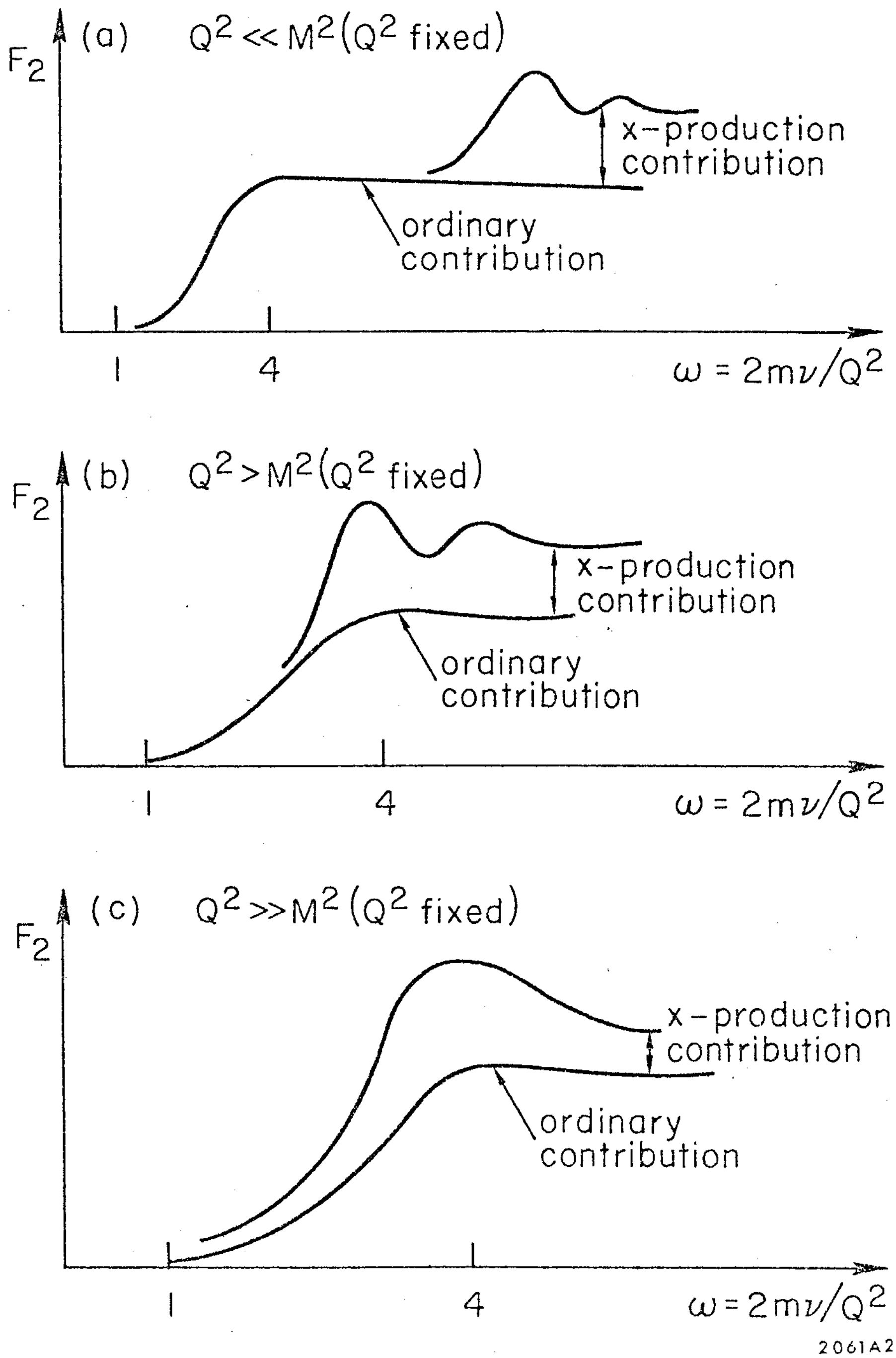
2061A3



F(ω)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
10
100
1000
ω = 2Mν/Q²
Kuti and Weisskopf.
Iizuka, Kobayashi, and Nitto.
$F_V^{\bar{\nu}p}(\omega)$
$F_V^{\nu p}(\omega)$
$F_V^{\nu p}(\omega)$
$F_V^{\bar{\nu}p}(\omega)$

206181

Fig. 3



(a) $Q^2 \ll M^2$ (Q^2 fixed)
F_2
x-production contribution
ordinary contribution
1
4
$\omega = 2m\nu/Q^2$
(b) $Q^2 > M^2$ (Q^2 fixed)
F_2
x-production contribution
ordinary contribution
1
4
$\omega = 2m\nu/Q^2$
(c) $Q^2 \gg M^2$ (Q^2 fixed)
F_2
x-production contribution
ordinary contribution
1
4
$\omega = 2m\nu/Q^2$
2061A2

Fig. 4