SLAC-PUB-1024
(TH) and (EXP)
March 1972

THRESHOLD RELATIONS FOR INELASTIC SCATTERING OF POLARIZED LEPTONS FROM POLARIZED NUCLEONS*

Jean Cleymans†

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

ABSTRACT

Threshold relations, following from scaling and duality, are obtained for inelastic scattering of polarized leptons from polarized nucleons.

(To be submitted to Physical Review)

---

* Supported in part by the U. S. Atomic Energy Commission.

† NATO Fellow

I. INTRODUCTION

A recent extension[1] of the Bloom-Gilman[2] threshold relations from inelastic electron scattering to inelastic neutrino scattering has given non-trivial results. The approach proposed by Bloom and Gilman to obtain information on inelastic form factors starting from the elastic form factors seems to be orthogonal to naive quark parton models[3]. For instance, the quark parton model (or the light cone commutators of Fritzsch and Gell-Mann[4]) imply the following relation[5] between the scaling functions:

$$-12\left(F_1^{\gamma p}(\omega) - F_1^{\gamma n}(\omega)\right) = F_3^{\nu p}(\omega) - F_3^{\nu n}(\omega) \tag{1}$$

while the threshold relations predict[6]:

$$2\,\frac{F_A(0)\,\dfrac{m_A^4}{m_V^4}\,(\mu_p - \mu_n)}{\mu_p^2 - \mu_n^2}\left(F_1^{\gamma p}(\omega=1) - F_1^{\gamma n}(\omega=1)\right) = F_3^{\nu p}(\omega=1) - F_3^{\nu n}(\omega=1)\,. \tag{2}$$

The coefficient in front of the left hand side bracket of Eq. (2) equals, experimentally, approximately -3 which should be compared to the factor -12 appearing in (1).

As a second example of discrepancy, the vector contributions to $F_1^{\bar{\nu} p}$ does not equal the axial vector contribution, as one would expect from the quark parton model or from the Fritzsch-Gell-Mann light cone commutators.

Finally, as a more dramatic consequence of the threshold relations, one has[1]:

$$\frac{F_i^{(\nu p)}(\omega = 1)}{F_i^{(\bar{\nu} p)}(\omega = 1)} = 0 \quad \text{for } i = 1, 2, 3 \tag{3}$$

while this ratio is very unlikely to be zero in any version of the parton model.

The above mentioned three examples make it clear that it is of interest to look for further predictions of the Bloom-Gilman approach in order to find other tests of duality in inelastic lepton scattering and, possibly, to compare them with corresponding quark parton model predictions. To this end we will investigate here deep inelastic scattering of polarized leptons from polarized nucleons since experimental results are expected to come out on this process in the near future.

The plan of this paper is as follows: in Section II we fix our notations, in Section III we derive the threshold relations and calculate the asymmetry, in Section IV we discuss our results.

II. NOTATIONS

We will only consider the spin dependent structure functions in detail. For the rest we will follow the notations, normalizations and metric already introduced in reference 1.

The Lorentz invariant structure functions $G_1(q^2, \nu)$ and $G_2(q^2, \nu)$ are defined in the following way:

$$W_{\mu\nu}(p, q, s) = \int \frac{d^4x}{4\pi m_p} e^{iq\cdot x} < ps \,|\, \left[J_\mu(x), J_\nu(0)\right] | \, ps > = W_{\mu\nu}^{[S]} + W_{\mu\nu}^{[A]} \tag{4}$$

$J_\mu(x)$ is the electromagnetic current, $W^{[S]}_{\mu\nu}$ is a symmetric tensor in μ, ν while $W^{[A]}_{\mu\nu}$ is antisymmetric. Only $W^{[A]}_{\mu\nu}$ is of interest here:

$$W^{[A]}_{\mu\nu}(p, q, s) = i\,\epsilon_{\mu\nu\lambda\sigma}\, q^\lambda s^\sigma \frac{G_1(q^2, \nu)}{m_p^2} + i\epsilon_{\mu\nu\lambda\sigma}\, q^\lambda (p\cdot q s^\sigma - s\cdot q p^\sigma) \frac{G_2(q^2, \nu)}{m_p^4} \; . \tag{5}$$

We have normalized the polarization vector s^σ to $s^2 = -m_p^2$. With the above definition, $G_1(q^2, \nu)$ and $G_2(q^2, \nu)$ have the same dimension as $W_1(q^2, \nu)$ and $W_2(q^2, \nu)$.

A useful, equivalent, way of rewriting (5) is:

$$W^{[A]}_{\mu\nu} = \frac{1}{4m_p^2}\, \bar{u}(p, s) \Big\{ G_1(q^2, \nu)(\gamma_\mu \not{q}\, \gamma_\nu - \gamma_\nu \not{q}\, \gamma_\mu) + G_2(q^2, \nu) \frac{1}{m_p} \Big[2\not{q}\,(q_\mu \gamma_\nu - q_\nu \gamma_\mu) + 2i\sigma_{\mu\nu} q^2 \Big] \Big\} u(p, s) \; . \tag{6}$$

In terms of $G_1(q^2, \nu)$ and $G_2(q^2, \nu)$ the difference in cross sections for nucleons polarized in a direction parallel or anti-parallel to the incident, longitudinally polarized, lepton is:

$$\frac{d^2\sigma(\uparrow\uparrow)}{d\Omega dE'} - \frac{d^2\sigma(\uparrow\downarrow)}{d\Omega dE'} = \frac{4\alpha^2}{q^2} \frac{E'}{E m_p} \left[G_1 \, (q^2, \nu)\,(E + E' \cos\theta) + G_2(q^2, \nu) \frac{q^2}{m_p} \right] \tag{7}$$

Carlson and Wu-Ki Tung[7] have pointed out that, in order to determine $G_2(q^2, \nu)$, it is more interesting to consider also the difference of cross sections where the target polarization vector is perpendicular to the momentum of the incident lepton:

$$\frac{d^2\sigma(\uparrow\leftarrow)}{d\Omega dE'} - \frac{d^2\sigma(\uparrow\rightarrow)}{d\Omega dE'} = \frac{4\alpha^2}{q^2}\,\frac{E'}{Em_p}\left[G_1(q^2,\nu) + G_2(q^2,\nu)\,\frac{2E}{m_p}\right]\sin\theta \quad . \tag{8}$$

In the next section we will derive threshold relations for the functions $G_1(q^2,\nu)$ and $G_2(q^2,\nu)$ in the scaling region.

III. THRESHOLD RELATIONS

By repeating the arguments leading to the scaling of $m_p W_1(q^2,\nu)$ and $\nu W_2(q^2,\nu)$, one obtains the following limits for $G_1(q^2,\nu)$ and $G_2(q^2,\nu)$:[8]

$$\lim_{\substack{\nu\to\infty \\ -q^2\to\infty \\ \omega=\frac{2m_p\nu}{-q^2}\ \text{fixed}}} \nu G_1(q^2,\nu) \equiv g_1(\omega) \tag{9}$$

$$\lim_{\substack{\nu\to\infty \\ -q^2\to\infty \\ \omega=\frac{2m_p\nu}{-q^2}\ \text{fixed}}} \frac{\nu^2 G_2(q^2,\nu)}{m_p} \equiv g_2(\omega) \tag{10}$$

Since it appears, experimentally, that scaling sets in earlier in the variable[2] ω', we will assume, naturally, that the same phenomena will also be true for the functions $\nu G_1(q^2,\nu)$ and $\nu^2 G_2(q^2,\nu)$. We are thus led to consider, following the original analysis of Bloom and Gilman[2], the following finite energy sum rules:

$$\int_1^{\omega_t'} d\omega' g_1(\omega') = \frac{2m_p}{-q^2} \int d\nu \, \nu G_1^{\text{elastic}}(q^2, \nu) \tag{11}$$

$$\int_1^{\omega_t'} d\omega' g_2(\omega') = \frac{2}{-q^2} \int d\nu \, \nu^2 G_2^{\text{elastic}}(q^2, \nu) \tag{12}$$

where $\omega_t' = 1 + \frac{W_t^2}{-q^2}$ and $W^2 = (p+q)^2$; the index t means that we work close to threshold. The sum rules (11) and (12) can be written down both for proton and for neutron.

Inserting the known elastic nucleon form factors in the right hand side of equations (11) and (12), we obtain:

$$\int_1^{\omega_t'} d\omega' g_1(\omega') = \tfrac{1}{2} \frac{(q^2 + 4m_p^2) G_M^2(q^2) - 8m_p^2 G_M(q^2) G_E(q^2)}{q^2 - 4m_p^2} \tag{13}$$

$$\int_1^{\omega_t'} d\omega' g_2(\omega') = \frac{q^2}{2} \frac{G_E(q^2) G_M(q^2) - G_M^2(q^2)}{q^2 - 4m_p^2} \, . \tag{14}$$

As in references 1 and 2, we now take the derivative of equations (13) and (14) with respect to q^2. We then take the limit $-q^2 \to \infty$. This leads to the following set of threshold relations:

$$\frac{g_1^p(\omega = 1)}{g_2^p(\omega = 1)} = \frac{\mu_p}{1 - \mu_p} \tag{15a}$$

$$\frac{g_1^p\,(\omega=1)}{g_1^n\,(\omega=1)} = \left(\frac{\mu_p}{\mu_n}\right)^2 \tag{15b}$$

$$\frac{g_1^n\,(\omega=1)}{g_2^n\,(\omega=1)} = -1\ . \tag{15c}$$

The subscripts p and n specify whether we consider the proton's structure function or the neutron's. The following assumptions, concerning the asymptotic behavior of the elastic form factors, have been made in deriving the threshold relations (15):

$$G_E^p(q^2) \simeq \frac{G_M^p(q^2)}{\mu_p} \simeq \frac{G_M^n(q^2)}{\mu_n} \simeq \frac{1}{\left(1-\frac{q^2}{m_V^2}\right)^2} \tag{16}$$

and $G_E^n(q^2)$ was neglected compared to $G_M^n(q^2)$.

A quantity of interest is the asymmetry, defined as:

$$A = \frac{\frac{d^2\sigma(\uparrow\uparrow)}{d\Omega dE'} - \frac{d^2\sigma(\uparrow\downarrow)}{d\Omega dE'}}{\frac{d^2\sigma(\uparrow\uparrow)}{d\Omega dE'} + \frac{d^2\sigma(\uparrow\downarrow)}{d\Omega dE'}}\ . \tag{17}$$

In terms of the structure functions W_1, W_2, G_1 and G_2, the asymmetry has the following form:

$$A = \frac{q^2}{2EE'm_p}\;\frac{G_1(q^2,\nu)(E+E'\cos\theta) + G_2(q^2,\nu)\,\frac{q^2}{m_p}}{2W_1(q^2,\nu)\sin^2\frac{\theta}{2} + W_2(q^2,\nu)\cos^2\frac{\theta}{2}} \tag{18}$$

In the scaling region we can rewrite the asymmetry as:

$$A_S(\omega, x) \equiv \lim_{\substack{\nu \to \infty \\ -q^2 \to \infty \\ \omega \text{ fixed}}} A = -2 \frac{g_1(\omega)\, x(2-x)}{F_2(\omega)\left[2 - x(2-x)\right]} \tag{19}$$

where $x = \nu/E$. In (19) we have made use of the Callan-Gross relation[9] $\omega F_2(\omega) = 2F_1(\omega)$.

We thus obtain the result:

$$A_S(\omega = 1,\ x = 1) = -1 \tag{20}$$

which means that, in this limit, we expect a maximal negative asymmetry.

IV. DISCUSSION

One way to remove the existing discrepancies, mentioned in the introduction, between the quark parton model and the threshold relations of Bloom and Gilman is to notice that at $\omega = 1$ the impulse approximation to the quark model breaks down. This would mean that predictions of the quark parton model are expected to be correct at values of ω larger than one only. The correct answer can only be given by future experimental data on inelastic neutrino scattering.

ACKNOWLEDGMENTS

It is a pleasure to thank Professor Sidney Drell for the hospitality extended to the author at SLAC.

Financial support from NATO and from the Université Catholique de Louvain is gratefully acknowledged.

Thanks are also due to Professor F. Gilman and to Dr. C. H. Llewellyn Smith for interesting discussions.

REFERENCES

1. J. Cleymans and R. Rodenberg, Threshold Relations for Inelastic Neutrino-Nucleon Interactions (Aachen, Germany, preprint), Phys. Rev., to be published.

2. E. Bloom and F. Gilman, Phys. Rev. Letters 25, 1140 (1970), Phys. Rev. D4, 2901 (1971). See also, F. Gilman, Invited talk at the Tel-Aviv Conference 1971, SLAC-PUB-896.

3. This was first pointed out by C. H. Llewellyn Smith, lectures delivered at the International Summer Institute in Theoretical Physics, DESY, July 1971.

4. H. Fritzsch and M. Gell-Mann in "Broken Scale Invariance and the Light Cone", Gordon and Breach (1971).

5. C. H. Llewellyn Smith, Nucl. Phys. B17, 277 (1970).

6. Our notations are the same as those of reference 1.

7. C. E. Carlson and Wu-Ki Tung, Spin Dependent Inelastic Electron Scattering, EFI71-46.

8. A. J. G. Hey and J. E. Mandula, Light Cone Analysis of Spin Dependent Deep Inelastic Electron Scattering, CALT-68-342.

9. C. Callan and D. Gross, Phys. Rev. Letters 21, 311 (1968).