e^{+}\text{-} e^{-} \text{ ANNIHILATION TO HADRONS}^* \\

D. M. Kitson \\
Stanford Linear Accelerator Center \\
Stanford University, Stanford, California 94305 \\

Invited Talk \\

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Introduction

In the next few years we shall see come into operation substantial new additions to $e^+e^-$ colliding beam rings, the C.E.A. bypass project (2-4 GeV), the Stanford Spear project (1.5-4 GeV), the new Novosibirsk project (~3 GeV), and the DESY (2-4 GeV) rings.

Our present knowledge of the total cross sections for $e^+e^- \rightarrow$ hadrons is summarized in Fig. 1 where the total cross section is plotted against $2E$, $E$ being the energy of one beam and $2E$ being the center-of-mass (c.m.) energy. The dotted line is the so-called "point Dirac cross section", the theoretical cross section for the production of a pair of point Dirac particles of integer charge. It is presently believed that the asymptotic total cross section will be close to this "point Dirac cross section".

The main features shown in Fig. 1 are the very large cross-sectional peaks due to the production of the $\rho$, $\omega$, and $\phi$ mesons. The areas under the peaks are roughly proportional to $(4\pi/\gamma^2)$, the coupling constant.

It is no accident that these large peaks are the only colliding beam features investigated to date with any precision. The rate $R$ at which events are observed in colliding beams are,

\[ R = fL\sigma \]

where $f$ is the triggering and solid angle efficiency for observing an interaction, $L$ is the "luminosity", $\sigma$ the cross section. $f$ is typically $\sim 1/4$ and $L$ for the Novosibirsk and Orsay rings about $10^{32}\text{cm}^{-2}\text{per hour}$. For Adone, the Italian (2 X 1.2)GeV rings the luminosity peaks at $10^{33}\text{cm}^{-2}\text{per hour}$. Therefore relatively easily measureable cross sections on the low energy machines are above 100 nanobarns, and for Adone above 10 nanobarns (roughly the point cross section). Hopefully the next
generation of machines is going to do much better.

To date our main knowledge consists of the following well measured processes from the lower energy machines.¹

\[ e^+ + e^- \rightarrow \rho \rightarrow 2\pi \] (around \( \rho \) peak)

\[ e^+ + e^- \rightarrow \omega \rightarrow 2\pi \] (around \( \omega \) peak)

\[ e^+ + e^- \rightarrow \phi \rightarrow K^+ + K^- \] (around \( \phi \) peak)

\[ K^0_L + K^0_s \]

From Frascati² we have information on total cross sections and partial information on the \( c^+ + c^- \rightarrow \) hadron channels from 1.4-2.4 GeV c.m.

As the \( \rho, \omega, \phi \) resonances are well understood and have been well covered at various conferences I shall not discuss them further¹,³. I shall confine the discussion to the kinds of information at higher energies that we have and that we can hope for in the next few years.

Let me now discuss, in turn, the areas of interest. To give focus to the discussion I will give some naive theoretical models and expectations to show what physics we may be sensitive to. As quark models have generally been most successful I shall have no hesitation in using them. This of course neither implies that I believe in the actual existence of quarks or "point" constituents but simply that these models appear to be good physics guides.

**Total Cross sections for Hadron Production**

Fig. 2 shows the standard diagram used by everybody to describe the annihilation process. A photon interacts with two "point-Dirac" quarks,
alternatively partons etc., which dress themselves and appear in the
lab. This description is what we should expect for "time-like" photons
knowing what we do about "space-like" photons in deep inelastic scat-
ttering. Asymptotically the total cross section is expected to be the point
Dirac cross section \( \pi c^2 \beta^3 / 3 E^2 \). Possibly we can invoke "duality" and
expect the total cross section in the low energy resonance region to
average out to this value.

Frascati\(^2\), as shown in Fig. 1, and on a finer scale in Fig. 3\(^2\)
approximately confirms these expectations of a "point cross section",
giving total cross sections about 1.5 times the point cross section.

**SU\(_3\) Ratios for Two-body Final States**

The photon has U-spin zero and therefore if SU\(_3\) was not broken
members of the same U-spin should be produced equally abundantly, and
we would predict:

\[
\begin{align*}
\sigma(e^+ e^- \rightarrow \pi^+ \pi^-) &= \sigma(e^+ e^- \rightarrow K^+ K^-) \\
\sigma(e^+ e^- \rightarrow \pi^0 \pi^0) &= \sigma(e^+ e^- \rightarrow \Sigma^+ \Sigma^+) \\
\sigma(e^+ e^- \rightarrow N\bar{N}) &= \sigma(e^+ e^- \rightarrow \Xi_0 \Xi^0) \\
\sigma(e^+ e^- \rightarrow \Delta^- \bar{\Delta}^-) &= \sigma(e^+ e^- \rightarrow \Omega^- \bar{\Omega}^-)
\end{align*}
\]

At present these ratios are unknown.

It is clear from all our experience with proton machines and with
photons at SLAC that SU\(_3\) is strongly broken dynamically and that strange
particles are produced considerably less abundantly than non-strange
particles.

A simple model of SU\(_3\) breaking suggested from these hadron observa-
tions is shown in Fig. 4.
It is assumed that in strong interactions $\Lambda \bar{\Lambda}$ pairs are produced with amplitude $f$ of that for $p\bar{p}$ and $n\bar{n}$ pairs. One would estimate $f$ to be $\sim 0.3$.

In this model assuming $f \sim 0.3$:

$$\frac{\sigma(K^+K^-)}{\sigma(\pi^+\pi^-)} = \frac{(1/3 + 2/3 f)^2}{(1/3)^2} \approx 0.27$$

$$\frac{\sigma(\Sigma^+\Sigma^+)}{\sigma(p\bar{p})} \approx \frac{(1/3 + 2/3 f + 2/3 f^2)^2}{(1/3 + 2/3 + 2/3)^2} = 0.2$$

Doubly-strange production ($2 \Lambda \bar{\Lambda}$ pairs) is down by a factor $\approx 0.2 f^2$.

$$\frac{\sigma(\Xi^0\Xi^0)}{\sigma(N\bar{N})} \approx \frac{(1/3 f + 2/3 f^2 + 1/3 f^2)^2}{(1/3 + 2/3 + 1/3)^2} = 0.4$$

Triply-strange production ($3 \Lambda \bar{\Lambda}$ pairs) is down by a factor $\approx 0.3 f^4$

$\approx 2.5 \times 10^{-3}$.

Certainly such a broken $SU_3$ model is far closer to our expectations than are the unbroken $SU_3$ predictions of approximately equal production of strange and non-strange hadrons.

Of course not only the cross sectional ratios but the actual cross sections and their energy dependence is of great interest. Study of the form factors requires very detailed theoretical models and I will omit most discussion of form factors from this talk. Experimentally the earlier results on $e^+e^+\rightarrow p\bar{p}$ giving a value of $(0.5 \pm 0.2)10^{-33} cm^2$ for the cross section at $2 \times 1.05 GeV$, or a form factor of 0.2 have now considerably better statistics which confirm the earlier value. The channel $e^+e^-\rightarrow \pi^+\pi^-$ in this range has form factors that appears to lie somewhat above those predicted from the tail of the $\rho$ meson and have values of about $0.25-0.5$. Fig. 5 shows the results of last December and improved
results should become available soon.

\textbf{SU}_6 \text{ Quark Model Predictions for Quasi Two-Body Particle Production}

If we assume that spin-spin forces are small then the quark model predicts ratios for the production of particles in the pseudoscalar family (\(\pi\)-octet) and vector family (\(\rho\)-octet).

Fig. 6 shows the assumed model. In this model the photon forms an initial quark pair and then the pair "dresses" itself with an additional pair. On the assumption of small spin-spin forces we can immediately project out the various states \(\pi\), \(\rho\), etc.

The tabulation of ratios relatives to the \(\pi\) is shown.

\[
\begin{array}{ccc}
\pi^+ & \pi^+ & 1 \\
\rho^+ & \rho^+ & 5 \\
\pi^+ & \pi^+ & 2 \\
\eta^0 & \pi^0 & 2 \\
\rho^0 & \pi^0 & 2/9 \\
\rho^0 & \eta^0 & 2
\end{array}
\]

These ratios which do not include production of the higher \(\text{SU}_3\) families such as \(\pi^+ A^+_2\), already show the great complexity of final states to be expected from \(e^+ e^-\) annihilation.

Using the same model for \(\text{SU}_3\) breaking as was given in the previous section we would generate a similar set of ratios for processes such as \(K^\pm K^\mp\), down by a factor of about four from their non-strange \(U\) spin partners.

On the above basis, as other \(\text{SU}_3\) families are undoubtedly formed, we can set up an asymptotic upper limit to \(e^+ e^- \rightarrow 2\pi\). This limit gives \(\sigma(\pi^+ \pi^-)\) to be less than about 1/16 of the total hadronic cross section.
Experimentally the production of pion pairs seems to be of this order at the energies of Adone (~ 2 x 1GeV).

Other Resonance Production Processes

We already know from the Frascati results\textsuperscript{2} at (2-2.5GeV c.m. energy) that the final states are more complex than the above set and we expect $\pi^\pm A_2^\pm$, $\pi A$, etc. pairs to be relatively copiously formed either in the original process or as secondary products of initial $\pi \rho$, $\rho \rho$, etc., states.

Very little is known about these ratios although we have model predictions from various authors\textsuperscript{5} for processes of the type shown in Fig. 7, where a $\rho$ meson is produced as a virtual particle and then decays to a complex final state.

Multiplicities and "transverse" Momentum Distributions

We now know a great deal empirically about high energy hadron-hadron interactions from a "statistical" or "inclusive" point of view. Probably the most striking fact is that only small transverse momenta or $P_\perp$'s occur with any appreciable frequency. However, as has been pointed out\textsuperscript{9}, it is not clear how this restriction to small $P_\perp$'s is to be interpreted for colliding beam processes. The two extreme descriptions are shown diagrammatically in Fig. 8. Description a) leads to the initial formation of two fast recoiling hadronic centers which then interact to give a typical hadronic $P_\parallel$, $P_\perp$ phase distribution. Multiplicities will be the same as for hadron-hadron collisions (with low initial angular momentum corresponding to small impact parameters) at the same c.m. energies. Defining the parallel axis as having the direction of the highest energy particles the transverse $P_\perp$'s will have their usual hadronic distribution
strongly peaked at low values.

For process b) the \(\gamma\)-ray forms an excited "fireball" of hadronic matter (fireball, resonance, etc.) that evaporates off mesons with a limiting temperature. For this process the multiplicity \(N\) will be given by:

\[
N \approx \frac{2E}{\left\langle P_\perp \right\rangle} \approx \frac{(E/2)}{3 kT_0} \approx \frac{(2E)}{480\text{MeV}}
\]

and will grow linearly with c.m. energy.

There will of course be a whole spectrum of intermediate models where more than two initial centers may be formed.

Fig. 9 shows schematically the phase space distributions for the two center, three center, and single center or limiting temperature case. At Frascati the average charged multiplicity is about 3.5, which would be predicted from either of these two extreme models. The Boson group at Frascati is developing a rough breakdown of events into the various channels in the range \(2 \times 600\text{MeV}\) to \(2 \times 1.2\text{GeV}\). A very rough tabulation of their results with no error assignments would indicate the following percentages for the various channels.

<table>
<thead>
<tr>
<th>Channel</th>
<th>% of Cross section</th>
</tr>
</thead>
<tbody>
<tr>
<td>((\pi^+ \pi^-))</td>
<td>10%</td>
</tr>
<tr>
<td>((\pi^+ \pi^- \eta^0))</td>
<td>35% (varies strongly over the energy range)</td>
</tr>
<tr>
<td>((2\pi^+ 2\pi^-))</td>
<td>35%</td>
</tr>
<tr>
<td>((3\pi^+ 3\pi^- \text{ and } 3\pi^+ 3\pi^- + \eta^0))</td>
<td>5%</td>
</tr>
</tbody>
</table>

Experiments over the next few years should be able to eliminate (or prove) the hypothesis that the multiplicity grows linearly with the c.m. energy. To prove unambiguously that the two "jet" model is correct will
be hard even for the next generation of \( e^+e^- \) machines. The \( P_\perp \)'s involved will be constrained kinematically by energy and momentum conservation to be not much larger than the predicted \( P_\perp \) distributions, and therefore the experiments may not be unambiguous. For clear cut experiments we may need colliding beam experiments at above 10GeV.

If we assume mechanism (a) i.e., that the annihilation process starts by creating an original pair of high energy particles that then interact to give the observed hadrons, we can conclude the following.

The original pair of particles will in general give rise to the two leading particles with the highest \( P_\parallel \) and thus these particles can be identified. If we use the same quark model that we used for two-body and quasi two-body production we would view the process as proceeding through the production of a pair of particles which would then "pionize" as shown in Fig. 10. This model predicts the identical ratios of strange to non-strange particles for the leading particles in multi-body processes as we found previously in quasi two-body processes. The central or low \( P_\parallel \) particles would be expected on anybody's model to have the same low strange to non-strange ratios as those observed in high energy hadron-hadron interactions.

In some theories the leading particles are postulated to be bare nucleons and anti-nucleons and for these theories "leading" nucleons and anti-nucleons should predominate over leading pions. If such indeed was the case it would be most exciting, but I think most of us would expect leading \( p\bar{p} \) pairs to be infrequent.

Production and Observation of New Resonances

Resonances can be observed in colliding beams for: a) formation experiments where the total or partial cross section peaks at the c.m.
energy corresponding to the rest mass of the resonance,

\[ e^+e^- \rightarrow X^0 \rightarrow \text{hadrons} \]

This is the process responsible for the very distinctive peaks from \( \rho, \omega, \text{and } \phi \) production. Such a resonance must have \( J^P \) of \( 1^- \) identical to the photon quantum numbers; b) production experiments, where the resonance is found to occur in a detailed analysis of the final hadronic states.

In the Frascati measurements\(^7\) there is no obvious evidence in the total cross section measurements for the formation of new Vector mesons. Fig. 11 shows the results of the Barbarino et al., group at Frascati which indicates the possibility of formation of a resonance around 1600MeV in the "four charged pion only" channel.

The resonance is only observed in the channel for which \( e^+e^- \rightarrow 2\pi^+ + 2\pi^- \) and not in any other channel. As there is no magnetic field to measure momentum the method of analysis of the results which uses only angular information is interesting. From Monte-Carlo calculations 30% of the events of the type \( e^+e^- \rightarrow 2\pi^+ + 2\pi^- + \pi^0 \) or \( \pi^0 \)'s, will be consistent with the hypothesis of being "four pion only" events using the constraints provided by energy and angular momentum conservation. For the other 70% of the \( (2\pi^+ 2\pi^- \pi^0) \) events no solution consistent with the hypothesis of "four pion only" with real positive energies for all particles exists. Thus for a typical case for \( 2 \times E \) of 1.5GeV, 14 events with four charged prongs were observed and 11 events were consistent with being "four-charged pions only". Therefore for the three events which did not give a physical solution, and therefore had more than four pions, we would expect to find one spurious "four-charged pion only" event, and thus the number
of real events would be 11 minus this one spurious event or 10 events (with the appropriate statistical error). This data must be treated with some reserve as the statistics are not good. However more data will soon be available. We should expect that any channel will show a rise at threshold followed by a drop off. The Frascati data shows, if taken literally, a very sharp drop off in cross section and would probably not be consistent with a simple threshold effect. We can use the total cross section data to set an upper limit to the strength of the coupling of this resonance to the photon. If we assume from inspection of Fig.'s 1 and 3 that the resonance has a width of the order of twice the $\rho$ meson width and a peak cross section less than or equal to the order of one times point Dirac, we would find the coupling to the photon to be less than or equal to 10% of the coupling of the $\rho$ meson to the photon.

In "production" experiments it may well turn out that colliding beams constitute a particularly rich source of certain mesons. In this area we have no data and we will hopefully learn more in the next few years.

$\gamma - \gamma$ Processes

Because of time considerations I will omit almost all discussion of the possibilities from the collisions of the virtual $\gamma$-rays from one beam with the virtual $\gamma$-rays from the other beam. The cross sections shown in Fig. 8 will grow to be very large. Kinematically and experimentally these processes are separable. Hadrons with charge conjugation number plus one will be formed in "$\gamma-\gamma$" processes, complementing the hadrons with charge conjugation number minus one formed via $e^+e^-$ annihilation. Disentangling these events is likely to constitute a hard, second round for our new colliding beam accelerators.
Conclusions

The results from Orsay and Novosibirsk have provided us with excellent information on the production and decays of the $\rho$, $\omega$, and $\phi$ mesons.

At higher energies, in the region $2 \times 800$ MeV to $2 \times 1.3$ GeV we now have information on total cross-sections, multiplicities of charged particles, and a rough breakdown into various channels including the $\pi^+\pi^-$ channel, the $p\bar{p}$ channel, and the $\pi^+\pi^-$ channel.

At higher energies which will soon be available, it is clear that colliding beams are going to provide us with a very interesting state of "hadronic matter" similar but not identical to that we see in hadron-hadron interactions. We shall certainly learn much about "hadronic matter", and if nature is kind, (and machine builders provide the promised luminosities) we shall learn much about the various resonant states of hadrons.
REFERENCES

1. c.f. for example, Perez y Jorba for a summary of early ACO work,
Daresbury 1969 Photon Conference, P. 213;
V. Sidorov, Daresbury 1969 Photon Conference, p. 227

2. c.f. for example, Carlo Bernardini. Review of e^+e^- results at the
Cornell 1971 Photon Conference, p. 38

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4. c.f., for instance, S.D. Drell, Rapporteur Report, Elementary Particles, Amsterdam


7. G. Barbarino, F. Ceradini, M. Conversi, M. Grilli, E. Iarocci, M. Nigro,
L. Paoluzi, R. Santonico, P. Spillantini, L. Trasatti, V. Valente,
R. Visentin and G. T. Zorn: "Observations of a Broad Peak in the
Production of Four Charged Pions by e^+e^- Collisions around 1.6 GeV"
and private communication.
FIGURE CAPTIONS

Fig. 1: Cross-section for $e^+e^-$ Annihilation into Hadrons.

Fig. 2: Diagram for the Annihilation of $e^+e^-$ into a $q\bar{q}$ Pair where $q\bar{q}$, can be "quarks", "partons", nucleon anti-nucleons, etc.

Fig. 3: a) Total hadronic cross-section measurements at $2 \times 750$ MeV to $2 \times 1.2$ GeV.
     b) The same energy range for the channel $e^+e^- \rightarrow 2\pi^+ + 2\pi^-$.
     c) The same energy range for the channel $e^+e^- \rightarrow 2\pi^+ + 2\pi^- +$ neutrals.

Fig. 4: Simple model of SU3 breaking. (a) shows $\pi^+\pi^-$ pair production; (b) shows $K^+K^-$ production. It is assumed that a $\Lambda\bar{\Lambda}$ quark pair is produced in strong interaction with an amplitude $f$ less than a $n\bar{n}$ quark pair.

Fig. 5: Cross-sections for the channel $e^+e^- \rightarrow \pi^+ + \pi^-$ versus the extrapolated $\rho$ decay values. The "dot-dash" line is put in as a rough mean of the experimental results.

Fig. 6: Production of pseudo-scalar or vector mesons. The appropriate SU6 and spin projection gives the amplitudes for the various combination $\pi^\pm \pi^\pm$, $\rho^\pm \rho^\pm$, etc.

Fig. 7: Typical diagram for a complex final state formed via an intermediate $\rho$-meson.

Fig. 8: (a) Diagrammatic representation of $e^+e^-$ annihilating to two particles which then interact strongly to give typical "strong interaction" jets.
     (b) Representation where the $\gamma$ gives an excited "fireball" or resonance and this then boils off mesons at the limiting "hagedorn" temperature of 160 MeV.

Fig. 9: Diagrammatic guess as to the population of the $P_1$, $P_{11}$ phase space by particles assuming initial production of a) two "partons", b) three "partons", and c) many partons.
Fig. 10: A photon forms a $q\bar{q}$ pair which then "dresses" itself to produce a multi-hadron state.

Fig. 11: Cross-section for $e^+e^- \rightarrow \pi^+ + \pi^-$.

Fig. 12: Point cross-sections for $e^+e^- \rightarrow$ various channels compared with the "$\gamma-\gamma$" or $ee \rightarrow ee +$ various final states, calculated on the basis of an Equivalent Photon (E.P.) spectrum.
TOTAL COLLIDING BEAM \( e^+e^- \) CROSS SECTIONS TO HADRONS

\[ \sigma_{\text{TOTAL}} \text{ in } \mu \text{Barns} \]

\[ 2E \text{ (energy of beam)} \]

Fig. 1
Fig. 2

CROSS SECTION $\sigma \text{ (nb)}$
$e^+ e^- \rightarrow \pi^+ \pi^-$

Fig. 4
Fig. 5
Fig. 8
TWO CENTERS  THREE CENTERS

MULTI — CENTERS

Fig. 9
$e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$

Fig. 11
Fig. 12

\[ \sigma_T \ (\text{nb} = 10^{-33} \text{ cm}^2) \]

- \( e^+e^- \rightarrow \mu^+\mu^- \)
- \( ee \rightarrow ee\mu^+\mu^- \) (E.P.)
- \( ee \rightarrow ee\sigma \Gamma_\sigma = 400 \text{ MeV} \)
- \( e^+e^- \rightarrow \pi^+\pi^- \) (POINT-LIKE)
- \( ee \rightarrow ee\pi^+\pi^- \) (E.P.)

2E (GeV)