# DEEP INELASTIC ELECTROPRODUCTION, SCALING, AND FORM FACTORS OF THE NUCLEON IN A BREMSSTRAHLUNG MODEL* 

Héctor Moreno<br>Stanford Linear Accelerator Center Stanford University, Stanford, California 94305


#### Abstract

It is shown how one can account for the main features of the nucleon's elastic and inelastic form factors and of the electroproduction experiments in the framework of a simple bremsstrahlung model.


[^0]It is the purpose of this communication to describe how a simple semiclassical model of high-energy scattering suffices to account for (a) the elastic and inelastic nucleon electromagnetic form factors at large momentum transfers, and (b) Bjorken ${ }^{1}$ scaling in deep inelastic electroproduction. It provides also a natural framework to study the role of resonances in electroproduction ${ }^{2}$ and is a realization of previous considerations ${ }^{3-5}$ on the subject.

The basic idea of the model ${ }^{6,7}$ is that at high energy, a nucleon describes a classical path, parameterizable by its four-momentum, which remains constant throughout its motion except for sudden changes caused by large momentum transfer ${ }^{8}$ collisions.

The nucleon is coupled to a field, here taken to be massive, and neutral ${ }^{9}$ and referred to as the meson field. The assumption of constancy of the nucleon's four-momentum implies that only soft mesons should be considered, and no pairs allowed.

The neutral, soft character of the meson field guarantees that subsequent emissions and absorptions are uncorrelated. The deflections of the nucleon introduce current frequency components that induce meson bremsstrahlung. However, there is an overall exponential factor in the scattering amplitude which survives in the limit of no real meson emission. That factor measures how difficult it is for the nucleon to deflect without "breaking, " ${ }^{10}$ i.e., radiating, and will be identified as the depositary of the relevant physical information about the form factors.

In the context of the specific calculation described below, it is essential to enforce dominance of the infrared region ${ }^{11}$ by a cutoff procedure ${ }^{12}$ that is required to be consistent with the softness assumption of the meson exchanges. In choosing the cutoff, one is guided by the experimental behavior of the form factors. ${ }^{5}$

Two models will be considered: (a) scalar meson field and (b) vector meson field. One can deal with the two cases along the same lines and the results are similar.

## I. Scalar Meson Model

The S matrix elements for the process

$$
\mathrm{n} \text { mesons } \longrightarrow \mathrm{m} \text { mesons }
$$

in the presence of a classical current distribution when the meson field obeys the equation

$$
\begin{equation*}
\left(\square-\mu^{2}\right) \sigma=J \tag{1}
\end{equation*}
$$

is given by

$$
\begin{equation*}
S_{m n}={ }_{i n}\langle m|: e^{i \int J \sigma_{\mathrm{in}}}:|n\rangle_{\mathrm{in}} e^{-\mathrm{W} / 2} \tag{2}
\end{equation*}
$$

with

$$
\begin{align*}
& \mathrm{W}=\frac{1}{2} \frac{1}{(2 \pi)^{3}} \int \mathrm{~d}_{4} \mathrm{k} \delta\left(\mathrm{k}^{2}-\mu^{2}\right)|\widetilde{J}(\mathrm{k})|^{2}  \tag{3}\\
& \widetilde{\mathrm{~J}}(\mathrm{k})=\int \mathrm{e}^{\mathrm{i} k x} \mathrm{~J}(\mathrm{x}) \dot{d}_{4} \mathrm{x} \tag{4}
\end{align*}
$$

Taking now as source current

$$
\begin{equation*}
J(\mathrm{x})=\mathrm{g}\left\{\mathrm{~m}\left(\mathrm{E}_{\mathrm{p}}\right)^{-1} \delta\left(\overrightarrow{\mathrm{x}}-\left(\mathrm{E}_{\mathrm{p}}\right)^{-1} \overrightarrow{\mathrm{p}} \mathrm{x}^{0}\right) \theta\left(-\mathrm{x}^{0}\right)+\mathrm{m}^{\prime}\left(\mathrm{E}_{\mathrm{p}^{\prime}}\right)^{-1} \delta\left(\overrightarrow{\mathrm{x}}-\left(\mathrm{E}_{\mathrm{p}^{\prime}}\right)^{-1} \overrightarrow{\mathrm{p}}^{\prime} \mathrm{x}^{0}\right) \theta\left(\mathrm{x}^{0}\right)\right\} \tag{5}
\end{equation*}
$$

corresponding to the graph of Fig. 1, and where $\mathrm{p}^{\mu}=\left(\mathrm{E}_{\mathrm{p}}, \overrightarrow{\mathrm{p}}\right), \mathrm{p}^{\prime}{ }^{\mu}=\left(\mathrm{E}_{\mathrm{p}^{\prime}}, \overrightarrow{\mathrm{p}}^{\prime}\right)$ are the four-momenta of the incoming and outgoing particles; $m, m^{\prime}$ their rest masses and $g$ the meson-source coupling constant, one finds

$$
\begin{equation*}
\mathrm{W}=\frac{1}{2} \frac{\mathrm{~g}^{2}}{(2 \pi)^{3}} \int \mathrm{~d}_{4} \mathrm{k} \delta\left(\mathrm{k}^{2}-\mu^{2}\right)\left[\frac{\mathrm{m}^{\prime}}{\mathrm{kp}^{\prime}+\mathrm{i} \epsilon}-\frac{\mathrm{m}}{\mathrm{kp}+\mathrm{i} \epsilon}\right]^{2}+\mathrm{W}_{\mathrm{f}} \tag{6}
\end{equation*}
$$

with

$$
\begin{equation*}
\mathrm{W}_{\mathrm{f}}=\frac{\mathrm{g}^{2}}{2(2 \pi)^{3}} \int \mathrm{~d}_{4} \mathrm{k} \delta\left(\mathrm{k}^{2}-\mu^{2}\right)(2 \pi \mathrm{i})\left[\frac{\mathrm{m}^{\prime 2} \delta(\mathrm{kp})}{\mathrm{k} \mathrm{p}^{\prime}+\mathrm{i} \epsilon}+\frac{\mathrm{m}^{2} \delta(\mathrm{kp})}{\mathrm{kp}+\mathrm{i} \epsilon}\right] \tag{7}
\end{equation*}
$$

In the limit of forward scattering, $\mathrm{W} \rightarrow \mathrm{W}_{\mathrm{f}}$. One can simply subtract $\mathrm{W}_{\mathrm{f}}$ from Eq. (6) by referring all amplitudes to the forward case or set $W_{f}$ equal to zero by defining ${ }^{8,14}$ :

$$
\begin{equation*}
\int \mathrm{d}_{4} \mathrm{k} \delta\left(\mathrm{k}^{2}-\mu^{2}\right) \frac{\delta\left(\mathrm{kp}{ }^{\prime}\right)}{\mathrm{k} \mathrm{p}^{\prime}+\mathrm{i} \epsilon}=\lim _{\ell \rightarrow \mathrm{p}^{\prime}} \int \mathrm{d}_{4} \mathrm{k} \delta\left(\mathrm{k}^{2}-\mu^{2}\right) \frac{\delta\left(\mathrm{kp}^{\prime}\right)}{\mathrm{k} \ell+\mathrm{i} \epsilon}=0 \tag{8}
\end{equation*}
$$

since both k and $\mathrm{p}^{\prime}$ are time-like vectors.
By inserting parametric integral representations for the denominators in Eq. (6), one can easily bring $W$ to the form:

$$
\begin{equation*}
\mathrm{W}=-\frac{\mathrm{g}^{2}}{4 \pi} \mathrm{D}\left[1-\int_{\Lambda}^{\infty} \frac{\mathrm{d} \ell}{\ell^{2}-\mathrm{A}^{2}}\right] \tag{9}
\end{equation*}
$$

where

$$
\begin{align*}
& A^{2}=4 \lambda(\lambda+1)  \tag{10}\\
& \lambda=\frac{\left(m^{\prime}-m\right)^{2}}{4 m^{\prime}}-\frac{q^{2}}{4 m^{\prime} m^{\prime}}  \tag{11}\\
& \Lambda=2 \lambda+1  \tag{12}\\
& q^{2}=-Q^{2}=t=\left(p^{\prime}-p\right)^{2}  \tag{13}\\
& D=\int_{0}^{\infty} N_{1}(y) d y \tag{14}
\end{align*}
$$

It is here where one has to introduce a cutoff in order to prevent $D$ from becoming logarithmically divergent as a result of the lower limit of the integral in Eq. (14) being zero, which reflects the effect of the integration over all the on-mass shell momentum space of Eq. (6). As $\mu \neq 0$, there is no infrared divergence, ${ }^{11}$ only the ultraviolet one, and to be consistent with the soft meson
character of the model, one has to introduce a positive lower limit in the integral of Eq. (14):

$$
\begin{equation*}
\mathrm{D} \rightarrow \mathrm{D}\left(\mathrm{y}_{0}\right)=\int_{\mathrm{y}_{0}}^{\infty} \mathrm{N}_{1}(\mathrm{y}) \mathrm{dy}=\mathrm{N}_{0}\left(\mathrm{y}_{0}\right) \tag{15}
\end{equation*}
$$

W can be straightforwardly evaluated after this cutoff to be:

$$
\begin{equation*}
\mathrm{W}=-\frac{\mathrm{g}^{2}}{4 \pi} \mathrm{~N}_{0}\left(\mathrm{y}_{0}\right)\left\{1-\frac{1}{\sqrt{\lambda(\lambda+1)}} \ln (\sqrt{\lambda}+\sqrt{\lambda+1})\right\} \tag{16}
\end{equation*}
$$

If $\mathrm{y}_{0} \lesssim 1$, one can replace $\mathrm{N}_{0}\left(\mathrm{y}_{0}\right) \sim \frac{2}{\pi} \ln \mathrm{y}_{0} \quad$ and write

$$
\begin{equation*}
G\left(Q^{2}, \mathrm{~m}^{2^{2}}\right) \equiv \mathrm{e}^{-\mathrm{W} / 2}=\left(\mathrm{y}_{0}\right)^{\left.\frac{\mathrm{g}^{2}\langle }{4 \pi^{2}} \cdot 1-\frac{1}{\sqrt{\lambda(\lambda+1)}} \ln (\sqrt{\lambda}+\sqrt{\lambda+1})\right]} \tag{17}
\end{equation*}
$$

At this point, one must invoke experiment to determine $y_{0}$. It appears ${ }^{5,3,4}$ that the excitation form factors of the nucleon are universal functions of the ratio $Q^{2} / \mathrm{m}^{\prime^{2}}$ falling as a power of it for $Q^{2} \geq \mathrm{m}^{\prime 2}(\lambda>1)$. That fact suggests:

$$
\begin{equation*}
\mathrm{y}_{0} \underset{\lambda>1}{\sim}\left(\frac{\mathrm{Q}^{2}}{\mathrm{~m}^{2}}\right)^{-1} \tag{18}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{n}=\mathrm{g}^{2} / 4 \pi^{2} \tag{19}
\end{equation*}
$$

as the fall-off power.
Guided by Eq. (18), it is easy to establish a relation between $\lambda$ and $y_{0}$. An argument follows:

The role of the variable $\lambda$ in Eq. (10) and (12) is reminiscent of angular momentum. Writing

$$
\begin{equation*}
\lambda=m^{\prime} b_{0} \tag{20}
\end{equation*}
$$

where $b_{0}$ is an "impact parameter," one obtains from Eq. (11):

$$
\begin{equation*}
4 \mathrm{mb}_{0}=\left(1+\frac{\mathrm{Q}^{2}}{\mathrm{~m}^{\prime^{2}}}\right)+\left(\frac{\mathrm{m}^{2}-2 \mathrm{~m} \mathrm{~m}^{\prime}}{\mathrm{m}^{\prime^{2}}}\right) \tag{21}
\end{equation*}
$$

For $m^{\prime} \sim m, 4 \mathrm{mb}_{0} \sim Q^{2} / \mathrm{m}^{\prime^{2}}$ while in the limit $\mathrm{m}^{\prime 2}>\mathrm{m}^{2}(\lambda \gg 1)$, one has ${ }^{13}$

$$
\begin{equation*}
4 \mathrm{mb}_{\substack{0 \\\left(\mathrm{~m}^{\left.\prime^{2} \gg \mathrm{~m}^{2}\right)}\right.}}\left(1+\frac{\mathrm{Q}^{2}}{\mathrm{~m}^{\prime^{2}}}\right) \equiv \rho^{\prime}=\frac{\omega^{\prime}}{\omega^{\prime}-1} \tag{22}
\end{equation*}
$$

with

$$
\begin{equation*}
\omega^{\prime}=\frac{2 \mathrm{pq}}{\mathrm{Q}^{2}}+\frac{\mathrm{m}^{2}}{\mathrm{Q}^{2}}, \tag{23}
\end{equation*}
$$

the Bloom-Gilman variable. ${ }^{2}$ Comparison of Eq. (21) and (18) deems it consistent to set

$$
\begin{equation*}
y_{0}=\frac{1}{4 m b_{0}}=\frac{m^{\prime}}{4 \mathrm{~m} \lambda} \tag{24}
\end{equation*}
$$

which is the desired relation between $\mathrm{y}_{0}$ and $\lambda$.
Turning to electroproduction, consider the virtual forward Compton scattering diagram of Fig. 2, associated with a current distribution:

$$
\begin{align*}
J(x)= & g m^{\prime}\left(E_{p+q}\right)^{-1} \delta\left(\vec{x}-(\vec{p}+\vec{q})\left(E_{p+q}\right)^{-1} x^{o}\right)\left[1-\theta\left(x^{0}-T\right)-\theta\left(-x^{0}-T\right)\right] \\
& +g m\left(E_{p}\right)^{-1} \delta\left(\vec{x}-(\vec{p}+\vec{q})\left(E_{p+q}\right)^{-1} T-\vec{p}\left(E_{p}\right)^{-1}\left(x^{0}-T\right)\right) \theta\left(x^{0}-T\right) \\
& +\operatorname{gm}\left(E_{p}\right)^{-1} \delta\left(\vec{x}+(\vec{p}+\vec{q})\left(E_{p+q}\right)^{-1} T-\vec{p}\left(E_{p}\right)^{-1}\left(x^{o}+T\right)\right) \theta\left(-x^{o}-T\right) \tag{25}
\end{align*}
$$

$T$ is a parameter, related to the "lifetime" of the intermediate state as seen in the frame of the figure. Notice that $\tau \equiv 2 \mathrm{~m}^{\prime}\left(\mathrm{E}_{\mathrm{p}+\mathrm{q}}\right)^{-1} \mathrm{~T}$ is the relativistic interval between the virtual photon absorption and emission.

The corresponding $W$ (denoted from now on as $W_{e}$ ) is easily computed to be ${ }^{14}$ :

One can see from Eq. (26) that if $\tau=0$, corresponding to light-cone-related absorption and emission, then $\mathrm{W}_{\mathrm{e}}=0$. However, $\tau=0$ is not consistent with the model since $\mathrm{p}^{\prime 2}=\mathrm{m}^{{ }^{2}}>0$ and therefore $\tau>0$, i.e., the electromagnetic interactions must be related by time-like intervals.

The contribution to $W_{e}$ coming from the term 1 in the first square bracket of Eq. (26) is just $2 \mathrm{~W}, \mathrm{~W}$ defined by Eq. (6). The contribution from the $\exp \left[i k p^{\prime} \frac{\tau}{\mathrm{m}^{\dagger}}\right]$ term can easily be cast into the form

$$
\begin{equation*}
\mathrm{W}_{\mathrm{e} \tau}=\frac{\mathrm{g}^{2}}{8 \pi} \int_{1}^{\infty} \mathrm{d} \ell \mathrm{~N}_{0}(\mu \tau \ell)\left[\frac{1}{\ell^{2}}+\frac{(2 \lambda+1) \ell}{\left(\ell^{2}+\mathrm{A}^{2}\right)^{3 / 2}}-\frac{2}{\ell\left(\ell^{2}+\mathrm{A}^{2}\right)^{1 / 2}}\right] \tag{27}
\end{equation*}
$$

with

$$
\begin{equation*}
\mathrm{W}_{\mathrm{e}}=2\left(\mathrm{~W}+\mathrm{W}_{\mathrm{e} \tau}\right) \tag{28}
\end{equation*}
$$

and $\lambda, A^{2}$ defined as in Eq. (11) and (10).
Therefore

$$
\begin{equation*}
e^{-\frac{\mathrm{W}_{\mathrm{e}}}{2}}=\left(\mathrm{e}^{\left.-\frac{\mathrm{W}}{2}\right)^{2}\left(\mathrm{e}^{\left.-\frac{\mathrm{W}_{\mathrm{e} \tau}}{2}\right)^{2}} \equiv \mathrm{~g}^{2}\left(\mathrm{Q}^{2}, \mathrm{~m}^{\prime 2} ; \mu, \mathrm{m}\right)\right.}\right. \tag{29}
\end{equation*}
$$

One observes that Eq. (29) implies a factorization of $g\left(Q^{2}, \mathrm{~m}^{\prime 2} ; \mu, \mathrm{m}\right)$; and if $\lambda \gg 1$ :

$$
\begin{equation*}
\mathrm{g}\left(\mathrm{Q}^{2}, \mathrm{~m}^{\prime^{2}} ; \mu, \mathrm{m}\right)=\mathrm{G}\left(\mathrm{Q}^{2}, \mathrm{~m}^{\prime^{2}}\right) \mathrm{f}\left(\mu, \mathrm{~m}, \mathrm{~m}^{\prime}\right) \tag{30}
\end{equation*}
$$

where one has tacitly assumed $\tau$ to depend on $\mu, \mathrm{m}, \mathrm{m}^{\prime}$.
Assuming that the $\nu \mathrm{W}_{2}$ structure function of electroproduction is built up from resonance contributions only, one can write, ${ }^{3}$ in the narrow-width
approximation:

$$
\begin{equation*}
\nu \mathrm{W}_{2}\left(\mathrm{Q}^{2}, \nu\right)=\mathrm{Q}^{2} \sum_{\mathrm{f}} \mathrm{~g}^{2}\left(\mathrm{Q}^{2}, \mathrm{~m}_{\mathrm{f}}^{2}, \mu, \mathrm{~m}\right) \delta\left(\mathrm{m}^{\prime 2}-\mathrm{m}_{\mathrm{f}}^{2}\right) \tag{31}
\end{equation*}
$$

If one has an infinite number of closely spaced terms in the sum of Eq. (31) so that a level density $\rho\left(\mathrm{m}^{\prime 2}\right.$ ) can be defined, and recalling Eq. (17), (24), and (22), one can write in the limit $\mathrm{Q}^{2}>\mathrm{m}^{\prime 2} \gg \mathrm{~m}^{2}$ (equivalent to large $\rho^{\prime}$ or $2 \gtrsim \omega^{\prime} \gtrsim 1$ ), following the steps of Ref 3:

$$
\begin{equation*}
\nu \mathrm{W}_{2} \underset{\omega^{\prime} \rightarrow 1}{\sim} \frac{1}{\omega^{\prime}-1}\left(\frac{\omega^{\prime}-1}{\omega^{\top}}\right)^{2 \mathrm{n}} \tag{32}
\end{equation*}
$$

if

$$
\begin{equation*}
\rho\left(\mathrm{m}^{\prime 2}\right)=\frac{1}{\mathrm{~m}^{\mathrm{r}^{2}}} \mathrm{f}^{-2}\left(\mu, \mathrm{~m}, \mathrm{~m}^{\prime}\right) \tag{33}
\end{equation*}
$$

Equation (32) manifestly satisfies the Dreli-Yan-West relation ${ }^{15}$ between the behavior of the nucleon elastic form factor and $\nu \mathrm{W}_{2}$ as $\omega^{\dagger} \longrightarrow 1$.

In the strict Bjorken limit $Q^{2} \longrightarrow \infty, \omega=\frac{2 p q}{Q^{2}}$ fixed, one has $\lambda \longrightarrow \infty$, $4 \mathrm{mb}_{0} \longrightarrow \omega(\omega-1)^{-1}$ from Eq. (22). Therefore, $\mathrm{y}_{0}=(\omega-1) / \omega$. All the form factors are given by $G\left(Q^{2}, \mathrm{~m}^{\prime 2}\right) \equiv \exp [-\mathrm{W} / 2]=\exp \left\{\left(\mathrm{g}^{2} / 8 \pi\right) \mathrm{N}_{0}[(\omega-1) / \omega]\right\}$ and as long as Eq. (33) holds, one has:

$$
\begin{equation*}
\nu \mathrm{W}_{2} \sim \frac{1}{\omega-1} \exp \left[\mathrm{n} \pi \mathrm{~N}_{0}[(\omega-1) / \omega]\right] \tag{34}
\end{equation*}
$$

which for large $\omega$ falls like $(\omega-1)^{-1}$.
As scaling is experimentally observed, and it is very good near $\omega^{\prime} \lesssim 3$, one may use Eq. (33) and see how $\rho\left(\mathrm{m}^{2^{2}}\right.$ ) is related to $\tau$ in the limit $\mathrm{m}^{\prime 2} \gg \mathrm{~m}^{2}$ :

$$
\begin{equation*}
\rho\left(\mathrm{m}^{\prime 2}\right)=\frac{1}{\mathrm{~m}^{\prime^{2}}} \mathrm{e}^{\mathrm{W}_{\mathrm{e} \tau}} \tag{35}
\end{equation*}
$$

## II. Vector Meson Model

Vector mesons can be handled in a similar way as the scalar mesons. There are some modifications which will be pointed out.

Equations (1) to (4) are valid with the addition of Lorentz vector indices for the meson field and the current, provided the latter is conserved. The products $J \sigma,|\widetilde{J}(\mathrm{k})|^{2}$ are understood as $\mathrm{J}^{\mu} \sigma_{\mu} ; \widetilde{J}^{\mu}(\mathrm{k}) \widetilde{\mathrm{J}}_{\mu}^{*}(\mathrm{k})$. Equation (5) is changed by introducing $p_{\mu}, p_{\mu}^{\prime}$ instead of the $m, m^{\prime}$ coefficients. Then $\partial_{\mu} J^{\mu}=0$ and the current is conserved. Equation (6) will now read ${ }^{17}$ :

$$
\begin{equation*}
\overline{\mathrm{W}}=\frac{1}{2} \frac{\mathrm{~g}^{2}}{(2 \pi)^{3}} \int \mathrm{~d}_{4} \mathrm{k} \delta\left(\mathrm{k}^{2}-\mu^{2}\right)\left[\frac{\mathrm{p}_{\mu}^{\prime}}{\mathrm{k} \mathrm{p}^{\prime}+\mathrm{i} \epsilon}-\frac{\mathrm{p}_{\mu}}{\mathrm{kp}+\mathrm{i} \epsilon}\right]^{2}+\mathrm{W}_{\mathrm{f}} \tag{36}
\end{equation*}
$$

with $W_{f}$ given by Eq. (7). $W_{f}$ will be dropped from now on, appealing to the same arguments.

Inserting parametric integral representations for the denominators in Eq. (36), one finds:

$$
\begin{equation*}
\overline{\mathrm{W}}=-\frac{\mathrm{g}^{2}}{4 \pi} \mathrm{D}\left[1-\Lambda \int_{\Lambda}^{\infty} \frac{\mathrm{d} \ell}{\ell^{2}-\mathrm{A}^{2}}\right] \tag{37}
\end{equation*}
$$

with the definitions of Eq. (10) to (14). Cutting off the D integral as in Eq. (15), one is left with

$$
\begin{equation*}
\overline{\mathrm{W}}=-\frac{\mathrm{g}^{2}}{4 \pi} \mathrm{~N}_{0}\left(\overline{\mathrm{y}}_{0}\right)\left[1-\frac{2 \lambda+1}{\sqrt{\lambda(\lambda+1)}} \ln (\sqrt{\lambda}+\sqrt{\lambda+1})\right] \tag{38}
\end{equation*}
$$

$\therefore$. The terms in the square bracket of Eq. (38) are denoted by $F$ in the first paper of Reference 6. The form factor will be given by

$$
\begin{equation*}
\overline{\mathrm{G}}\left(\mathrm{Q}^{2}, \mathrm{~m}^{\prime}{ }^{\prime 2}\right) \equiv e^{-\frac{\overline{\mathrm{W}}}{2}}=e^{\frac{\mathrm{g}^{2}}{4 \pi} \frac{\mathrm{~N}_{0}\left(\overline{\mathrm{y}}_{0}\right)}{2} F(\lambda)} \tag{39}
\end{equation*}
$$

The behavior of $F(\lambda)$ as $\lambda$ increases is

$$
\begin{equation*}
F(\lambda) \xrightarrow[\lambda \gg 1]{ } \ln (4 \lambda)^{-1} \tag{40}
\end{equation*}
$$

Therefore, $\overline{\mathrm{y}}_{0}=$ constant (such that $\mathrm{N}_{0}\left(\overline{\mathrm{y}}_{0}\right)>0$ ) guarantees a behavior $\left(\mathrm{Q}^{2} / \mathrm{m}^{2}\right)^{-\gamma}$ for $\bar{G}\left(Q^{2}, m^{2}\right)$. That is the same choice of cutoff as Fried and Gaisser, ${ }^{6}$ and is the main difference in the scalar meson calculation.

Defining

$$
\begin{equation*}
\gamma=\frac{\mathrm{g}^{2}}{4 \pi} \frac{\mathrm{~N}_{0}\left(\overline{\mathrm{y}}_{0}\right)}{2} \tag{41}
\end{equation*}
$$

one finds that the $\lambda \gg 1$ expression for the form factor is

$$
\begin{align*}
\overline{\mathrm{G}}\left(\mathrm{Q}^{2}, \mathrm{~m}^{\prime^{2}}\right) & =(4 \lambda)^{-\gamma}=\left(\frac{\mathrm{m}^{\prime}}{\mathrm{m}}\right)^{-\gamma}\left(4 \mathrm{mb}_{0}\right)^{-\gamma} \\
\lambda & \gg 1  \tag{42}\\
& =\left(\frac{\mathrm{m}}{\mathrm{~m}^{\prime}}\right)^{\gamma}\left(1+\frac{\mathrm{Q}^{2}}{\mathrm{~m}^{\prime 2}}\right)^{-\gamma}
\end{align*}
$$

where use of Eq. (21) and (22) has been made.
For electroproduction, the $\mathrm{m}^{\prime}, \mathrm{m}$ coefficients of the terms appearing in
Eq. (25) are substituted by $p_{\mu}^{\prime}, p_{\mu}$. Equation (26) is replaced by:

$$
\begin{align*}
\overline{\mathrm{w}}_{\mathrm{e}} & =\frac{\mathrm{g}^{2}}{(2 \pi)^{3}} \int \mathrm{~d}_{4} \mathrm{k} \delta\left(\mathrm{k}^{2}-\mu^{2}\right)\left[1-\mathrm{c}^{\left.\mathrm{ikp} \frac{\tau}{\mathrm{~m}^{\prime}}\right]\left[\frac{\mathrm{p}_{\mu}^{\prime}}{\mathrm{k} \mathrm{p}^{\prime}+\mathrm{i} \epsilon}-\frac{\mathrm{p}_{\mu}}{\mathrm{kp+i} \epsilon}\right]^{2}}\right. \\
& =2\left(\overline{\mathrm{~W}}+\overline{\mathrm{W}}_{\mathrm{e} \tau}\right) \tag{43}
\end{align*}
$$

with $\bar{W}_{\mathrm{e} \tau}$ given by:

$$
\begin{equation*}
\overline{\mathrm{W}}_{\mathrm{e} \tau}=\frac{\mathrm{g}^{2}}{8 \pi} \int_{1}^{\infty} \mathrm{d} l \mathrm{~N}_{0}(\mu \tau \ell)\left\{\frac{1}{\ell^{2}}+\frac{(2 \lambda+1) \ell}{\left(\ell^{2}+\mathrm{A}^{2}\right)^{3 / 2}}-\frac{2(2 \lambda+1)}{\ell\left(\ell^{2}+\mathrm{A}^{2}\right)^{1 / 2}}\right\} \tag{44}
\end{equation*}
$$

One now has
and, therefore, in the limit $\lambda \gg 1, m^{\prime} \gg m$ and under the same conditions as
Eq. (31), one can write for all $\omega^{\prime}$ :

$$
\begin{equation*}
\nu \mathrm{W}_{2} \cong \frac{1}{\omega^{\prime}-1}\left(\frac{\omega^{\prime}-1}{\omega^{\prime}}\right)^{2 \gamma} \tag{46}
\end{equation*}
$$

if

$$
\begin{equation*}
\rho\left(\mathrm{m}^{\prime 2}\right)=\frac{1}{\mathrm{~m}^{\prime 2}}\left(\frac{\mathrm{~m}^{\prime}}{\mathrm{m}}\right)^{2 \gamma} \mathrm{e}^{\overline{\mathrm{W}}_{\mathrm{e}} \tau} \tag{47}
\end{equation*}
$$

Once again, the Drell-Yan-West relation is satisfied and in the Bjorken limit, $\nu \mathrm{W}_{2}$ is given by Eq. (46) with $\omega^{\prime} \longrightarrow \omega$.

## III. Summary

One can conclude from this paper that the bremsstrahlung model of nucleon high-energy scattering is appropriate to describe the qualitative features of the form factors of the nucleon and of inelastic electron-nucleon scattering.

The model provides a framework in which the interesting kinematical variable $\rho^{\prime}=\left(1+\mathrm{Q}^{2} / \mathrm{m}^{\prime 2}\right)$ plays an important role. That same variable appears very naturally in the parton model. ${ }^{3}$

On the other hand, and considering Eq. (35) and (47), one has a connection between the density of electromagnetically excited statcs of the nucleon and the relativistic interval $\tau$ between absorption and emission in virtual forward Compton scattering. $\tau$ depends on $\mathrm{m}^{\prime}$, and it is easy to see that a form like $\mu \tau \sim \frac{\mu}{\mathrm{m}}\left(\mathrm{m}^{\prime} / \mathrm{m}\right)^{\alpha}$,
where $\alpha$ is some constant, is consistent with narrow-width approximations ${ }^{16}$ if $\alpha=\epsilon-1$; $\epsilon>0$. Such a form allows both an approach to $\tau=0$ (light cone) with increasing $\mathrm{m}^{\prime}$ if $\alpha<0$, and a recession from it if $\alpha>0$. In any case, it is interesting to see that in the context of such a simple model, one has elements of relation to (a) the parton model, (b) the light-cone behavior, and (c) the resonant structure of the inelastic form factors of the nucleon.

## Acknowledgments

It is a pleasure to thank Professor S. D. Drell and SLAC for the warm hospitality extended to the author. He also wishes to show his deep gratitude to the institutions that support him: Centro de Investigación y de Estudios Avanzados del Instituto Politécnico Nacional and Consejo Nacional de Ciencia y Tecnologia, México. Many stimulating discussions with Dr. C. Domínguez, R. Suaya, and J. Willemsen are hereby acknowledged, as well as the efficient work of Mrs. V. Smoyer in typing the preprint. Thanks are also due to Drs. H. M. Fried and J. Pestieau for their valuable comments.

## References and Footnotes

1. J. D. Bjorken, Phys. Rev. 179, 1547 (1969).
2. E. D. Bloom and F. J. Gilman, Phys. Rev. Letters 25, 1140 (1970); F. J. Gilman, SLAC-PUB-842 (invited talk at the Symposium of Hadron Spectroscopy, Balatonfüred, Hungary (1970)).
3. H. Moreno and J. Pestieau, Preprint Centro de Investigación y de Estudios Avanzados del Instituto Politécnico Nacional, México (March 1971).
4. G. Domokos, S. Kovesi-Domokos, and E. Schonberg, Phys. Rev. D3, 1184 (1971).
5. M. Elitzur, Phys Rev. D3, 2166 (1971) and M. Elitzur, Phys. Rev. Letters 27, 895 (1971).
6. H. M. Fried and T. K. Gaisser, Phys Rev. 179, 1491 (1969); T. K. Gaisser, Phys. Rev. D2, 1337 (1970); H. M. Fried and K. Raman, Phys. Rev. D3, 269 (1971) ; H. M. Fried and T. K. Gaisser, Phys. Rev. D3, 224 (1971);
H. M. Fried and T. K. Gaisser, Brown University Preprint (1971);
H. M. Fried and H. Moreno, Phys Rev. Letters 25, 625 (1970).
7. H. Gemmel and H. A. Kastrup, Nucl. Phys. B14, 566 (1969); H. Gemmel and H. A. Kastrup. Z. Physik 229, 321 (1969).
8. One should not expect the model to be accurate at small momentum transfers, since in that kinematical region, the deflection might be due to several (not necessarily just one) very soft collisions. This may lead to "double counting" and it was pointed out by Professor R. Blankenbecler.
9. In previous papers, ${ }^{6}$ the field has been assumed to be massive, neutral, and of vector character. In this paper, two possibilities are explored: (a) scalar mesons and (b) vector mesons. One can then compare the two cases easily and, furthermore, find in the vector meson case the results of the first paper of Reference 6 for the elastic form factors in a generalized form for inelastic nucleonresonance processes.
10. R. P. Feynman, Phys. Rev. Letters 23, 1415 (1969) and Proceedings of the Third International Conference at High Energy Collisions, Stony Brook, 1969, edited by C. N. Yang et al. (Gordon and Breach, New York, 1969), p. 237.
11. F. Bloch and A. Nordsieck, Phys. Rev. 52, 54 (1937); D. Yennie, S. Frautschi, H. Suura, Ann. Phys., N. Y., 13, 379 (1961).
12. See first and last papers of reference 6 .
13. The variable

$$
1+\frac{Q^{2}}{m^{\prime 2}}=\rho^{\prime}=\frac{\omega^{\prime}}{\omega^{\prime}-1}=\frac{p(p+2 q)}{(p+q)^{2}} \cdot \text { with } \omega^{\prime}=\omega+\frac{m^{2}}{Q^{2}}
$$

was studied in reference 3, where it was shown to originate in a natural way in the parton model. It was suggested by the work of G. Cocho and J. Salazar, Phys. Rev. Letters 27, 892 (1971).
14. After subtraction of terms of the same type as $W_{f}$ in Eq. (7), which may be defined to be zero with the operational procedure of Eq. (8). That procedure defines the ambiguous product of the two $\delta$-functions of the same argument as the limit of the product of the two $\delta$-functions where one of them has its argument infinitesimally displaced. But then products of the type $\delta\left(\mathrm{k}^{2}-\mu^{2}\right) \delta(\mathrm{kp})$ are zero since both $k$ and $p$ are timelike and $k p \neq 0$.
15. S. D. Drell and T. M. Yan, Phys. Rev. Letters 24, 181 (1970); G. West, Phys. Rev. Letters 24, 1206 (1970). See also last paper of reference 6.
16. In the rest frame of the nucleon, $\mathrm{E}_{\mathrm{p}+\mathrm{q}}=\nu+\mathrm{m} \sim \nu(\mathrm{pq}=\mathrm{m} \nu) ; \mathrm{m}^{\prime}=\sqrt{(\mathrm{p}+\mathrm{q})^{2}}=$ $=\sqrt{\mathrm{m}^{2}+2 \mathrm{~m} \nu(1-\mathrm{x})} \mathrm{x}=\mathrm{Q}^{2} / 2 \mathrm{~m} \nu$. Therefore, $\mathrm{m}^{\prime} \sim \nu^{1 / 2}$ for x fixed $\neq 1$. Hence, if $m \tau \sim\left(\frac{m^{\prime}}{m}\right)^{\epsilon-1}, \epsilon>0$; then $t_{\text {life }} \sim \nu^{\epsilon / 2}$ which will be large in the Bjorken limit $\nu \longrightarrow \infty$, x fixed.
17. Bars on the symbols that appeared in the scalar meson calculation will denote the same quantities in the vector meson case.

## Figure Captions

1. Diagram for the form factor calculation. The deflection is taken to occur at the origin of space-time coordinates.
2. Diagram for forward virtual Compton scattering.


Fig. 1


Fig. 2


[^0]:    *Supported by Centro de Investigación y de Estudios Avanzados del Instituto Politécnico Nacional and Consejo Nacional de Ciencia y Tecnología, México and by the U. S. Atomic Energy Commission.

