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DEEP-INELASTIC ELECTROPRODUCTION*

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I shall discuss the following topics:

(1) Trouble with the kindergarten parton model.

(2) Remarks on "precocious scaling."

(3) Hadron final-states in electroproduction: a connection between vector-dominance, partons, and e^+-e^- annihilation.

(4) A calculation of νW_2 in perturbation theory by Gribov and Lipatov.

1. TROUBLE WITH PARTONS

The recent data[1] on W_{2n}/W_{2p} indicates that perhaps $W_{2n}/W_{2p} \to 0$ as $\omega = 2M\nu/Q^2 \to 1$. This combined with the data on neutrino cross sections begin to put naive parton-model calculations in trouble, a point emphasized by Nachtmann.[2] Let us assume the validity of the kindergarten parton calculations. Then σ_S/σ_T small requires J=1/2 partons to have the dominant contribution. Assume only J=1/2 partons of isospin 1/2 or 0. Call those of isospin 1/2 (u_i, d_i) and antipartons $(\bar{d}_i, \bar{u}_i)$ with

$$J_\mu^{weak}(x) = \sum_i \bar{u}_i(x)\, \gamma_\mu (1-\gamma_5)\, d_i(x) \tag{1.1}$$

The parton-model calculations give

$$\sigma_{tot}(\nu p) = \frac{G^2 s}{\pi} \sum_i \int_0^1 dx \left[D_i(x) + \frac{1}{3}\bar{U}_i(x)\right]$$

$$\sigma_{tot}(\nu n) = \frac{G^2 s}{\pi} \sum_i \int_0^1 dx \left[U_i(x) + \frac{1}{3}\bar{D}_i(x)\right] \tag{1.2}$$

etc.

where the differential probability of finding a parton d_i with fraction x of the proton's momentum (at infinite momentum) is

$$dP = \frac{dx}{x} D_i(x) \tag{1.3}$$

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with obvious notation for the other distribution functions, U, $\bar{D}$, $\bar{U}$. The average neutrino cross section is

$$\sigma_{ave} = \frac{1}{4}\left[\sigma_{\nu p} + \sigma_{\nu n} + \sigma_{\bar{\nu}p} + \sigma_{\bar{\nu}n}\right]$$

$$= \frac{G^2 ME}{\pi} \left\{\frac{2}{3}\int_0^1 dx\ F(x)\right\} \equiv \frac{G^2 ME}{\pi} a_{ave} \qquad (1.4)$$

where

$$F(x) = \sum_i \left[U_i(x) + D_i(x) + \bar{U}_i(x) + \bar{D}_i(x)\right] \qquad (1.4)$$

From experiment[3]

$$\frac{1}{2}(a_{\nu p} + a_{\nu n}) \cong 0.5 \pm .15 \qquad (1.6)$$

and from theory

$$\frac{1}{2}(a_{\bar{\nu} p} + a_{\bar{\nu} n}) \geq \frac{1}{6}(a_{\nu p} + a_{\nu n}) \qquad (1.7)$$

$$\therefore\ a_{ave} \geq \frac{2}{3}(0.5 \pm .15) \qquad (1.8)$$

From momentum conservation

$$\int_0^1 dx\ F(x) + |\text{isoscalar contribution}| = 1 \qquad (1.9)$$

and putting together (1.4), (1.8), and (1.9),

$$\int_0^1 dx\ F(x) > 0.5 \pm .15 \qquad (1.10)$$

In words, ~ 1/2 the proton momentum is carried by I=1/2 constituents. This fact will bound electroproduction cross sections from below. The electromagnetic current is

$$J_\mu = \sum_i \left(\bar{q}_i + \frac{1}{2}\right) \bar{u}_i \gamma_\mu u_i + \sum_i \left(\bar{q}_i - \frac{1}{2}\right)\left(\bar{d}_i \gamma_\mu d_i\right)$$

$$+ \text{(isotopic-scalar parton contribution)} \qquad (1.11)$$

The parton-calculation of electroproduction gives

$$\left(\bar{q}^2 + \frac{1}{4}\right)\int dx\, F(x) + |\text{isoscalar}| = \int dx \left[\frac{\nu W_{2p} + \nu W_{2n}}{2}\right] \approx .14 \tag{1.12}$$

where

$$\bar{q}^2 = \frac{\int dx \sum_i q_i^2 (U_i + \bar{U}_i + D_i + \bar{D}_i)}{\int dx \sum_i (U_i + \bar{U}_i + D_i + \bar{D}_i)} \tag{1.13}$$

and putting together (1.12) with (1.10) gives

$$\left(\bar{q}^2 + \frac{1}{4}\right) \leq \frac{.14}{.5 \pm .15} \sim 0.3 \pm 0.1 \tag{1.14}$$

Integer-charge partons require $\bar{q}_i \geq 1/2$; hence $\bar{q}^2 \geq 1/4$. That is not favored by the data, Eq. (1.14). On the other hand, fractional charge partons (quarks) are acceptable. But they are in trouble with the measured ratio W_{2n}/W_{2p} near threshold. Since in the quark model

$$\nu W_{2p} = \frac{4}{9} U(x) + \frac{1}{9} D(x) + \frac{1}{9} S(x) + (U, D, S \rightarrow \bar{U}, \bar{D}, \bar{S})$$

$$\nu W_{2n} = \frac{4}{9} D(x) + \frac{1}{9} U(x) + \frac{1}{9} S(x) + (U, D, S \rightarrow \bar{U}, \bar{D}, \bar{S})$$

we have

$$.25 \leq \frac{\nu W_{2n}}{\nu W_{2p}} \leq 4$$

At $x \sim 1.3$, the ratio is ~ 0.3, dangerously close to the limit. More precise νN and eN data could well be decisive for ruling out a large class of kindergarten models, perhaps all.

These considerations also apply to the calculations based on a free-field light-cone algebra a la Fritsch and Gell-Mann.[4]

2. PRECOCIOUS SCALING

Many people have remarked, occasionally with considerable wonder, at the speed at which the electroproduction structure function νW_2 approaches its scaling limit as Q^2, the squared photon mass, increases from zero. Already at $Q^2 \sim 1\ \text{GeV}^2$, the scaling limit is reached to considerably better than 20%, a feature hard to reconcile with pure dimensional analysis. This is called "precocious scaling," and the question may be put as to whether this is a reasonable expectation or not. I want to make a little argument that it isn't a surprise, provided we assume the Adler sum rule for neutrino-processes[5] is correct and that in the range $0 < Q^2 < 1\ \mathrm{GeV}^2$ it converges for $\nu \lesssim 5\ \mathrm{GeV}^2$ (as it does[6] for $Q^2 \approx 0$, in the Cabibbo-Radicati limit). We have

$$\int_0^{5\ \mathrm{GeV}} \frac{d\nu}{\nu} \left[\nu\ \beta_\nu^{\mathrm{odd}}(\nu, Q^2)\right] \approx 1 \tag{2.1}$$

and

$$\beta_\nu^{\mathrm{odd}} = (\beta^{\bar{\nu}p} - \beta^{\nu p})\ \mathrm{vector}$$

For low Q^2, the sum is contributed by elastic, s-wave, and resonances (Δ, N*(1512), N*(1688)); however, these disappear as Q^2 increases, roughly like $(1+Q^2/0.7)^{-4}$, a suppression $\sim$ a factor 30 by the time $Q^2 \sim 1\ \mathrm{GeV}^2$. Therefore, the continuum must already have replaced the resonances. Finally, for large Q^2, as long as the sum converges at $\nu \sim (\mathrm{const})\, Q^2$, as suggested by estimates of important longitudinal distances or of minimum-momentum-transfer, then

$$\int_1^{\mathrm{const}} \frac{d\omega}{\omega}\ \nu\ \beta^{\mathrm{odd}}(\omega, Q^2) \sim 1 \tag{2.2}$$

suggesting that $\nu\beta$ should have stabilized to something near the scale limit for $Q^2 \gtrsim 1\ \mathrm{GeV}^2$. We conclude that the important parameter characterizing the approach to scaling is $\langle r^2\rangle_{\mathrm{proton}}$, not the target mass.

3. HADRON FINAL STATES IN ELECTROPRODUCTION

Whether or not scaling should be precocious, the fact remains that it is. This, along with the Bloom-Gilman[7] "duality" between resonance-production and the deep inelastic limit, may contain an important principle for understanding the deep inelastic phenomenon itself. The principle is something like the correspondence principle in old-fashioned quantum mechanics. We are exploring new regions in ν, Q^2 space; they are shown in Fig. 1. Some are more familiar than others, for example the resonant and photoproduction regions. But in any case, the correspondence principle would say that we should have smooth transitions in the nature of the phenomena in passing from one of the regions to another including into the familiar regions. For a time the resonance and deep inelastic processes at large Q^2 were thought to be distinct. Now we think they are intimately related. We often tend to think of the phenomena at $\omega \sim 3$ and $\omega \gg 3$ in very different terms; yet a glance at the shape of νW_2 with ω suggests a very close connection. For large ω, we sometimes think of diffractive phenomena at large Q^2 differently from good old ρ-dominance at $Q^2=0$; yet precocious scaling connects the two over a small region of Q^2. Can the deep-inelastic dynamics really be discontinuous across that interval from $Q^2=0$ to $Q^2 \sim 1\ \mathrm{GeV}^2$??

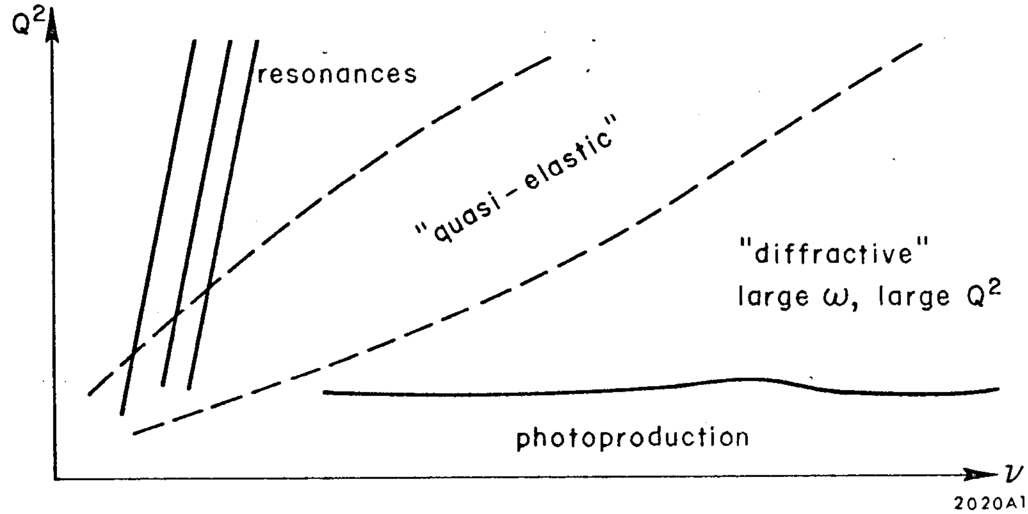

FIG. 1--Regions of ν - Q^2 space.

It is in this spirit that the following remarks are made, first on the relationship of secondary hadron distributions at small Q^2 and large Q^2, and secondly on the relationship of large ω electroproduction to vector-dominance, to the closely related process of $e^+e^- \to$ hadrons, and to the parton picture.

A. Secondary Hadron Production for Large Q^2

To proceed, we have to assume something, and I will assume the picture of short-range correlation in rapidity given in Frazer's lecture to this conference.[8] Thus for Q^2=0 and ν large, we have the distribution in rapidity y shown in Fig. 2.

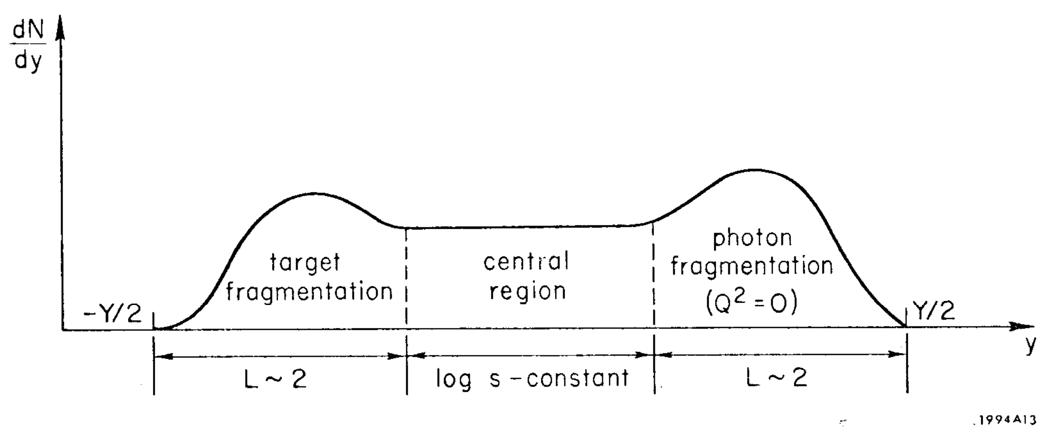

FIG. 2--Schematic rapidity distribution for high energy photoproduction.

According to the "correspondence principle," this picture should not change discontinuously as Q^2 is increased from 0 into the scaling region. Therefore, according to the idea of short-range correlation, only the photon fragmentation region changes, because only the nature of the photon has changed. To see how it changes, we keep Q^2 fixed and large and decrease ν. This suppresses the central plateau, and we may expect it to have disappeared when $\omega \sim 4$, because Pomeron exchange no longer dominates the ν-dependence of the total virtual photoabsorption cross section; certainly dominance of Pomeron exchange in σ_{tot} is a necessary condition for existence of a central plateau. Thus for $\omega \sim 4$, we have Fig. 3:

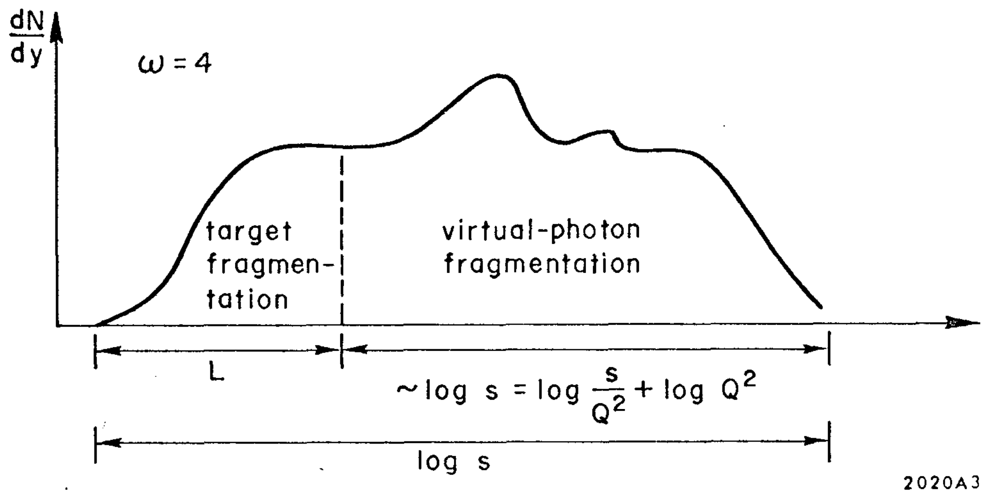

FIG. 3--Rapidity distribution for $\omega \sim 4$.

Because the length in rapidity of the photon-fragmentation region is a function of Q^2 only, we find its length is $\sim \log Q^2$ and for $\omega \gg 4$ we have Fig. 4

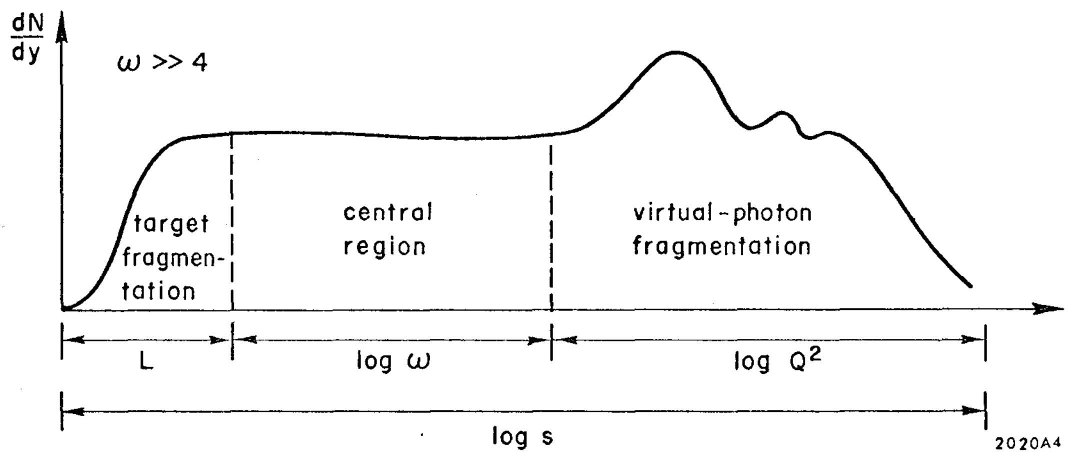

FIG. 4--Rapidity distribution for $\omega \gg 4$

with the conclusions that for $\omega \gtrsim 3$,

$$\bar{n} \sim (\text{const}) + \log \omega + f(Q^2) \ . \tag{3.1}$$

We see that, as far as hadron distributions are concerned, the phenomena at $\omega \sim 3$ are smoothly connected to $\omega \gg 3$. As ω decreases toward 1, photon and target regions interpenetrate; this is quite possibly controlled by s-channel dynamics, just as the case in pure hadron phenomena.

B. Vector Dominance and Colliding Beams

To study photon-fragmentation, we go back to large ω and connect it with phenomena at $Q^2 \sim 0$ using ideas of vector dominance. First, at $Q^2=0$, we use Gribov's simple picture (Fig. 5) for forward Compton scattering[9]:

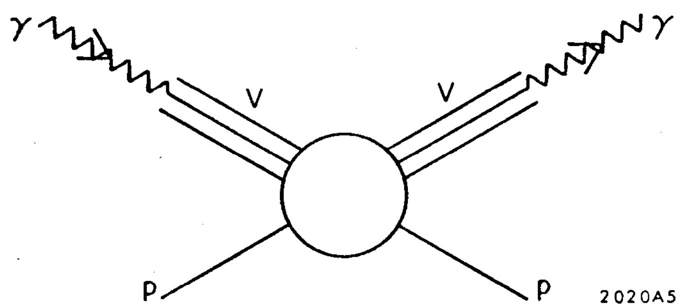

FIG. 5--Forward Compton amplitude.

Use old-fashioned perturbation theory:

$$F \sim \sum_m \frac{\langle 0|j_\perp|m\rangle \ \langle m|T|m\rangle \ \langle m|j_\perp|0\rangle}{(\Delta E_m)^2}$$

$$\Delta E_m = \frac{Q^2 + m^2}{2\nu}$$

and assume all states m are absorbed by the target geometrically:

$$F \sim i\nu \ (\pi R^2) \sum_m \frac{|\langle 0|j_\perp|m\rangle|^2}{(\Delta E_m)^2} \tag{3.2}$$

Thus F is related to the absorptive part of vacuum-polarization, and Gribov's formula gives

$$\sigma_{\gamma p} = \left(1 - Z_3^{\text{had}}\right) \pi R^2 \tag{3.3}$$

where

$$(1-Z_3) \sim \int_0^s ds' \sigma(s')_{e^+e^- \to \text{hadrons}} \tag{3.4}$$

= probability γ is hadron system

and

$$\pi R^2 = \text{absorption cross section of hadron system} \quad . \tag{3.5}$$

For virtual transverse γ's, we get, from the square of the vector propagator,

$$\sigma_\gamma^\perp(s) \sim \frac{R^2}{\alpha} \int_0^s ds' \left(\frac{s'}{s' + Q^2}\right)^2 \sigma(s')_{e^+e^- \to h} \tag{3.6}$$

Assume scale-invariance for the annihilation cross section

$$\sigma(s') \sim \frac{\text{const}}{s'} \tag{3.7}$$

The result is disastrous from the viewpoint of electroproduction scaling.

$$\sigma_\gamma^\perp \sim \pi R^2 \log \omega \tag{3.8}$$

One option is, of course, to abandon scaling in the annihilation process and have

$$\sigma(s') \sim s^{-2}; \quad \text{then} \quad \sigma_\gamma^\perp \sim Q^{-2} \quad . \tag{3.9}$$

But it is interesting to retain (3.7) and try to salvage the situation. About the only way to do this is to reduce the opacity of the target as seen by some class of intermediate hadron states created by the photon. The problem comes from the high-mass states; they are evidently the same states created in e^+e^- annihilation at high energy. They are, in a scale-invariant world, most likely hadron "jets" of high momentum and low multiplicity.[10] (Figure 6.)

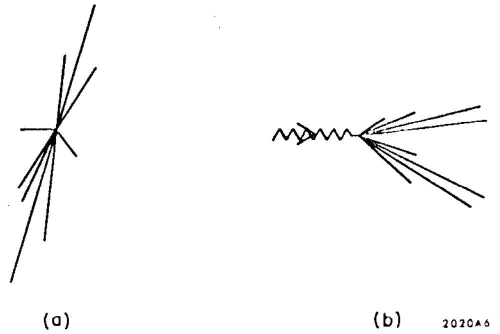

FIG. 6--Hadron production in e^+e^- annihilation: a) center-of-mass frame; b) lab frame

which in the lab are swept forward. In general, the jets have $\langle p_\perp \rangle \sim \sqrt{s'}$. Perhaps the target is transparent to such configurations except when the jet is aligned along the photon beam direction. (Figure 7.)

FIG. 7--"Aligned" jets: a) center-of-mass frame; b) lab frame.

The probability of such aligned configurations is $\sim \Delta\Omega \sim \theta^2 \sim \langle p_\perp \rangle^2/s'$, and when this factor is inserted into (3.6), we get

$$\sigma_\gamma(s) \sim \pi R^2 \int ds' \, \sigma(s') \left(\frac{s'}{s'+Q^2}\right)^2 \quad \left(\frac{\text{const}}{s'}\right) \sim \frac{\text{const}}{Q^2} \tag{3.10}$$

absorption of hadron system	probability γ is hadron system	probability hadron system is aligned

For real γ's, this picture predicts the spectrum of states coupled to the γ is, asymptotically,

$$\frac{dN}{dm^2} \sim \frac{1}{m^4} \tag{3.11}$$

leading to a constant $\sigma_{DD}^\perp$ for diffractive production of vector states. For virtual γ's

$$\frac{d\sigma^\perp}{dm^2} \sim \frac{1}{(Q^2+m^2)^2} \tag{3.12}$$

and

$$\sigma_{DD}^\perp(Q^2) \sim \frac{1}{Q^2} \qquad Q^2 \gg m_\rho^2 \tag{3.13}$$

In addition, $\langle p_\perp^2 \rangle$ is always small and the connection with the parton-model is very visible: if each hadron-jet produced in the e^+e^- annihilation is the descendant of a parton, then the configurations aligned along the beam direction, when transformed to the lab, contain a wee parton. According to Feynman's dogma[11] this is the only kind of parton the target will absorb.

So in this picture the photon fragmentation region contains a hadron jet, which is essentially the same jet as one would find in a high energy event of $e^+e^- \to$ hadrons. Thus the overall picture interlocks nicely and is compatible with the "correspondence principle." Indeed, a more accurate version of (3.12) would be

$$\frac{d\sigma}{dm^2} \sim \frac{\rho(m^2)}{(Q^2+m^2)^2} \tag{3.14}$$

with $\rho(m^2)$ schematically shown in Fig. 8. In the sense of duality it may, for spacelike (positive) Q^2, be a good approximation to replace $\rho(m^2)$ by a

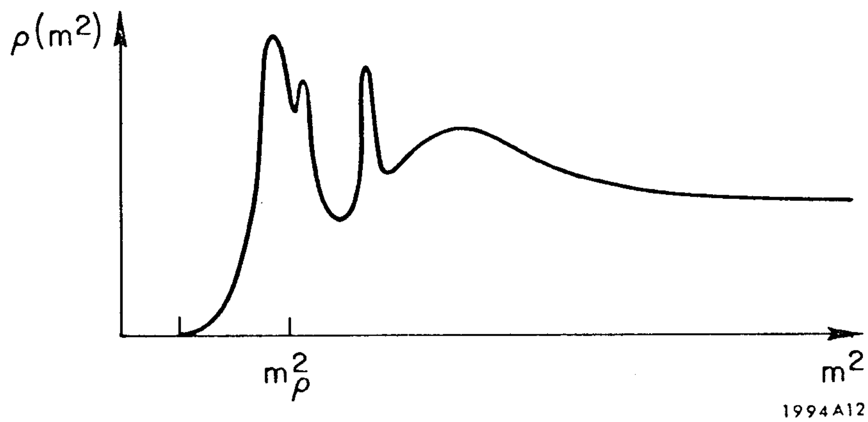

FIG. 8--Expected mass spectrum $\rho(m^2) = m^2\sigma(m^2)_{e^+e^-}$ of hadrons coupled to γ.

step-function with threshold $m_0^2 \lesssim m_\rho^2$. Thus

$$\sigma^1 \sim \frac{1}{Q^2+m_0^2} + \text{small corrections} \tag{3.15}$$

We then have "precocious scaling" with the scale for its onset set by $\sim m_\rho^2$, consistent with the argument in Section 2. We also see it is misleading to treat the ρ-dominant portion separately; it is an integral part of the scaling phenomenon.

Although this example is attractive to me, I don't believe it is a unique picture (see, e.g., Chou and Yang[12] and, in particular, Hwa and Lam[13]). It is meant to be an example of methodology as well as a possible option for how deep-inelastic hadron final states could appear.

4. DEEP-INELASTIC PHENOMENA IN PERTURBATION THEORY

Gribov and Lipatov[14] have just completed a study of the behavior of electroproduction structure functions in two field-theory models,[15] summed to all orders of $g^2 \log(Q^2/m^2)$; $g^2 \ll 1$. The theories are neutral pseudo-scalar (bare p, π^0) and neutral vector (bare p, ω^0). The important diagrams turn out to be t-channel ladder graphs, in which propagators and vertices are "exact," i.e., computed to all orders of $g^2 \log(k^2/m^2)$. For example, in the γ_5-theory the virtual Compton amplitude is obtained from the Bethe-Salpeter equation shown in Fig. 9;

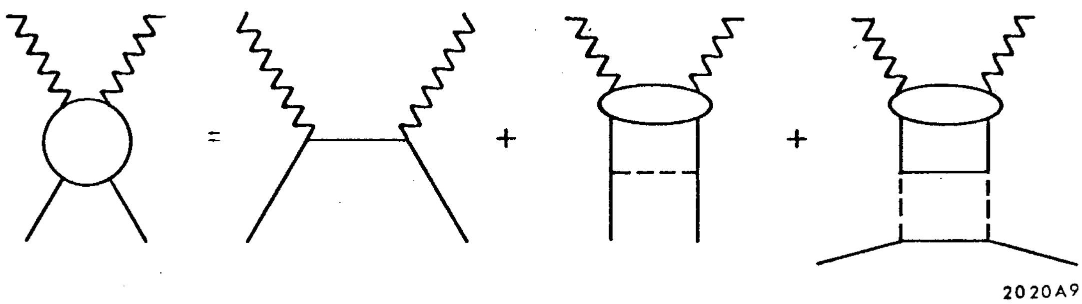

FIG. 9--Bethe-Salpeter equation for virtual Compton amplitude.

while for the vector case, the same diagrams, with dotted lines representing transverse vector mesons, apply. Two more, contributed by longitudinal mesons, must be included, as well as polarization bubbles on the external photon legs. In the scale limit, the integral equations may be solved by Mellin transforms.

The theory calculated to this order is not completely consistent because of the existence of the famous ghost in the photon propagator

$$d \sim \frac{1}{1 - \frac{g^2}{12\pi^2} \log \frac{k^2}{m^2}} \tag{4.1}$$

and Q^2 is restricted to values less than the ghost value. The results are phrased in terms of the "invariant charge"

$$\xi = \int_0^{Q^2} \frac{dk^2}{k^2}\, d(k^2) = \begin{cases} \log \left[1 - \frac{g^2}{16\pi^2} \log \frac{Q^2}{m^2}\right]^{-1} & \gamma_5 \text{ model} \\ \frac{3}{4} \log \left[1 - \frac{g^2}{12\pi^2} \log \frac{Q^2}{m^2}\right]^{-1} & \gamma_\mu \text{ model} \end{cases} \tag{4.2}$$

Among the conclusions are

1. $\sigma_S/\sigma_T \approx 0$.
2. $W_1 = W_1(\omega, \xi)$; i.e., does not scale.
3. Near $\omega=1$, the decrease of the elastic form factor is not compensated by the contribution of the inelastic channels in the γ_5 theory, but is overcompensated in the γ_μ theory.
4. Just as for real photons, in the vector theory $W_1 \sim \omega^p$ with $p > 1$ as $\omega \to \infty$, as a consequence of multiperipheral exchange of J=1 mesons.

5. The structure-functions for annihilation and scattering processes are simply related; for both γ_5 and γ_μ theories

$$W_1(\omega, \xi) = \frac{1}{\omega} W_1(1/\omega, \xi)_{\text{annihilation}} \tag{4.3}$$

6. The Callan-Gross sums are

$$\int_1^\infty \frac{d\omega}{\omega^3} W_1 = \begin{cases} \left.\begin{array}{ll} 6/7 + 1/7\, e^{(-7/3)\xi} & \text{p target} \\ 6/7 - 6/7\, e^{(-7/3)\xi} & \pi^o \text{ target} \end{array}\right\} & \gamma_5 \text{ model} \\ \left.\begin{array}{ll} (1/3 + 2/3\, e^{-2\xi})\, d^2(\xi) & \text{p target} \\ (1/3 - 1/3\, e^{-2\xi})\, d^2(\xi) & \omega \text{ target} \end{array}\right\} & \gamma_\mu \text{ model} \end{cases} \tag{4.4}$$

It is hard to assess the physical implications of all these results, especially because of the presence of the ghost and the restriction $g^2 \ll 1$. But in any case the work is a considerable technical accomplishment and may well teach us quite a bit.

REFERENCES

1. H. Kendall, Proceedings of the 1971 International Symposium on Electron and Photon Interactions at High Energies (Cornell University, to be published).
2. O. Nachtmann, Orsay preprint.
3. I. Budagov, D. Cundy, C. Franzinetti, W. Fretter, H. Hopkins, C. Manfredotti, G. Hyatt, F. Nezrick, M. Nikolic, T. Novey, R. Palmer, J. Pattison, D. Perkins, C. Ramm, B. Roe, R. Stump, W. Venus, H. Wachsmuth, and H. Yoshiki, Phys. Letters 30B, 364 (1969).
4. H. Fritsch and M. Gell-Mann, Proceedings of the 1971 Coral Gables Conference on Fundamental Interactions at High Energy, Coral Gables, Florida, Jan. 20-22, 1971, M. Dal Cin, G. J. Iverson, and A. Perlmutter, editors (Gordon and Breach Science Publishers, New York, 1971).
5. S. Adler, Phys. Rev. 143, 1144 (1966).
6. S. Adler and F. Gilman, Phys. Rev. 156, 1598 (1967).
7. E. Bloom and F. Gilman, Phys. Rev. Letters 25, 1140 (1970).
8. See also, W. Frazer, L. Ingber, C. Mehta, C. Poon, D. Silverman, K. Stowe, P. Ting, and H. Yesian, Rev. Mod. Phys. (to be published).
9. V. Gribov, "High Energy Interactions of Gamma Quanta and Electrons with Nuclei" (October 1969); Fourth Winter Seminar on the Theory of the Nucleus and the Physics of High Energies, Ioffe Institute of Engineering Physics, Acad. Sci., U.S.S.R., Feb. 5-17, 1969. See also S. Brodsky and J. Pumplin, Phys. Rev. 182, 1794 (1969); K. Gottfried and D. Yennie, Phys. Rev. 182, 1595 (1969).
10. See J. Bjorken, Proceedings of the 1971 International Symposium on Electron and Photon Interactions at High Energies (Cornell University, to be published).

11. R. P. Feynman, Phys. Rev. Letters 23, 1415 (1969).
12. T. T. Chou and C. N. Yang, SUNY Stony Brook preprint (to be published).
13. R. Hwa and C. Lam, SUNY Stony Brook preprint (to be published). This paper in fact carries out for the diffraction-excitation and "pulverization" picture much of the line of argument we use.
14. V. Gribov and L. Lipatov, Phys. Letters 37B, 78 (1971).
15. See also P. Fishbane and J. Sullivan, Phys. Letters 37B, 63 (1971).