

SIMPLE DIFFRACTION CONSIDERATIONS AND RELATIONS BETWEEN σ ,
SLOPE AND "ELASTIC" ELECTROPRODUCTION OF VECTOR MESONS*

Yehuda Eisenberg**

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

The expected characteristics of electroproduction of vector mesons are examined by utilizing simple diffraction considerations. In the Bjorken limit a constant transparency assumption predicts that both σ_T and ρ^0 production slopes behave like $\sim 1/q^2$. Agreement with existing experiments is found and predictions for the dependence of the scaled ρ^0 production parameters on both q^2 and energy W are made.

(Submitted to Phys. Rev. Letters.)

* Work supported by the U. S. Atomic Energy Commission.

** On leave from the Weizmann Institute, Rehevoth, Israel.

The inelastic electron-proton scattering experiments¹ in which total cross sections for the reaction

$$\gamma_V + p \rightarrow \text{hadrons} \quad (1)$$

are measured, were recently supplemented by coincidence measurements in which the "elastic" production of ρ^0 mesons was observed²:

$$\gamma_V + p \rightarrow \rho^0 + p \quad (2)$$

γ_V in reactions (1) and (2) is the virtual photon exchanged in the inelastic electron scattering. Its energy ν in the laboratory system and (space-like) squared mass q^2 are related to E (the incident electron energy), E' (the scattered electron energy), and θ (the scattering angle) by $\nu = E - E'$ and $q^2 \cong 4EE' \sin^2 \frac{\theta}{2}$. The invariant mass W of the final state is then given by: $W^2 = 2M\nu + M^2 - q^2$, where M is the nucleon mass. In photoproduction experiments with real photons one finds³ that reaction (2), in the limit $q^2 = 0$, is almost purely diffractive. For finite q^2 photons this property is likely to persist. Therefore, in this paper we apply an essentially classical diffraction picture to reactions (1) and (2). It turns out that such a description is useful and indicates which of the variables of the problem may be expected to remain constant and in what way — in analogy with σ_{tot} , $\sigma_{\text{tot}}^{\text{el}}$, the slope B and the forward cross section A of hadrons and real photon physics.

We consider diffraction scattering off a disk with radius R and transparency ϵ ($\epsilon = 0$ means an opaque disk). For such a case the differential cross section is given by⁴:

$$\frac{d\sigma}{dt} \approx A \cdot e^{-Bt} \quad , \quad B \approx R^2/4 \quad (3)$$

while the total elastic, $\sigma_{\text{tot}}^{\text{el}}$, and total cross sections are given by:

$$\sigma_{\text{tot}}^{\text{el}} = \pi R^2 (1 - \epsilon)^2 \approx A/B \quad (4)$$

and

$$\sigma_{\text{tot}} = 2\pi R^2 (1 - \epsilon) \quad . \quad (5)$$

From (3) - (5) we further get

$$\sigma_{\text{tot}}^{\text{el}} / \sigma_{\text{tot}} = \frac{1}{2} (1 - \epsilon) \quad , \quad B = \frac{\sigma_{\text{tot}}}{16\pi} \cdot \left(\frac{\sigma_{\text{tot}}}{\sigma_{\text{tot}}^{\text{el}}} \right) \quad . \quad (6)$$

Generally speaking,⁴ in hadron reactions (3) - (6) describe the experimental data rather well at high energies (above the resonance region) with about constant ϵ (namely, almost constant ratio of elastic to total cross sections, about 15 percent) and almost constant R (about 1 fermi). For real photons³ the situation is very similar ((4) and (5) above need, of course, to contain VDM factor³ $g_{\gamma\rho}^2 \simeq 3 - 5 \cdot 10^{-3}$).

In considering inelastic electron scattering and electroproduction it may be very useful to keep the above picture in mind. Naturally, in general one expects both R and ϵ to be functions of ν and q^2 (or W). However, it is reasonable to conjecture that as in hadron and real photon physics, ϵ remains constant (or nearly constant) for virtual photons as well. This is equivalent to assuming that the ratio of the "elastic" cross section, reaction (2), to the total cross section for reaction (1) is constant independent of ν and q^2 . We further assume that (3) - (6) above hold in electroproduction, namely that reaction (2) is diffractive and that (2) and (1) are coupled to each other like the elastic and total cross sections in hadron and (real) photon physics.⁵ (We do not expect our picture to be valid at low ω regions where the neutron and proton cross sections are quite different.¹) Let us now examine in more detail the consequences of the above assumptions.

First, we note that the virtual photon-nucleon cross section for transversely polarized photons, $\sigma_{\text{T}}(\nu, q^2)$ and longitudinal (scalar) photons, $\sigma_{\text{S}}(\nu, q^2)$ are

related to the scaling variable $\omega = 2M\nu/q^2$ (or $\omega' = 1 + W^2/q^2$) via the relations⁶:

$$2MW_1(\omega') = \frac{W^2 - M^2}{4\pi^2 \alpha} \sigma_T(\nu, q^2) \quad ,$$

$$W_2(\omega') = \frac{W^2 - M^2}{2M} \cdot \frac{q^2}{q^2 + \nu^2} \cdot \frac{\sigma_T + \sigma_S}{4\pi^2 \alpha} \quad (7)$$

where W_1 and W_2 are the usual structure functions. Since experimentally¹ $\sigma_S/\sigma_T \simeq 0.18$, we shall consider only σ_T and identify it with σ_{tot} defined by (5). Scaling¹ in ω' occurs at $q^2 \gtrsim 1.0 \text{ GeV}^2$ and $W \gtrsim 2.0 \text{ GeV}$. Thus we shall consider ω' mainly and note that $2MW_1(\omega')$ is scaled already at low energies.

From (7) it is clear that in the scaling region, where W_1 and W_2 do not depend on ν and q^2 separately but only on ω' , the dimensionless quantity σ'_T defined as

$$\sigma'_T = (W^2 - M^2) \sigma_T = (W^2 - M^2) 2\pi R^2(1 - \epsilon) \propto W_1(\omega') \quad (8)$$

obeys the scaling law and is a function of ω' only. However, if our conjecture is correct, ϵ is constant for all ω' and thus $(W^2 - M^2) \cdot R^2$ must scale. Thus, in the large ω' regions ($\omega' \gtrsim 6$) where the inelastic reactions are expected to be diffractive,¹ we are led from Eq. (3) to define:

$$B' = (W^2 - M^2) \cdot B \propto W_1(\omega') \quad (9)$$

which is also a scaled quantity. From (3) and (4) we find two more scaling functions:

$$\sigma'^{\text{el}}_T = (W^2 - M^2) \cdot \sigma^{\text{el}}_T \propto W_1(\omega') \quad (10)$$

$$A' = (W^2 - M^2)^2 \cdot A \propto [W_1(\omega')]^2 \quad (11)$$

In relations (9) - (11) we have defined scaled cross sections and slopes and we predict that they would depend only on ω' , in the way specified above.

Before comparing these predictions with the meager experimental material available at present, we wish to make a few remarks. First, we note that (8) follows from the definition of σ_T and only (9) - (11) are predictions which follow from the above conjecture of constant ϵ . Further, from Eq. (6) we note that even if ϵ would not turn out to be constant, the slope B would still be related to σ_T and σ_{el}/σ_T . Specifically, we remark that in our picture the slope of ρ^0 production in reaction (2) must change⁷ with q^2 and W via relation (9) above if its production is diffractive.

There is a certain amount of ambiguity in our treatment because of the definitions of the cross sections (7). We followed the Hand definitions.⁶ If we would use Gilman's cross sections,⁶ our scaled quantities would have to be modified slightly ($(\nu^2 + q^2)$ would replace $(W^2 - M^2)$ in relations (8) - (11)). At present we have no way of examining which definition would be preferred.

Finally, we examine the behavior of the cross section in the Bjorken⁸ limit (ν and $q^2 \rightarrow \infty$ and finite ratio ν/q^2). If, in this limit, $\nu \cdot W_2(\omega) \rightarrow \text{constant}$ ($W_1/\omega \rightarrow \text{constant}$), we get from (8) - (10) (neglecting logarithmic terms⁷) that both the cross sections and slopes behave like $\sim \frac{\omega}{\omega-1} \cdot \frac{1}{q^2}$ in the Hand definition⁶ and very similarly with Gilman's cross sections: $\sigma, B \sim \left(\frac{\nu}{2M} \omega + 1\right)^{-1} \cdot \omega \cdot \frac{1}{q^2}$. Thus, we obtain that both σ_T and B go to zero like $1/q^2$ (or faster) in the Bjorken limit and will not correspond to scattering with a fixed radius (say the proton⁷ radius, R_p). In this respect our results are similar to those of Griffith.⁷ Actually for dimensional reasons, since W_1 scales, σ_T would have to behave⁷ in the Bjorken limit like $1/(\text{Energy})^2$ and thus go to zero like $\sim 1/q^2$. One expects, of course, $q^2 \cdot \sigma_T$ to scale and be finite for finite ω . We can achieve the condition of $\sigma_T \sim \frac{1}{q^2}$ and maintain constant R ($R \rightarrow R_p$) in the Bjorken limit, by giving up the assumption of fixed ϵ and demanding that $(1 - \epsilon) \sim \frac{1}{q^2}$ (namely,

complete transparency ($\epsilon \rightarrow 1$) as $q^2 \rightarrow \infty$). This would bring about a different asymptotic behavior than the one outlined above: The slope B would become constant and the drop of the total elastic cross section would be faster, $\sigma_T^{el} \sim 1/q^4$ (by (4) - (6) above we also get: $\sigma_T^{el}/\sigma_T \sim 1/q^2$). It is worth noting that for both fixed or variable ϵ the forward cross section would behave like: $A \sim 1/q^4$.

The comparison with the experimental material that now exists can be best illustrated by considering the scaled quantities (8) - (11). In Fig. 1a we show σ_T^{el} and B' , as function of ω' , and in the same graph $2MW_1(\omega')$ is also shown.¹ As remarked above, σ_T^{el} is identical to the $2MW_1$ curve by the definition (7). It seems that within the errors, σ_T^{el} and B' follow the W_1 curve rather nicely. The $q^2 = 1.2 \text{ GeV}^2$ Cornell² point yields $B' \approx 60$ while for $\omega' = 8.5$ we expect $B' \approx 15$. However, one may argue that at present the error on B' for this particular point is still too large for any conclusions.

In Fig. 1b we show A and, for comparison, $[2MW_1(\omega')]^2$. (A for the Cornell data was estimated by extrapolating the data to $t=t_{\min}$ and its errors from the spread of the dipion t-distribution near the ρ^0 mass as given in Ref. 2. B was calculated from A and σ_T^{el} by utilizing relation (4).) The data is very preliminary and no firm conclusions can be reached. However, in general there seems to be agreement between the measured points and our predictions (points with $q^2 \leq .3 \text{ GeV}^2$ which are too far away from the scaling region are not included in Fig. 1 since they are expected to deviate considerably from the universal W_1 curves).

We thus conclude that our simple picture seems to agree with present experiments and that it will be an illuminating and useful one if future experiments will also support it. It relates in a simple way the observed decrease of the

slope B (or photon size⁷) with q^2 to the scaling mechanism and predicts that B would depend not only on q^2 but also on W (or ν) in the finite energy regions. In the Bjorken⁸ limit both B and σ_T^{el} would go to zero like $\sim 1/q^2$ if ϵ remains constant. Alternatively, we can have $(1 - \epsilon) \sim 1/q^2$ with $B \rightarrow$ constant. In such a case we would have $\sigma_T^{el} \sim 1/q^4$ with $(\sigma_T^{el}/\sigma_T) \sim 1/q^2$. In both cases $A \sim 1/q^4$. Finally, if the effective⁵ VDM factor g_γ^2 would behave asymptotically^{5,7} like $1/q^2$, one could achieve scaling in a VDM picture with fixed R and ϵ . This would bring about a situation similar to our conjectured one: $\sigma_T^{el} \sim 1/q^2$ and fixed σ_T^{el}/σ_T . However, B would be constant and $A \sim 1/q^2$.

Discussions with G. Chadwick, S. Drell and F. Gilman concerning various aspects of the present work are greatly appreciated.

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5. At low q^2 , as in $q^2=0$, the ρ^0 alone in the VDM picture accounts almost completely for σ_T^{el} . It is quite probable that at high q^2 ($q^2 \gg M^2$) the ρ^0 cross section will no longer constitute the entire "elastic" cross section, σ_T^{el} , and that higher mass states having the photon quantum numbers (if such states exist) would have to be added to it. In this case our σ_T^{el} would have to include all such states. Note also that the effective (integrated over all masses) VDM coupling might behave in such a case like $g_\gamma^2 \sim 1/q^2$, avoiding the scaling difficulties that one usually encounters in the ρ^0 VDM treatment.^{2,7} We are grateful to J. Bjorken for calling our attention to this point.
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FIGURE CAPTION

1. The scaled total cross section, σ_T^{el} , slope B' and forward intercept A' in reaction (2) as a function of ω' . Data is from Ref. 2. For comparison we show (a) $2MW_1(\omega')$ and (b) $\left[2MW_1(\omega')\right]^2$ from Ref. 1, to which the respective scaled quantities should be proportional (dashed curve above $\omega'=20$ is a guessed extrapolation of existing W_1 data). The scale was chosen to give rough correspondence between the data and W_1 (i. e., $\sigma_T^{\text{el}}/\sigma_T = 15$ percent) maintaining relation (4), $\sigma^{\text{el}} = A/B$. The numerical values of A and B were estimated from the published event distributions given in Ref. 2, when no actual number was given. Most of the experimental data is preliminary.

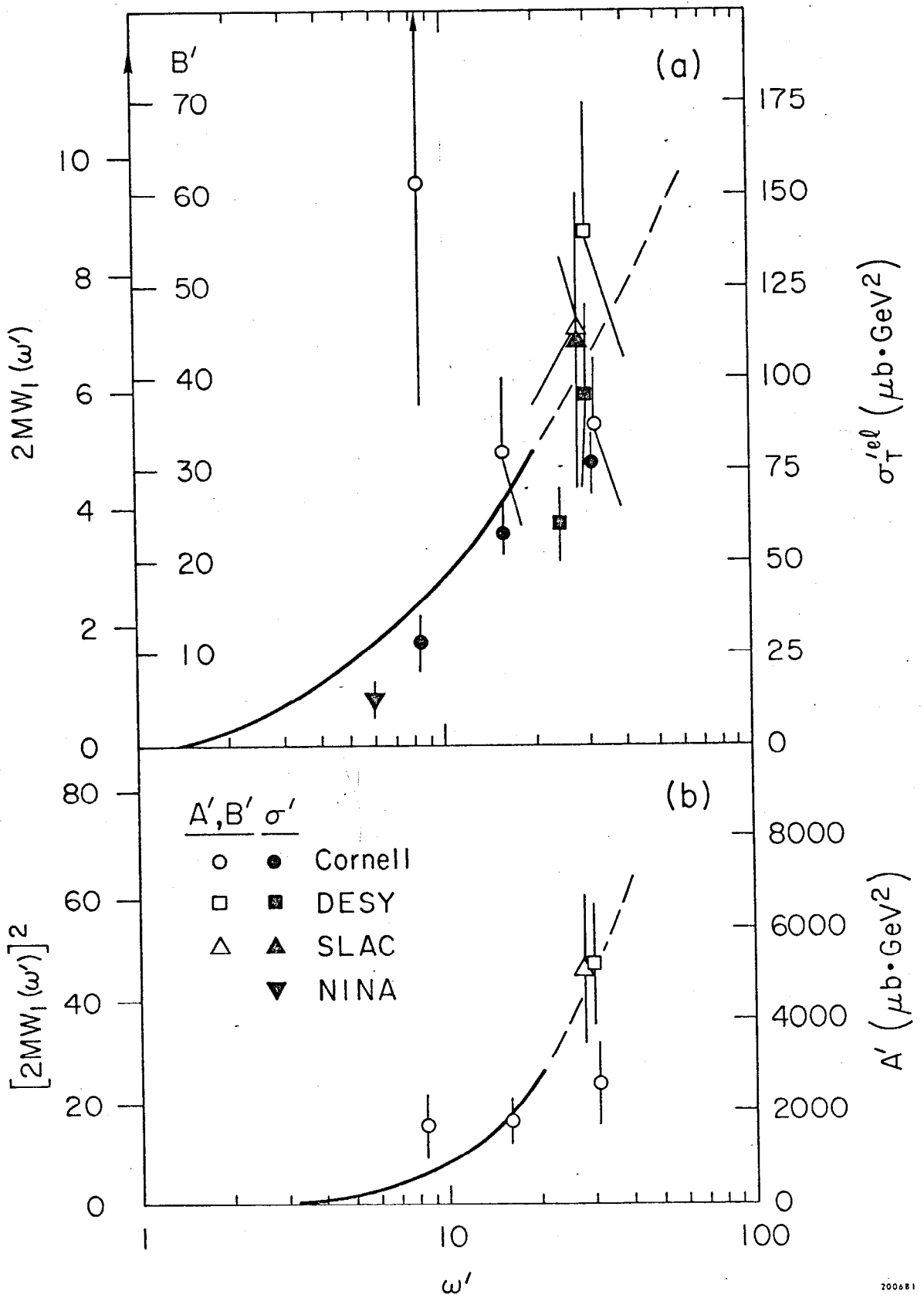


Fig. 1