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# BREMSSTRAHLUNG MODEL CALCULATION OF HIGH ENERGY NUCLEON-NUCLEON SCATTERING*

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ABSTRACT

A model calculation of nucleon-nucleon scattering is presented, which results in an amplitude that in the elastic high energy limit realizes the Wu-Yang idea regarding the relation between the elastic form factor of the nucleon and the differential cross section; and which is consistent with the behavior of the differential cross section for the production of nucleon resonances. The calculation makes use of results obtained with the same model in the study of electroproduction.

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This paper presents a calculation of nucleon-nucleon scattering at very high energies using a model which has proved useful[1] to describe deep inelastic electroproduction, scaling and the nucleon's elastic and inelastic form factors.

The basic idea of the model is that at high energies the nucleon trajectories can be parameterized by the four-momentum, which remains constant except for sudden changes due to hard collisions.

The nucleon is coupled to a neutral, massive and soft[2] vector meson field. The probability that this field adjusts itself in such a way that no real mesons are radiated upon the nucleon's collision has been previously calculated[1] and shown to be of the right form to account for the nucleon's form factors. If additionally the nucleon possesses a spectrum of excited states of an appropriate density[3] the scaling behavior of the $\nu W_2$ structure function of the electroproduction experiments can also be accounted for.

These ideas will be applied here to study the very high energy scattering of two nucleons, under the following considerations and assumptions:

(a) An inelastic collision between two nucleons occurs when they come into close range of one another at high energy. The very strong interaction excites the nucleons to resonant states which are the outgoing scattered particles. These later decay, producing two jets of particles. In addition one can have the quasi-elastic production of soft bremsstrahlung particles.[4] In this paper only the inelastic collisions corresponding to the excitation process will be considered.

(b) The probability amplitude for the production of two resonant states of masses $m_1$, $m_2$ and four-momenta $P_{1\mu}$, $P_{2\mu}$ (with $P_1^2 = m_1^2$, $P_2^2 = m_2^2$) in the $s \gg m_1^2$, $m_2^2$ limit is proportional to the product of the density of states corresponding to those masses. The total amplitude for such a process, up to an overall factor, is obtained by multiplying the densities times the matrix element for no real meson bremsstrahlung,[1] given by exp $\{-W/2\}$ with

$$W = \frac{1}{2} \frac{1}{(2\pi)^3} \int d_4k \ \delta(k^2 - \mu^2)\, \tilde{J}^{\mu}(k)\, \tilde{J}_{\mu}^{*}(k) \tag{1}$$

$$\tilde{J}_{\mu}(k) = \int e^{ikx} J_{\mu}(x)\, d_4x \tag{2}$$

$\mu$ is the meson mass and $J_{\mu}(x)$ is the conserved current distribution corresponding to the process.

(c) The spectrum density has the form consistent with the scaling behavior of the deep inelastic electroproduction structure functions.[1]

(d) The decay of the resonant states can be treated independently of the collision process, and proper account of it should not greatly affect the results of (b).

The current distribution corresponding to the scattering $p_1p_2 \rightarrow P_1P_2$ of Figure 1 is:

$$\begin{aligned} J_{\mu}(x) = g \Big\{ & p_{1\mu}(E_{p_1})^{-1} \ \delta(\vec{x} - (E_{p_1})^{-1}\, \vec{p}_1 x^o)\ \theta(-x^o) \\ & + P_{1\mu}(E_{P_1})^{-1} \ \delta(\vec{x} - (E_{P_1})^{-1}\, \vec{P}_1 x^o)\, \theta(x^o) \\ & + p_{2\mu}(E_{p_2})^{-1} \ \delta(\vec{x} - (E_{p_2})^{-1}\, \vec{p}_2 x^o)\, \theta(-x^o) \\ & + P_{2\mu}(E_{P_2})^{-1} \ \delta(\vec{x} - (E_{P_2})^{-1}\, \vec{P}_2 x^o)\, \theta(x^o) \Big\} \end{aligned} \tag{3}$$

where g is the meson-current coupling constant and $p_i^{\mu} = (E_{p_i}, \vec{p}_i)$, $P_i^{\mu} = (E_{P_i}, \vec{P}_i)$; $p_i^2 = m^2$, $P_i^2 = m_i^2$; $i = 1, 2$.

Thus $m_i$ are the invariant masses of the observed jets; m is the rest mass of the nucleon.

W can be written in the form

$$W = W_f + \frac{g^2}{2(2\pi)^3}\int d_4k\ \delta(k^2-\mu^2)\left\{\left[\frac{p_1}{kp_1+i\epsilon}-\frac{P_1}{kP_1+i\epsilon}\right]^2 + \left[\frac{p_2}{kp_2+i\epsilon}-\frac{P_2}{kP_2+i\epsilon}\right]^2 + \left[\frac{p_1}{kp_1+i\epsilon}-\frac{P_2}{kP_2+i\epsilon}\right]^2 + \left[\frac{p_2}{kp_2+i\epsilon}-\frac{P_1}{kP_1+i\epsilon}\right]^2 - \left[\frac{p_1}{kp_1+i\epsilon}-\frac{p_2}{kp_2+i\epsilon}\right]^2 - \left[\frac{P_1}{kP_1+i\epsilon}-\frac{P_2}{kP_2+i\epsilon}\right]^2\right\} \tag{4}$$

with

$$W_f = \frac{g^2}{2(2\pi)^3}\int d_4k\ \delta(k^2-\mu^2)\ (2\pi i)\left\{\frac{m^2}{(kp_1+i\epsilon)}\ \delta(kp_1) + \frac{m^2}{(kp_2+i\epsilon)}\ \delta(kp_2) + \frac{m_1^{\ 2}}{(kP_1+i\epsilon)}\ \delta(kP_1) + \frac{m_2^{\ 2}}{(kP_2+i\epsilon)}\ \delta(kP_2)\right\} \tag{5}$$

In the limit of forward elastic scattering $W \rightarrow W_f$. $W_f$ will be dropped from now on by the referral of all amplitudes to the forward scattering case.[5]

The total amplitude A(s, t, u) will be proportional to:

$$A(s,\ t,\ u) \sim \rho(m_1^{\ 2})\ \rho(m_2^{\ 2}) \exp\{-W/2\} \tag{6}$$

where $\rho(m_i^2)$ is the density of states with mass $m_i$.

W can be easily written in the form

$$W = - \frac{g^2}{4\pi} D \left[ F(\lambda) + F(\tilde{\lambda}) + F(\xi) + F(\tilde{\xi}) - F(\sigma) - F(\tilde{\sigma}) \right] \qquad (7)$$

with

$$F(x) = 1 - \frac{2x+1}{\left[x(x+1)\right]^{1/2}} \ln \left| (x)^{1/2} + (x+1)^{1/2} \right| \qquad (8)$$

$$\lambda = \frac{(m_1 - m)^2 - t}{4 m_1 m} \qquad (9)$$

$$\tilde{\lambda} = \frac{(m_2 - m)^2 - t}{4 m_2 m} \qquad (10)$$

$$\xi = \frac{(m_1 - m)^2 - u}{4 m_1 m} \qquad (11)$$

$$\tilde{\xi} = \frac{(m_2 - m)^2 - u}{4 m_2 m} \qquad (12)$$

$$\sigma = \frac{s - (m_1 + m_2)^2}{4 m_1 m_2} \qquad (13)$$

$$\tilde{\sigma} = \frac{s - (2m)^2}{4 m^2} \qquad (14)$$

$$s = (p_1 + p_2)^2 = (P_1 + P_2)^2 \qquad (15)$$

$$t = (P_1 - p_1)^2 = (P_2 - p_2)^2 \qquad (16)$$

$$u = (P_2 - p_1)^2 = (P_1 - p_2)^2 \tag{17}$$

$$D = \int_{y_0}^{\infty} N_1(y)\,dy = N_0(y_0); \quad y_0 > 0 \tag{18}$$

and where $N_1$, $N_0$ are Bessel functions of the second kind.

The $y_0$ lower limit in the integral of Eq. (18) should rigorously be zero. However it is shifted to a positive number in order to cut off the ultraviolet logarithmic divergence of Eq. (4). This is necessary to be consistent with the assumed soft character of the neutral meson field. $y_0$ will be chosen to be a constant such that $N_0(y_0)$ is positive.[6]

With the definition

$$\gamma \equiv \frac{g^2}{4\pi} \quad \frac{N_0(y_0)}{2} > 0 \tag{19}$$

Eq. (6) can be written as:

$$A(s,t,u) \sim \rho(m_1{}^2)\rho(m_2{}^2) \exp\left\{\gamma\left[F(\lambda) + F(\tilde{\lambda}) + F(\xi) + F(\tilde{\xi}) - F(\sigma) - F(\tilde{\sigma})\right]\right\} \tag{20}$$

Various limits of Eq. (20) will be discussed:

(1) For elastic scattering $m_1 = m_2 = m$. Equation (20) reduces to the result of Fried and Gaisser:[7]

$$A_{elastic} \sim \exp\left\{2\gamma(F(\alpha) + F(\beta) - F(\eta))\right\} \tag{21}$$

with $\alpha = -t/4m^2, \beta = -u/4m^2, \quad \eta = (s - 4m^2)/4m^2$.

For fixed t and $s \rightarrow \infty$ , $F(\beta) - F(\eta) \rightarrow 0$ and

$$A_{elastic} \underset{\substack{s \rightarrow \infty \\ t \text{ fixed}}}{\sim} \exp\left[2\gamma F(\alpha)\right] = G^2(t) \tag{22}$$

Since G(t),the elastic form factor,[7] is $\exp\{\gamma F(\alpha)\}$. Therefore Fried and Gaisser pointed out that the model realizes the Yang-Wu idea[8] relating the elastic differential cross section to the fourth power of the form factor.

When x is large $F(x) \rightarrow \ln(4x)^{-1}$; and the elastic form factor behaves as

$$G(t) \underset{-t \gg m^2}{\sim} \left(\frac{-t}{m^2}\right)^{-\gamma} \tag{23}$$

Experimentally a value of $\gamma = 2$ is consistent with the data.

(2) For the inelastic scattering case and in the kinematical region $s \gg m_1^2, m_2^2$; $m_1 \gg m$, $m_2 \gg m$, one has $\lambda$, $\tilde{\lambda}$, $\xi$ , $\tilde{\xi}$, $\sigma$, $\tilde{\sigma}$ all large and taking the values:

$$4\lambda \rightarrow \frac{m_1}{m} (1 - t/m_1^2) \tag{24}$$

$$4\tilde{\lambda} \rightarrow \frac{m_2}{m} (1 - t/m_2^2) \tag{25}$$

$$4\xi \rightarrow \frac{m_1}{m} (1 - u/m_1^2) \tag{26}$$

$$4\tilde{\xi} \rightarrow \frac{m_2}{m} (1 - u/m_2^2) \tag{27}$$

$$4\sigma \rightarrow \frac{s}{m_1 m_2} \tag{28}$$

$$4\tilde{\sigma} \to \frac{s}{m^2} \tag{29}$$

Hence Eq. (20) takes the form:

$$A(s,t,u) \underset{\substack{s \gg m_1^2,\, m_2^2 \\ m_1,\, m_2 \gg m}}{\sim} \prod_{j=1}^{2} \left[ \left(\frac{m}{m_j}\right)^{\gamma} \rho(m_j^2) \right] \prod_{i=1}^{2} \left[ \frac{\frac{s}{m_i^2}}{\left(1 - \frac{t}{m_i^2}\right)\left(1 - \frac{u}{m_i^2}\right)} \right]^{\gamma} \tag{30}$$

Of course $s + t + u = 2m^2 + m_1^2 + m_2^2$. When $m_1$, $m_2$ are kept fixed and $s$, $-t$, $-u$ are separately large, Eq. (30) becomes:

$$A(s,t,u) \underset{\substack{s \to \infty \\ |-t| \to \infty \\ |-u| \to \infty \\ m_i \text{ fixed}}}{\sim} \left\{ \prod_{j=1}^{2} \left[ (mm_j)^{\gamma} \rho(m_j^2) \right] \right\} \left(\frac{s}{tu}\right)^{2\gamma} \tag{31}$$

Showing the appearance[7,9] of the Krisch variable (ut/s). For $s \gg m_i^2$, $|u| \gg m_i^2$ and $t$, $m_i$ fixed one has

$$A(s,t,u) \underset{\substack{s,\, |u| \gg m_i^2 \gg m^2 \\ t,\, m_i \text{ fixed}}}{\sim} A(t) \sim \prod_{j=1}^{2} \left[ \left(\frac{m}{m_j}\right)^{\gamma} \rho(m_j^2) \right] \cdot \prod_{i=1}^{2} \left(1 - \frac{t}{m_i^2}\right)^{-\gamma} \tag{32}$$

The density $\rho(m_i^2)$ consistent with scaling in electroproduction has been shown to be[1]:

$$\rho(m_i^2) = \frac{1}{m_i^2} \left(\frac{m_i}{m}\right)^{2\gamma} e^{\overline{W}_i} \tag{33}$$

where $\gamma$ is given by Eq. (19), and $\overline{W}_i$ is an expression related to the de-excitation of the resonant state.[10] As such, the $\exp\{\overline{W}_i\}$ will be incorporated to the undetermined proportionality factor in the amplitude according to assumption (d).

Substitution of Eq. (33) into Eq. (30), (32) and use of the experimental value $\gamma = 2$ greatly simplifies the mass dependent coefficient, which reduces to $(m)^{-4}$, a constant.

Therefore Eq. (32) shows that in this case:

$$A(s,t,u) \xrightarrow[\substack{s,\ |u| \gg m_i^2 \gg m^2 \\ t,\ m_i \text{ fixed}}]{} A(t) \sim \left(1 - \frac{t}{m_1^2}\right)^{-2} \left(1 - \frac{t}{m_2^2}\right)^{-2} \qquad (34)$$

(3) When $m_1 \sim m$, $m_2 \sim m$, $s \gg m_i^2$ and $|u|$, $|t|$ are large so that $\lambda$, $\tilde{\lambda}$, $\xi$, $\tilde{\xi}$ are large one has:

$$4\lambda \rightarrow \frac{m_1}{m} \left(- \frac{t}{m_1^2}\right) \qquad (35)$$

$$4\tilde{\lambda} \rightarrow \frac{m_2}{m} \left(- \frac{t}{m_2^2}\right) \qquad (36)$$

$$4\xi \rightarrow \frac{m_1}{m} \left(- \frac{u}{m_1^2}\right) \qquad (37)$$

$$4\tilde{\xi} \rightarrow \frac{m_2}{m} \left(- \frac{u}{m_2^2}\right) \qquad (38)$$

$$4\sigma \rightarrow \frac{s}{m_1 m_2} \qquad (39)$$

$$4\tilde{\sigma} \rightarrow \frac{s}{m^2} \qquad (40)$$

Eq. (20) takes the form:

$$A(s,t,u) \underset{\substack{s \gg m_i^2 \\ m_i \sim m \\ |u|\,,\,|t| \gg mm_i}}{\sim} \prod_{j=1}^{2} \left[ \left(\frac{m}{m_j}\right)^{\gamma} \rho(m_j^2) \right] \prod_{i=1}^{2} \left[ \left( \frac{\frac{s}{m_i^2}}{\frac{t}{m_i^2} \quad \frac{u}{m_i^2}} \right) \right]^{\gamma} \tag{41}$$

Use of Eq. (33) with $\gamma=2$ simplifies the mass dependent coefficient once again; and in the $|t|$, $m_i$ fixed; $s \to \infty$ case one has

$$A(s,t,u) \xrightarrow[\substack{s \to \infty \\ |t| \gg mm_i,\ \text{fixed} \\ m_i\ \text{fixed}}]{} A(t) \sim \left(-\frac{t}{m_1^2}\right)^{-2} \left(-\frac{t}{m_2^2}\right)^{-2} = G\left(\frac{t}{m_1^2}\right) G\left(\frac{t}{m_2^2}\right) \tag{42}$$

consistent with the idea of Elitzur[11] who studied $pp \to pp^*$ assuming the right-most side of Eq. (42).

The model presented here is not expected to provide an accurate description of the collision process when the energy and momentum transfers are small, and this can be seen in the behavior of the elastic form factor $\exp\{\gamma F(\alpha)\}$, $\alpha = -\frac{t}{4m^2}$. When $\alpha$ is not very large one should use $F(\alpha)$ as given by Eq. (8), and although $F(0) = 0$ as it should, the fit to the form factor provided by $\exp\{\gamma F(\alpha)\}$ with $\gamma=2$ is not accurate in the region $|t| \sim m^2$. Nevertheless Elitzur finds that an expression of the type of Eq. (42) is reasonable using the experimental information precisely in such a kinematical region.

Consideration of Eqs. (30), (33), (41) with $\gamma=2$ shows that for a hard collision (i.e., one involving a large momentum transfer and/or a large excitation energy, and $s \gg m_i^2$) one can write:

$$A(s,t,u) \sim \prod_{i=1}^{2} \frac{G(T_i)G(\mathcal{T}_i)}{G(\Sigma_i)} \tag{43}$$

where

$$T_i = 1 - \frac{t}{m_i^2} \tag{44}$$

$$\mathcal{T}_i = 1 - \frac{u}{m_i^2} \tag{45}$$

$$\Sigma_i = T_i + \mathcal{T}_i \tag{46}$$

and where $G(x) = x^{-2}$ is the functional form of the elastic form factor of the nucleon. One may conjecture that Eq. (43) should remain true even in the region where the model is not accurate if one substitutes for G the experimental functional form. For example, the phenomenological dipole fit corresponds to the form:

$$\begin{aligned} G(t)_{\text{Dipole}} &= \left(1 - \frac{t}{.71}\right)^{-2} = \left(\frac{.71}{m^2}\right)^2 \left[\left(1 - \frac{t}{m^2}\right) - \left(1 - \frac{.71}{m^2}\right)\right]^{-2} \\ &= \frac{.65}{\left[\left(1 - \frac{t}{m^2}\right) - .195\right]^2} \simeq \frac{\left(\frac{2}{3}\right)}{\left[\left(1 - \frac{t}{m^2}\right) - \left(1 - \sqrt{\frac{2}{3}}\right)\right]^2} \end{aligned} \tag{47}$$

Therefore, the conjecture with a dipole fit states that for $s \gg m_i^2$, Eq. (43) will hold with

$$G(A) = \frac{.65}{(A - .195)^2} \sim \frac{\frac{2}{3}}{\left[A - \left(1 - \sqrt{\frac{2}{3}}\right)\right]^2} \tag{48}$$

In order to have a comparison with experiment one can study the case of elastic proton-proton scattering with the help of Eqs. (43) - (46) and the dipole form of G(A) given by Eq. (48). Figure 2 shows a set of theoretical curves calculated with these equations where the ordinate is the ratio of $X = (d\sigma/dt)/(d\sigma/dt)_{t=0}$ to the fourth power of G(t). The abscissa is the absolute value of the momentum transfer measured in $GeV^2$. The curves are labelled by the incident momentum in the laboratory measured in GeV/c.

Figure 2 also shows the $\theta_{CM} = 90^o$ curve and, as a dashed line, the asymptotic value of $X/G^4(t)$ as $s \to \infty$ at that angle. That value is $2^8$.

Figure 3 is the set of the corresponding experimental curves.[12] The most obvious difference between the model and the experiment is the theoretical absence of the dip at about $t = -1.2\ GeV^2$, which is present for incident momenta larger than 7 GeV/c.[13]

It is convenient to notice the combination of variables

$$K = \frac{(1 - t/m^2 - a)(1 - u/m^2 - a)}{(2 - t/m^2 - u/m^2 - a)} \tag{49}$$

appearing in Eq. (43). The parameter $a = 0.195$ if G(x) is given by Eq. (48) and zero if $G(x) = x^{-2}$. When $|t|, |u| \to \infty$, $m^2 K \to ut/s = \beta^2 p_\perp^2$, the Krisch variable.[9] Recently Leader and Pennington[14] and Ball and Pinsky[15] have found variables which also reduce to the Krisch variable in some limit.

Pinsky[16] has followed the idea of Odorico[17] and studied the dips in pp elastic scattering as zeroes of the amplitude in the u, t plane; such zeroes occurring on some lines for which his kinematical variable (essentially the Krisch variable) takes appropriate constant values.

In the same spirit, and since K given by Eq. (49) is intimately related with the Krisch variable, one may (in the context of the model) attribute the presence of a dip at $t = -1.2\ \mathrm{GeV}^2$ in the elastic pp scattering case to a zero of the amplitude (which should arise in the undetermined overall factor) occurring at some value of K.

A line of constant K is a hyperbola in the u, t plane with asymptotes $u = -(K + a - 1)m^2$ and $t = -(K + a - 1)m^2$. Its two vertices are located at $(v_\pm, v_\pm)$ with $v_\pm = \{(1 - K - a) \pm [K(K + a)]^{1/2}\} m^2$.

An asymptotic zero will be present at $-t = 1.2\ \mathrm{GeV}^2 \sim \frac{3}{2} m^2$ setting $K = \frac{5}{2} - a$. Then one expects to find a zero at $90^\circ$ in the center of mass at $s = 4m^2 - 2v_-$, i.e., $s = 12m^2$ if $a = 0$ or $s = 11.8m^2$ if $a = 0.195$. The zero will rapidly approach its asymptotic position upon increasing s. The value $s = 12m^2$ corresponds to an incident momentum in the laboratory of about 4.6 GeV/c.

Figure 2 shows that for such an incident momentum the value of $X/G^4(t)$ at $90^\circ$ is in the vicinity of $2^7$. This construction suggests that the appearance of zeroes in the elastic scattering amplitude could be related to the approach of $X/G^4(t)$ to an asymptotic value at $\theta_{CM} = 90^\circ$.

The amplitude (43) is symmetric under interchange $u \leftrightarrow t$. This is natural since the model does not incorporate spin or isospin. When one talks about pp or nn elastic scattering one should in principle talk about the five independent helicity amplitudes.[18] Nevertheless, if one assumes that at high energies one has helicity conservation only the nonflip helicity amplitude $f_{++,++}$ will be important[19] and that is symmetric about $90^\circ$ for pp or nn. Therefore, in this context the amplitude (43) should be considered as the dominant amplitude in the process.

When one considers $pn \to pn$ one can have the total isospin equal to zero or one. In the latter case $f^I_{++,++}$ is symmetric about $90^\circ$ and in the former anti-symmetric. Therefore one can have an asymmetry about $90^\circ$ due to interference between these amplitudes. However at high energies this asymmetry is expected to disappear[20] and Eq. (43) should again describe the process.

For inelastic processes one should take the same attitude and expect that the expressions appearing in this paper, arising from a very simplified model, describe the main trend of the experimental facts. It will be interesting to see if they can be useful to parameterize the data of high energy nucleon-nucleon scattering that will soon be available from the new accelerator experiments.


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## References and Footnotes


1. H. Moreno, to be published in Phys. Rev.

2. Vector mesons are used here for simplicity. As shown in reference 1, one can also work out a model of scalar mesons. The vector meson soft model was introduced and studied by H. M. Fried and T. K. Gaisser, reference 7 below. A scalar meson model was used by H. Gemmel and H. A. Kastrup, Nucl. Phys. B14, 566 (1969) to study elastic pp scattering.

3. H. Moreno and J. Pestieau, Preprint Centro de Investigación y de Estudios Avanzados del Instituto Politécnico Nacional, México (March, 1971). To be published in The Physical Review.

4. Real massive vector meson bremsstrahlung is relevant in the limit of infinite energy. See H. M. Fried and T. K. Gaisser, Phys. Rev. D4, 3330 (1971).

5. Or by defining the ambiguous product of two $\delta$-functions of the same argument present in the integrand of Eq. (5) as the limit of the product where one of the $\delta$-functions has its argument infinitesimally displaced. Then products of the type $\delta(k^2 - \mu^2)\ \delta(kp)$ are zero since k and p are timelike and kp can not be zero.

6. This is equivalent to the cutoff of H. M. Fried and T. K. Gaisser,[7] and was also utilized in reference 1.

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10. See Eq. (44) of Reference 1; in the limit $\lambda \gg 1$.

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12. J. V. Allaby et al., Phys. Letters 34B, 431 (1971). Reproduced with the permission of North Holland Publishing Company.

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19. $f_{++,++}$ stands for the amplitude of scattering with the two nucleons in states of helicity $+\frac{1}{2}$ initially and finally.

20. T. T. Wu and C. N. Yang; loc. cit.

## Figure Captions

1. Space time diagram of a nucleon-nucleon collision as considered in the model.

2. Theoretical curves for $X/G^4(t)$, $X = (d\sigma/dt)/(d\sigma/dt)_{t=0}$. They have been calculated for the elastic scattering case with Eqs. (43) - (46) and Eq. (48). The curves are labelled by the incident momentum in the laboratory measured in GeV/c. Also shown are the $\theta_{CM} = 90^o$ curve and, as a dashed line, the asymptotic value ($s \to \infty$) of $X/G^4(t)$ at that angle.

3. The experimental curves corresponding to Figure 2, reproduced from J. V. Allaby et al., Phys. Letters 34B, 431 (1971) (with the permission of North-Holland Publishing Company).

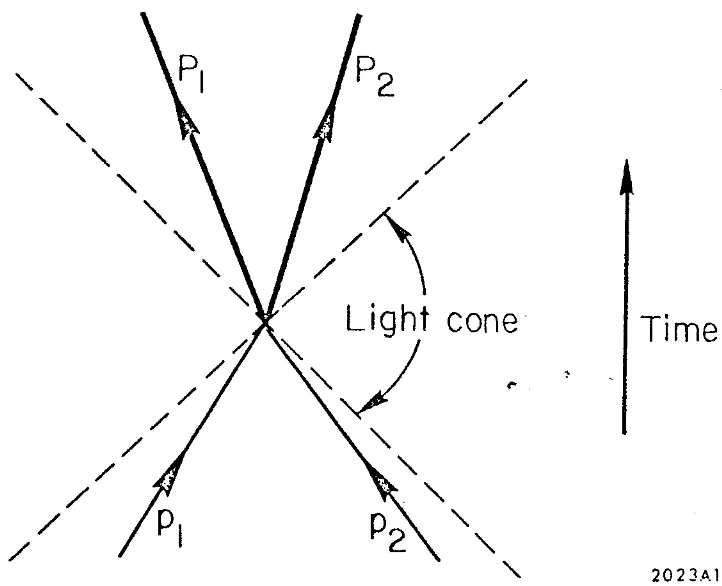


Fig. 1

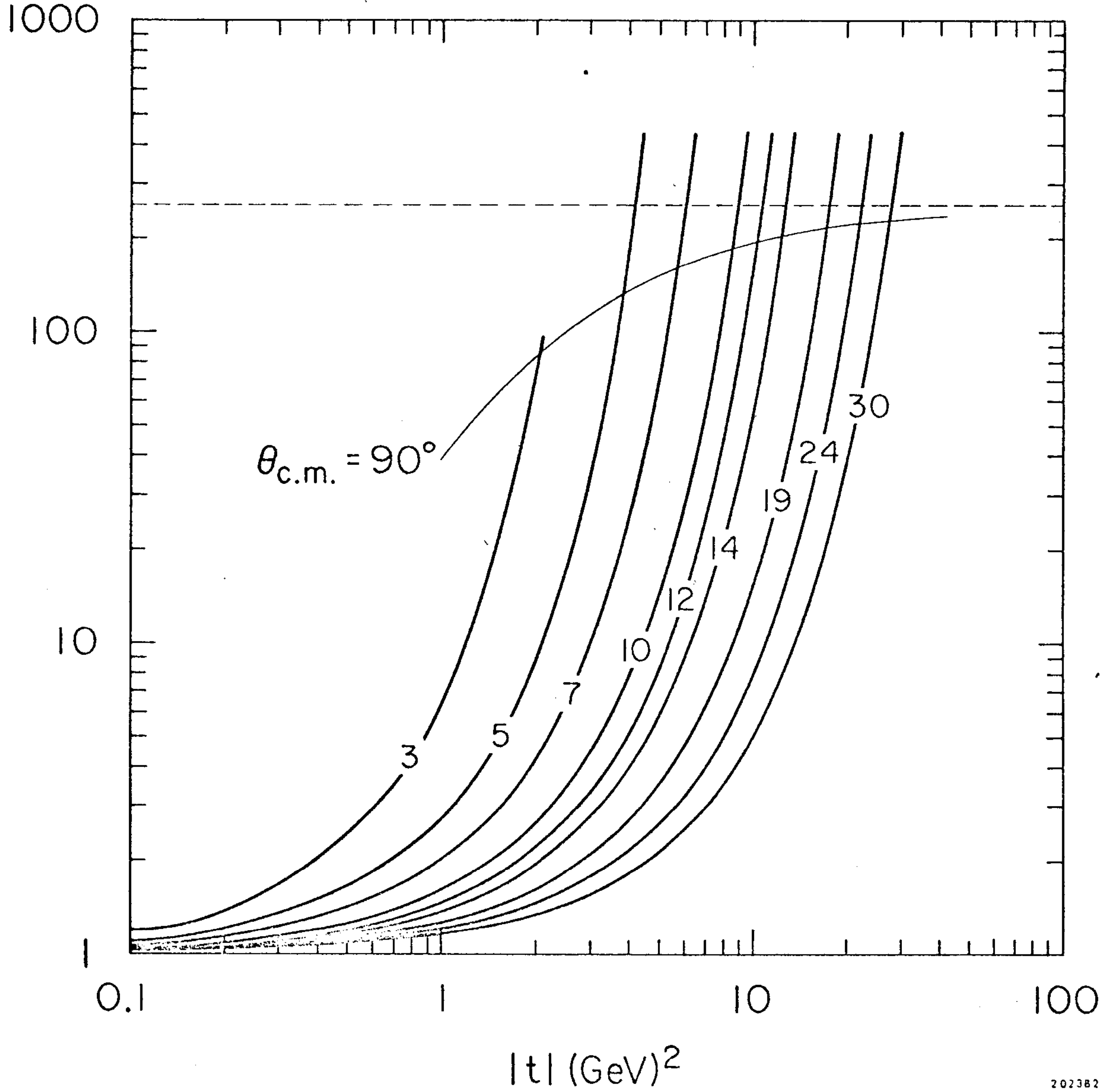
1000
100
10
1
0.1
1
10
100
$\frac{X}{G^4(t)}$
$|t|\ (\mathrm{GeV})^2$
$\theta_{c.m.} = 90°$
3
5
7
10
12
14
19
24
30
202382

Fig. 2

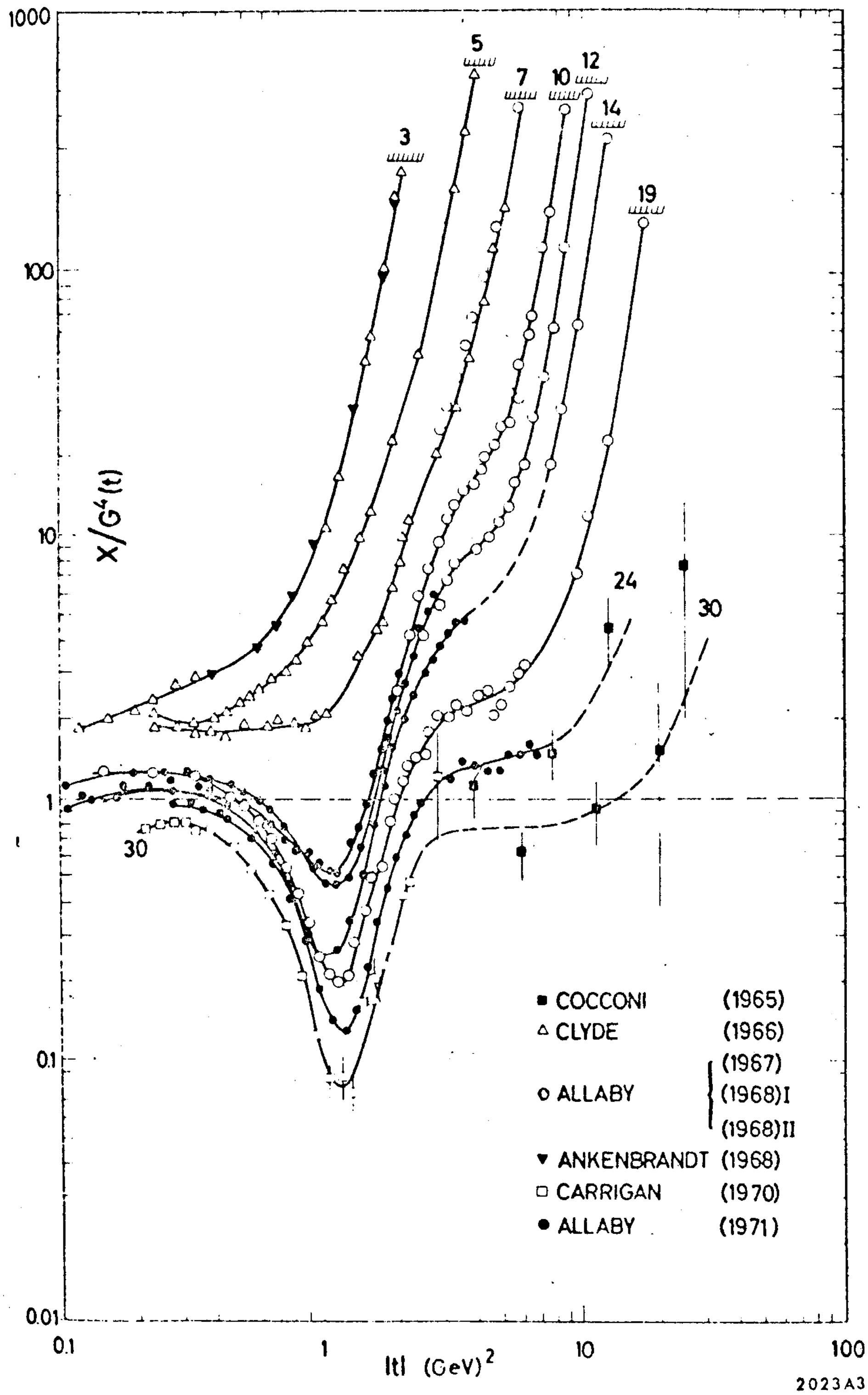
1000
100
10
1
0.1
0.01
0.1
1
10
100
X/G⁴(t)
|t| (GeV)²
3
5
7
10
12
14
19
24
30
30
■ COCCONI (1965)
△ CLYDE (1966)
○ ALLABY (1967) (1968)I (1968)II
▼ ANKENBRANDT (1968)
□ CARRIGAN (1970)
● ALLABY (1971)
2023A3

Fig. 3