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GAUGE CONDITIONS IN DUAL RESONANCE MODELS*

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ABSTRACT

Simplifying the arguments of Chang and Mansouri, we show that the Virasoro conditions in dual resonance models are related to geometrical and mechanical properties of classical two-dimensional media.

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In dual resonance models there exist in general a class of gauge conditions[1] which reflect invariance properties of the underlying dynamics. It is generally held that these conditions lead to the elimination, or decoupling, of ghost states from the physical amplitudes. Recently[2] it has been pointed out that they can also be derived from a geometrical principle reminiscent of that in general relativity. The main point of this argument is that the dynamics of the dual resonance model, if represented in terms of a two-dimensional elastic medium (a "world sheet") embedded in space-time, should not depend on the internal parameterization of the medium. When scalar external fields act on the periphery of the boundary, this leads to the Virasoro condition that the external particles must be tachyons with $\alpha' m^2 = -1$, where α' is the slope of the trajectories. Though disappointing, it is not a surprising result since an incoming momentum q_μ will excite internal harmonic oscillator modes in the direction q_μ, which are ghost modes for timelike q_μ.

In the present note we would like to present a simple discussion of these points in terms of a classical picture. Some generalizations will also be attempted.

We start from the free Lagrangian density[3]

$$L = \frac{1}{2} \partial_\alpha x^\mu \partial^\alpha x_\mu , \qquad \partial_\alpha = \partial/\partial\zeta^\alpha ,$$

$$\zeta^\alpha = (\zeta^0, \zeta^1) = (\tau, \xi) \qquad (1)$$

Often we will also use the notation $\partial x^\mu/\partial\tau = \dot{x}^\mu$, $\partial x^\mu/\partial\xi = x'^\mu$. Consider first a Euclidean metric for both x^μ and ζ^α. The metric tensor of the internal 2-space is given by

$$g_{\alpha\beta}(\zeta) = \partial_\alpha x^\mu \partial_\beta x_\mu \qquad (2)$$

whereas the tensor surface element $d\sigma^{\mu\nu} = \sigma^{\mu\nu} d^2\zeta$ of the 2-space as embedded in the external 4-space is given by

$$\sigma^{\mu\nu} = \partial(x^\mu, x^\nu)/\partial(\zeta^0, \zeta^1) \tag{3}$$

We can then write Eq. (1) as

$$L = \frac{1}{2} g^\alpha_\alpha = \frac{1}{2} (g_{00} + g_{11}) \tag{4}$$

Next we take another Lagrangian L' which is a natural candidate for being independent of internal parameterization, namely the surface area element

$$L' = (\sigma^{\mu\nu}\sigma_{\mu\nu})^{1/2} = g^{1/2},$$
$$g = \det g_{\alpha\beta} = g_{00}g_{11} - (g_{01})^2 \tag{5}$$

Let us now demand that the solutions to Eq. (1) make L numerically equal to L' so that they will have a geometrical, parameter-independent meaning. This is an essential restriction because L itself is invariant only under conformal transformations which are a subgroup of general coordinate transformations. Equating Eqs. (4) and (5) we obtain

$$\frac{1}{4} (g_{00} - g_{11})^2 + (g_{01})^2 = 0 \tag{6}$$

from which follows

$$\frac{1}{2} (g_{00} - g_{11}) = g_{01} = 0$$

or

$$T_{\alpha\beta} = 0 \tag{7}$$

for all ζ^α.

Here the $T_{\alpha\beta}$ are the energy-momentum tensor

$$T_{00} = -T_{11} = \frac{1}{2}(g_{00} - g_{11}), \quad T_{01} = T_{10} = g_{01},$$

$$\partial^{\alpha} T_{\alpha\beta} = 0 \tag{8}$$

Equation (7) is the Virasoro gauge condition for the free system, and are compatible with the equation of motion

$$\partial_{\alpha}\partial^{\alpha} x^{\mu} = 0 \tag{9}$$

due to the continuity Eq. (8).

Conversely, it can readily be seen that a solution to Eqs. (9) and (7) is a solution to the Euler equation for L'. Thus such a solution represents a minimal surface,[2] and the action integral becomes equal to the surface area.

We will next consider a finite surface S, at whose boundary acts a distribution of external forces k_{μ}. We can choose the coordinates ζ^{α} so that the boundary corresponds to $\xi = 0$. Equation (7) then should be supplemented with the boundary condition

$$x^{\mu\prime} = k^{\mu} \qquad (\xi = 0) \tag{10}$$

This driving force may be derived from an interaction Lagrangrian

$$L_{int} = -V(x(\zeta))\,\delta(\xi),$$

$$k_{\mu} = -\partial V/\partial x^{\mu} \tag{11}$$

Equation (11) will have a coordinate-independent meaning if we have in addition

$$g_{00} = \dot{x}^{\mu}\dot{x}_{\mu} = 1, \qquad (\xi = 0) \tag{12}$$

since then we can multiply L_{int} by $\sqrt{g_{00}}$ without changing its value.

On the other hand, Eq. (7) must remain valid everywhere in the interior of S. As we approach the boundary, consistency of Eqs. (7) and (10) then demands

$$k^2 = g_{11} = g_{00} = 1 \;;$$

$$k^\mu \dot{x}_\mu = g_{01} = 0 \quad (\xi = 0) \tag{13}$$

or

$$\partial V(x)/\partial \tau = 0$$

along the boundary. Thus the external forces must be "quantized" and perpendicular to the boundary, which must lie on an equipotential surface. The meaning of these conditions becomes clear if we compare the system to a film of soap water. The action integral corresponds to the potential energy of the film, which has unit strength per unit area. The surface tension is therefore unity per unit length. In order to maintain equilibrium, the external forces applied to the boundary must obviously match the surface tension. This is the content of Eq. (13).

In going over to a Hamiltonian formalism and eventually to quantization it would be appropriate to change the internal metric by the formal substitution $\xi \longrightarrow i\xi$. At the same time we recover the external Minkowskian metric (+---). Equation (7) now reads[4]

$$T_{\alpha\beta} = 0; \quad T_{00} = -T_{11} = \frac{1}{2}\,(g_{00} + g_{11}) = \frac{1}{2}\,(\dot{x}_\mu \dot{x}^\mu + x'_\mu\, x'^\mu),$$

$$T_{01} = g_{01} = \dot{x}_\mu\, x'^\mu \tag{14}$$

whereas Eq. (13) is replaced by

$$k_\mu k^\mu = g_{11} = -g_{00} = -1$$

$$k_\mu\, \dot{x}^\mu = \dot{V} = 0 \qquad (\xi = 0), \tag{15}$$

Eq. (12) being unchanged. The last point reflects the physical assumption that τ is the proper time for the boundary world line which is timelike. Equation (15) means that the forces must be orthogonal to the four-velocity $\partial x^{\mu}/\partial\tau$, hence k^{μ} is spacelike.

Equation (15) is not identical with the Virasoro condition $-q^2 = 1/\alpha'$, which equals $2\pi\hbar$ in our convention. For k is the force, or momentum per unit length, continuously distributed along the boundary. On the other hand, q is a discrete quantum mechanical impulse which is related to k only on the average. As a matter of fact, conditions like (7) and (12) become meaningless in quantum theory because of the infinite zero-point fluctuations of the operators. What has been done[1] to circumvent the difficulty is essentially to expand x^{μ} and $T_{\alpha\beta}$ into Fourier components, and retain as subsidiary conditions only the positive frequency parts $T^{(+)}_{00}(n)$, $n = 1, 2, \cdots$, of the energy momentum tensor which lower the internal excitation energy:[5]

$$\left[T^{(+)}_{00}(n) - T_{00}(0)\right]\psi = 0, \qquad T_{00}(0)\psi = 0 \tag{16}$$

where $T_{00}(0)$ is nothing but the total Hamiltonian with an infinite zero-point energy subtracted out. (The reason for taking the difference of T's in Eq. (16) is to eliminate the interaction term from the constraint equations.) The correspondence between classical and quantum mechanical constraints is therefore obscured, especially as it concerns Eq. (15). Nevertheless one can make the following observation. If a hadron in its ground state is viewed as a string of length ℓ forming a boundary at $\tau = \pm\infty$, its total momentum p^{μ} will be related to the classical force k^{μ} across the boundary by

$$p^{\mu} = \ell k^{\mu} \qquad \text{or} \qquad |p| = \ell\,|k| = \ell \tag{17}$$

On the other hand the uncertainty principle suggests $|p| \ell$ Ch, C = 0(1) so that

$$|p|^2 \sim C^2 h \tag{18}$$

which agrees with the Virasoro condition if C = 1.

The above considerations make us realize that it would be very difficult to modify the dual resonance model so as to turn the tachyon into a bona fide particle. At the same time, however, it is not at all clear why the general covariance requirements should result in the elimination of ghosts.

Finally we will discuss some generalizations. When the external interaction is electromagnetic rather than scalar, the general covariance is automatic as is obvious from[6]

$$L_{int} = e\, A_\mu(x(\xi))\, \dot{x}^\mu\, \delta(\xi) \tag{19}$$

Therefore we do not need the condition (12). The force k^μ in Eq. (10) is now the Lorentz force

$$k^\mu = eF^{\mu\nu}\, \dot{x}_\nu \tag{20}$$

which in conjunction with $T_{\alpha\beta} = 0$ leads to

$$\dot{x}^\mu \dot{x}_\mu = -(eF^{\mu\nu}\dot{x}_\nu)(eF_{\mu\rho}\dot{x}^\rho) \tag{21}$$

This implies a constraint on the strength of the electromagnetic field. Suppose $\dot{x}^\mu$ is timelike. Then Eq. (21) means

$$(e\vec{E}_0)^2 = 1 \tag{22}$$

where $\vec{E}_0$ is the electric field in the rest frame of $\dot{x}^\mu$. Since $\vec{E}_0^2 - \vec{B}_0^2 \equiv \lambda$ and $\vec{E}_0 \cdot \vec{B}_0 \equiv \mu$ are Lorentz invariant, one cannot satisfy Eq. (22) for any choice of $\dot{x}^\mu$ unless

$$e^2\lambda \le 1, \quad |\mu| \le |\vec{E}_0|(\vec{E}_0^2 - \lambda)^{1/2} = e^{-2}(1 - e^2\lambda)^{1/2} \tag{23}$$

Thus $e^2\lambda = 1$ is a natural upper limit. In the practically interesting case of $\alpha' = 1\ \mathrm{GeV}^{-2}$, this corresponds to a potential gradient of $1\ \mathrm{GeV}^2/2\pi\hbar c = 2.4$ GeV/fermi. It is the same value that obtains for k_μ from Eq. (15).

We do not find an obvious constraint if $\dot{x}^\mu$ is lightlike.[7] But the properties of a point charge moving with light velocity are not so obvious either.

The gauge conditions (16) remain unchanged in the presence of electromagnetic interactions except that the velocity $\dot{x}^\mu$ and the canonical momentum π^μ differ by an interaction term. Therefore all constraint equations will explicitly depend on the interaction, although their commutator algebra may not be altered.

The above model is not really interesting because it does not lead to a correct dual theory for arbitrary photon mass. For a more satisfactory theory one must generalize the basic picture.[8] Nevertheless, let us explore the consequences of the curious condition (23). Assume that a hadron contains two charges e located at the two ends of the elastic "string" of length r. Equation (23) for each charge then reduces to

$$e^2/r^2 \le 1\ \mathrm{GeV}^2/2\pi\hbar c$$

or

$$e^2/r \le (e^2/2\pi\hbar c)^{1/2}\ \mathrm{GeV} \tag{24}$$

which gives a minimum size $r_0 = 4.2 \times 10^{-15}$ cm. If there are magnetic charges g we can apply the same argument by interchanging E and B; r_0 should be larger by a factor g/e = 137/2, or $r_0 = 2.9\times 10^{-13}$ cm, approximately the electron classical radius.[9] [Equation (22) does not distinguish between the different relative sings of the charges, but we have taken the same sign (repulsion) for intuitive reasons. The string will be stretched by the electromagnetic forces to a length $\geq r_0$.]

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REFERENCES

1. M. A. Virasoro, Phys. Rev. D1, 2933 (1970).

2. L. N. Chang and F. Mansouri, to be published.
See also T. Takabayashi, Prog. Theor. Phys. 44, 1429 (1970) and Univ. Nagoya preprint DPNU-14 (1971). O. Hara, Nihon University (Tokyo) preprint NUP-A-71-5 (1971). T. Goto, Prog. Theor. Phys. 46, 1560 (1971).

3. We largely follow the notations of Y. Nambu, Phys. Rev. D4, 1193 (1971) and Ref. 2.

4. If we just make the substitution in Eq. (6) we get

$$(g_{00} + g_{11} + 2g_{01})(g_{00} + g_{11} - 2g_{01}) = 0$$

which means

$$\frac{\partial x_\mu}{\partial u_+} \frac{\partial x^\mu}{\partial u_+} = 0 \quad \underline{\text{or}} \quad \frac{\partial x_\mu}{\partial u_-} \frac{\partial x^\mu}{\partial u_-} = 0,$$

$$u_\pm = (\tau \pm \xi)/\sqrt{2}$$

Equation (14) amounts to demanding both of the above relations simultaneously. This is dictated by the boundary condition we consider here.

5. T_{01} does not lead to new conditions. An alternative procedure (Ref. 2) is to take for normal modes periodic running waves going only in one direction (except for the center-of-mass mode). In this case the sign of frequency and that of wave number are uniquely related. Physically it corresponds to a rubber band picture rather than a rubber string.

6. For an approximately dual theory see Ref. 3.

7. Clavelli and Ramond [L. Clavelli and P. Ramond, Phys. Rev. D4, 3098 (1971).] show the consistency of the Virasoro conditions when the external particles are the zero mass vector states on the tachyon trajectory. This

case is different from the present one by the absence of an A^2 term in the Hamiltonian, or the presence of the same in the Lagrangian in comparison with Eq. (19). The extra term, however, will be both coordinate independent and effectively zero if the boundary line is always lightlike. But then we get from Eqs. (10) and (14) $k^2 = g_{11} = -g_{00} = 0$ in agreement with these authors.

8. The Drummond-Rebbi model (I. T. Drummond, CERN Preprint TH 1301 (1971) revised version, C. Rebbi, Nuovo Cimento Letters 23, 967 (1971).) may be adapted to the vector currents. But this model still has bad features. [Y. Nambu and J. Willemsen, to be published.]

9. If we take this point seriously, α' and the electron mass m_e are related by $\alpha'(m_e c^2)^2 = (2/\pi)(e^2/\hbar c)^3$. $1/\sqrt{\alpha'}$ is then predicted to be 1.027 GeV.