SLAC-PUB-1003
(TH) and (EXP)
January 1972

EXPLORATION OF SU(3) SYMMETRY OF BARYON REGGE RESIDUES*

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ABSTRACT

The SU(3) symmetry of baryon Regge poles is explored by comparing the angular distributions of backward meson-baryon reactions. Several indications of such symmetry are found. In particular, it is discovered that one can generate the observed highly-structured angular distribution for $\pi^+ p \to p\pi^+$ from the featureless one for $K^+ p \to pK^+$, simply by assuming SU(3) symmetry, and taking exchange degeneracy in $K^+ p$ scattering into account. No parameters are involved.

(Submitted to Phys. Rev. Letters)

*Work supported in part by the U. S. Atomic Energy Commission, and in part by the National Science Foundation.

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We are seeking evidence in backward 0^- meson - $\frac{1}{2}^+$ baryon scattering for SU(3) symmetry of baryon Regge residues. A number of reflections of this symmetry, one of them rather striking, have been found.[1]

As is well-known, the residue of an ordinary resonance in the process (state i) $\rightarrow$ (state j) obeys close to exact SU(3) symmetry, provided the barrier penetration factor $(q_i q_j)^{\ell+1/2}$ is first divided out.[2] (Here q_i is the c.m. momentum in state i.) It is then natural to conjecture that the partial wave residue of a Regge pole is also SU(3)-symmetric, once the exact same quantity $(q_i q_j)^{\ell+1/2}$ is removed. Thus, we express this residue as[3] ($W \equiv \sqrt{u}$ is the energy carried by the Regge pole)

$$\left[\frac{\beta_{ij}(W)}{s_0^{\ell+1/2}}\right] (q_i q_j)^{\ell+1/2} \quad , \tag{1}$$

with

$$\beta_{ij}(W) = \left[gC_i\, \gamma(W)\right] \cdot \left[gC_j\, \gamma(W)\right] \quad . \tag{2}$$

Each bracket in (2) expresses a Regge vertex in terms of an overall coupling strength g, SU(3) Clebsch-Gordan coefficient C, and SU(3)-invariant W-dependence γ. In order for $\left[\beta/s_0^{\ell+1/2}\right]$ to obey SU(3), the scale constant s_0, which will appear in the characteristic Regge energy-dependence $(s/s_0)^{\alpha-1/2}$, must be SU(3)-invariant. (For octets of Regge poles, the vertex $[gC\gamma]$ is replaced by $\left[g(C_{id}\, d + C_{if}\, f)\, \gamma(W)\right]$, to accommodate the two independent octet-octet-octet couplings. The C_d and C_f are Clebsch-Gordan coefficients, and we normalize $d+f \equiv 1$.)

If $\beta_{ij}(\sqrt{u})$ and s_0 obey the assumed symmetry, then different backward meson-baryon reactions governed by u-channel exchange of Regge poles in the same SU(3) multiplet will have almost identical angular distributions at any given energy. For example, if $\pi^+p \to p\pi^+$, $K^-n \to \Lambda\pi^-$, and $K^-n \to \Sigma^0\pi^-$ are all dominated by the N_α Regge pole, their u-dependences should be rather similar. That they are indeed similar is evident in Figs. 1-3.

As a quantitative test of the symmetry, we have used a simple model based on N_α and Δ_δ poles (plus weak cuts which are unimportant for the present considerations) to describe the reactions just mentioned and $\pi^-p \to p\pi^-$. Except for the d/f ratio of the α octet, all parameters of the model are determined by fitting $\pi^\pm p \to p\pi^\pm$.[4] Assuming SU(3), one can then predict $d\sigma/du$ for $K^-n \to \Lambda\pi^-$, apart from the overall normalization, which depends on d. A fit to the data at 3.9 GeV/c[5] yields the value $d \simeq 0.8$. The corresponding theoretical curve, which compares quite favorably with the data, is shown in Fig. 1.[6] With d now fixed, one may make a parameter-free prediction for the reaction $K^-n \to \Sigma^0\pi^-$. The predicted angular distribution for 3.0 GeV/c is compared with the data[7] in Fig. 2. The agreement is quite reasonable.[8]

The successes represented by Figs. 1-2, besides suggesting that the N_α and Δ_δ residues do indeed possess SU(3) symmetry, give some evidence for factorization of the octet u-channel amplitude. A priori, there are three independent octet amplitudes. Suppose, however, that all three can be expressed in terms of two vertices, d-type and f-type. Then one can determine them from just the two reactions $\pi^+p \to p\pi^+$ and $K^-n \to \Lambda\pi^-$ (having previously used $\pi^-p \to p\pi^-$ to fix the decuplet amplitude), and then go on, as we did, to successfully predict the cross section for $K^-n \to \Sigma^0\pi^-$.

Now how well does SU(3) work when not only the external particles are changed, as above, but in addition the exchanged Regge pole is replaced by another from the same multiplet? Consider the reactions $K^+p \to pK^+$ and $\pi^+p \to p\pi^+$. The first of these has an undistinguished, rather flat backward peak,[9] while the second has a steep backward peak, deep dip, and secondary maximum. Is it possible, by constructing a simple picture of each reaction and applying SU(3), to generate the highly-structured π^+p angular distribution from the featureless one for K^+p?

In the exotic reaction $K^+p \to pK^+$, one expects the Regge poles to contribute in strongly exchange degenerate pairs, the predominant pair being $\Lambda_\alpha(1115)$ and $\Lambda_\gamma(1520)$.[10] Given SU(3), any contribution from a $\Sigma_\alpha - \Sigma_\gamma$ pair will look the same as that from $\Lambda_\alpha - \Lambda_\gamma$, so let us pretend, for simplicity, that d has the value d=1/2 for which only the Λ pair couples. Then, neglecting the demonstrably small decuplet contribution, the invariant amplitudes for K^+p are proportional to

$$\left[\left(1 + e^{-i\pi\bar{\alpha}_\Lambda}\right) + \left(1 - e^{-i\pi\bar{\alpha}_\Lambda}\right)\right] \left(\frac{s}{s_0}\right)^{\bar{\alpha}_\Lambda(u)} , \tag{3}$$

where $\bar{\alpha}_\Lambda \equiv \alpha_\Lambda - \frac{1}{2}$. The Λ_α and Λ_γ contributions are, respectively, the $(1+e^{-i\pi\bar{\alpha}_\Lambda})$ and $(1 - e^{-i\pi\bar{\alpha}_\Lambda})$ pieces of this expression.

From the observed properties of the 1520 MeV resonance, the Λ_γ trajectory is mostly an SU(3) singlet.[2] Thus it has no analogue in π^+p. The Λ_α, of course, turns into the N_α when we go to π^+p. Thus, the application of SU(3) to (3) yields π^+p amplitudes proportional to $\left(\bar{\alpha}_N \equiv \alpha_N - \frac{1}{2}\right)$

$$\left[1 + e^{-i\pi\bar{\alpha}_N}\right] \left(\frac{s}{s_0}\right)^{\bar{\alpha}_N(u)} . \tag{4}$$

Squaring (3) and (4), we find that

$$\frac{(d\sigma/du)_{\pi^+p}}{(d\sigma/du)_{K^+p}} = C \cos^2 \frac{\pi\bar{\alpha}_N(u)}{2} \left(\frac{s}{s_0}\right)^{2(\alpha_N-\alpha_\Lambda)} , \tag{5}$$

where C is a constant depending on Clebsch-Gordan coefficients. Since α_N and α_Λ are parallel, $(s/s_0)^{2(\alpha_N - \alpha_\Lambda)}$ does not depend on u.

The new 5 GeV/c K^+p data[11] may be described by[11] $(d\sigma/du)_{K^+p} = 28\ e^{4.2u}$ $\mu b/GeV^2$, with u in GeV^2. Thus, from (5) one predicts that at the same energy

$$(d\sigma/du)_{\pi^+p} = 344\ e^{4.2u} \cos^2 \frac{\pi\bar{\alpha}_N(u)}{2}\ \mu b/GeV^2 \quad . \tag{6}$$

The normalization factor 344 has been computed with d taken as $\frac{1}{2}$, and s_0 as 1 GeV^2, a round number suggested by the slope of the K^+p peak. Note that at 5 GeV/c, $(s/s_0)^{2(\alpha_N - \alpha_\Lambda)} = 5.5$, a large SU(3)-breaking effect arising from the relatively small α_N, α_Λ splitting.

The prediction (6) is compared with the π^+p data at 5.2 GeV/c in Fig. 3. The observed u-dependence is reproduced with striking success. The steepness of the backward peak, the general shape of the secondary maximum, and the height of the secondary maximum relative to points in the backward peak, are all correctly given. Of course, (6) undershoots the bottom of the dip, because the N_α vanishes at this point and we have omitted the Δ_δ background. The overall normalization, the only thing which depends on d, also comes out correctly. To be sure, if instead of 0.5 one takes d to be 0.8, the normalization will shift upward by a factor of 1.7. However, this could be compensated for by choosing a larger s_0; also, it would shift downward again if one used the earlier K^+p measurements[9] as input to (5). The most important thing is the π^+p u-dependence, which does turn out to be just $\cos^2 \pi\bar{\alpha}_N/2$ times that for K^+p.[12]

A check on the consistency of the SU(3)-symmetric picture emerges from the determination of the d-value implied by the process $\pi^-p \rightarrow \Lambda K^o$. This reaction can involve exchange of $\Sigma_\alpha(1190)$, $\Sigma_\gamma(1670)$, $\Sigma_\delta(1385)$, and $\Sigma_\beta(1765)$.

The d-value for the α octet can be found if one can isolate the Σ_α contribution, and compare it to the N_α term which dominates $\pi^+ p \to p\pi^+$.

Applying SU(3) to the Δ_δ contribution measured in $\pi^- p \to p\pi^-$, one estimates that the Σ_δ contribution to $\pi^- p \to \Lambda K^o$ at 6 GeV/c and u=0 is 6% of the measured rate.[13] From exchange degeneracy requirements and SU(3), the Σ_β contribution is quite a bit smaller still.[1] Thus, Σ_α and Σ_γ must be supplying most of the observed cross section. Now if duality diagrams are anywhere near right, the Σ_γ is not small compared to the Σ_α. Since we lack an accurate estimate of this Σ_γ background, we simply eliminate it by going to the u-value where $\alpha_{\Sigma_\gamma}(u) = -3/2$, and the Σ_γ contribution vanishes because of a wrong-signature nonsense zero. From its Chew-Frautschi plot, $\alpha_{\Sigma_\gamma} = -3/2$ at $u = -0.61\ \mathrm{GeV}^2$, so we compare the $\pi^- p \to \Lambda K^o$ rate (6.2 GeV/c data[13]) with the $\pi^+ p \to p\pi^+$ rate (5.2 GeV/c data[9]) at this u. Neglecting at first the Σ_δ and Σ_β, the comparison follows the principles that led to relation (5), and includes a correction for the energy mismatch based on Regge power law behavior. It yields the result d=0.66. One can refine this value by including the Σ_δ, making some guesses about how Σ_α and Σ_δ interfere. This leads to the modified result d=0.63. Obviously, the Σ_δ has very little effect, so we make no attempt to defend our particular guesses. We observe that the value just found, 0.63, is reassuringly close to the 0.8 obtained previously by fitting $K^- n \to \Lambda\pi^-$. (The values of d deduced from various meson-baryon coupling constants differ by more than this.[14])

Further work within the SU(3)-symmetric scheme is in progress. In particular, we are trying to understand the very large, positive polarization in $\pi^- p \to \Lambda K^o$.[13] If one assumes strong exchange degeneracy in exotic reactions such as $K^+ p \to pK^+$, and SU(3), then the sign of this polarization may be used to determine the relative sign of the δ-decuplet and α-octet residues. Unfortunately, the so-determined sign contradicts that deduced from the 180^0 elastic πN cross sections.[15] We take this to suggest that exchange degeneracy is somewhat broken, and are proceeding on that assumption.[16]

In closing, we note that better, higher-energy data, on more reactions, would be very valuable for the further testing of SU(3) symmetry and exchange degeneracy. For example, the processes $K^+n \rightarrow pK^o$ and $K^+p \rightarrow pK^+$ involve different admixtures of the same exchange-degenerate $\Lambda_\alpha - \Lambda_\gamma$ and $\Sigma_\alpha - \Sigma_\gamma$ pairs, whose contributions look the same if SU(3) is true. Thus, these processes should have similar angular distributions. However, it has recently been reported[17] that at 2 GeV/c, the charge exchange cross section looks completely unlike that for $K^+p \rightarrow pK^+$. It will be extremely interesting to see whether $(d\sigma/du)_{K^+n \rightarrow pK^o}$ takes on the shape of $(d\sigma/du)_{K^+p \rightarrow pK^+}$ as the momentum is raised toward the Regge regime of 5 GeV/c and above. (If this does happen, the charge exchange rate will provide new information on the value of d, as would the resolving of the uncertainty over the $K^+p \rightarrow pK^+$ normalization.)

More generally, data at several higher energies for reactions which have so far been studied only at 3 or 4 GeV/c ($K^-n \rightarrow \Lambda\pi^-$, $\pi^-p \rightarrow \Sigma^-K^+$, etc.) will permit a meaningful test of Regge pole energy-dependence. Angular distributions for reactions which have not been studied even at moderate energies ($K^-p \rightarrow \Xi^0K^0$ and many others) will make possible new tests of the predictions of SU(3) symmetry of Regge residues.

One of us (F.H.) would like to thank Professor J. Finkelstein, Dr. F. Halzen, Professor G. Kane, Dr. C. Michael, and Dr. C. Schmid for very helpful discussions. The other (B.K.) likewise thanks Dr. M. Einhorn, Dr. R. Field, Jr., and Professor J. D. Jackson. He is especially grateful to Professor V. Barger for the conversation which stimulated the attempt to generate the $\pi^+ p \rightarrow p\pi^+$ data from those for $K^+ p \rightarrow pK^+$. Both authors thank Dr. R. F. Peierls for making the computing facilities of Brookhaven National Laboratory available during an early part of this work.

REFERENCES AND FOOTNOTES

1. A more complete discussion, with details, will be presented elsewhere.
2. E. Flaminio et al., Brookhaven preprint BNL 14572 (March 1970).
3. Kinematical factors which are suggested by analyticity requirements and involve little symmetry breaking are also included.
4. F. Hayot and B. Kayser, Il Nuovo Cimento 3A, 45 (1971).
5. D. Crennell et al., Phys. Rev. Letters 23, 1347 (1969).
6. This favorable result is confirmed by a residue comparison made by J. Frøyland, F. Halzen, and B. Petersson, CERN preprint TH. 1255.
7. R. Barloutaud et al. (SABRE collaboration), Nucl. Phys. B26, 557 (1971).
8. Exploratory pole model fits to the $\bar{K}N \rightarrow \Sigma\pi$ reactions, incorporating SU(3) constraints, have been given by V. Barger, Coral Gables Conference on Fundamental Interactions at High Energy, January, 1969 (Gordon and Breach, New York, 1969).
9. W. Baker et al., Phys. Letters 28B, 291 (1968).
10. A successful Λ_α-Λ_γ model for K^+p has been presented by V. Barger, Phys. Rev. 179, 1371 (1969).
11. CERN-Orsay-Ecole Polytechnique-Stockholm Collaboration, Phys. Letters (to be published). We thank Dr. M. Davier for making the data, and the exponential fit to them, available to us.

12. F. Halzen, in Proceedings of the 6th Rencontre de Moriond, March, 1971, has calculated the N_α and Λ_α residues from the cross sections for $K^- n \to \Lambda\pi^-$ and $K^+ p \to pK^+$, respectively, and found them to be related according to SU(3). His analysis, unlike ours, includes a non-negligible N_γ in $K^- n \to \Lambda\pi^-$, but this should not make too much difference. Thus his result and ours are confirmations of the same basic SU(3) picture.

13. W. Beusch et al., Nucl. Phys. B19, 546 (1970).

14. R. Field, Jr., and J. D. Jackson, Phys. Rev. D4, 693 (1971), Fig. 4.

15. E. Berger and G. Fox, Nucl. Phys. B26, 1 (1971).

16. The $\Sigma_\alpha - \Sigma_\gamma$ model for $\pi^- p \to \Lambda K^0$ of V. Barger, D. Cline, and J. Matos (Phys. Letters 29B, 121 (1969)) implies a complete breakdown of strong exchange degeneracy.

17. D. Davies et al., UCLA preprint.

FIGURE CAPTIONS

1. Differential cross section for $K^- n \rightarrow \Lambda \pi^-$ at 3.9 GeV/c. The shape of the theoretical curve follows from SU(3), with no parameters. The overall normalization has been adjusted to give the best fit to the data, and corresponds to $d=0.8$. Data from Ref. 5.

2. Parameter-free SU(3) prediction for the $K^- n \rightarrow \Sigma^0 \pi^-$ cross section at 3.0 GeV/c. Data from Ref. 7.

3. Prediction of the angular distribution for $\pi^+ p \rightarrow p \pi^+$ at 5.2 GeV/c, obtained by applying SU(3) symmetry to the angular distribution for $K^+ p \rightarrow p K^+$ at the same energy. No parameters have been adjusted. Data for $\pi^+ p$ from Ref. 9.