SLAC-PUB-1019
(TH) and (EXP)
February 1972

FINAL STATE HADRONS IN DEEP INELASTIC PROCESSES AND COLLIDING BEAMS*

J. D. Bjorken
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

(1971 International Symposium on Electron and Photon Interactions at High Energies, Cornell University, Ithaca, New York, August 23-27, 1971.)

*Work supported by the U. S. Atomic Energy Commission.

PROLOGUE

Ken Wilson at the beginning of his talk pleaded incompetence as a rapporteur; an assertion he quickly disproved. I likewise plead incompetence, but I can prove it. In the 1967 photon conference I had the session on deep inelastic electroproduction, which had only two contributed papers. Those were the good old days. But inadvertently I neglected to mention either one. So I would like to mention one of them, by H. A. Kastrup,[1] [who is one of the pioneers in the field of dilatations and scale invariance] on a bremsstrahlung model for the deep inelastic electroproduction. Assuming the mesons to be emitted according to a Poisson distribution, the probability none are emitted is $e^{-\bar{n}} \sim G_M^2(Q^2)$; thus $\bar{n}(S, Q^2) \sim -\log G_M^2 (Q^2)$. In a related paper submitted to this conference by Fishbane and Sullivan,[2] they look at massive QED, and obtain the following conclusions:

1. Scaling is broken; νW_2 is sharply peaked at $\omega - 1$.
2. $\bar{n} \sim \log^2 Q^2$.
3. The emitted mesons are Poisson distributed.
4. $\bar{n} \propto -\log F^2 (Q^2)$, with F the elastic form factor.

This talk will concentrate on inclusive processes in colliding-beams, deep-inelastic electroproduction, and some other reactions. The main reason for this is that the statistical accuracy in deep-inelastic experiments with detection of final state hadrons is likely to be limited for some time to come. Also, because there are many competing open channels at the interesting energies the number of events per exclusive channel will be small. There are exceptions of course, but perhaps the best clues to the underlying dynamics will come from studies of the particle spectra, multiplicities and the like. It is fortunate that our understanding of inclusive processes in ordinary hadron-hadron collisions and in photoproduction are progressing[3] with great rapidity, so that one has a normalization point in terms of which to determine what kind of behavior is ordinary and what is extraordinary in the case of the deep inelastic. We shall not try to give definitive theoretical answers to any questions. That is no doubt too hard. Our goal will be to try to catalogue various options on how the data could behave, based on considerations which are as model-independent as possible.

The outline of this talk will be first to discuss colliding-beam processes at very high energy, then deep-inelastic electroproduction inclusive channels, a few words on small photons and diffraction dissociation of virtual photons, and finally the role of the parton-model in deep-inelastic process.

I. Colliding Beams

We consider only the one-photon annihilation channel and look at the inclusive cross-section

$$d\sigma_i/dpd\Omega = [A_i(p) + B_i(p)\cos^2\theta] \qquad (1.1)$$

for finding a hadron of type i of momentum p emerging from the collision at angle θ relative to the beam. It satisfies the sum rules

$$\int d\sigma_i = \int (d\sigma_i/dpd\Omega)\, dpd\Omega = \bar{n}_i\sigma_{tot}(s) \qquad (1.2)$$

$$1/E_{cm}\int E_i d\sigma_i = \varepsilon_i\sigma_{tot} \qquad (1.3)$$

where ε_i = fraction of C.M.S. energy carried off by species i. Therefore (1.4)

$$\sum_i \varepsilon_i = 1 \qquad (1.5)$$

It follows that from measurement of single-particle distributions at large angles both $\bar{n}$ and σ_{tot} may be determined.

What is the nature of $d\sigma/dp$? The extreme cases are

(a) Identify it with $d\sigma/dp_\perp \sim e^{-ap_\perp}$ in ordinary collisions.

(b) Identify it with $d\sigma/dp_\parallel \sim f(p_\parallel/E_{cm})$.

Case (a), which Brodsky and I looked at,[4] leads to the remarkable prediction of a constant mean energy per particle; therefore

$$\bar{n} \sim E_{cm}/\langle E_{ave}\rangle \sim 3\sqrt{Q^2} \qquad (1.6)$$

which would be spectacular at high energies.

Case (b), most popular theoretically, follows from assuming that dimensional analysis can be used to determine the cross-section behavior for the process. In electroproduction the scaling behavior of νW_2 does follow from dimensional analysis.

The prediction is, for high enough Q^2 and $|p| \gg m_i$

$$d\sigma_i/dpd\Omega = (c/Q^2)(1/p)\,[a_i(2p/\sqrt{Q^2}) + b_i(2p/\sqrt{Q^2})\cos^2\theta]. \qquad (1.7)$$

There is no general argument that states that if νW_2 obeys scaling in electroproduction, then (1.7) follows. To see this consider electron-uranium scattering:

$$e^- + U \to e^- + \text{hadrons} \quad .$$

If the hydrogen data scales, undoubtedly νW_2 for uranium does as well. However, even if

$$e^+ + e^- \to \bar{U} + \text{anything}$$

exhibits scaling-behavior, such a fact is very likely irrelevant to physics. However, this doesn't mean that, <u>within</u> the context of models, use of crossing might not be reasonable. [See, e.g. the diagrams of Figure 1.] This was first considered by Drell, Levy and Yan[5] in their parton-model and by many other since.[6]

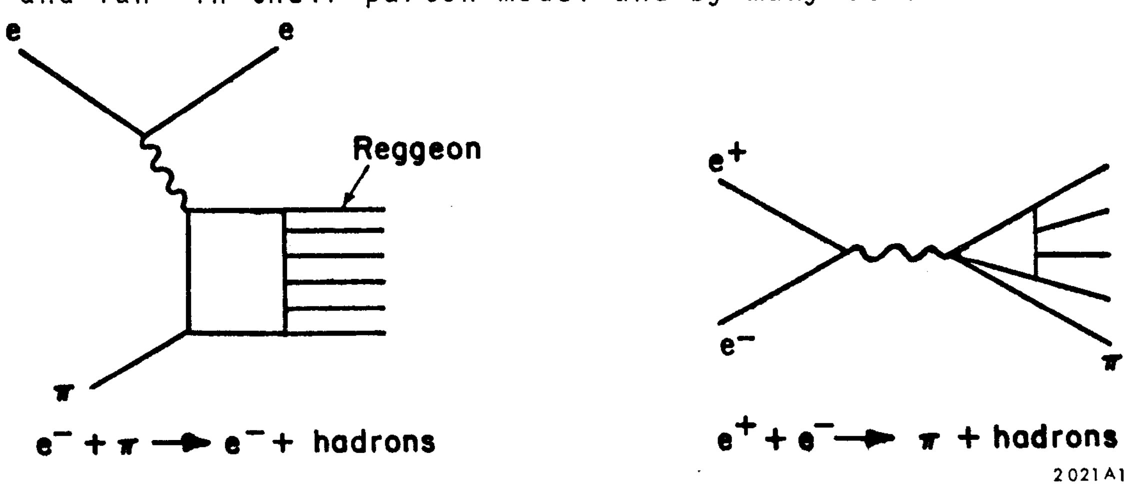


Fig. 1

Moffat and Snell,[7] in a contribution to this conference, show that starting with a reasonable Regge-like formula for $e^- + \pi \to e^-$ + hadrons, one gets, using crossing, a reasonable formula for $e^-e^+ \to \pi$ + hadrons.

If, for simplicity, we average (1.7) over angles and sum over hadron types i

$$p(d\sigma/dp) = \sigma_{tot} f(2p/\sqrt{Q^2}) \quad (p \gg m) \tag{1.8}$$

then

$$\int dx f(x) = 2 \tag{1.9}$$

$$\int dx/x \; f(x) = \bar{n} \tag{1.10}$$

which serves to normalize our ideas about the particle-distribution. Another normalization at $x \sim 1$ comes from what I will call the inclusive-exclusive connection. In electroproduction this is the Drell-Yan-West or Bloom-Gilman relation connecting behavior of νW_2 for $\omega \sim 1$;[8]

$$\nu W_2 \sim (\omega - 1)^n \tag{1.11}$$

to the behavior of resonance form-factors

$$F(q^2) \sim (q^2)^{-p} \tag{1.12}$$

the relation being

$$n = 2p - 1 \quad . \tag{1.13}$$

The inclusive-exclusive connection is equivalent to the statement that the extrapolation of the inclusive distribution into the resonance region (defined as the interval of p for which the mass of the unobserved system is less than some fixed number, say 2 GeV) gives a contribution of the same order as the sum of resonances, independent of external conditions such as angle or beam energy:

$$\int_{\text{resonances}} dp(dN/dp)_{\substack{\text{extrapolated}\\ \text{inclusive}}} \approx \Sigma\,(\text{resonances}) \qquad (1.14)$$

That this is reasonable follows quite generally:

1. Assume the matrix-element for the inclusive distribution can be smoothly extrapolated into the resonance region.

2. Because the mass of the unobserved system is small (say <2 GeV), only a finite number of channels and partial waves should contribute in this region. Assume this is so, and decompose the matrix element into these partial waves.

3. Put Breit-Wigner enhancement factors on the resonant waves. These modify the matrix elements by a finite factor.

This procedure implies the conclusion of (1.14): the ratio of resonance-signal to background is always O(1), i.e. is not systematically dependent on external conditions. Thus if the behavior of $e^+e^- \to \pi^+\pi^-$ is typical of resonant channels and $F_\pi \sim 1/Q^2$ we get Figure 2a. If $F_\pi \sim 1/Q^4$, p(dN/dp) might well look quite similar to νW_2 (Figure 2b), especially if f(0) → const. and $\bar{n} \sim \log Q^2$. This multiplicity growth is not implied by scaling, however.

Which resonances are likely to contribute? This has been studied by several people.[9] Kramer, Uretsky and Walsh[9] use ρ-dominance for this purpose; an immediate conclusion is that G-even final states dominate G-odd. There should be predominantly an even number of π's in the final state. They consider the channels ππ, ρρ, πω, πA1, πA2, the latter three giving the

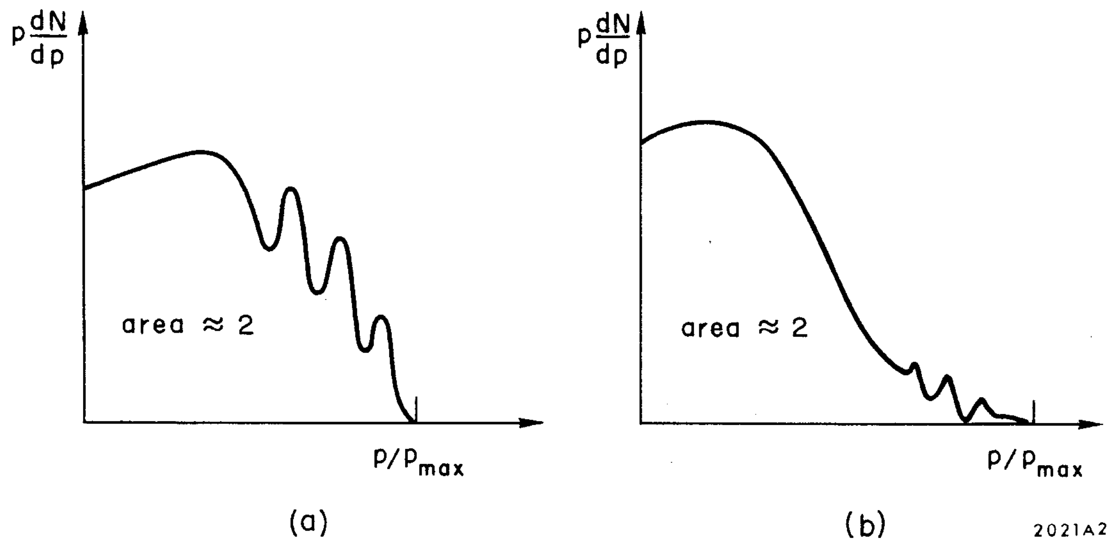


Fig. 2

dominant contribution and of the order of the total Frascati total cross section. Layssac and Renard[9] have estimated still other channels. Of course the ρ is very far off-mass-shell and the conclusions correspondingly fragile. But if these channels do dominate, the single-particle spectrum should show considerable resonant structure. Up to 25% of the single-π spectrum could be contributed by the resonance bumps.

The inclusive-exclusive connection evidently can be used even if the distribution does not exhibit scaling. For example in the case of option (a), the statistical model, we would conclude the pion form factor falls faster than any power of Q^2. For this reason that option looks relatively unattractive.

Finally, in option (b), a large fraction of the secondaries in a given event have large $p \gg 350$ MeV. It is unlikely these are emitted in random directions. Much more likely is that they line up in jets in order that as many pairs have low invariant mass as possible.[10] At very high energies we have, event by event, secondary distributions looking very much like those in, say, $\pi\pi$ scattering but with almost arbitrary orientation of the collision-axis.

II. Inclusive Electroproduction

To discuss the inclusive hadron distributions we shall accept concepts now popular in hadron physics: short-range correlation in rapidity and limiting fragmentation. We describe phase space by the variables $p_\perp$ and

$$y = \frac{1}{2} \log \left(E + p_{\parallel}\right)/\left(E - p_{\parallel}\right) = \log\left(E + p_{\parallel}\right)/\sqrt{p_\perp^2 + m^2} \qquad (2.1)$$

where y is the rapidity-variable of Wilson[11] and Feynman[12]

$$d^3p/E = d^2p_\perp dy \qquad (2.2)$$

The phase volume available for π's has a shape at 10 GeV shown in Figure 3, taken from the SLAC-LRL-Tufts Bubble chamber experiment. The length of phase space varies with $p_\perp$, but roughly is log s. The major virtue of y is that, under a longitudinal boost, $y \to y + \omega$; it displaces. Also, on the average, large Δy between two phase-points (secondary particles) means large invariant mass or subenergy suggesting important correlations only exist for small Δy. This is formally expressed very nicely by Mueller,[11] in terms of cluster expansions[15] (see also Amati *et al.*,[12] Wilson,[13] and R. P. Feynman[14]) of inclusive distribution

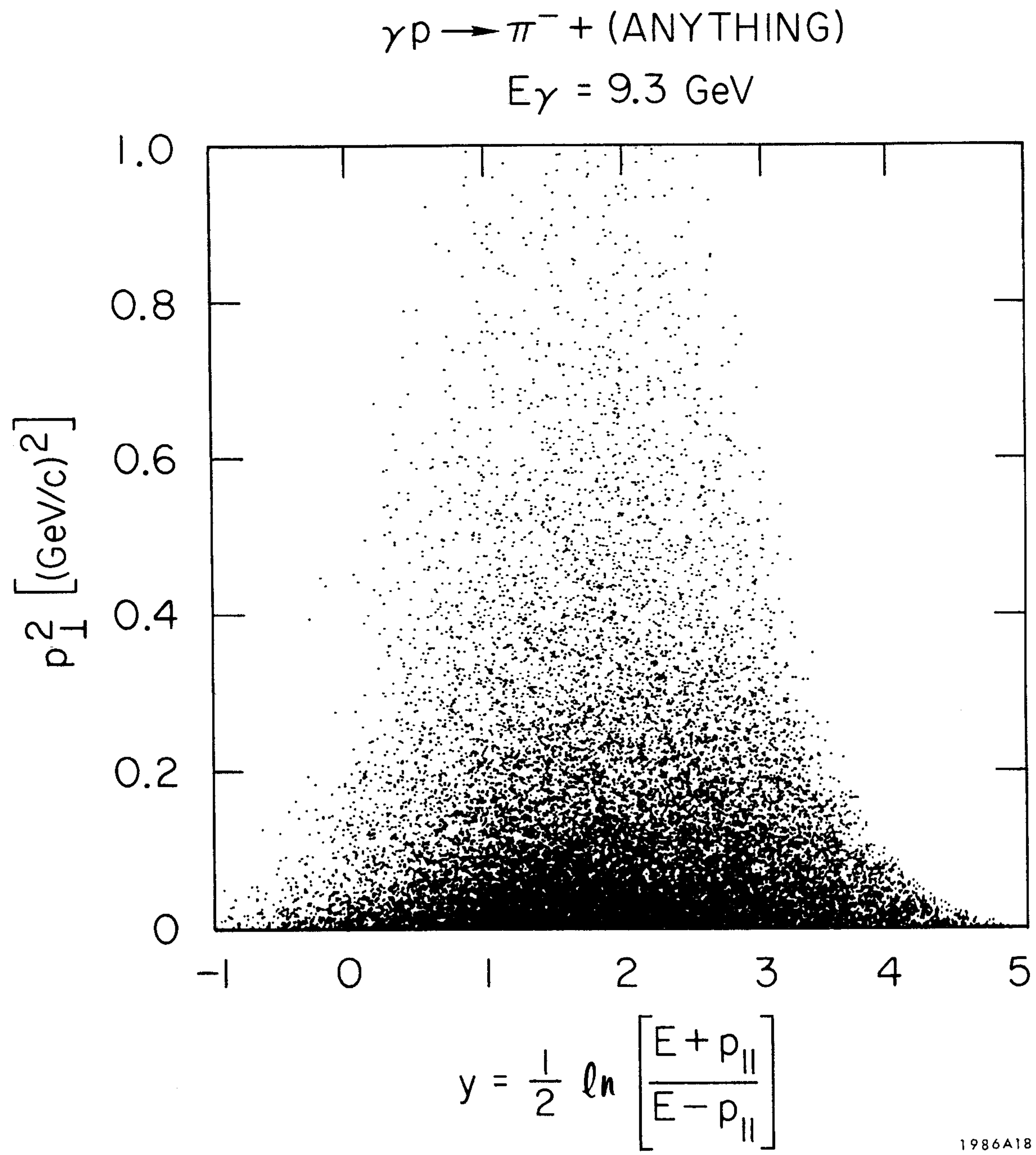

$\gamma p \to \pi^- + (\text{ANYTHING})$
$E\gamma = 9.3$ GeV
1.0
0.8
0.6
0.4
0.2
0
$p_\perp^2 \left[(\text{GeV/c})^2\right]$
-1
0
1
2
3
4
5
$y = \frac{1}{2} \ln \left[\frac{E+p_\parallel}{E-p_\parallel}\right]$
1986A18

Fig. 3

functions. With short-range correlation only, we expect the phase points to be distributed

1. with uniform density far from the boundary

2. with a density near a boundary depending only on the distance from the boundary and the nature of the boundary (target-associated or projectile associated).

Important is the "correlation length" L beyond which there is no correlation. One might expect Reggeistically[11] the correlations to behave as $s^{-1/2} \sim e^{-1/2 \log s}$, suggesting $L \sim 2$. Castagnoli <u>et al</u>.[16] also estimated this by looking at the distribution dN/dy for isotropic decay of a "fireball" into pions of low momentum. dN/dy is approximately gaussian, with full width at half maximum of 2.2 units. Also take two low-energy pions located in phase space as shown in Figure 4a. $y = \pm\frac{1}{2} \log (1+\cos 45^0/1-\cos 45^0) \approx \pm 1$.

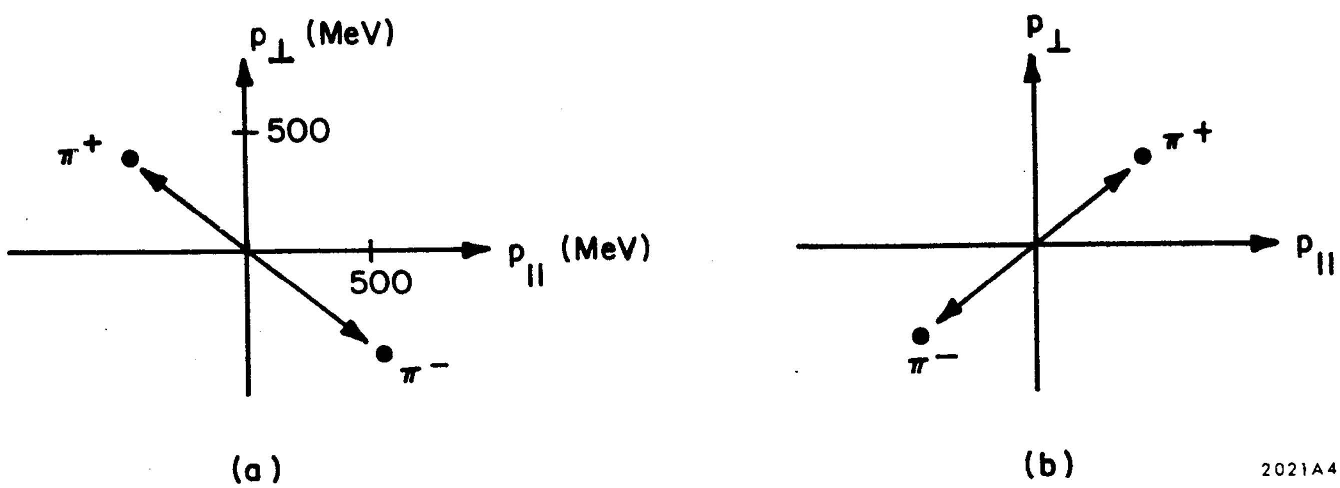


Fig. 4

If they scatter through 90^{0} (Figure 4b) then $\Delta y = 2$. So within a correlation-length, pions (but not K or p,$\bar{p}$) have mobility, and we conclude that because of short-range mobility, pion distributions should not vary rapidly over distances $\Delta y << L$ (except possibly near boundaries). So in hadron scattering processes at extremely high energy there are three regions (Figure 5).

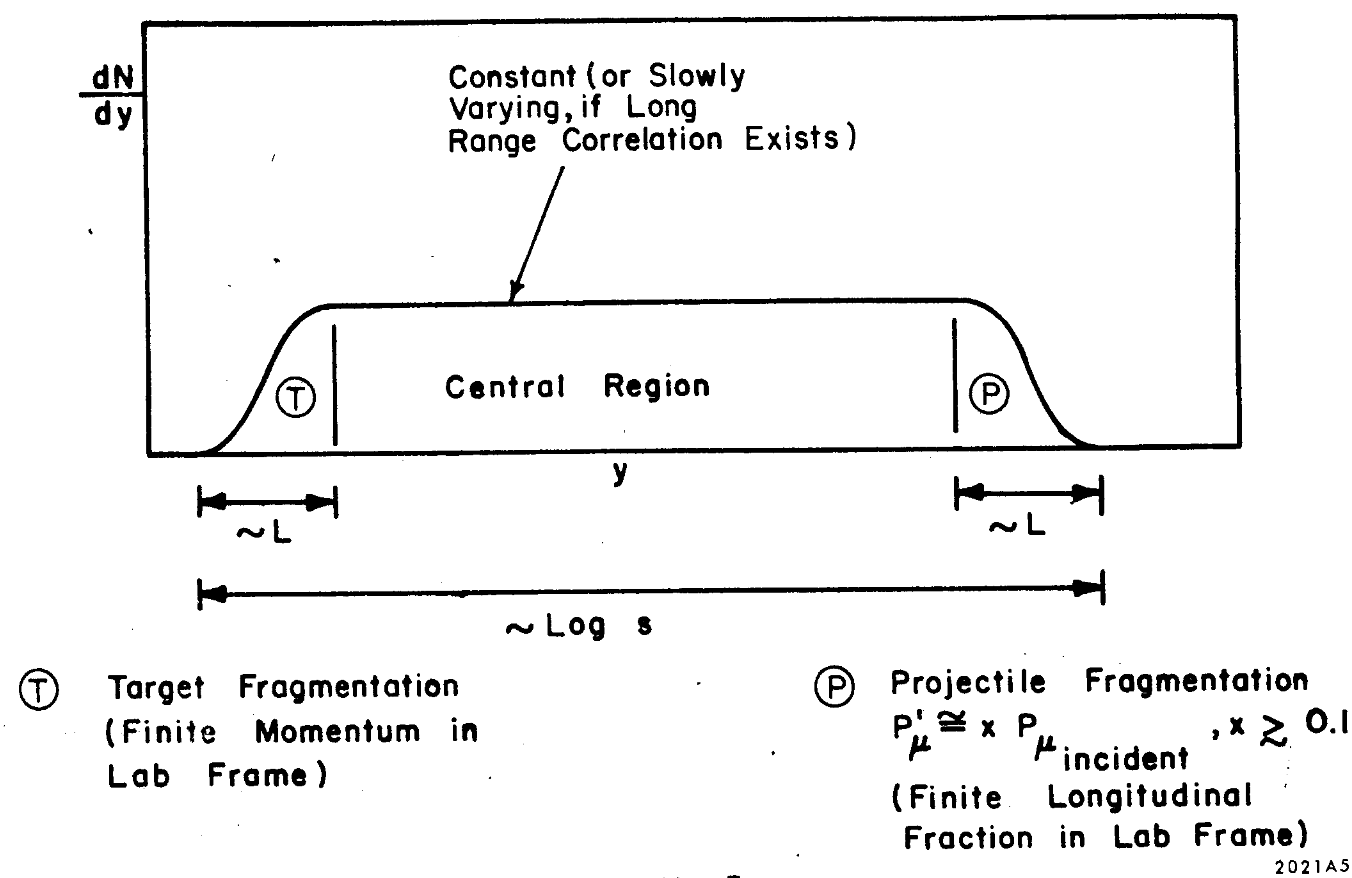


Fig. 5

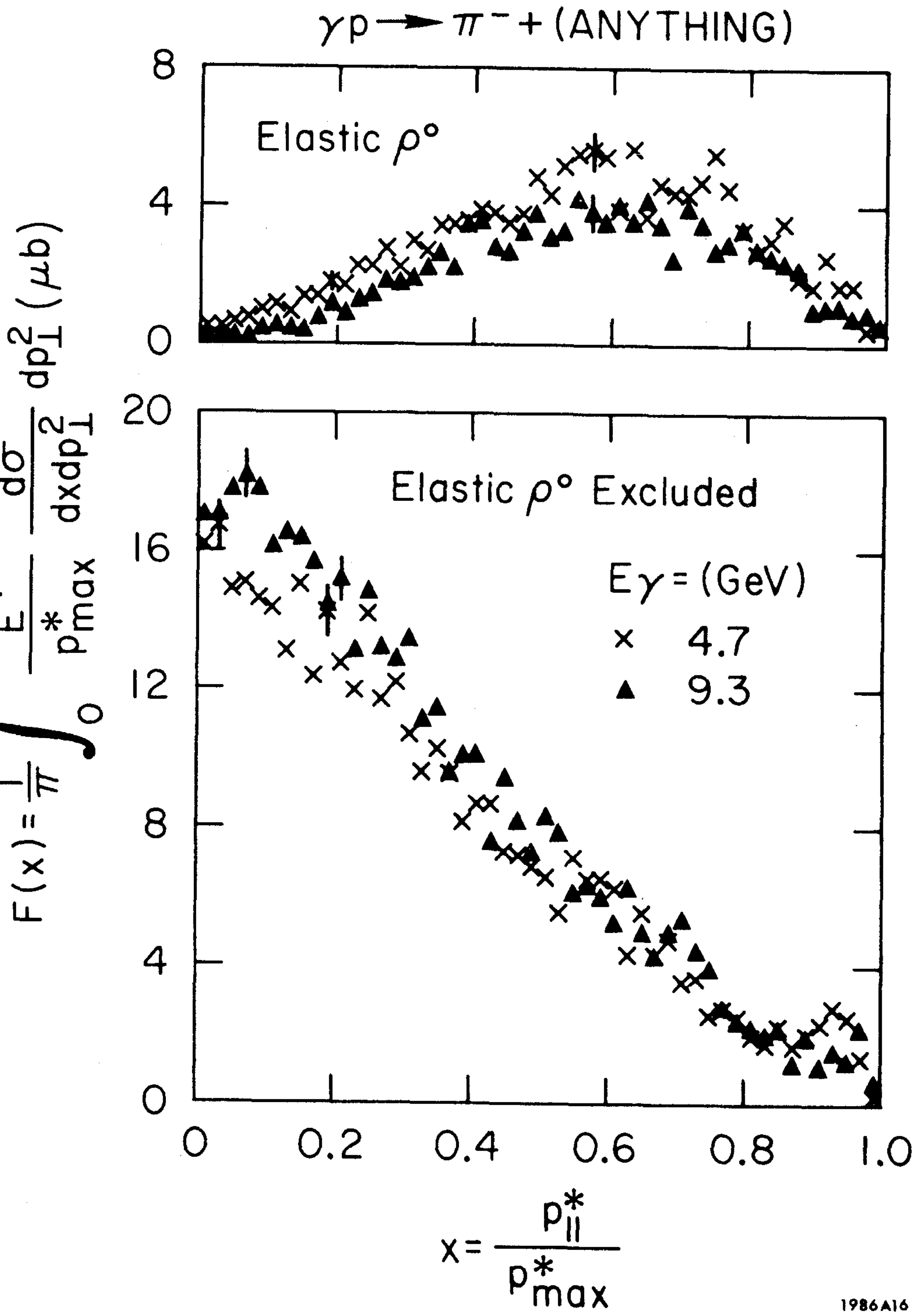
In the projectile-fragmentation region, the inclusive-exclusive connection [resonance signal/inclusive noise = 0(1)] described before leads to the prediction[14]

$$dN/dxdp_{\perp}^{2} = (1-x)^{1-2\alpha(p_{\perp}^{2})} \qquad (2.3)$$

where $\alpha(t)$ is the Regge-intercept describing the energy-dependence of the two-body resonance processes. For photoproduction we expect $\alpha(t)\approx 0$. The data (Figure 6) show the scaling behavior with $\alpha(0) \sim 0$, but $\alpha(t)$ falling [Figure 7] in contradistinction to the behavior in the exclusive channels. The distribution in the target-region should be independent of photon energy, and is verified experimentally. It should be independent of projectile as well, but the normalization differs by a factor ~2.3 from that of K^+p as reported to this conference. Optimistically this may be because the energy is too low.

We will now assume these concepts are applicable to hadron electroproduction as well and need the distribution as function of ν and Q^2, or better $s = W^2$ and Q^2. Start at s very large and $Q^2 = 0$ and increase Q^2. How does the distribution change? Only the projectile has changed, so the assumption of short-range correlation in rapidity implies only the distribution in the projectile fragmentation region, and possibly its size, changes. How big is the projectile-fragmentation region? We now keep Q^2 fixed and large (>>1 GeV^2) and decrease s until projectile communicates with target. This happens when $\omega \approx (s/Q^2) \lesssim 4$; when $\nu W_{2p} \neq \nu W_{2n}$ for example (Figure 8a). Therefore the length of the projectile fragmentation region is (Figure 8b)

$$\sim \log s = \log \omega + \log Q^2 \sim \log Q^2 \ .$$



γp → π⁻ + (ANYTHING)
Elastic ρ°
Elastic ρ° Excluded
Eγ = (GeV)
× 4.7
▲ 9.3
F(x) = 1/π ∫₀^∞ E*/p*max dσ/dxdp⊥² dp⊥² (μb)
x = p*∥ / p*max
0
4
8
12
16
20
0.2
0.4
0.6
0.8
1.0
1986A16

Fig. 6A

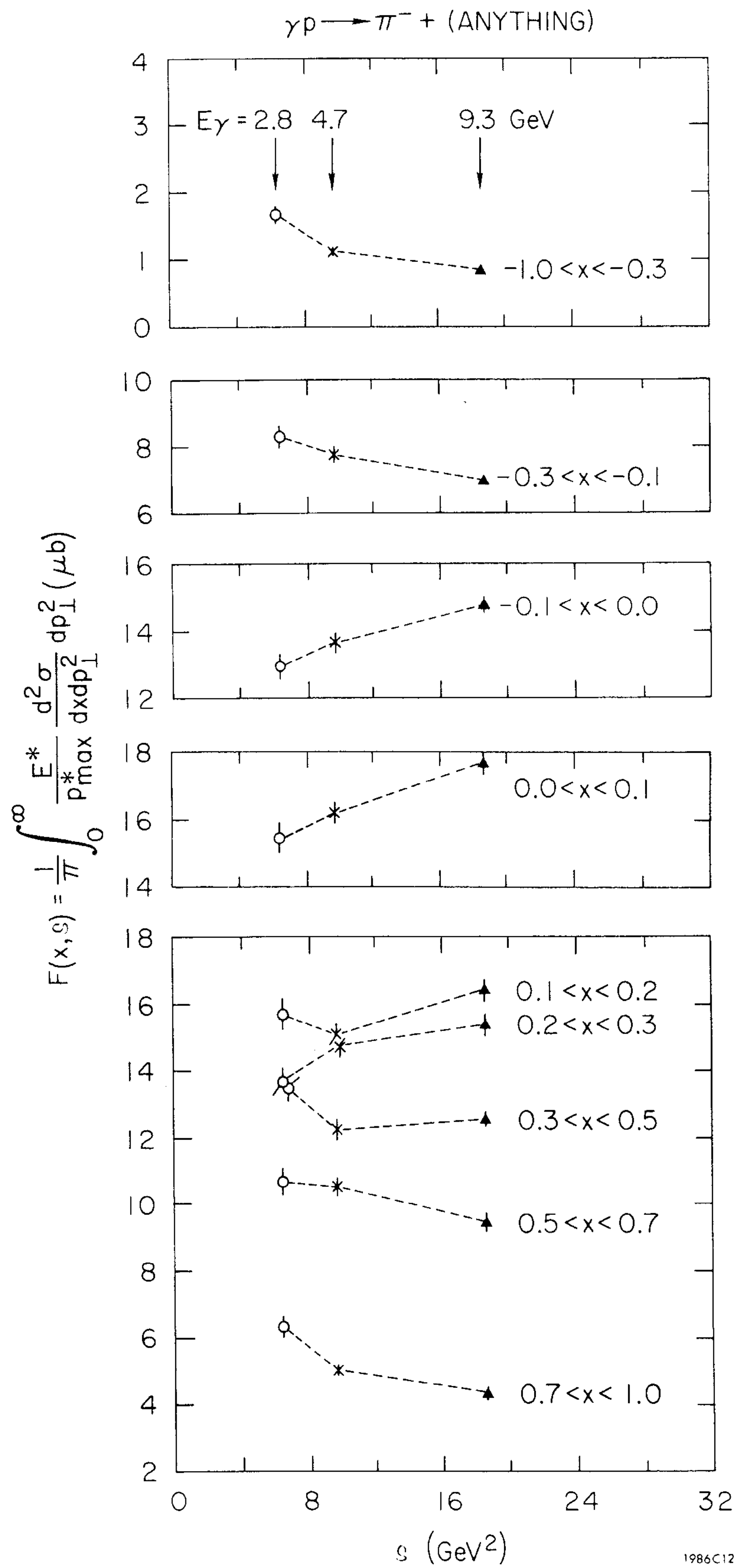

γp → π⁻ + (ANYTHING)
Eγ = 2.8
4.7
9.3 GeV
−1.0 < x < −0.3
−0.3 < x < −0.1
−0.1 < x < 0.0
0.0 < x < 0.1
0.1 < x < 0.2
0.2 < x < 0.3
0.3 < x < 0.5
0.5 < x < 0.7
0.7 < x < 1.0
F(x,s) = 1/π ∫₀^∞ E*/p*max d²σ/dxdp⊥² dp⊥² (μb)
s (GeV²)
0
8
16
24
32
1986C12

Fig. 6B

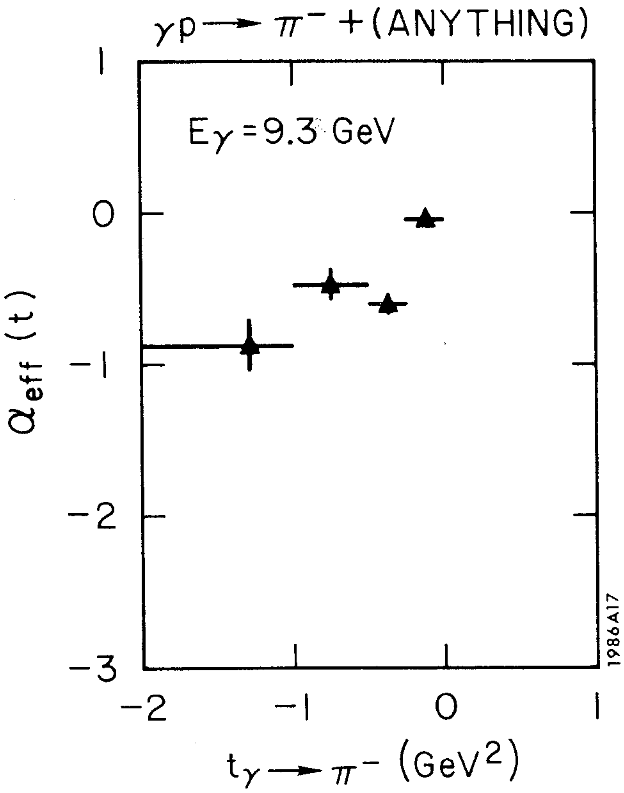


Fig. 7

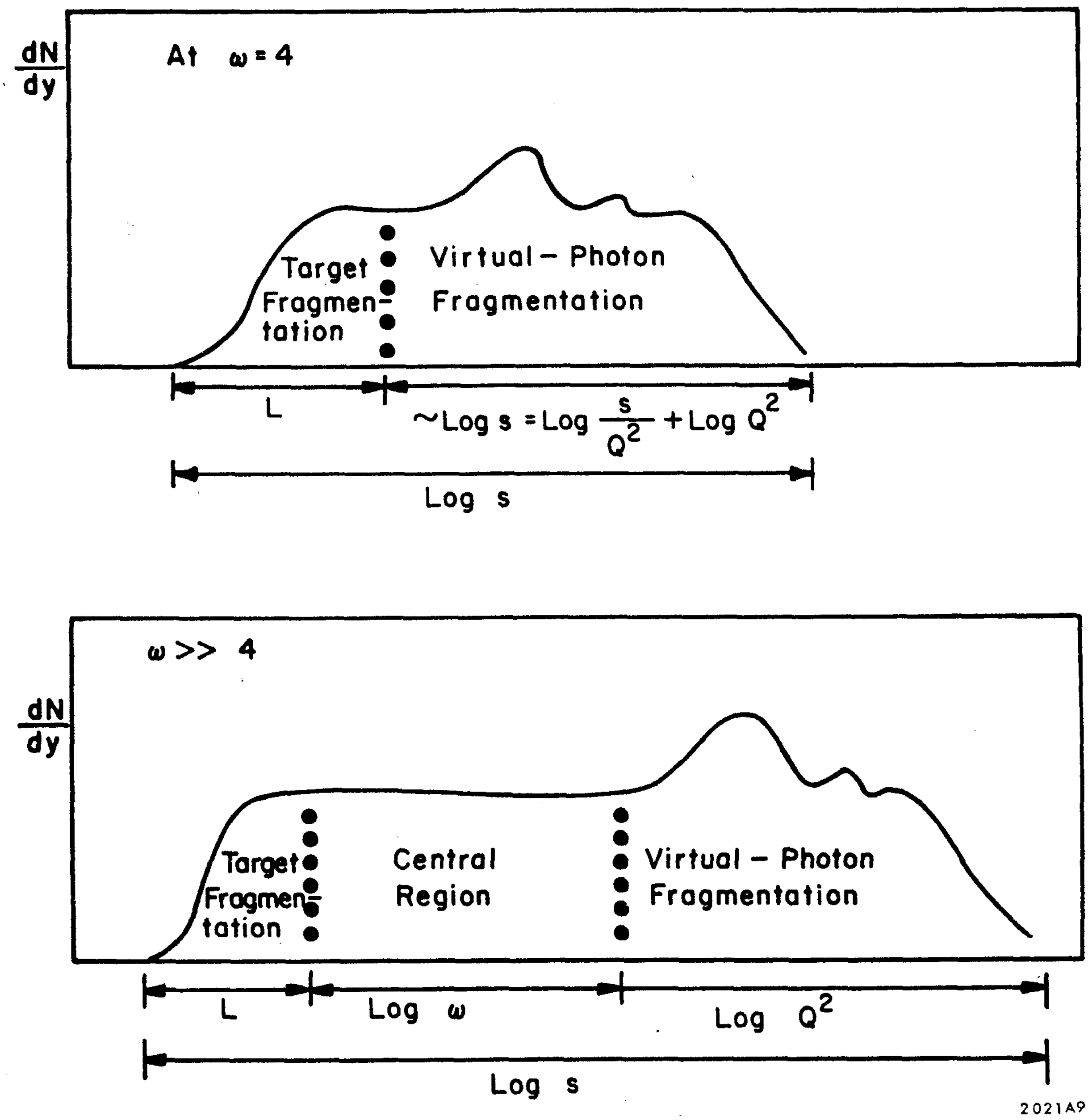
We also conclude that because for $\omega \gg 4$ the distribution outside the photon fragmentation region is independent of Q^2, the mean multiplicity is

$$\bar{n}(\omega, Q^2) \sim c \log \omega/4 + \bar{n}(4, Q^2) \qquad (2.4)$$

with

$$c \sim 1.1 \pm 0.2 \quad , \qquad (2.5)$$

the number believed[17] to describe the central density in ordinary collisions. This picture also probably implies $\nu W_2 > 0$ as $\omega \to \infty$. When ω is $\gtrsim 1$ both fragmentation regions merge, and a description



dN/dy
At ω = 4
Target Fragmentation
Virtual – Photon Fragmentation
L
~Log s = Log s/Q² + Log Q²
Log s
ω >> 4
dN/dy
Target Fragmentation
Central Region
Virtual – Photon Fragmentation
L
Log ω
Log Q²
Log s
2021A9

Fig. 8

in terms of resonance production, as pursued by Domokos, Kovesi-Domokos and Schonberg,[18] may well be the best way to proceed.

These general arguments encompass many similar considerations presented to this conference.[19-21] But they tell us very little about what to expect in the photon fragmentation region. The options include

1. $\bar{n}(4, Q^2)$ is a finite number, even when $Q^2 \to \infty$:

$$\bar{n} = \text{constant} + \log \omega \qquad (2.6)$$

This came out of the Drell-Levy-Yan model[22] and some early multi-peripheral calculations.[23] In my heart I know it's wrong.

2. $\bar{n}(4, Q^2)$ increases as a power of Q^2; this is in the spirit of a model of Chou and Yang.[24] They call this photon pulverization. However they assert

$$\bar{n}(\omega, Q^2) \to s^{\alpha(\omega)} \qquad (2.7)$$

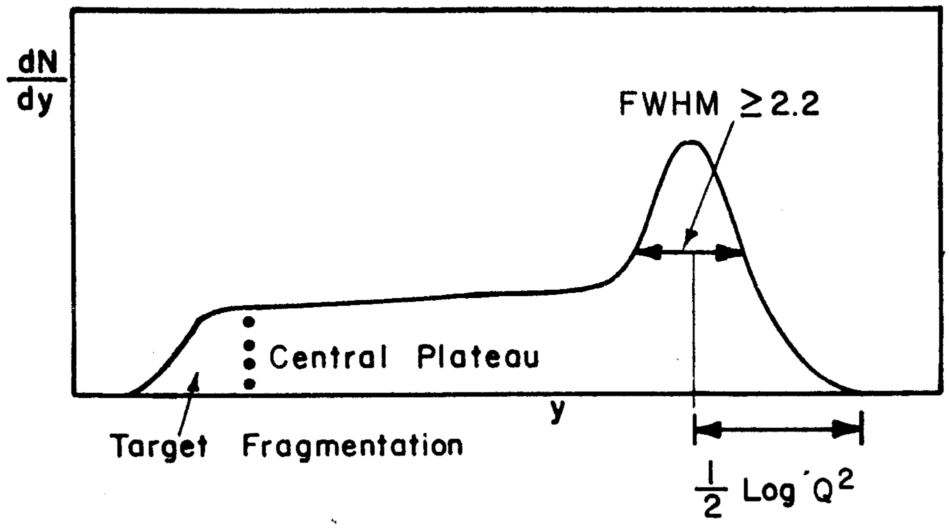
with

$$\alpha(\omega) \to 0 \text{ as } \omega \to \infty$$
$$\alpha \leq 1/2 \qquad (2.8)$$

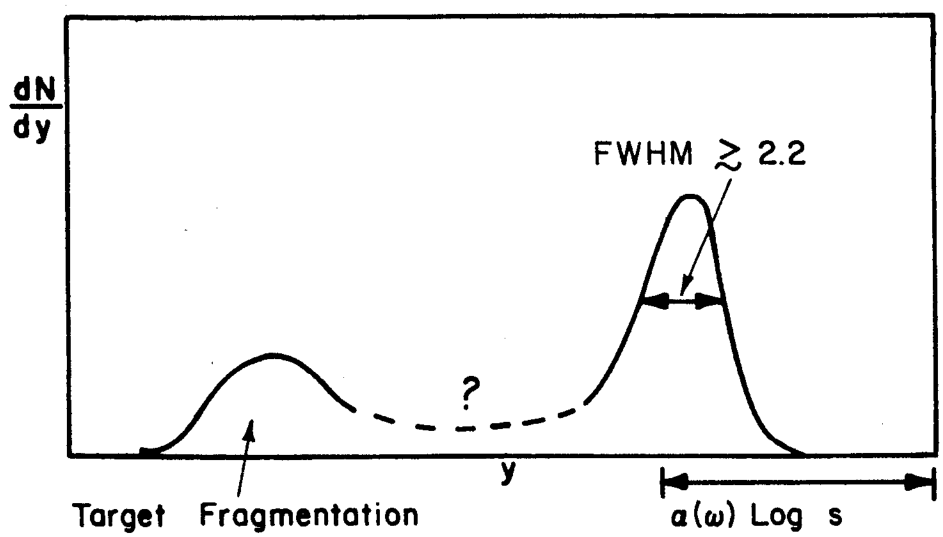
Within our ground rules of short-range order, a fireball (gaussian in y, FWHM ~2.2, area = $\bar{n} \approx 3\sqrt{Q^2}$) located in the middle of the fragmentation region would nicely mesh with the statistical model, option (a), in the colliding beam process. In the Chou-Yang model, the position of the fireball is a distance

$$\log n \simeq \alpha(\omega) \log s \qquad (2.9)$$

from the boundary. This violates the condition of short-range order in rapidity, which is not implied by the droplet model. In certain parts of Stony Brook, the central plateau is below sea level.



(a) Photon Pulverization With Limiting Fragmentation



(b) Chou – Yang Pulverization

2021A10

Fig. 9

3. $\bar{n}(4, Q^2)$ grows logarithmically with Q^2. Perhaps even

$$\bar{n}(4, Q^2) \sim c \log Q^2 \tag{2.10}$$

so that
$$\bar{n}(\omega, Q^2) \sim c[\log \omega + \log Q^2] = c \log s \tag{2.11}$$

just as for the usual processes. Within this option we divide into two more possibilities:

3a) High $p_\perp$ in fragmentation region

3b) Ordinary $p_\perp$ in fragmentation region

Option (3b) is favored in parton-models, especially some multiperipheral or dual models. I will discuss it in the context of the kindergarten parton-model[25] at the end of this talk. But option (3a) can be made plausible as well, as follows.

The phenomenon of leading particle is well-established in $\pi^- p$ and pp collisions; one particle with the quantum-numbers of the projectile emerges, on the average, with half the incident momentum and a flat momentum distribution. (Figure 10)

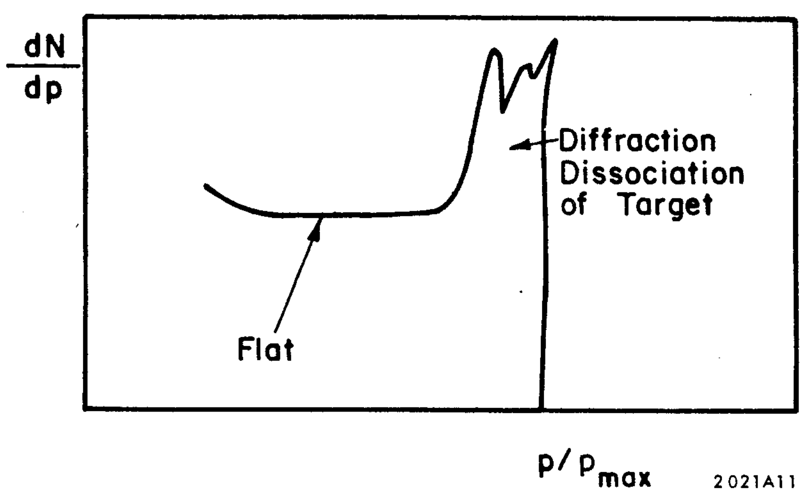


Fig. 10

In real γ-p collisions ρ-dominance suggests that there should be many leading ρ's. Indirect evidence for this exists in the data of the SLAC-LRL-Tufts experiment which shows a correlation of the azimuthal distribution of leading photoproduced π's with the photon polarization vector[26] (Figure 11). But perhaps a cleaner test lies in measurement of the distribution of momentum of photoproduced ϕ's. It should be flat and rather large. The formula is

$$d\sigma/dp \cong 1/k\ (\pi\alpha/\gamma_\phi^2)\sigma_{\phi N}\ . \tag{2.12}$$

Now make the photon virtual, and continue to use vector dominance, but not ρ, ω, ϕ dominance. We shall need the spectrum of masses of $J = 1^-$ states coupled directly to the photon. This is directly measured in experiments on coherent production from nuclei. For large Q^2 let us guess this using dimensional analysis. That has worked before:

$$(d/dm^2)\sigma^{Diff} \sim (1/Q^4)f(M^2/Q^2) \qquad (s \to \infty;\ Q^2 \text{ fixed}) \tag{2.13}$$

For example $$f \sim (Q^2/m^2 + Q^2)^2$$

$$\sigma_T^{\text{total diffractive}} = (1/Q^2) \int dy f(y) \quad \begin{pmatrix} s \to \infty \\ Q^2 \text{ fixed} \end{pmatrix} \tag{2.14}$$

The total diffractive component is roughly proportional to

$$\sigma_T^{total}(Q^2) \approx \text{const.}/Q^2 \tag{2.15}$$

just as in ordinary processes. Equation (2.13) implies

$$\langle m^2 \rangle = [Q^2 \int dy\ yf(y)/\int dy\ f(y)] \propto Q^2 \tag{2.16}$$

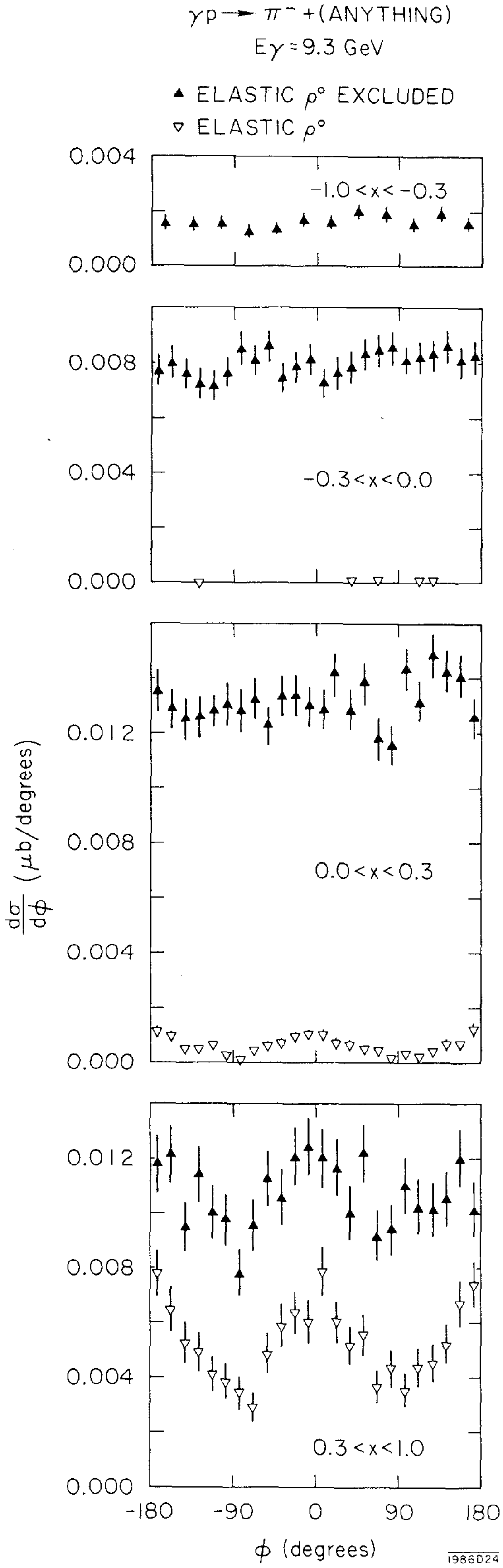

γp → π⁻ + (ANYTHING)
Eγ = 9.3 GeV
▲ ELASTIC ρ° EXCLUDED
▽ ELASTIC ρ°
-1.0 < x < -0.3
-0.3 < x < 0.0
0.0 < x < 0.3
0.3 < x < 1.0
dσ/dφ (μb/degrees)
0.000
0.004
0.008
0.012
-180
-90
0
90
180
φ (degrees)
1986D24

Fig. 11

What are these massive states coupled to the photon? They are the same as produced in the colliding-beam reaction; here we clearly should take the low-multiplicity option (b) of jets. Quite often the jet will be aligned perpendicular to the direction of q, leading to high $p_\perp$ as shown in Figure 12a. A simple estimates gives, for $x = p_{||}/\nu$

$$\langle p_\perp^2 \rangle \sim 0.1\ \mathrm{GeV}^2 + x^2 Q^2 (a + b \cos^2\theta) \qquad (2.17)$$

where a and b could be of order unity x<1/2 but is probably smaller. ϕ is the angle between the plane of the leptons and the plane defined by q and $p_\perp$.

---

* If Gribov's picture of diffractive processes[28] is at all applicable, these results could emerge in a rather subtle way. Assuming constant absorption of a state of any mass m on the target, Gribov obtains for real γ's

$$\sigma_\perp \to (1 - Z_3)_{had}\sigma_o \propto \int_o^s dm^2 \sigma(m^2)_{e^+e^- \to \text{hadrons}} \qquad (A1)$$

and for virtual γ's

$$\sigma_\perp(Q^2) \propto \int_o^s dm^2 \sigma(m^2)(m^2/m^2 + Q^2)^2 \qquad (A2)$$

For $\sigma(m^2) \propto m^{-2}$ as we presume, this leads to much too large a value for $\sigma_\perp(Q^2)$:

$$\sigma_\perp(Q^2) \propto \log s/Q^2 \qquad (A3)$$

not $\sim Q^{-2}$, as expected for the total electroproduction. Therefore the picture is inconsistent. This inconsistency can be removed if one considers again the nature of the state m as two jets, and presumes that the target is opaque to the jets only when the axis is approximately parallel to q. The probability of this is

$\sim\langle p_\perp^2\rangle/m^2$, turning (A2) into (2.13). This results interlocks nicely with the parton picture. However even a small residual opacity of the target to transversely oriented jets $\propto s^{-1}$ would lead to a fraction of the diffractively produced states (of order ω^{-1}) possessing the two-jet structure. An experimental search would seem justified.

III. Small Photons

High $p_\perp$ is suggested from another direction as well, as discovered by Cheng and Wu.[27] Consider again the case of $s \to \infty$ at fixed Q^2. Under these circumstances, we expect vector dominance to work and the γ to convert to some hadronic system V and that system V then interacts with the target. What Cheng and Wu found, in the context of QED, is that if one looks at the internal structure of V in the two-dimensional transverse configuration space the energetic partons are concentrated in a region of order

$$\Delta X_\perp \sim 1/\sqrt{Q^2} \qquad (3.1)$$

Another way of seeing this is from the estimate of important distances in the light cone commutator discussed by Treiman and Wilson. There again $\Delta X_\perp \lesssim 1/\sqrt{Q^2}$; indicating that if the initial hadron system is mismatched transversely with the final by more than $\Delta X_\perp \sim 1/\sqrt{Q^2}$, there is little overlap. The reason for the difference from real photons lies in the energy denominators of the virtual states, which are

$$\Delta E \sim (Q^2 + m^2)/2\nu \qquad (3.2)$$

Thus Δt and Δz decrease with increasing Q^2, and the matter, which originates at a point, has less time to spread transversely before

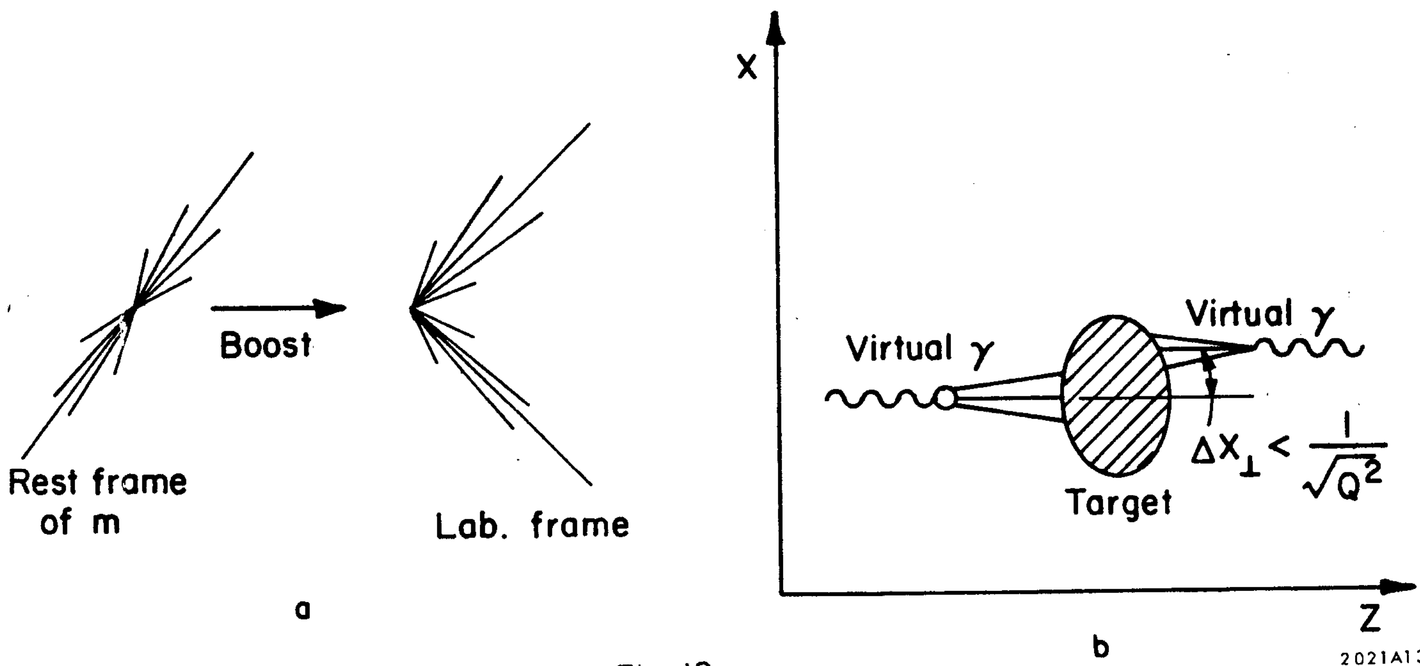


Fig. 12

interacting (Figure 12b). What are the consequences?

1. The diffraction peak for the ρ-electroproduction should broaden with increasing Q^2, but not more than a factor 4. Writing $d\sigma/dt \sim e^{at}$, a as a function of Q^2 is shown in Figure 13.

2. The absorption cross section on a nucleon is now πR^2, not $4\pi R^2$; therefore the absorption mean-free-path is increased by a factor $\lesssim 4$. Therefore the effective number of nucleons contributing should increase sharply as Q^2 increases; i.e.

$$\sigma(e + A \to e + \rho^0 + A; Q^2)/\sigma(e + p \to e + \rho^0 + p)$$

$$= \text{increasing function of } Q^2 \qquad (3.3)$$

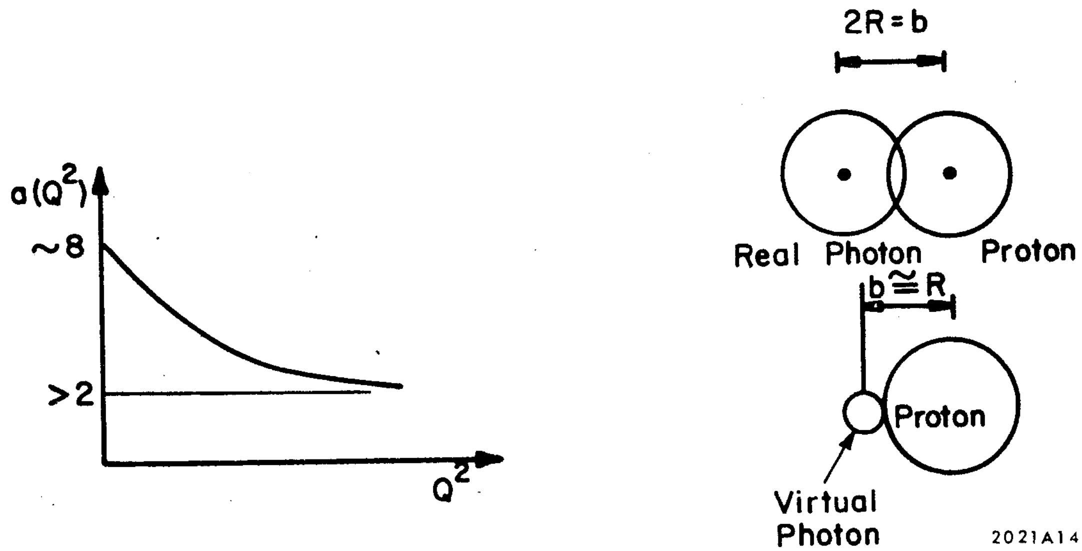


Fig. 13

3. Because the photon is small, the target fragmentation may decrease somewhat. A cannonball does more damage than a bullet. But this depends on the existence of long range order in rapidity.

IV. Other Exclusive Processes

The most accessible channel, single-π electroproduction, has several interesting features. Certainly the Q^2 dependence of the sharp forward peak in $d\sigma/dt$ tells us much about the pion form factor. One of the most interesting questions is what will happen between the resonance region ($s < 4$ GeV^2) and Regge region ($\omega \gtrsim 10$; that is $s > 10\ Q^2$). As Q^2 increases the gap widens, and the duality connections between Regge-tail and resonances become more remote, (Figure 14 Frishman, Rittenberg, Rubinstein, and Yankielowicz[29] suggest that

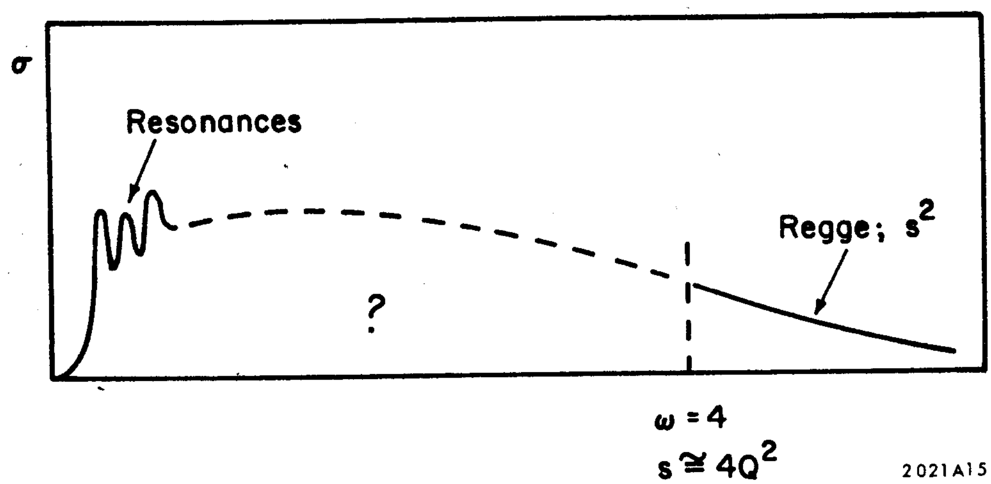


Fig. 14

this exclusive channel is dominated by a commutator on the light cone. This is a weak argument, as the authors themselves show. From the expression

$$A \sim \int d^4x\; e^{iq.x} \langle p'|[j_\mu(x),\; \phi_\pi(0)]|p\rangle\, \theta(x_0) \qquad (4.1)$$

analysis of the factor $e^{iq.x}$ [assuming no rapid oscillations in the commutator] leads to the important distances being dominated by the light cone. Rewriting it as follows

$$A = \int d^4x e^{-ip_\pi \cdot x} \langle p'|[j_\mu(x), \phi_\pi(0)]\, p\rangle \, \theta(x_0) \qquad (4.2)$$

gives the opposite conclusion. Nevertheless the answer appears reasonable and doesn't violate common sense. They say the invariant amplitudes A_i should satisfy

$$A_i = s^p f(\omega, t) \qquad (4.3)$$

Roy[30] studies the pion electroproduction in a manner analogous to the study of the elastic form factor by Drell and Yan,[8] using parton ideas. This is related to comments made by Feynman earlier in this conference. Roy concludes that at fixed ω

$$\sigma_{tot}(s, Q^2) \sim F_\pi(Q^2) F_N(Q^2) g(\omega)$$

consistent with (4.3).

There is an embarrassment in this channel with regard to the inclusive-exclusive connection discussed before. For fixed Q and $s \to \infty$ we expect Regge asymptotics to still apply, so that

$$\sigma^{\perp}_{(\gamma p \to \pi N,\ \pi\Delta,\ \text{etc.})} \propto s^{-2} \qquad (4.4)$$

(We consider transverse photons only) This implies for the inclusive distribution

$$dN/dx \propto (1 - x) \qquad (4.5)$$

where $x = (p/\nu)_{lab} \sim 1$. If this is an extrapolation of the entire differential cross section, then

$$d\sigma/dx \approx \text{const.}(1 - x)/Q^2 \qquad (4.6)$$

according to scale invariance. Then the inclusive-exclusive connection implies

$$\sigma^{\perp}(\gamma p \to \pi N,\ \pi\Delta,\ \text{etc.}) = \text{const.}/Q^2 s^2 \tag{4.7}$$

which looks much too big. Probably the scaling contribution falls as a larger power of (1 - x) not connected to the leading trajectory but to those which build the dotted part of the curve in Figure 14 while the exclusive processes fall more rapidly with Q^2 than given by Eq. (4.7).

V. The Parton Model and Inclusive Processes at High $p_\perp$

In this section I will describe this subject in the context of work done in collaboration with John Kogut and Sam Berman.[25] Many of the conclusions have been reached by many others,[5,19,20,31,32] and especially in the work of Drell and Yan.[32]

Longitudinal phase space grows as log s. The entire phase space grows as $\int(d^3p/E) \propto s$. Even at the CERN-ISR there are only a paltry 6 correlation lengths of longitudinal phase space in which to study inclusive distributions. It would appear that the high p_T phase space is worth a great deal of attention; indeed our experience with the deep inelastic phenomena shows that there exists processes which populate the high-p_T region of lepton phase space with a relatively large number of particles, and the process is not uninteresting.

What about ordinary hadron collisions, say pp collisions? Both the observations, and Hagedorn's thermodynamic model for $p_\perp$ distributions suggest a fall faster than any power. Look at the $p_\perp$ distribution expected for pp collisions $pp \to \pi$ + anything and $\theta_{cm} = 90^o$, $E_{mc} = 400$ GeV (Figure 15). No experimentalist wants to measure a curve that is vertical. But is there really so much nothing out there? The answer is very likely no. In pp scattering, just the deep inelastic electromagnetic scattering itself provides a mechanism for populating the region, as emphasized by Berman and Jacob.[33] On Figure 15 is shown the function $4\pi\alpha^2/p_\perp^4$, the natural scale for the electromagnetic scattering. Even giving away

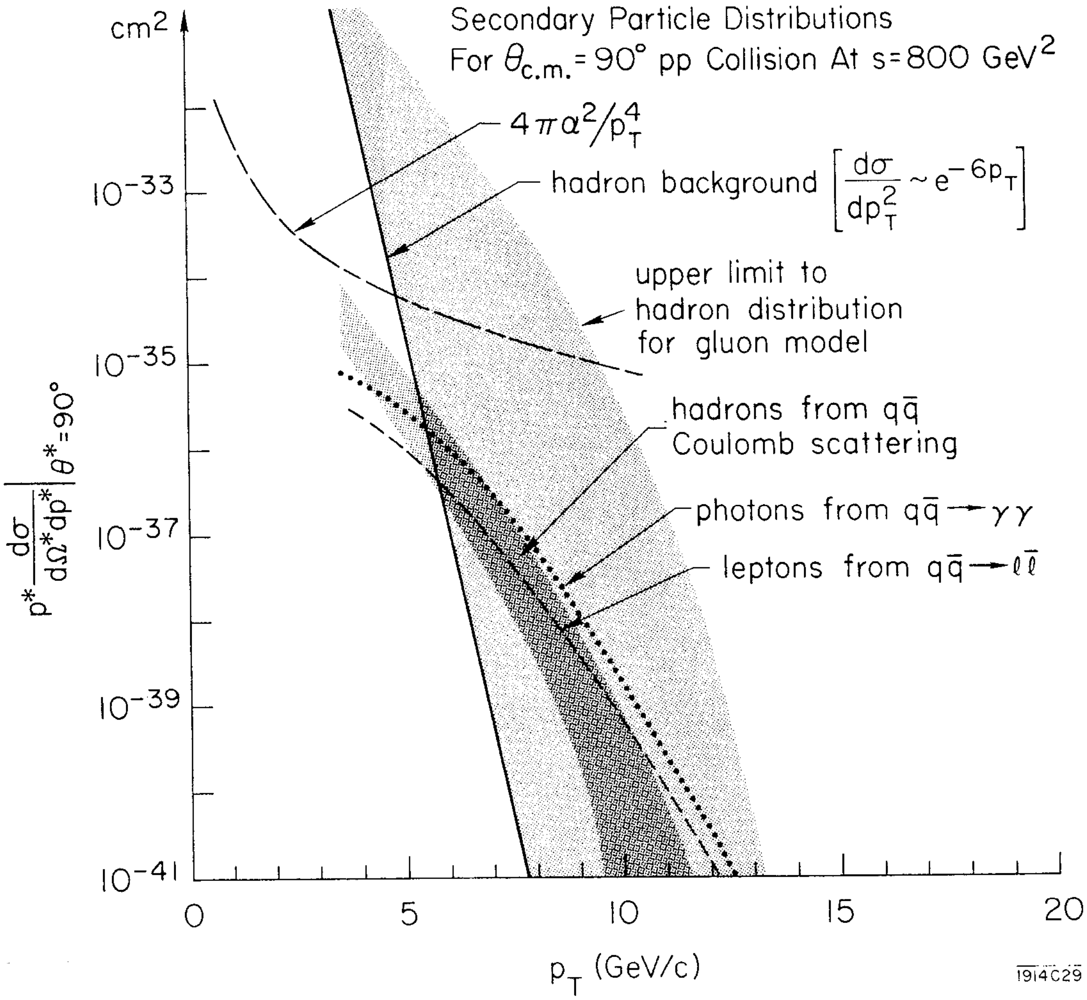


Fig. 15

several powers of 10 for distributing the $p_\perp$ of the photon among several final hadrons, we get something far above the Hagedorn curve. The problem is to estimate it better. Considering that our ignorance is measured by factors of 10^4, even crude models can help. It is in this spirit we use the parton model to attempt a description.

Recall from Ken Wilson's lecture that, to everyone but Ken, partons are massless point particles, quanta of some H_0, suitable for describing the constitution of very energetic real particles during scattering processes. At such high energy there are three elements to the description:

1. Decomposition of the incident particle into its beam of constituent partons. Because of the time dilation effect there is supposed to be no mutual interaction of the partons during the collision.

2. The collision itself. This is presumed to be simple at such high energies. In quantum electrodynamics the charged partons simply acquire a Coulomb phase as they pass by each other. Also possible in QED is exchange of partons in those regions of phase space for which the distributions of projectile and target overlap. This is the mechanism Feynman assumes predominates in hadron collisions. Yang's view is more similar to the Coulomb scattering.

3. The third, most difficult, stage of the picture is to find the overlap of the produced parton configuration with the system of final hadrons which is observed.

Now, a deep inelastic process will be defined as one in which a few incident partons collide violently with exchange of large $p_\perp$. We call this an elementary process; the simplest is two-body scattering via trilinear couplings, which we adopt. Clearly one can generalize to elementary production processes, etc., but these we shall here ignore, arguing that they are no doubt even more rare than the scatterings. It is useful to catalogue all possible generic processes for leptons, hadrons, and photons. These are to be calculated with point vertices (Figure 16).

We now return and discuss the three stages of the process again.

For deep inelastic processes, it is reasonable to assume the beams of incident partons scatter incoherently from each other

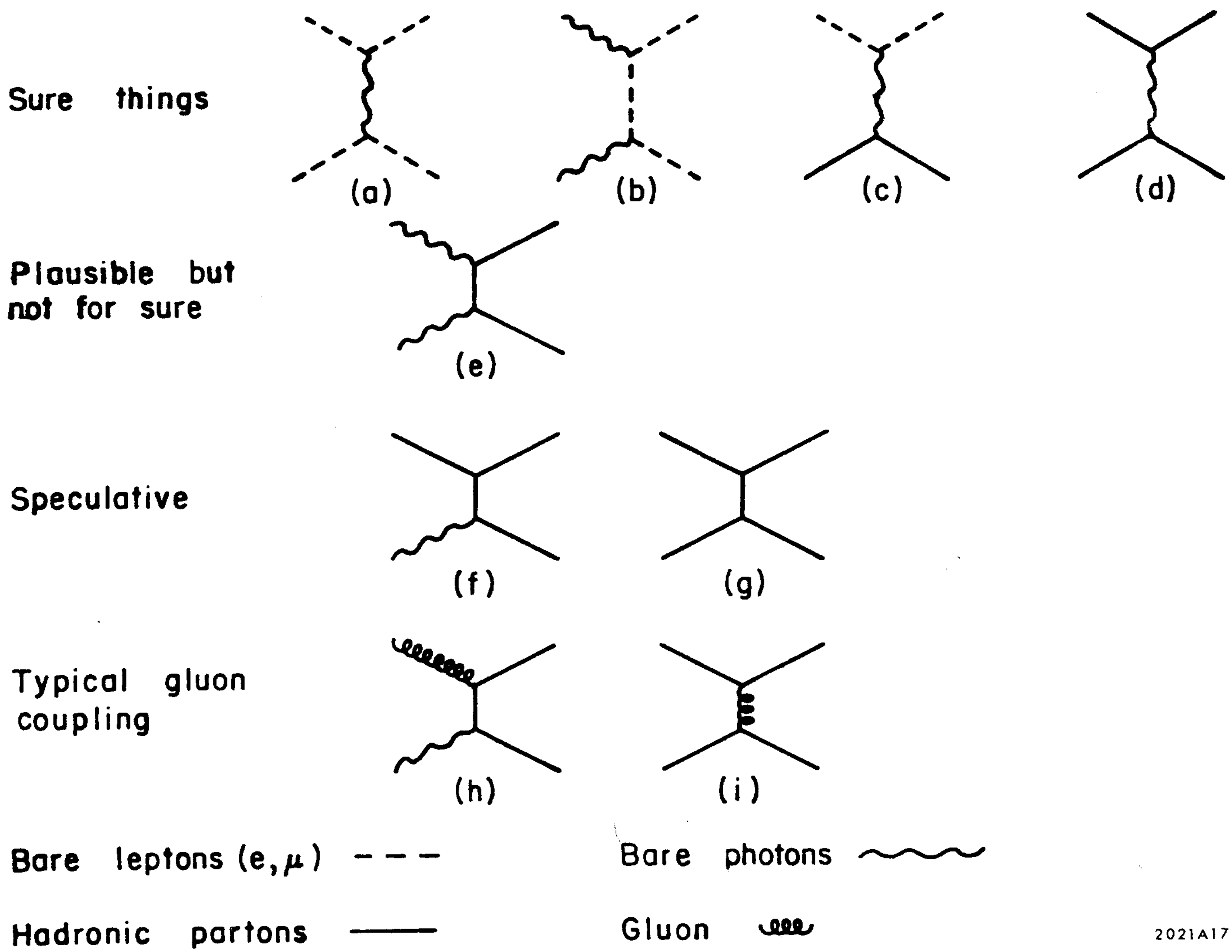

Sure things
(a)
(b)
(c)
(d)
Plausible but not for sure
(e)
Speculative
(f)
(g)
Typical gluon coupling
(h)
(i)
Bare leptons (e, μ)
Bare photons
Hadronic partons
Gluon
2021A17

Fig. 16

because of the large $p_\perp$ exchange. All we need is the inclusive distribution function for the partons in the incident beam. Feynman tells us what to write down: the number dN_{ai} of partons of type i having fraction x in dx of the momentum of the incident hadron a is given by

$$dN_{ai} = F_{ai}(x)\ dx/x \tag{5.1}$$

This gives the momentum spectrum of the incident parton beam. The point cross section for elementary process (a), applicable to electroproduction, has a δ-function in it, so that the inclusive cross section with lepton observed directly measures $F_{ai}(x)$

$$\nu W_2 = \sum_i e_i^2 F_{ai}(x) = F(x) \tag{5.2}$$

In this case, one doesn't have to cope with stage three, understanding the relationship of final hadrons and intermediate partons. However in general one does, and to do this it is convenient to picture the configurations of partons in momentum space. We take three processes

$$pp \to \text{hadrons}$$

$$ep \to e + \text{hadrons}$$

$$e^- + e^+ \to \gamma \to \text{hadrons}$$

Before the collision, the configurations are shown in Figure 17. The mean $\langle p_\perp \rangle$ in the initial state is taken small ~300 MeV. Immediately after the deep inelastic collision, we get the configuration in Figure 18. How do these evolve? Partons being eigenstates of H_0, aren't stable. With ordinary low $p_\perp$ mechanisms dominating parton division, after a while we get Fig. 19. At this point each of these

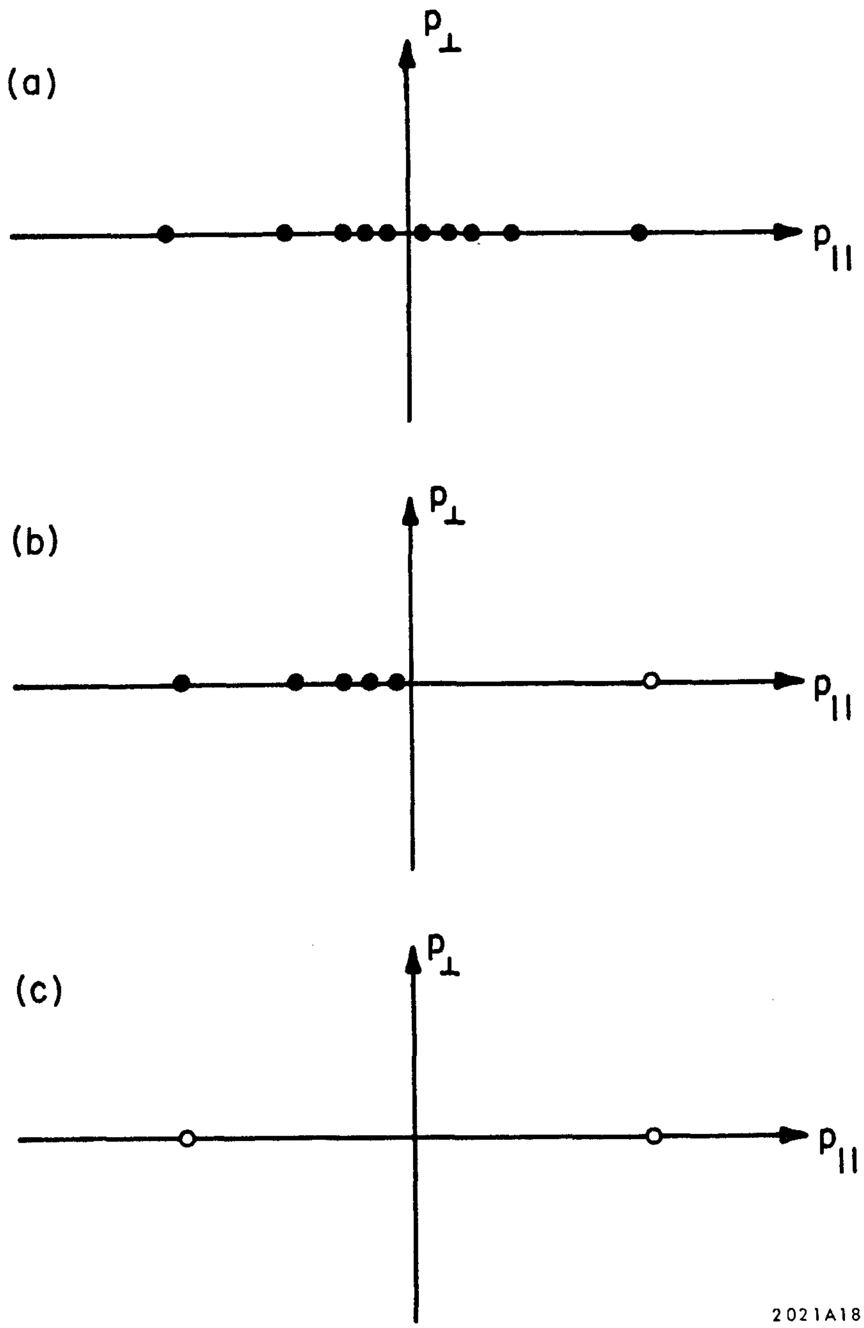


Fig. 17

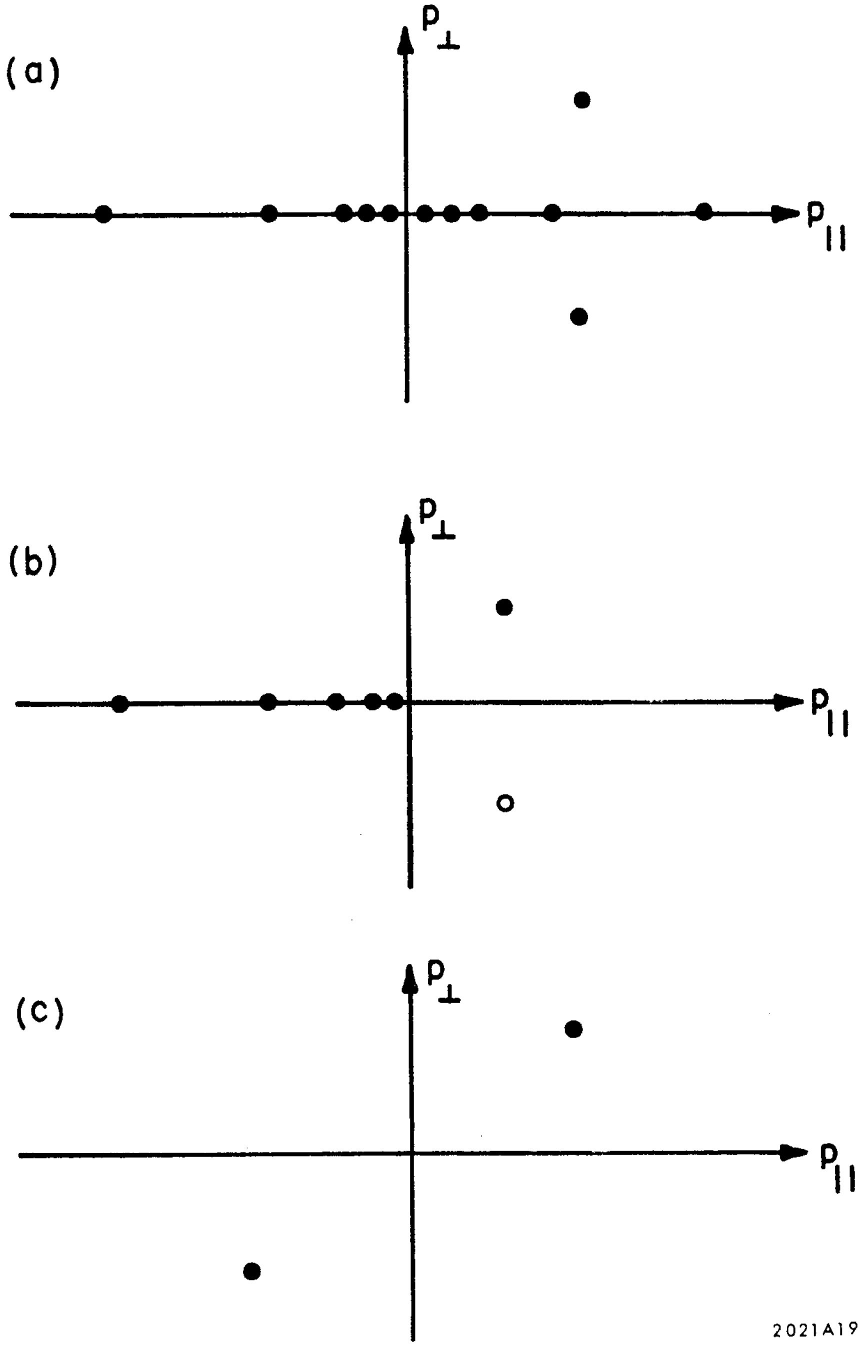


Fig. 18

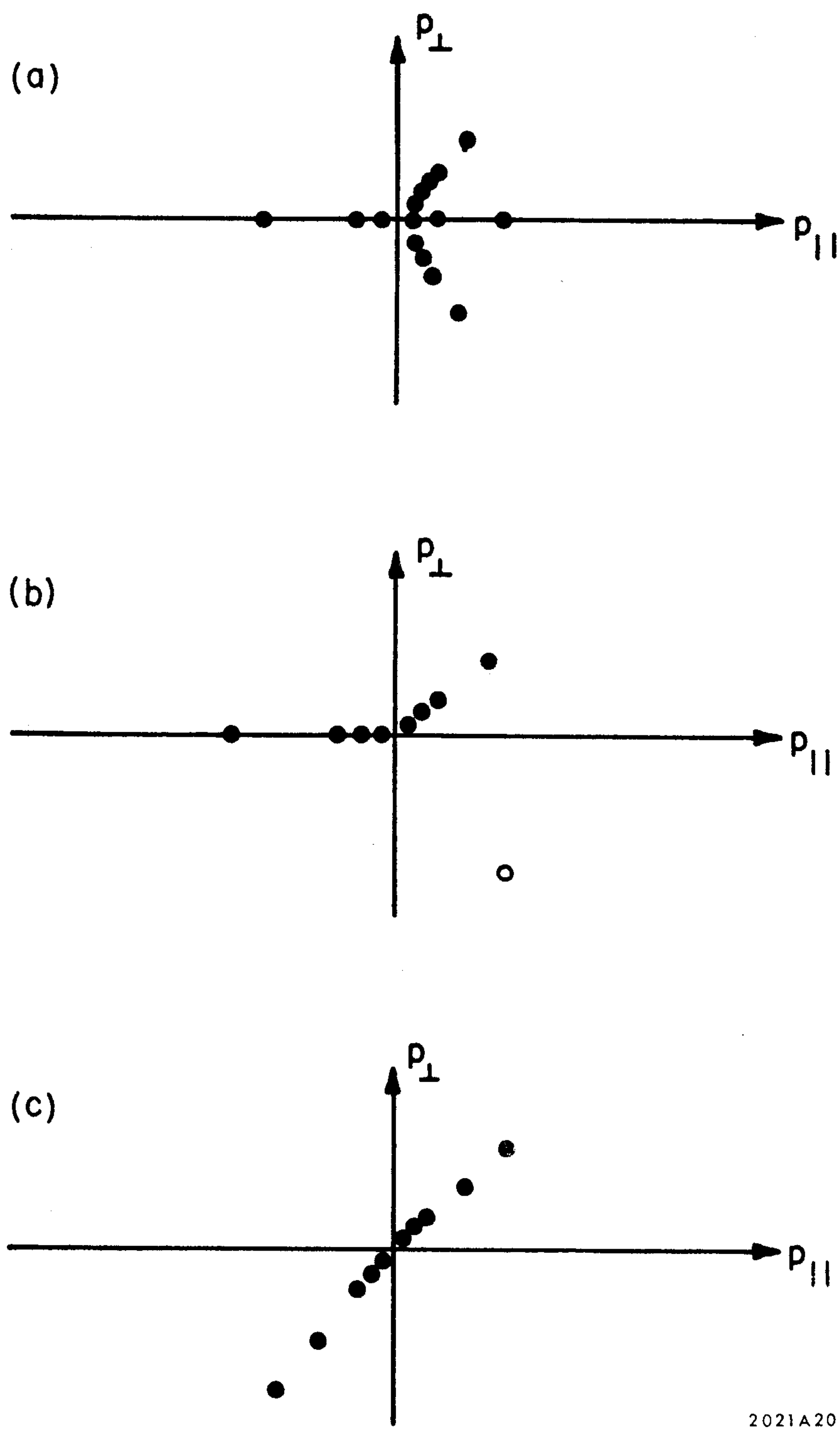


Fig. 19

jets might be expected to reach an equilibrium configuration quite similar to an ordinary final state configuration (Approximate dx/x, low $p_\perp$, etc.). At this point, we use Feynman's conjecture that it is okay to assume the hadron distribution is similar to the parton distribution, so that Figure 19 represents the hadron distribution as well. Before going on, we see that the manifestation of the deep inelastic collision is the phenomenon of multiple cores, in the language of cosmic ray physics. By measuring the energy and angles of the cores one can reconstruct the parton-parton elementary processes and directly measure their matrix elements. Provided the process exists, NAL and CERN-ISR energies are sufficiently high to clearly observe such a phenomenon.

To compute distribution functions requires one more step. This could be called environmental independence, factorization, or limiting fragmentation for partons.[25,31,32] Because of short range order, we may expect that the processes determining the hadron distributions in the leading, high $p_\perp$ ends of the jets are dependent only on the parent parton. Thus the secondary distributions for colliding beams would be expected to be very similar to the distributions in electroproduction, which in turn would be expected to be independent of ω. This conclusion and in fact much of this picture has been independently found by Feynman.

With the jet picture (option b) clearly the applicable one in colliding beams, it is natural to assume the same kind of inclusive distribution as before: the probability dP_{ia} of finding hadron a emerging from parton i with fraction x of its momentum in dx is

$$dP_{ia} = (dx/x)G_{ia}(x) \qquad (5.3)$$

These functions are universally applicable. In the quark model, the charge 2/3 p-quark contributes four times as much, so that it will generally dominate, and one distribution function $G_a(x)$ should suffice at the factor-of-two level of accuracy.

This hypothesis allows one to compute inclusive distribution functions for all processes in terms of the colliding-beam function. Given the sum rules and the inclusive-exclusive connection, there is not too much arbitrariness in the choice of G.

An important property of the inclusive distributions is the scale invariance which follows from dimensional analysis

$$E(d\sigma/dp_\perp^2 dp\ ) = (4\pi\alpha^2/p_\perp^4)\mathcal{F}(x_1,x_2) \tag{5.4}$$

$$\text{with } x_1 = p_B \cdot p_C / p_A \cdot p_B = -u/s;\ x_2 = p_A \cdot p_C / p_A \cdot p_B = -t/s \tag{5.5}$$

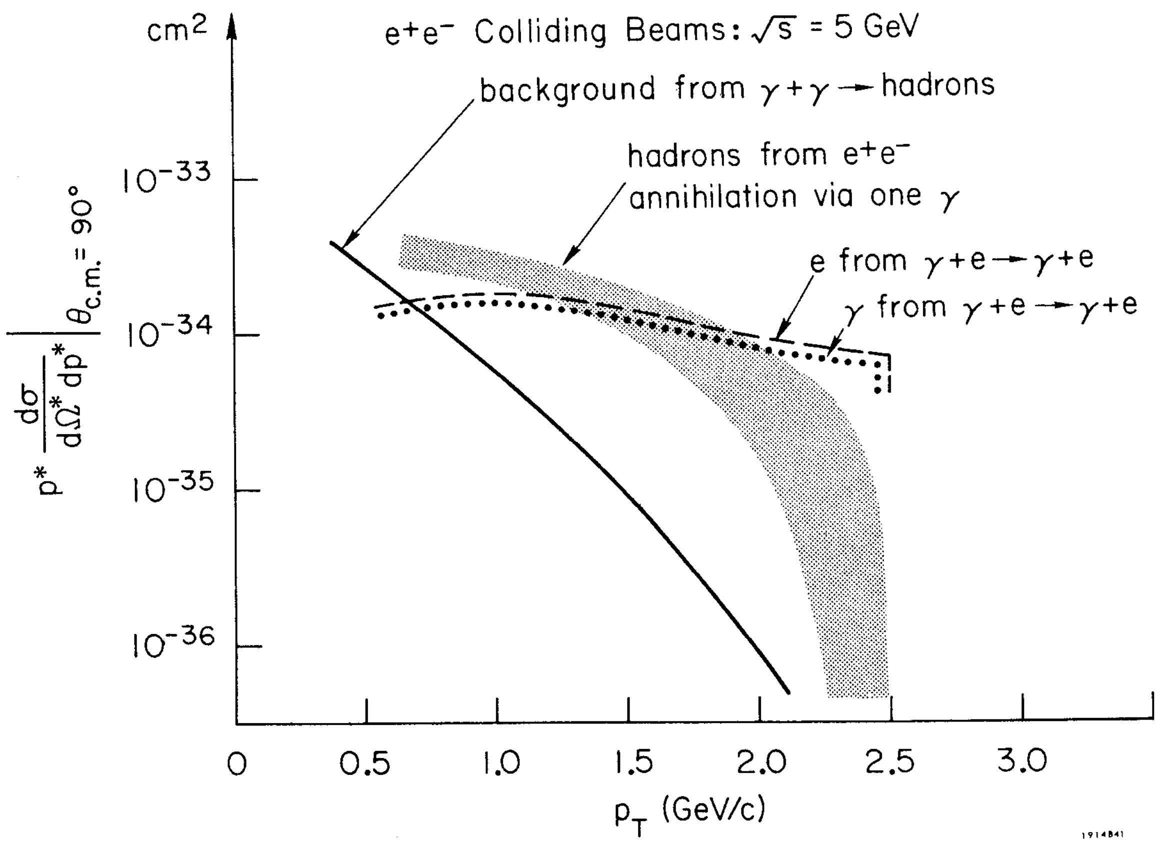
two dimensionless variables ($x_1 + x_2 \leq 1$). We survey all generic process $A + B \rightarrow C$ + anything and compute $\mathcal{F}$ for several of them using quarks for partons. Among the results are

a) 90° pp scattering: The inclusive distribution is shown in Figure 15. If one believes the Wu-Yang picture[34] the QED estimate should be multiplied by $\sim 10^4$, corresponding to replacing photon exchange by exchange of a J = 1 gluon with coupling constant O(1). The distribution is for the sum over all hadron types.

b) Colliding beams: the inclusive distributions at 90° are shown in Figures 20 and 21. The one photon contribution always dominates at high $p_\perp$, independently of energy, and the yields are quite big.

c) For $E_{CM} = 2\nu$

$$p(dN/dp)\big|_{\substack{\text{colliding}\\ \text{beams}}} \cong 2p(dN/dp)\big|_{\substack{\text{electro-}\\ \text{production}}} \tag{5.6}$$



cm²
e+e− Colliding Beams: √s = 5 GeV
background from γ+γ → hadrons
hadrons from e+e−
annihilation via one γ
e from γ+e → γ+e
γ from γ+e → γ+e
10−33
10−34
10−35
10−36
p* dσ/dΩ* dp* | θc.m. = 90°
0
0.5
1.0
1.5
2.0
2.5
3.0
pT (GeV/c)

Fig. 20

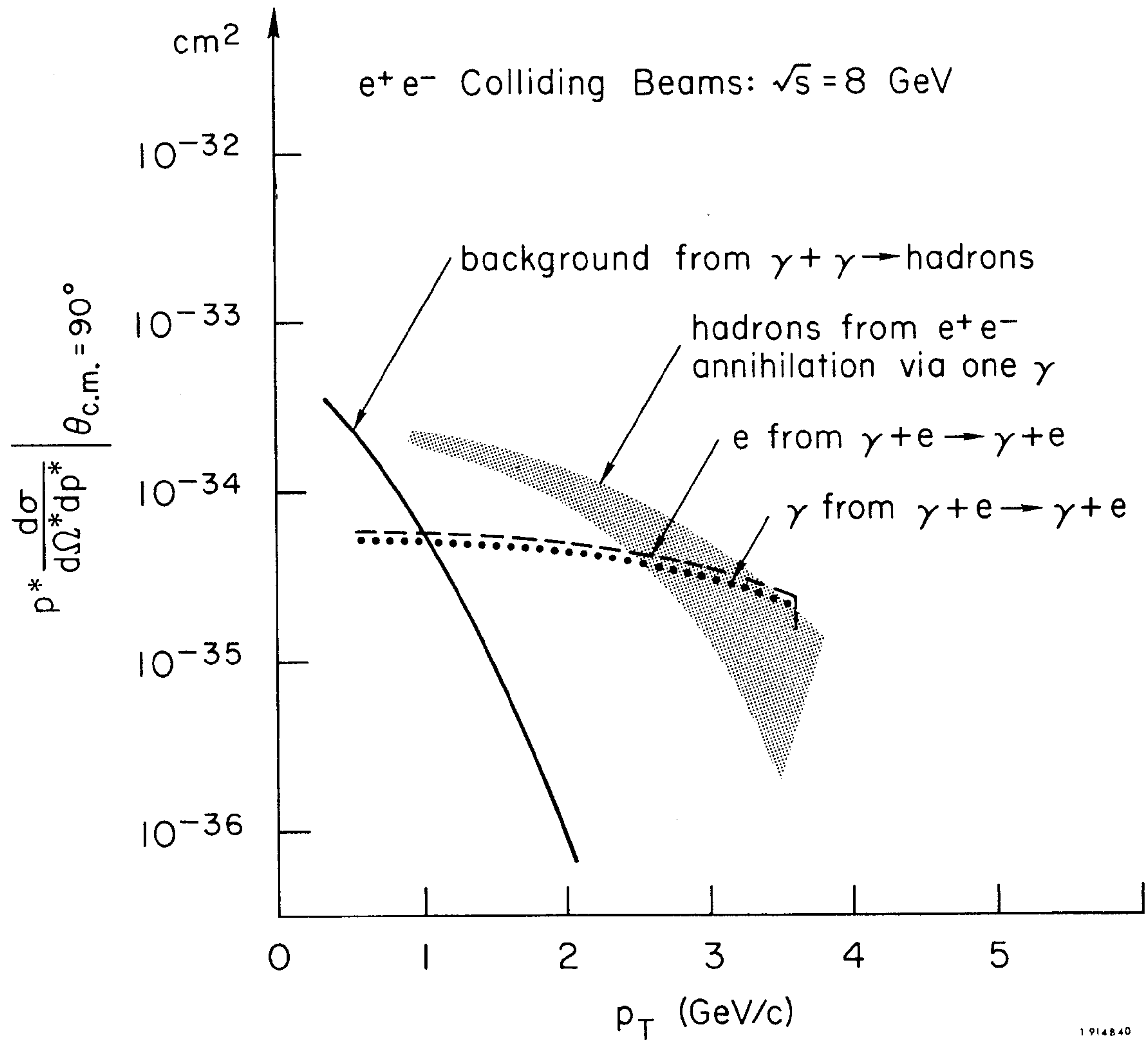

cm2
e+ e− Colliding Beams: √s = 8 GeV
10−32
10−33
10−34
10−35
10−36
background from γ + γ → hadrons
hadrons from e+ e−
annihilation via one γ
e from γ + e → γ + e
γ from γ + e → γ + e
p* dσ/dΩ*dp* |θc.m. = 90°
0
1
2
3
4
5
pT (GeV/c)
1914B40

Fig. 21

Also for J = 1/2 partons

$$dN/dpd\Omega = 3/16\pi(dN/dp)(1 + \cos^2\theta) \quad (5.7)$$

VI. Deep Inelastic Compton Scattering and $pp \to \mu^+\mu^- + h$

Among the deep inelastic inclusive processes of the class above is the process

$$\gamma + p \to \gamma + \text{hadrons} \quad (6.1)$$

which is to proceed by the conjectured elementary process in Figure 16e.[35] The Santa Barbara group[36] has searched for it using a SLAC bremsstrahlung beam at 20.5 GeV, and observing the produced photons in a bank of lead glass shower counters. The data at 20.5 GeV is shown in Figure 22. The problem is the background γ's from π^0 decay. At sufficiently high p_T this should be negligible but at 20.5 GeV it is not. The π^0 background is assumed equal to the recent measurements of π^+ inclusive spectra, and the authors convince themselves that contributions from $\omega \to \pi^0 + \gamma$ and $\eta \to 2\gamma$ are negligible. There is a hint of an excess, but no definitive statement that the inelastic γ process exists. They plan to run again with means to positively identify the π^0.

Finally, the deep inelastic inclusive process measured by the Columbia-BNL group[37]

$$pp \to \mu^+\mu^- \text{ hadrons}$$

has created much attention.[38,39,40] There are three fits to the data reported in this conference.

Kuti and Weiss kopf[41] use a kindergarten quark-parton calculation as described in the previous section, using the elementary

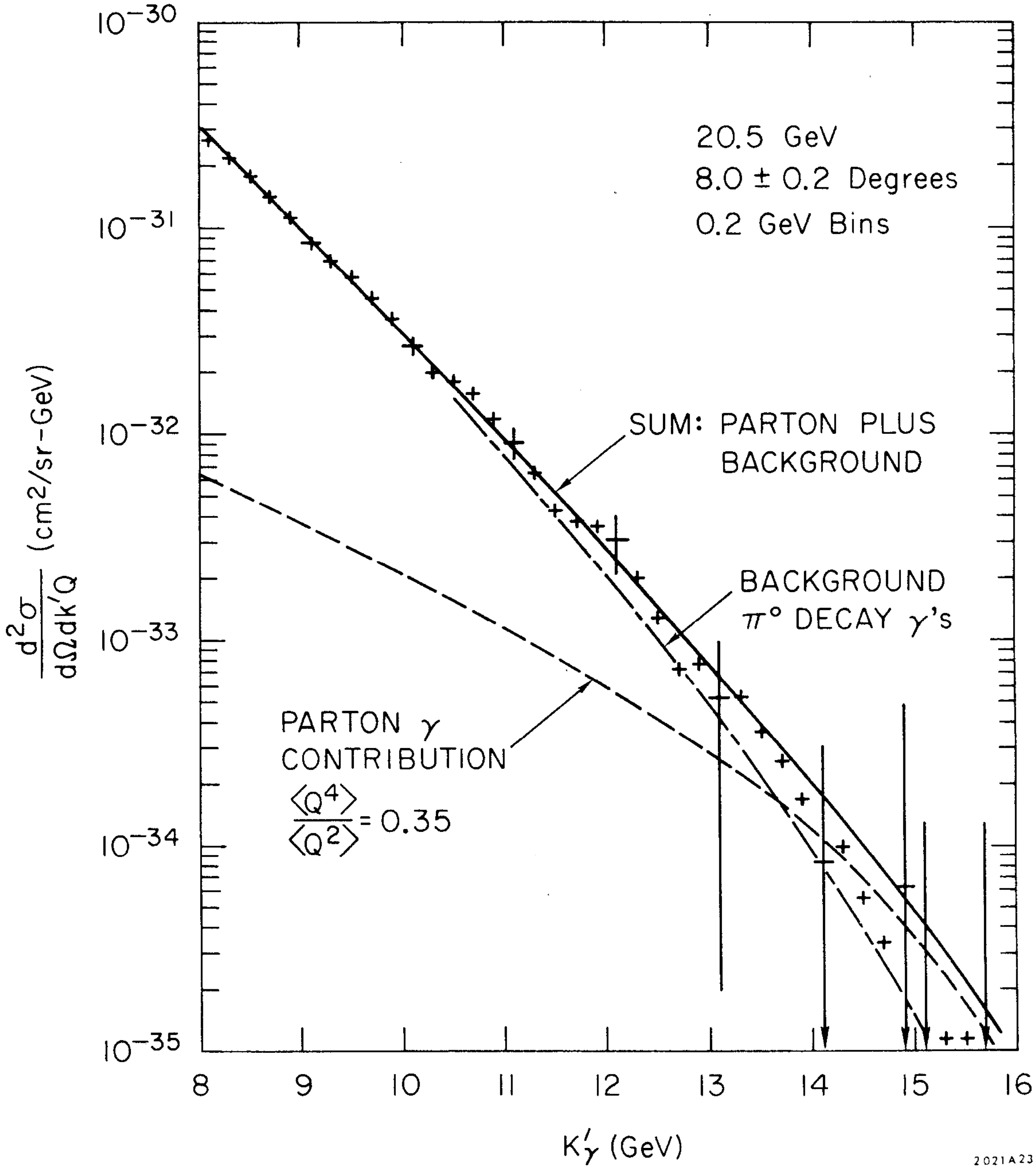

20.5 GeV
8.0 ± 0.2 Degrees
0.2 GeV Bins
SUM: PARTON PLUS BACKGROUND
BACKGROUND
π° DECAY γ's
PARTON γ CONTRIBUTION
⟨Q⁴⟩/⟨Q²⟩ = 0.35
d²σ/dΩdk'Q (cm²/sr-GeV)
K'γ (GeV)
10⁻³⁰
10⁻³¹
10⁻³²
10⁻³³
10⁻³⁴
10⁻³⁵
8
9
10
11
12
13
14
15
16
2021A23

Fig. 22

process of $q\bar{q}$ annihilation described by Drell and Yan.[39] Kogerler[42] uses a one-parameter Mueller-type Regge analysis,[8] while Landshoff and Polkinghorne[43] use two Landshoff-Polkinghorne diagrams (Figure 23), again there is only one free parameter.

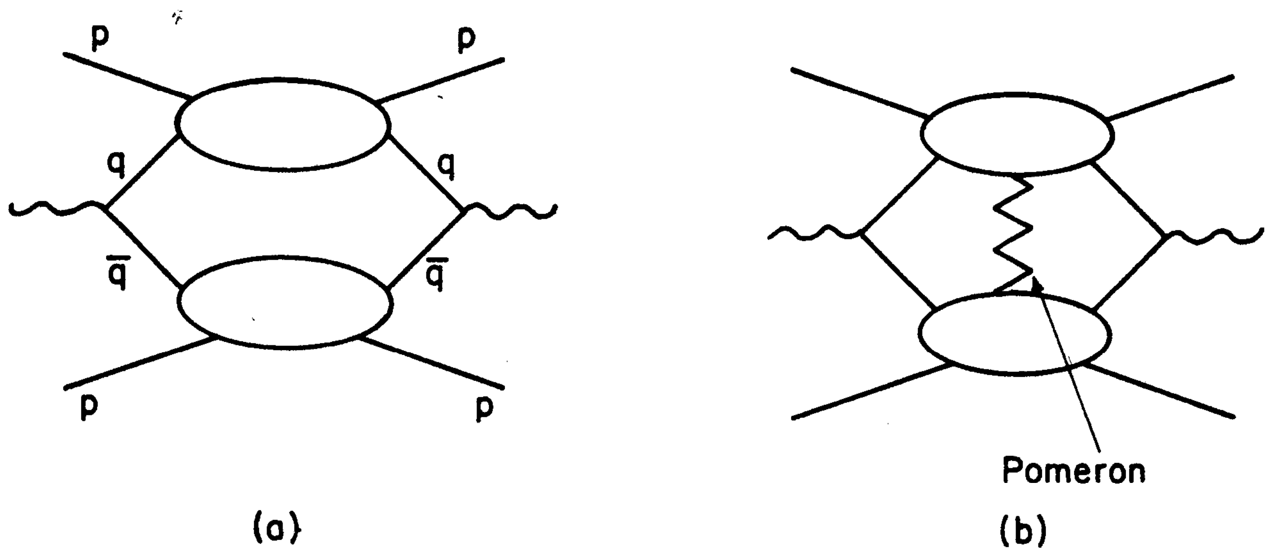
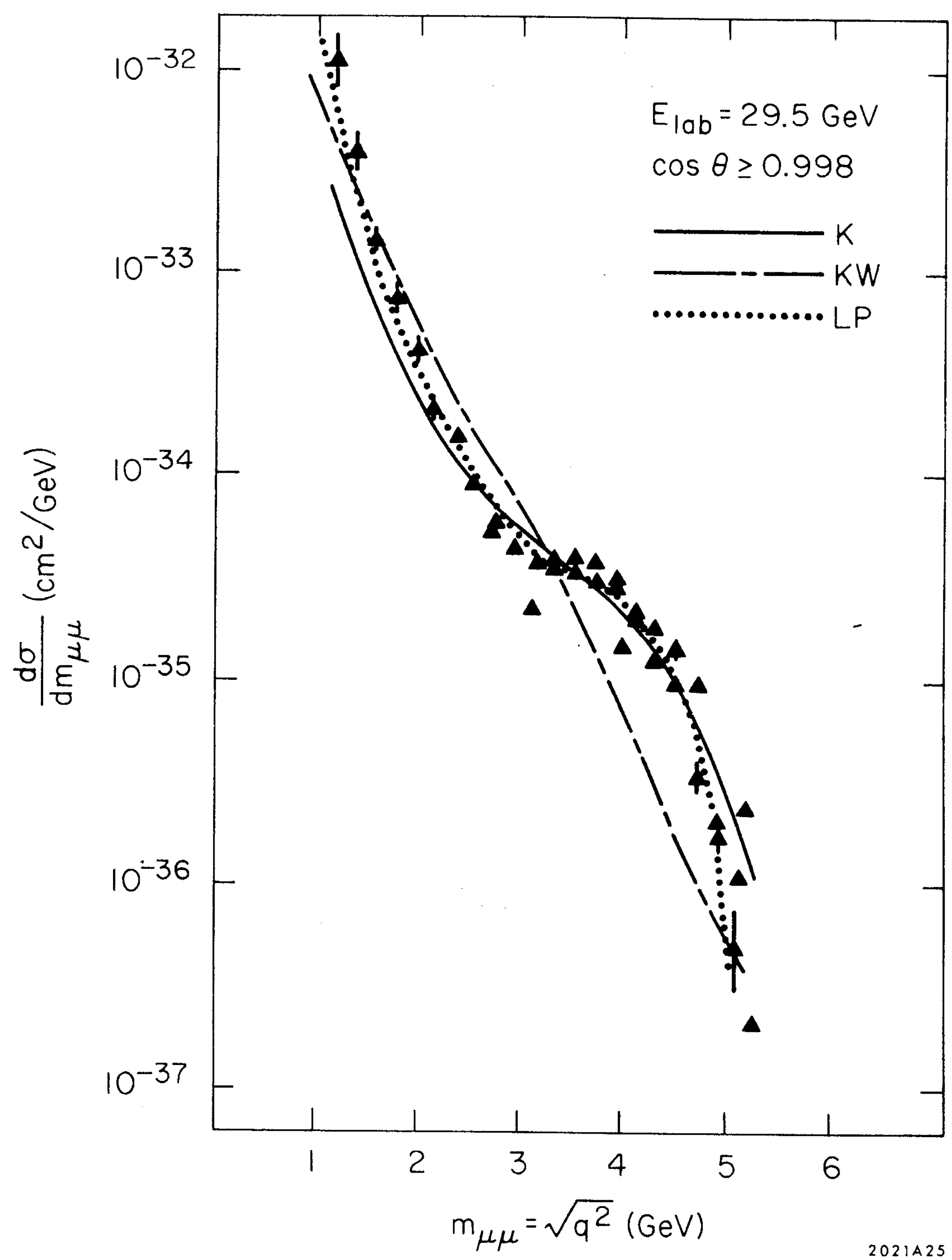


Fig.23

The fits are shown in Figure 24. I'm impressed at how well everyone is able to fit such a complex process when we still don't understand the proton form factor properly.

VI. Ken Wilson's Mysterious Paper[44]

In this paper, which has no equations and is short, Wilson warns against accepting too easily the popular supposition (which I've made) that on the light cone the free field propagation



E_{lab} = 29.5 GeV
cos θ ≥ 0.998
K
KW
LP
10^{-32}
10^{-33}
10^{-34}
10^{-35}
10^{-36}
10^{-37}
$\frac{d\sigma}{dm_{\mu\mu}}$ (cm^2/GeV)
1
2
3
4
5
6
$m_{\mu\mu} = \sqrt{q^2}$ (GeV)
2021A25

Fig. 24

or massless free quarks is the correct description. There is no theoretical evidence in support of that and quite a bit against. Based on his matchless experience in grappling with the diseases of renormalizable field theory, he suggests that in the long run the theory will succeed, and not look at all like the free-field case. He visualizes a parton model of very massive partons but coupled in a way depending only on ratios of masses so that a scaling behavior could emerge. So if all parton masses are changed by a factor, along perhaps with a change in interaction strength--there is no change in the physics. Such perfect scaling would imply a mass spectrum from zero to ∞. Wilson then goes to second order in degree of speculation and suggests it be broken at the light end of the spectrum with minimum masses m_π, m_k, m_N, etc., giving rise to scale invariance violations when Q^2 is of order m_i^2. Wilson guesses such a region of violation exists for $8 < Q^2 < 30\ \mathrm{GeV}^2$ corresponding to lots of $\bar{N}$ production.

Such a phenomenon will be welcome when it is found that the Adler sum rule for neutrino processes fails, and the dispersion integral gives 1 while the right-hand side is 2. One would then conjecture that a new piece should be added onto the structure functions with a large threshold in ν. But I'd bet it would correspond to something more interesting than antibaryon production.

ACKNOWLEDGMENTS

I wish to thank Tung Mow Yan, Mikkel Johnson, and Jack Uretsky for valuable help in preparing this contribution, and Haim Harari, Luca Canesihi, R. P. Feynman, and Leo Stodolsky for help in the physics.

REFERENCES

1. H. A. Kastrup, Phys. Rev. 147, 1130 (1966).
2. P. M. Fishbane and J. D. Sullivan, contribution no. 151 to this conference. See also H. Fried and H. Moreno, Phys. Rev. Lett. 25, 625 (1970); H. Fried and T. Gaisser Phys. Rev. D1, 224 (1971).
3. See the review article submitted to Rev. Mod. Phys. by W. R. Frazer, L. Ingber, C. H. Metha, C. H. Poon, D. Silverman, K. Stowe, P. D. Ting, and H. J. Yesian (1971).
4. J. D. Bjorken and S. Brodsky, Phys. Rev. D1, 1416 (1970).
5. S. D. Drell, D. J. Levy and T. M. Yan, Phys. Rev. 187, 2159 (1969), and Phys. Rev. D1, 1617 (1970).
6. See for example, J. Pestieau and P. Roy, Phys. Lett. 30B, 483 (1969), N. Cabibbo, G. Parisi and M. Testa, Lettere al Nuovo Cimento 4, 35 (1970), J. Manassah and S. Matsuda, contribution no. 141 to this conference.
7. J. W. Moffat and V. G. Snell, Contribution no. 101 to this conference.
8. S. D. Drell and T. M. Yan, Phys. Rev. Lett. 24, 181 (1970). G. West, Phys. Rev. Lett. 24, 1206 (1970). E. Bloom and F. Gilman, Phys. Rev. Lett. 25, 1140 (1970). R. P. Feynman's notebooks, A. H. Mueller, Phys. Rev. D2, 2963 (1970).
9. G. Kramer, J. L. Uretsky and T. Walsh, Phys. Rev. D3, 719 (1971); R. Gatto, in Proceedings of the International Symposium on Electron and Photon Interactions at High Energies (Springer, Berlin, 1965), p. 106; J. Layssac and F. Renard, Lett. Nuovo Cimento, 1, 197 (1971); M. Vaughn and P. Polito, Lett. Nuovo Cimento, 1, 74 (1971).

10. S. D. Drell, D. J. Levy and T. M. Yan, Phys. Rev. 187, 2159 (1969), and Phys. Rev. D1, 1617 (1970); N. Cabibbo, G. Parisi and M. Testa, Lett. Nuovo Cimento, 4, 35 (1970); J. D. Bjorken and S. J. Brodsky, Phys. Rev. D1, 1416 (1970).

11. A. H. Mueller, Phys. Rev. D4, 150 (1971).

12. D. Amati, A. Stanghellini and S. Fubini, Nuovo Cimento 26, 896 (1962). For a review see S. Fubini in Strong Interactions and High Energy Physics, ed. R. G. Moorhouse (Plenum Press, New York, 1964).

13. K. Wilson, Acta Physica Austriaca 17, 37 (1963).

14. R. P. Feynman, Phys. Rev. Lett. 23, 1415 (1969); and High Energy Collisions (Gordon and Breach, New York, 1969), and unpublished notebooks.

15. K. Huang, Statistical Mechanics (New York, Wiley, 1963).

16. Castagnoli, Cortini, Frazinetti, Manfredini and Moreno, Nuovo Cimento 10, 1539 (1953).

17. L. W. Jones et al., Phys. Rev. Lett. 25, 1679 (1970); N. F. Bali et al., Phys. Rev. Lett. 25, 557 (1970).

18. G. Domokos, S. Kovesi-Domokos, and E. Schonberg, Phys. Rev. D3, 1184 (1971) and contribution no. 173 to this conference.

19. H. Terazawa, contribution no. 158 to this conference.

20. J. Stack, contribution no. 163 to this conference.

21. H. Satz and D. Schildknect, contribution no. 218 to this conference.

22. S. D. Drell, D. J. Levy and T. M. Yan, Phys. Rev. Lett. 22, 744 (1969).

23. S. S. Shei and D. M. Tow, Phys. Rev. Lett. 26, 470 (1971).

24. T. T. Chou and C. N. Yang, to be published.

25. S. Berman, J. D. Bjorken and J. Kogut, contribution no. 314 to this conference.

26. Spin dependence is discussed by H. Abarbanel and D. Gross, contribution no. 135 to this conference.

27. H. Cheng and T. T. Wu, Phys. Rev. 183, 1324 (1969).

28. V. N. Gribov, "High Energy Interactions of Gamma Quanta and Electrons with Nuclei" (October 1969), preprint from the Fourth Winter Seminar on the Theory of the Nucleus and the Physics of High Energies, February 5-17, 1969, Ioffe Institute of Engineering Physics, Acad. Sci., USSR. See also S. J. Brodsky and J. Pumplin, Phys. Rev. 182, 1794 (1969), K. Gottfried and D. R. Yennie, Phys. Rev. 182, 1595 (1969).

29. Y. Frishman, V. Rittenberg, H. R. Rubinstein and S. Yankielowicz, Phys. Rev. Lett. 26, 798 (1971).

30. P. Roy, contribution no. 105 to this conference.

31. R. P. Feynman's notebooks (unpublished and unseen).

32. S. D. Drell and T. M. Yan, Phys. Rev. Lett. 24, 855 (1970).

33. S. Berman and M. Jacob, Phys. Rev. Lett. 25, 1683 (1970).

34. T. T. Wu and C. N. Yang, Phys. Rev. 137, B 708 (1965).

35. J. D. Bjorken and E. Paschos, Phys. Rev. 185, 1975 (1969).

36. Caldwell, Elings, Kendall, Morrison, Murphy and Worceter.

37. J. Christenson, G. Hicks, L. M. Lederman, P. Limon, B. Pope, and E. Zavaltini, Phys. Rev. Lett. 25, 1523 (1970).

38. S. Berman, D. J. Levy and T. L. Neff, Phys. Rev. Lett. 23, 1363 (1969).

39. S. D. Drell and T. M. Yan, Phys. Rev. Lett. 25, 316 (1970).

40. G. Altarelli, R. Brandt and G. Preparata, Phys. Rev. Lett. 26, 42 (1971).

41. J. Kuti and V. F. Weisskopf, Contribution no. 178 to this conference.

42. R. Kogerler, contribution no. 129 to this conference.

43. P. V. Landshoff and J. C. Polkinghorne, contribution no. 90 to this conference.

44. K. G. Wilson, contribution no. 247 to this conference.