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RADIATIVE PROBLEMS AND QUANTUM ELECTRODYNAMICS*

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INTRODUCTION

This review is concerned with two important areas of electromagnetic interaction physics. The first is the traditional area of quantum electrodynamics. Recent advances in both the precision measurement and theoretical areas have now set the testing ground of the basic validity of QED at the few part per million level; in fact, at order α^3 in perturbation theory. The highlights of these advances are discussed here. High energy tests of QED utilizing e^+e^- colliding beams have now set the possible cutoff level of QED to beyond the 2-6 GeV level; these tests are discussed extensively in the colliding beam section of this conference.

The main focus of this talk, however, is on a new area of electromagnetic physics: the possibility of measuring the processes $\gamma+\gamma \to X$ via colliding beam or Coulomb target experiments. In particular, measurements of the processes $e^{\pm}e^- \to e^{\pm}e^- X$ will permit extraordinary checks on the electromagnetic interactions of hadrons involving two (real or virtual) photons, and the mapping out of the pure electromagnetic production of the C =+ hadronic spectrum. Thus the usefulness of the electron-positron facilities can effectively be doubled by these processes, which are complimentary to normal C =-1 hadronic production. Despite the newness of this area of physics, over 30 papers have been recently published, and this review talk is meant to be an introduction to this literature.

RECENT ADVANCES IN QUANTUM ELECTRODYNAMICS

The Anomalous Moment of the Electron

The comparison between the theoretical calculation and experimental measurement of the anomalous magnetic moment of the electron is the most critical test of the quantum electrodynamics, since it is the only sufficiently precise check on the validity of the renormalization procedure and the perturbation expansion through sixth order in e. Happily, the theoretical calculation and experimental measurements of a_e have now each been performed to 3 ppm accuracy and the crucial comparison can now be made.

The final result of the Wesley-Rich[1] spin precession measurement is

$$a_e^{exp} = 0.001\ 159\ 6577(35)$$
$$= \frac{1}{2}\left(\frac{\alpha}{\pi}\right) - 0.328479\left(\frac{\alpha}{\pi}\right)^2 + 1.68 \pm 0.33\left(\frac{\alpha}{\pi}\right)^3$$

where for purposes of comparison with the theoretical coefficient of $(\alpha/\pi)^3$ we have used $\alpha^{-1}_{N\ QED} = 137.03608(26)$ from the Taylor, Parker, Langenberg[2] adjustment of the fundamental constants, and the Schwinger and Sommerfield-Petermann results from second and fourth order perturbation theory.

The theoretical calculations of the sixth order coefficient are now complete thanks to a remarkable and beautiful calculation of the three photon vertex correction performed by Levine and Wright.[3] The total result from theory is

$$a_e^{th} = \frac{1}{2}\left(\frac{\alpha}{\pi}\right) - 0.328479\left(\frac{\alpha}{\pi}\right)^2 + 1.46 \pm 0.25\left(\frac{\alpha}{\pi}\right)^3$$

which is in extraordinary agreement with the precision measurement. If we consider that the gyromagnetic ratio g of the electron need not generally have started out with the Dirac value of 2, then apparently we are seeing agreement between theory and experiment for g at the ninth significant figure!

There are basically four separate components to the $(\alpha/\pi)^3$ order calculation of a_e.

-0.154 ±0.009	(1)	The insertion of second order vacuum polarization loops into the fourth order electron vertex [diagrams 32-38 of Fig. 1] evaluated by Brodsky and Kinoshita,[4] Calmet and Perottet.[5]
+0.0554	(2)	The insertion of fourth order vacuum polarization loops into the second order electron vertex

		[diagrams 29-31] evaluated by Mignaco and Remiddi,[6] and Brodsky and Kinoshita.[4]
$+0.36 \pm 0.04$	(3)	The light by light scattering contribution [diagrams 39-40] evaluated by Aldins, Brodsky, Dufner, and Kinoshita.[7]
1.20 ± 0.20	(4)	The 28 distinct non-fermion loop diagrams [1-28] evaluated by Levine and Wright.
$1.46\left(\frac{\alpha}{\pi}\right)^3$ ± 0.25		

Some particularly elegant features of the Levine and Wright calculation are: (a) automated trace computations and reduction to Feynman parametric form via symbolic manipulation of strings using Fortran subroutines;[8] (b) a renormalization procedure in which counter-terms are constructed such that infrared divergent contributions never have to be explicitly evaluated (also used in Ref. 4); and (c) a systematic gaussian quadrature integration procedure.

Independent calculations of the light-by-light and the non-fermion loop contributions are of course still required, and such calculations are in progress by Kinoshita (who has checked the results of step (a) of Levine and Wright's work), P. Carroll (using the Sommerfield self-energy method) and others. Also, a counter-revolution back to old-fashioned perturbation theory — using the infinite momentum frame — seems to be occurring. The calculation of the lowest order contribution — see especially Bjorken, Kogut, and Soper,[9] Chang and Ma,[10] and Foerster[11] — is extremely simple and elegant and provides considerable insight into the nature of the Feynman claculations. Whether such calculations will prove advantageous for performing higher order calculations is a worthwhile question.

The sixth order quantum electrodynamic theory of the muon anomalous moment (from the diagrams of Fig. 1a) is also now complete:

$$
\begin{aligned}
a_\mu^{QED} - a_e^{QED} &= 1.09426 \frac{\alpha^2}{\pi^2} && \text{second order electron v.p.}^{12} \\
&- \frac{1}{45} \frac{m_e^2}{m_\mu^2} \frac{\alpha^2}{\pi^2} && \text{second order muon v.p.}^{13} \\
&- 2.42 \pm 0.20 \frac{\alpha^3}{\pi^3} && \text{second order electron v.p.}^{4,14} \\
&+ 4.31 \pm 0.04 \frac{\alpha^3}{\pi^3} && \text{fourth order electron v.p.}^{4,12} \\
&+ 18.4 \pm 1.1 \frac{\alpha^3}{\pi^3} && \text{electron loop light-by-light}^{7} \\
&= 1.09426 \frac{\alpha^2}{\pi^2} + (20.3 \pm 1.3) \frac{\alpha^3}{\pi^3} = 616(1) \times 10^{-8} \quad .
\end{aligned}
$$

The hadronic ρ, ω, ϕ contribution to the muon moment[15] adds an additional $6.5 \pm 0.5 \times 10^{-8}$ $[\sim 6\, \alpha^3/\pi^3]$. (This contribution is a factor of order m_e^2/m_μ^2 smaller for a_e.) This value, which is obtained directly from an integral over $\sigma_{e^+e^- \to \gamma \to \text{hadrons}}$ (see Fig. 2), could be changed somewhat by the presence of further resonances like the ρ', or structure at low s in the annihilation cross sections not given by the usual vector meson fits. A cross section of order

$$\sigma_{e^+e^- \to \gamma \to \text{hadrons}} = \frac{10^{-31}}{s} \text{ cm}^2 \text{GeV}^2 \quad \text{for} \quad s \gtrsim 1 \text{ GeV}^2$$

indicated by the Frascati and Novosibirsk data adds only 0.6×10^{-8}.

Thus the most significant theoretical ignorance concerning the muon continues to be the nonrenormalizable weak interaction contribution. In the usual theory with intermediate vector bosons (Fig. 3a) and a unitarity or ξ-limiting cutoff, the result is of order[16] $-1 \times 10^{-8} \sim -0.8 \times 10^{-8}$. A comprehensive discussion of the corrections from four-point fermi interactions (Fig. 3b) has been given by C. Jarlskog.[18] The present comparison of theory (combining $a_e^{QED} + a_{\rho\omega\phi}^{had}$) and experiment[17] is

$$a_\mu^{expt} - a_e^{expt} = 652(32) \times 10^{-8}$$

$$a_\mu^{theory} - a_e^{theory} = 623(2) \times 10^{-8}$$

An expected improvement of the experimental error in a_μ to the level of $\pm 2 \times 10^{-8}$ will be an important check on the hadronic and weak contributions to the lepton electromagnetic interactions, as well as a basic check of muon electrodynamics.

Lamb Shift Tests in QED[19]

The fundamental and historic Lamb shift tests of quantum electrodynamics have recently undergone major changes, resulting in significant quantitative improvement in the comparison of experiment with theory.

As is well known, the $S_{1/2}$-$P_{1/2}$ separation in hydrogenic atoms (which is, of course, zero in the Dirac theory) is sensitive to short-range modifications of the electron's Coulomb interaction with the nucleus. By far, the most important effect is the order α modification of the Coulomb potential induced by the emission and subsequent reabsorption of a single photon by the electron (which, being effectively an electron size correction, weakens the s-state binding, causing a net $2S_{1/2}$-$2P_{1/2}$ separation of ~ 1079 MHz in H) and the vacuum polarization correction (which increases the binding and causes a corresponding decrease by 27 MHz in H). As indicated by the Bethe logarithm contribution, the dependence of the order α self-energy correction to the electron is nonanalytic in the $0(Z\alpha)^2$ binding corrections, and the energy shift must be expanded in the form[20]

$$\Delta E_n^{(2)} = \frac{\alpha(Z\alpha)^4 m}{n^3} \Big[C_{41} \log(Z\alpha)^{-2} + C_{40} + C_5(Z\alpha) + C_{62}(Z\alpha)^2 \log^2(Z\alpha)^{-2} + C_{61}(Z\alpha)^2 \log(Z\alpha)^{-2} + C_{60}(Z\alpha)^2 + \ldots\Big]$$

Recently, however, Erickson[21] has presented a technique which avoids this expansion and yields results approximately valid to all orders in $Z\alpha$; the results agree with the Disederio and Johnson[22] high $Z\alpha$ Lamb shift calculations and, by construction, the Erickson-Yennie[20] results for the terms through C_{61}. The uncertainty in the value of C_{60} can then be reduced by nearly a factor of 10.

The experimental precision, however, also requires careful consideration of the fourth order radiative corrections. In this case, it is sufficient to consider only first order effects in the Coulomb field so that knowledge of the electron vertex form factors in fourth order is sufficient. The important question is the value for

$$\Delta E_n^{(4)}(n, j, \ell) = \delta_{\ell 0} \frac{4(Z\alpha)^4 m}{n^3} m^2 \left. \frac{dF_1}{dq^2} \right|_{q^2=0}$$

due to the order α^2 effective rms radius of the Dirac form factor

$$\langle r^2 \rangle^{(4)} = 6 \left. \frac{dF_1^{(4)}}{dq^2} \right|_{q^2=0} \quad ;$$

higher terms in q^2 produce corrections of order $\pi Z\alpha$ smaller.

In 1970 Appelquist and I[23] reported a recalculation of the $q^2=0$ slope of the Dirac form factor in four order and found the result

$$m^2 \left. \frac{dF_1^{(4)}}{dq^2} \right|_{q^2=0} = [0.48 \pm 0.07] \frac{\alpha^2}{\pi^2}$$

(using analytic and numerical techniques similar to those used in the sixth order a_e calculation) which differed from previous calculations[24] by an overall sign and individual differences in noninfrared remainders of the crossed and corner vertex graphs (see Fig. 4). More recently there have been additional numerical and extremely arduous analytic calculations which have confirmed graph by graph the new results and removed the numerical integration uncertainty. The final result is[25]

$$\left. \frac{m^2\, dF_1^{(4)}}{dq^2} \right|_{q^2=0} = \left(\frac{\alpha}{\pi}\right)^2 \left[-\frac{4819}{5184} - \frac{49}{72}\, \zeta(2) + 3 \log 2\, \zeta(2) - \frac{3}{4}\, \zeta(3) \right]$$

$$= \left(\frac{\alpha}{\pi}\right)^2 [0.470]$$

as obtained in a dispersion calculation by Barbieri, Mignaco, and Remiddi,[26] and A. Peterman[27] [crossed graph] and J. Fox[28] [crossed graph]. Graphs (2) and (4) of Fig. 4 were also calculated numerically by de Rafael, Lautrup and Peterman.[29] This result leads to an increase of .34 MHz $\left[Z^4 (2/n)^3\, \delta_{\ell 0}\right]$ compared to previous compilations and accounts for the discrepancy with experiment reported at the Daresbury Conference.[30] The agreement of theory (see Tables 1 and 2) and experiment is now quite acceptable, and is in especially good agreement with the more recent measurements of Robiscoe and Cosens, although there is considerable scatter between various experiments. (The deuterium results include a revised experimental value for the deuteron radius.[21]) As Peterman has emphasized, the experiments and theory are now so precise that α can be determined to 2 ppm from the Cosens-Vorburger measurement by the $(2P_{3/2}\text{-}2S_{1/2})$ separation in H:

$$\alpha_{CV}^{-1} = 137.03570(27)$$

compared with the TPL value[2]

$$\alpha_{NQED}^{-1} = 137.03608(26)$$

A very significant development in the experimental Lamb shift situation is the successful use of an "interference narrowing" method to significantly reduce the natural line width. The first measurement[31]

$$\Delta E(3S_{1/2}\text{-}3P_{1/2})_H = 314.810 \pm 0.052 \text{ MHz}$$

is 8 times more precise than previous measurements. The theoretical value is 314.894 ± 0.009 MHz. Extensions of this technique are expected to lead to a greatly improved value for the n=2 transition, and a further degree of sensitivity in checking quantum electrodynamics.

During the past two years there has also been an experimental improvement in the measurements by the hyperfine splitting of muonium (μ^+e^-) and positronium. Increasingly precise values for the hyperfine splitting of ground state muonium have been obtained by the Yale and Chicago groups.

The latest measurement from Yale[32] (conducted at low pressures in the argon and krypton) is $\Delta\nu_{exp}$ = 4463.311(12) MHz (2.7 ppm).

A very novel method patterned after Ramsey's molecular beam technique, which greatly reduces the effective resonance line width, has been recently used by Telegdi's group at Chicago to obtain an extraordinary precise value[33]:

$\Delta\nu_{exp}$ = 4463.3013(40) MHz (0.9 ppm).

This may be compared with the theoretical prediction[19]

$$\Delta\nu_{th} = 4463.323(19)\ \text{MHz} \qquad (4.1\ \text{ppm})$$

which uses α^{-1} = 137.03602(21) and the critical value

$$\mu_\mu/\mu_p = 3.183347(9)\ \text{MHz}$$

as obtained from high precision measurements of the muon precession frequency in various liquid environments.[34] This value is also in agreement with the elegant determination of μ_μ/μ_p from the Zeeman structure of muonium performed recently by the Chicago group.

The most recent Yale value for the hfs of positronium,[35] which again is an extremely fundamental pure electrodynamic quantity, is

$$\Delta\nu_{exp} = 203.397 \pm .004\ \text{GHz}$$

compared with

$$\Delta\nu_{th} = 203.415\ \mathrm{GHz} \pm \text{corrections of order } \alpha^2 \ .$$

Perhaps the key problem is utilizing the very precise experimental measurements for the pure QED atoms (e^+e^- and μ^+e^-) to determine α or check theory is the lack of knowledge of the α^2 corrections beyond the $\alpha^2 \log \alpha^{-1}$ terms recently evaluated by Fulton, Owen, and Repko.[39] A central reason why such calculations are so formidable is that the calculations have always had to proceed starting from the ladder approximation to the Bethe-Salpeter equation. In fact, it becomes clear that the ladder approximation is unnatural for bound state problems if we consider that the Dirac fine structure formula for $m_1/m_2 \rightarrow 0$ does not come from the ladder approximation but requires the entire set of irreducible cross graph kernels.[40]

It is thus encouraging that alternative methods which effectively sum all ladder and cross graph contributions into an effective potential form have been developed. The simplest technique is the eikonal approximation[41] which automatically sums the lowest range part of the photon exchange graphs; this leads to covariant and elegant bound state spectra, which in fact give the fine structure in positronium correctly. The dangers in this method have been emphasized by Nandy and Sawyer[42] who note that a crucial s-state correction to the fine structure formula for two spinless particles (arising from short-range seagull pieces) is missing in the eikonal procedure. Recently, however, Todorov[43] has presented a "quasi-potential" equation approach which can systematically correct the eikonal spectra; in particular, the correct spin zero fine structure result is obtained.

A related method has already been used by Grotch and Yennie[44] to obtain recoil corrections to bound state spectra, and by Fronsdal and Huff[45] to evaluate the Lamb shift for general mass conditions; hopefully these non-ladder-approximation methods will lead to similar progress for the positronium and muonium hyperfine problems.

In the case of the hydrogen hfs, the main uncertainty continues to be the proton polarization contribution. Using the present theory of radiative corrections applied to the most accurate experimental results yields $\delta_p = 2.5 \pm 4.0$ ppm. Using the Cottingham rotation, the contribution can be related to cross sections for inelastic scattering of polarized electrons by polarized protons, and so hopefully we will be able to have a definite determination of the contribution by the next conference. Meanwhile, positivity conditions on the deep inelastic structure functions have been used by de Rafael[36] to bound the contribution; one finds

$$\delta_p = \delta_p^{(1)} + \delta_p^{(2)}$$

where

$$0 < \delta_p^{(1)} < 2 \text{ ppm}$$

(estimated by Drell and Sullivan[37] using the low energy theorem) and

$$|\delta_p^{(2)}| < 2.7 \text{ ppm} \quad ,$$

which de Rafael obtains using the measured spin-independent structure functions W_1 and W_2 (assuming R, the ratio of longitudinal and transverse currents is reasonably small). This may be compared with the value

$$\delta_p = 2.5 \pm 4.0 \text{ ppm}$$

obtained using

$$\alpha_{NQED}^{-1} = 137.03608(26) \quad ,$$

calculated QED corrections, and the incredible accurate hfs measurements.[19,38]

Other Tests of QED

The extraordinary agreement of theory and experiment in the precision tests leads to limits of the order of 5 GeV (on a negative metric photon modification) from a_μ and about 300 MeV from a_e. The beautiful colliding beam high momentum transfer checks of QED from $e^+e^- \to e^+e^-$, $\mu^+\mu^-$, and γ-γ are discussed in Professor Bernardini's report to this conference. Nauenberg[46] has emphasized that a modification of the photon propagator from a heavy photon with mass ~ 8 GeV would be difficult to reconcile with the observed scaling behavior of the SLAC-MIT deep inelastic e-p scattering data. Limits on, and evidence for, exotic leptons which can be derived from the a_μ^{th}-a_μ^{exp} comparison have been considered carefully by de Rafael, Lautrup, and Peterman.[19] Angular correlations from the decay of heavy leptons or intermediate vector bosons which could be produced in e^+e^- annihilation have been recently discussed by Y. S. Tsai.[47]

THE TWO PHOTON PROCESS FOR PARTICLE PRODUCTION BY COLLIDING BEAMS

General Features and the Equivalent Photon Approximation

The remainder of this review will be concerned with a very new area of particle physics — the exciting potential to measure photon-photon scattering cross sections,

$$\gamma + \gamma \to \text{hadrons}$$

for both real and virtual photons. The main technique which will be discussed is the two photon process in colliding electron-electron or electron-positron beams (see Fig. 6)

$$e^\pm e^- \to (e^\pm e^- \gamma\gamma) \to e^\pm e^- X$$

in which the virtual photons annihilate into a C =+ leptonic or hadronic state.

The general Feynman diagrams for the fourth order process are illustrated in Fig. 6. The dominant and most interesting contribution is from the C =+ diagram 6(a). The C = -1 contribution of 6(b) can be computed from lower order $e^+e^- \to$ annihilation data. (In most cases it may be suppressed by kinematic restrictions.) Other fourth order diagrams give completely negligible contributions. The exciting potential of diagram (a) is that the process $\gamma\gamma \to X$ is involved for both real and virtual photons and any C = + hadronic or leptonic system.

I hope to make it clear that rather than being a dreadful background for the usual one photon e^+e^- annihilation experiments, the two photon process has the potential for greatly extending our knowledge of the electromagnetic interaction of hadrons. Later, I will also discuss other techniques which can also lead to information on two photon physics.

The first work on the two photon process actually dates back to 1934 when Williams[48] and Landau and Lifshitz[49] considered the process of pair production by colliding charged particles. A typical cross section which can be derived using the Weizsäcker-Williams approximation is the total asymptotic cross section for muon pair production[50-52]:

$$\sigma_{ee \to ee\mu^+\mu^-} = \frac{112\,\alpha^4}{9\pi} \frac{1}{m_\mu^2} \log^2 E/m_e \log E/m_\mu \tag{1}$$

[Only the leading logarithmic terms are kept here.] The most surprising feature of this result is that, even though it is a fourth order cross section, it actually equals the one photon cross section for muon pair production at $E_\pm = 1$ GeV

$$\sigma_{ee \to (\gamma) \to \mu^+\mu^-} = \frac{\pi\alpha^2}{3E^2}$$
$$= \frac{2 \times 10^{-32}\ \mathrm{cm}^2}{E^2\ (\mathrm{GeV})^2}$$

and continues to grow as $\log^3 E$ compared to the E^{-2} behavior of the lowest order result. This is perhaps the most dramatic example of Cheng and Wu's[53] general remark that the asymptotic behavior of higher contributions may be completely different and indeed of controlling importance compared to the lower order terms. Of course, for the muon pair production, there is no experimental confusion between the one and two photon mechanisms: in the former the muons are collinear; in the 2 photon case, the final state consists of (near-forward) scattered electrons and generally noncollinear muons.

In fact the feature of logarithmically increasing $e^{\pm}e$ total cross sections in fourth order is quite general even for hadron production. To see this we can utilize the equivalent photon approximation, which gives a simple and intuitive approximate expression for the $ee \to eeX$ cross section in terms of the cross section for real photon annihilation:

$$\sigma_{ee \to eeX} = \int_0^E \frac{d\omega_1}{\omega_1} N(\omega_1) \int_0^E \frac{d\omega_2}{\omega_2} N(\omega_2) \qquad \sigma_{\gamma\gamma \to X}(s)$$

where at large energies

$$N(\omega) \simeq \frac{2\alpha}{\pi} \left[\frac{E^2 + (E-\omega)^2}{2E^2}\right] (\log E/m_e) \tag{2}$$

This form,[54] in fact, rigorously gives the leading total cross section behavior of $m_e/E \to 0$. The complete expression for the fourth order cross section for the production of a final state X with C = + has the general form

$$d\sigma = \left(\frac{\alpha}{2\pi^2}\right)^2 \frac{1}{E^2} \int \frac{d^3p_1'}{E_1'} \int \frac{d^3p_2'}{E_2'} \left(\frac{1}{k_1^2 k_2^2}\right)^2 \times$$

$$\left[2p_1^\mu p_1^\nu + \frac{1}{2} k_1^2 g^{\mu\nu}\right] \left[2p_2^\alpha p_2^\beta + \frac{1}{2} k_2^2 g^{\alpha\beta}\right] \frac{1}{8} M^+_{\mu\alpha} M_{\nu\beta} \, d\widetilde{\Gamma} \ ,$$

and

$$M_{\mu\nu} = i\int d^4x\, e^{-ik_1x} \langle X|T^*\left(J_\mu(x), J_\nu(0)\right)|0\rangle ,$$

$$d\widetilde{\Gamma} \equiv (2\pi)^4\, \delta^4(k_1+k_2 - p_X)\, d\Gamma \quad , \qquad (3)$$

where $d\Gamma$ is the invariant phase space of the state X. In the forward lepton angle limit, in which the virtual spacelike photons of momentum k_1 and k_2 may be taken as real, the integrand satisfies the relation

$$\lim_{\substack{k_1^2\to 0\\ k_2^2\to 0}} \frac{1}{8} M^+_{\mu\alpha} M^{\mu\alpha}\, d\widetilde{\Gamma} = (2\omega_1)(2\omega_2)\, d\sigma_{\gamma\gamma\to X} , \qquad (4)$$

where $d\sigma_{\gamma\gamma}$ is the corresponding cross section of the production of the state X by two oppositely directed photons of energy ω_1 and ω_2. The equivalent photon method is derived by taking only the transverse current contribution in the C =+ diagrams, and expanding the kinematics of the $\gamma+\gamma\to X$ cross section about $\theta'_1, \theta'_2=0$. The familiar log E/m_e factors come from the integration over the scattering angles of the scattered electrons and are each of the form (for small θ')

$$\int_0 \frac{d\theta'^2}{\theta'^2 + m_e^2/E^2}$$

Thus roughly half of the cross section comes from electrons scatter at an angle smaller than $\sqrt{m_e/E}$ (~ 1.3° or 20 mrad at 1 GeV).

If we consider the case where the electrons scatter within an angle $0 < \theta < \theta_{max}$, (e.g., $\theta_{max} \lesssim 100$ mrad), then the equivalent photon approximation result (2), with N replaced by[50]

$$N(\omega, \theta_{max}) = \frac{\alpha}{\pi} \left\{ \frac{E^2+E'^2}{E^2} \left(\ln \frac{E\,\theta_{max}}{2m_e} - \frac{1}{2} \right) + \frac{(E-E')^2}{2E^2} \left(\ln \left(\frac{2E'}{E-E'} \right) + 1 \right) + \frac{(E+E')^2}{2E^2} \ln \frac{2E'}{\left[(E-E')^2 + EE'\,\theta^2_{max} \right]^{1/2}} \right\} ,$$

may be used rigorously to order θ^2_{max}. This is easily understood since neglected terms from longitudinal-scalar currents, phase space approximations in $\sigma_{\gamma\gamma \to X}$, and the C = -1 fourth order amplitude (Fig. 6b) have no $d\theta^2/(\theta^2 + m_e^2/E^2)$ singularities. The contributions from the neglected C = -1 diagrams and longitudinal-scalar currents are thus of order θ^2_{max}. Other versions of the equivalent photon method have been used by Kessler's group[57] and Greco.[52] [The application of the above method to the case of ee → eeee is only approximately correct since the electron mass scale enters in s and the C = -1 diagram; one also needs to consider interference terms (see Baier and Fadin, Ref. 51).]

As a general note on the validity of Eq. (2), we note that it yields Eq. (1) and agrees with a complete analytic evaluation of all log E/m_e and log E/m_μ terms in $\sigma_{ee \to ee\mu^+\mu^-}$ given in a very impressive calculation by Baier and Fadin.[51] The agreement of Baier and Fadin's work with that of Brodsky, Kinoshita and Terazawa (Ref. 50) using the complete equivalent photon spectrum is extremely good. The asymptotic formula for $ee \to ee\mu^+\mu^-$ has also been derived using Cheng-Wu techniques by Greco.[52]

<u>Thus in the case of forward detected leptons one has precisely the requirements of colliding photon beams, each lepton producing a spectrum of forward, spacelike but nearly real, spin-averaged photons with spectrum</u> $\frac{\alpha}{\pi} N(\omega, \theta_{max})$.

For total cross sections, $\sigma_{\gamma\gamma \to X}$ is a function of only $s \simeq 4\omega_1\omega_2$ and we may integrate over ω_1-ω_2, the momentum of the $\gamma\gamma$ system relative to the lab ee system[58,50]:

$$\sigma_{ee \to eeX}(E) \cong 2\left(\frac{\alpha}{\pi}\right)^2 \log^2 E/m_e \int_{s_{min}}^{4E^2} \frac{ds}{s}\, f\left(\frac{\sqrt{s}}{2E}\right) \sigma_{\gamma\gamma \to X}(s)$$

where $f(x) = (2+x^2)^2 \log \frac{1}{x} - (1-x^2)\,(3+x^2)$.

Thus we see that the 2 photon total cross section is sensitive to small s in the γ-γ process, and invariably increases with energy and becomes competitive with the one photon cross section when the s/s_{min} and log factors [$\log E/m_e \sim 7.6$ at $E = 1$ GeV] compensate for the additional factor of α^2/π. Higher order QED effects can increase the logarithmic dependence at the expense of additional factors of α/π.

The first calculations of hadron production in colliding electron rings were done in 1960 by Low,[58] who derived the above equation for the case of $\gamma\gamma \to \pi^0$ to determine the π^0 lifetime, and also by Calogero and Zemach[59] who considered a special kinematic situation for $\pi^+\pi^-$ pair production. However, the rates involved seemed unmeasurable at that time and no further work was done. Recently, however, three groups [Kessler's group at College de France;[57] Budnev, Balakin and Ginzburg at Novosibirsk;[60] Kinoshita, Terazawa and myself[61,50] (Cornell-SLAC)] independently realized that the two photon process is important for present and planned storage rings, not only as a serious background process to normal annihilation experiments, but also as a process of its own intrinsic interest.

The simplest examples of the two photon cross sections are shown in Fig. 7, for $ee \to ee\mu^+\mu^-$, $\pi^+\pi^-$, π^0, η^0. In the case of pion pair cross sections, the Born approximation (structureless point pion) is used. For reference, the asymptotic formula[50,51] is

$$\sigma_{ee \to ee\pi^+\pi^-} \xrightarrow[m_e/E \to 0]{} \frac{16\alpha^4}{9\pi} \frac{1}{m_\pi^2} \log^2 E/m_e \log E/m_\pi$$

The cross section shown in the figure (using the complete equivalent photon (E.P.) spectrum), is quite large, crossing the 1γ cross section at $E_\pm = 1.5$ GeV.

In the next section we shall discuss some fundamental tests of pion physics which can certainly be done with the luminosity of Adone $\left[\sim 10^{31}\ \text{cm}^{-2}\ \text{sec}^{-1}\right]$ and the new machines under construction. The narrow resonance (such as π^0 and η^0) cross sections are the simplest to compute since

$$\sigma_{\gamma\gamma \to \pi^0} = \frac{8\pi^2\ \Gamma_{\pi^0 \to 2\gamma}}{m_\pi}\ \delta(m_\pi^2 - s)$$

(with $\Gamma_{\pi^0 \to \gamma\gamma} = 8.61 \pm 1.7$ eV and $\Gamma_{\eta \to 2\gamma} \cong 1.0$ keV, $m_\eta = 0.549$ GeV). In fact, this is also the simplest 2γ cross section to compute exactly from the complete fourth order expression. The comparison of the E.P. result with the exact calculation,[50] assuming $\phi_\pi \tilde{F}_{\mu\nu} F^{\mu\nu}$ coupling, with or without ρ-pole form factors, is shown in Fig. 8. The conclusion is that the variation of the results from theoretical uncertainties in the effective photon mass dependence is greater than the errors from the equivalent photon approximation. In general, the equivalent photon result should be within 30% accuracy for total cross sections if the leptons are not detected. It should be emphasized, however, that the equivalent photon method is accurate to order θ^2_{max} when both leptons are constrained to forward angles.

In general, we have the capability of understanding the couplings of two photons to C =+ hadronic systems. Because of the inverse dependence on s_{min}, however, the most practical applications are concerned with low mass resonances, and near-threshold processes.

A further refinement possible in the studies of the $\gamma\gamma \rightarrow$ hadron process is that the linear polarization of each photon is in the scattering plane of the respective lepton. Thus we have the added possibility of detecting polarization information, viz. scalar vs. pseudoscalar ($\underset{\sim}{E}^2 - \underset{\sim}{H}^2$ vs. $\underset{\sim}{E} \cdot \underset{\sim}{H}$) amplitudes (corresponding to parallel vs. antiparallel helicities) by measuring the correlation of the electron scattering planes. The parities of $\gamma\gamma$ produced resonances can thus be obtained. This is of course a precise analogue of the Yang method of using double Dalitz decay of the π^0 for its parity determination. The complete isotopic spin dependence of $\gamma\gamma \rightarrow \pi\pi$ will require, of course, measurements of the $\pi^0\pi^0$ channel.

Kinematics of the two photon process

Before proceeding further into models for various hadron channels we should note some kinematical features which are unique to the 2 photon process (see Fig. 9). Because the photons are roughly collinear with the electron-positron beam, a produced pion pair lies approximately in a plane with the incident beam. This coplanarity condition turns out, however, not to be very severe.[50,51] The equivalent photon approximation cannot be used to obtain such a distribution, and we have performed a complete numerical evaluation of the $ee \rightarrow ee\pi^+\pi^-$ $C=+$ diagram to obtain the distribution for the angle ψ between the planes the pions make with the beam. The result[50] (see Fig. 10) shows that at 1 GeV, typically 40-50% of the events have $\psi > 12^\circ$ [which is the typical index of noncoplanarity used in experiments to identify multihadron ($n > 3$) events].

We should emphasize the following point: The equivalent photon method is applicable to $ee \rightarrow ee\pi^+\pi^-$ differential cross sections such as $\frac{d\sigma}{ds\,d\theta_+}$ or $\frac{d\sigma}{d\Omega_+ d\theta_-}$ but not to "over-determined" cross sections like $\frac{d\sigma}{d\Omega_+ d\Omega_-}$ or $\frac{d\sigma}{d^3k_+ d^3k_-}$ which constrain the angles of the electrons in the phase space integrations. Here $\theta_{\pm}$ are the angles of $\pi_{\pm}$. This point has also been emphasized by Cheng and Wu[53] who have performed a beautiful analytic evaluation of such cross sections for the C=+ diagrams (assuming, however, only transverse current contributions). Because the relative angle ϕ of the electron scattering plane can be controlled, it can be expected that polarized photon information can be controlled in such an experiment.

Because the momenta of the two photons are generally different, the $\gamma\gamma$ c.m. frame is moving relative to the lab, and the pion pair will generally not be collinear; typically the transformation from the $\gamma\gamma$ c.m. to lab frame leads to pion pairs swept forward or backward along the beam. An isotropic distribution in the $\gamma\gamma$ c.m. leads to a pion angular distribution[50]

$$\frac{d\sigma_{ee\to ee\pi\pi}}{ds\, d\Omega_1} \propto \frac{1}{s^2}\, \frac{1}{1-\beta^2_{max}\cos^2\theta}$$

where $\beta_{max} = (1-s/4E^2)/(1+s/4E^2)$ is the maximum pion velocity allowed kinematically. This peaking is perhaps not as severe as one might first conclude: Approximately one-half of the cross section occurs for $\theta \gtrsim (2s/E^2)^{1/4}$. Typically, only $\sim 1/2$ of the cross section is lost relative to phase space if one pion is required to come out at angles larger than 45^o. On the other hand, the nonisotropic components from the pole terms in the $\gamma\gamma$ c.m. cross section are much more severely cut off. Detailed calculations of various distributions have been presented by Kessler's group,[57] Baier and Fadin,[51] and by Kinoshita, Terazawa, and myself.[50]

Strong interaction modifications of $\gamma\gamma \to \pi^+\pi^-$

Probably the two photon experiment of most basic interest and most practical to perform is that for $\gamma\gamma \to \pi^+\pi^-$. The amplitude is extremely interesting since it is a 4 point function in which the interactions of two of the particles, the photons, are known. Possible tests involving the crossing relation to the Compton amplitude and the exact nature of the low energy theorem for $k_1, k_2 \to 0$ make the $\langle 0|T^*\left[J_\mu(x), J_\nu(y)\right]|\pi^+\pi^-\rangle$ amplitude especially fascinating.

There are many additional reasons for its great theoretical importance:

1. Unitarity to order e^2 requires[50, 62, 63]

$$\mathrm{Im}\ F^J_{\gamma\gamma\to\pi\pi} \propto \left(F^J_{\gamma\gamma\to\pi\pi}\right)^* F^J_{\pi\pi\to\pi\pi}$$

   This relation holds for each amplitude of given (even) angular momentum J (in the photon-photon center-of-mass system) and isotopic spin, if s is in the region of elastic $\pi\pi$ scattering $\left(4m_\pi^2 < s < 16m_\pi^2\right)$. Accordingly, a Watson-type theorem holds and the phase of $F^J_{\gamma\gamma\to\pi\pi}$ must be the same as $F^J_{\pi\pi\to\pi\pi}$. This gives us the opportunity to learn a great deal about π-π physics.

2. One is particularly concerned with the s-wave in π-π scattering, especially whether there is an $\epsilon(760)$ or σ enhancement. The σ is expected to show up as an enhancement in the $\gamma\gamma\to\pi\pi$ s-wave. The measurement of the $ee\to ee\pi\pi$ obtained by tagging the energy of forward scattered leptons and measurements of the relative angular distribution of the pions is an incredibly background-free technique for measuring this component. Measurements of the $ee\to ee\pi\pi/ee\to ee\mu\mu$ ratio may be a considerable aid in avoiding normalization problems. Further, the measurement of relative ϕ angles allows a separation of the two helicity amplitudes which contribute to $\gamma\gamma\to\pi^+\pi^-$. In the total cross section $\sigma_{ee\to ee\pi\pi}$, the simplest type of σ enhancement leads

to cross sections 2 to 4 times larger than the Born cross section estimate (see Fig. 11).

3. The $\gamma\gamma \rightarrow \pi^+\pi^-$ s-wave parallel helicity amplitude must satisfy the low energy theorem; continued to the Compton amplitude for $s \rightarrow 0$ it approaches the Thomson limit. It is an interesting general question how smooth the extrapolation is from s=0 to the threshold region $s > 4m_\pi^2$.

4. Unitarity relates only the phases of the $\gamma\gamma \rightarrow \pi\pi$ and $\pi\pi \rightarrow \pi\pi$ amplitudes; in general, their ratio can be an ărbitrary real polynomial which could cause zeros in the partial wave cross section for $\gamma\gamma \rightarrow \pi\pi$. This point has been especially emphasized by Yndurain.[64]

5. Goble and Rosner[65] have made the following interesting conjecture: The maximum of the very broad σ enhancement in π-π scattering occurs at approximately 750 MeV. The s-wave amplitude is, however, required by PCAC to vanish at s=0. Thus the actual position of the s-matrix pole occurs at a considerably lower mass [~(560-200 i) MeV]. This of course assumes that there is no further structure in the s-wave amplitude in the low s region. However, the unitarized $\gamma\gamma \rightarrow \pi\pi$ s-wave amplitude is constrained to the value $-\alpha/m_\pi$ by the low energy theorem. This enhances the low s region and shifts the maximum of the enhancement to about 450 MeV (see Figs. 12-15). Thus the maximum of the π-π cross section can occur at an energy much lower in $\gamma\gamma$ annihilation; an experimental confirmation of this conjecture would be very good evidence that the current algebra constraint is working.

A complete dispersion theoretic phenomenological description of the $\gamma\gamma \to \pi\pi$ cross section has been discussed by Lyth[62] and with further improvements by Yndurain,[64] and Carlson and Tung.[63] The elements of this analysis for the ℓ=0 and ℓ=2 partial waves which dominate at low s (say for $s < 100\ m_\pi^2$) consist of

1. A right-hand cut contribution which is related by unitarity to the $\pi\pi$ amplitudes; these can be parameterized in terms of σ and ρ^0 resonances.

2. A left-hand cut contribution from the t and u channel Born (single π exchange) and multipion or Regge exchange. Simple parameterizations of the low energy data, in terms of the σ parameters and the t, u channel coupling strengths can be given.

Estimate of Multihadron Production in the Two Photon Process

It is desirable to have at least a rough estimate of the multihadron production cross section via the two photon process. A simple argument based on the equivalent photon method and a duality-type approach to $\sigma_{\gamma\gamma \to \text{hadrons}}$ has been presented in Ref. 50.

The general components of the total cross section $\sigma_{\gamma\gamma \to \text{hadrons}}$ consist of

1. the contribution of narrow C=+ resonances (π^0, η, η', etc.)
2. two pion production, starting at the threshold $s_{th} = (2m_\pi)^2$ modulated by the even ℓ resonances and enhancements in the π-π system, e.g., ϵ, f_0
3. the contribution of resonances which decay into other hadronic systems, e.g., A_2, δ

and finally

4. a nearly flat asymptotic component which may be estimated by factorization of the cross section at high energy (universal Pomeron coupling) or ρ dominance to be

$$\sigma^{\text{asympt.}}_{\gamma\gamma\to\text{had.}} \cong \frac{\left(\sigma^{\text{asympt.}}_{\gamma p}\right)^2}{\sigma^{\text{asympt.}}_{pp}} \cong 0.3\ \mu\text{b}$$

for large s.

From a duality point of view we can consider $\sigma_{\gamma\gamma\to\text{had.}}(s)$ as being essentially equal to $\sigma^{\text{asympt.}}_{\gamma\gamma\to\text{had.}}(s)$ with the C=+ resonances modulating the asymptotic value. Thus very roughly the multibody cross section might be expected to average out to 0.3 μb starting at a threshold s_{th} for s of order $(3m_\pi)^2$ or $(4m_\pi)^2$. The results for the total cross section are shown in Fig. 11.[50] The cross section for producing three or more hadrons is seen to be comparable in magnitude to the two pion production cross section discussed previously.

We may also obtain a simple estimate of the cross section for the process $ee \to ee\rho^o\rho^o \to ee\pi^+\pi^-\pi^+\pi^-$ via ρ-dominance. For $s \geq (2m_\rho)^2$ we expect[50]

$$\sigma_{\gamma\gamma\to\rho\rho} \cong \left(\frac{e}{2\gamma_\rho}\right)^4 \sigma_{\rho\rho\to\rho\rho} \sim \left(\frac{1}{300}\right)^2 (10\ \text{mb}) \sim 0.1\ \mu\text{b}.$$

This gives $\sigma_{ee\to ee\rho\rho} \sim 2\times 10^{-34}\ \text{cm}^2$ at 2 GeV per beam.

In addition to the above channels, a particularly interesting narrow pseudoscalar resonance, which can have a surprisingly large 2γ coupling, is the $\eta^{o\prime}$ or $X^o(960)$. Various estimates based on SU(3), and current algebra predict[60] the $\Gamma_{X_0\to 2\gamma}$ decay rate anywhere from 5 keV to 80 keV. Experimental determination[70] of the branching ratio and upper limit on the total width yields an upper limit of about 100 keV. Using[50]

$$\sigma_{ee \to ee\eta} = 2.5 \times 10^{-35}\ \mathrm{cm}^2\ \Gamma_{\eta' \to 2\gamma}\ (\mathrm{keV})$$

$$\hookrightarrow \pi^+\pi^-\pi^+\pi^-\pi^o, \text{ etc. at } E_\pm = 1 \text{ GeV}$$

we can have a multipion cross section as large as 2×10^{-33} cm^2 (about 1/10 $\sigma_{ee \to \mu^+\mu^-}$) from this one channel alone. Note in the case of heavy resonances the distribution of pions is (probably) fairly close to isotropic. The η' cross section is also rising fairly fast in the 1-2 GeV region.

We also should note, however, that the above estimates for the $\gamma\gamma$ coupling to the $\epsilon\,(\sigma)$ assume that it is the dominant resonance in the $\gamma\gamma \to \pi^+\pi^-$ system and lead to a decay width of order 50 keV. Finite energy sum rules which incorporate the and f in the s-channel, various assumptions for the cross channel Regge exchange and single vector dominance coupling, lead to predictions for the couplings of f(1260), ϵ(750), A_2(1300), and δ(960):

$$\left.\begin{aligned} \Gamma_{f \to \gamma\gamma} &= 5.7 \text{ keV} \\ \Gamma_{\epsilon \to \gamma\gamma} &= 22.2 \text{ keV} \end{aligned}\right\} \quad \text{Schrempp-Otto, Schrempp and Walsh}^{66}$$

and

$$\left.\begin{aligned} \Gamma(f \to \gamma\gamma) &= 0.8 \text{ keV} \\ \Gamma(\epsilon \to \gamma\gamma) &= 6 \text{ keV} \\ \Gamma(\delta \to \gamma\gamma) &= 50 \text{ keV} \\ \Gamma(A_2 \to \gamma\gamma) &= 0.3 \text{ keV} \end{aligned}\right\} \quad \text{Bramon and Greco}^{67}$$

The numerical predictions of Bramon and Greco for $\sigma_{ee \to eeX}$ is shown in Fig. 16. Kleinert, Staunton, and Weiss[68] have made the estimate $\Gamma_{\epsilon \to \gamma\gamma} = 5 \text{ keV } (1-d_s)^2$. Their result depends on the dilation dimension of the space component of the vector current d_s, with the quark model giving d_3=3.

All of such estimates involve considerable guess work but it is reasonable to expect multipion production in $ee \to eeX$ of order $\frac{1}{2} \times 10^{-32}\ \mathrm{cm}^2$ at energy/beam above ~ 1.5 GeV, i.e., comparable or greater than the $e^+e^- \to \mu^+\mu^-$ rate. It is clear that progress in separating and studying the annihilation and two photon processes will require tagging of the lepton beams.

We should also mention other radiative backgrounds to the annihilation experiments. As emphasized by A. Litke,[71] the process of hard photon radiation: $e^+e^- \to X + \gamma$ primarily from $e^+e^- \to \gamma + \gamma$ ($\gamma \to \rho \to \pi^+\pi^-$) causes an appreciable rate of coplanar noncollinear pion pairs. The coplanarity angle here is probably $\sqrt{m_e/E}$ in contrast to the less severe result for $ee \to ee\pi^+\pi^-$. Purely leptonic events of the type $ee \to \ell\ell\gamma$ and $ee \to ee\ell\ell$ of course also need to be carefully avoided. A large total cross section for $ee \to eeee$ and $ee \to eeeeee$ proportional to m_e^{-2} may be estimated from formula (2) (see Baier and Fradkin,[51] and Serbo[72]); however, the experimental rates are severely cut down by the experimental conditions on s_{min} and minimum angle requirements. Extensive calculations of these processes have also been made recently by Brown and Muzinich.[73] Sandweiss[74] has computed in detail the angular distribution of scattered leptons in the two photon process for various situations.

Test of Unitarity and CP Invariance with $\gamma\gamma$ Colliding Beams

A fundamental test of unitarity and CP invariance can be performed by measuring $\sigma_{\gamma\gamma \to \pi^+\pi^-}$ and $\sigma_{\gamma\gamma \to \pi^0\pi^0}$ (via an $ee \to ee\pi\pi$ experiment) at the invariant mass $s = M^2_{K_S}$. De Rafael[75] has noted that the decay rate

$$\Gamma_{K_S \to 2\gamma} = \frac{M_{K_S}}{16\pi} |A|^2 \geq \frac{M_{K_S}}{16\pi} \mathrm{Im}^2 A$$

may be bounded from below by unitarity, since to order αG, only the two pion intermediate state can contribute to the imaginary part of the decay amplitude.

This assumes CP invariance. Thus Im A can be bounded using the known I=0 and I=2 amplitudes for $K_S \to \pi\pi$ and the I=0 and I=2 J=0 cross sections for $\sigma_{\gamma\gamma \to \pi^+\pi^-}$ at $s = M^2_{K_S}$. The comparison with a measurement of the $K_S \to 2\gamma$ rate will be a unique test of unitarity since the intermediate state spectrum is so simple.

We also note that a study of the cross section $\sigma_{\gamma\gamma \to \pi^+\pi^-\pi^0}$ at the K_L mass may be helpful in understanding the 3π intermediate state contribution to the $K_L \to \mu^+\mu^-$ puzzle.[76] In this case one is also interested in the virtual photon amplitude.

Test of Current Algebra and Soft-Pion Theorems Using $ee \to ee\pi$

Colliding beam measurements of the process $2\gamma \to$ hadrons would seem to be ideal testing ground for the soft pion predictions of current algebra since pure mesonic systems can be studied without the usual complications of nucleon targets, and further, the $ee \to eeX$ cross section particularly emphasizes the low s, threshold regions.

The general result of PCAC is that hadronic amplitudes such as that of π-π scattering vanish as $k^\mu_\pi \to 0$. On the other hand, current algebra predictions for processes involving two photons are strikingly different: (a) the Compton amplitude must satisfy the low energy theorem, and (b) the $\gamma\gamma\pi^0$ amplitude has a non-zero value for $k^\mu_{\pi^0} \to 0$ (the Adler anomaly[77]); this contribution is provided by a triangle graph due to a quark or nucleon loop.

Recently Terazawa[78] has shown that the successive use of PCAC and reduction formulae allows one to relate the amplitude for $\gamma\gamma \to n(\pi^+\pi^-)$ at threshold (s=0) back to the $\gamma\gamma \to \pi^+\pi^-$ amplitude. An interesting feature is that the low energy theorem is satisfied as a convergence of Weinberg's first sum rule.

Terazawa's estimate for 2 charged pair production at E = 1 GeV is

$$\sigma_{ee \to 2\pi^+ 2\pi^-} = 3.5 \times 10^{-36}\ \mathrm{cm}^2$$

compared with the soft pion result

$$\sigma_{ee \to \pi^+\pi^-} = 1.4 \times 10^{-33}\ \mathrm{cm}^2$$

at the same energy. A very severe drop in cross section as one approaches the threshold value in s is thus predicted. A similar result has also been obtained by Goble and Rosner.[65]

The case of $\gamma\gamma$ annihilation into an odd number of soft pions has been discussed by Aviv, Dass, and Sawyer.[79] As we have noted, the amplitude for $\gamma\gamma \to \pi^o$ is especially interesting in current algebra since it involves the anomalous divergence of the axial current. The triangle fermion loop diagram yields an effective $\phi_\pi \widetilde{F}_{\mu\nu} F^{\mu\nu}$ $\pi^o\gamma\gamma$ coupling which is finite in the soft pion limit and presumably accounts for the observed decay rate. Aviv et al. have used an effective Lagrangian method to generalize the fermion loop calculation to obtain the amplitudes for $\gamma\gamma \to$ odd number of pions. The main conclusion of their calculation is that the amplitude[80] for $\gamma\gamma \to \pi^o\pi^o\pi^o$ vanishes whereas[79] the amplitude for $\gamma\gamma \to \pi^o\pi^+\pi^-$ is finite in the soft pion ($k^\mu_{\pi o} \to 0$) limit, and is directly related to the $\gamma\gamma \to \pi^o$ amplitude from the Adler anomaly.

It would be very interesting to learn whether the finite result for $\gamma\gamma \to \pi^o\pi^+\pi^-$ is correct, or whether the amplitude is actually zero as in T. Yao's[81] work; the latter result is also a consequence of an argument by Ken Wilson[82] that the Adler anomaly is expected to be a c-number on the basis of short distance scale-invariance, in contrast to the q-number prediction of Aviv et al.

These most interesting results will, however, be nearly impossible to test if we only dare apply the prediction to the very near threshold region in s, since phase space rapidly cuts down the counting rate.

A possible solution to this dilemma is to develop predictions of the soft-pion theory in which the critical condition for validity is $k^{\mu}_{\pi^+} \to 0$, not $\Sigma k^{\mu}_{\pi} \to 0$.[83] The case of $\gamma\gamma \to \pi^+$ + all would seem to be interesting in this respect. Further, the behavior of the amplitude for $\gamma\gamma \to \pi^+\pi^-\pi^o$ for $k^{\mu}_{\pi^o} \to 0$ could still be interesting at reasonably large s, say $s \lesssim 6m_{\pi}$, where phase space is not small. With enough events one can measure the slope and asymptotic values for individual $k_{\pi} \to 0$. A systematic analysis of all such possibilities with model calculations of breaking terms is very much needed.

Photon-photon scattering with virtual photons

Although most of the virtual photon experiments to be discussed in this section require high luminosities and may not be possible for several years, it is interesting to contemplate what eventually could be measured.

The complete expression for the cross section $ee \to eeX$ for the general C =+ state has already been discussed in the section on kinematics. At large lepton scattering angles, one is measuring the general properties on the two photon amplitude

$$M_{\mu\nu}(k_1, k_2) = \int d^4k \; e^{ik\cdot x} \langle X | T^*\left(J_{\mu}(x), J_{\nu}(0)\right) | 0 \rangle$$

for arbitrary photon masses k_1^2, k_2^2. The case of $X = \pi^+\pi^-$ is very interesting and is discussed in the next section.

If an inclusive sum over all states X is performed, the cross section is related to the absorptive part of the $\gamma\gamma \to \gamma\gamma$ amplitude $W^{\mu\nu,\sigma\tau}(k_1, k_2)$. By hermiticity, parity, angular momentum conservation and time reversal invariance these may be decomposed in terms of 8 invariant structure functions $W_i(k_1^2, k_2^2, k_1 \cdot k_2)$ which may be defined using a tensor basis or helicity amplitudes.[73, 63] Six of these functions can be determined by measuring the outgoing leptons in $ee \to eeX(had)$ events. Clearly the quantities $W_i(k_1^2, k_2^2, k_1 \cdot k_2)$ are very fundamental; as far as rate is concerned, however, one is replacing the equivalent photon factor of $\log E^2/m_e^2$ by $\log (q_{max}^2/q_{min}^2)$ for each lepton required to scatter at large angles (assuming a constant behavior for W_i). Eventually one can hope that there will be complete large angle electron tagging systems which will allow long term accumulation of total hadron production data. We note also that account must be made of the contribution of $C = -1$ bremsstrahlung fourth order diagrams, which in the case of large angle lepton scattering, are not negligible.

$\gamma\gamma$ scattering and the pion mass difference

An elegant test of the current algebra calculation of the $\pi^+ - \pi^0$ mass difference has been discussed by T. M. Yan.[84] He observed that the integrand in the Cottingham formula for the one photon contribution to the self-energy of the pion, after rotation of the d^4q contour to spacelike photons, is nearly the same amplitude measured in $ee \to ee\pi^+\pi^-$ when $q_1^2 = q_2^2 = -Q^2$ (see Fig. 17).

Of course, the Cottingham formula requires the amplitude

$$\langle \pi^+ | J_\mu J_\nu | \pi^+ \rangle$$

whereas the colliding beam experiment provides

$$\langle 0 | J_\mu J_\nu | \pi^+ \pi^- \rangle \quad .$$

If we work near threshold in the pion's kinematics, the soft pion procedure allows us to ignore the difference of $+k_\pi$ and $-k_\pi$, and the two matrix elements are equal (and can be expressed in terms of the propagators of the vector and axial currents). Thus we have the potential to unravel the integrand in the self-energy expression

$$m_{\pi_+}^2 - m_{\pi_0}^2 = \frac{3\alpha}{4\pi} \int_0^\infty F(Q^2)\, dQ^2$$

by measuring

$$d\sigma_{\gamma\gamma \to \pi^+\pi^-}(q_1=Q^2,\ q_2=Q^2)$$

for s near threshold, $k_\pi \cdot q$ small.

It would be very interesting to measure this cross section even for small Q^2 for the $\pi^+\pi^-$ state, although in general one wishes to know the fall-off in Q^2 and check the convergence.[92] In principle, one is checking whether the mass difference is in fact electromagnetic. Using PCAC and current algebra the result of Dass et al.,[85] Terazawa,[78] and Yan[84] for the integrand is

$$F(Q^2) = \frac{1}{F_\pi^2} \int_{(3m_\pi)^2}^{\infty} dm^2\ \frac{\rho_V(m^2) - \rho_A(m^2)}{m^2 + Q^2}$$

in terms of the spectral functions of the vector and axial current. At $Q^2=0$, $F(0)=1$ is a requirement of the low energy theorem and is the content of Weinberg's first sum rule.[78]

Inelastic Electron Scattering on a Photon Target

One of the intriguing experiments made possible by the two photon process is inelastic electron-photon scattering, the analogue of inelastic electroproduction in which the target is a photon rather than a nucleon (see Fig. 18). Such experiments have been proposed independently by T. Walsh[86] and Kinoshita, Terazawa and myself.[87] In this case one lepton is detected at small angles providing as usual an equivalent beam of photons $k_1^2 \sim 0$, and the other is detected at a large angle and energy loss, providing the virtual photon $k_2^2=q^2$. The $\lim k_1^2 \to 0$ reduces the eight invariant structure functions in the C = + process to two.

In parallel with the ep case we define inelastic form factors W_1 and W_2 by

$$(2\pi)^2 (2P_0) \sum_n \langle p|J_\mu(0)|n\rangle \langle n|J_\nu(0)|p\rangle (2\pi)^4 \delta^4(q+P-P_n)$$

$$= -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1(Q^2,\nu)$$

$$+\left(P_\mu - \frac{(P\cdot q)\, q_\mu}{q^2}\right)\left(P_\nu - \frac{(P\cdot q)\, q_\nu}{q^2}\right) W_2(Q^2,\nu)$$

The kinematics are shown in Fig. 18. For our case $P^2=m^2 = 0$ and

$$\frac{d\sigma_{e+\gamma \to e+\text{all had.}}}{dQ^2 d\nu} = -\frac{2\pi\alpha^2}{(Q^2)^2}\left[W_2(Q^2,\nu)(1-y) + W_1(Q^2,\nu)\frac{y^2Q^2}{2\nu^2}\right]$$

where $y=\nu/P\cdot P_\gamma$, $q^2=-Q^2$, and $\nu=q\cdot P_\gamma$. We then may use the equivalent photon spectrum $\int \frac{d\omega}{\omega} N(\omega,\theta_{max})$ to obtain the ee → eeX cross section. If we assume the real photon converts into a hadron spectrum with an efficiency $\epsilon_{\gamma\to\text{had.}} = \frac{1}{300}$, then one obtains

$$W_i^\gamma \sim \epsilon\, W_i^{\text{had.}}$$

Using the parton model of Bjorken and Paschos,[88] one can estimate the total cross section:

$$\sigma_{ee\to eeX} \cong \frac{4\alpha^3}{Q^2_{min}} (\log E\theta_{max}/m_e) \log(E/(E-\nu_{min})) \log(2E^2/\nu_{min}) F_2^\gamma$$

$$\cong 5\times 10^{-35}\ \text{cm}^2$$

at $E=2.5$ GeV, $\nu_{min}=1.5$ GeV, $Q^2_{min}=0.17\,\text{GeV}^2$. Here we have estimated $F_2^\gamma = \epsilon_{\gamma\to\text{had}} F_2^{\text{boson}} \sim 0.3/300$ in the scaling region. We note that it is possible that scaling sets in at a lower value of Q^2 than in the proton case. Also, the counting rate can be doubled by using symmetric electron detectors.

Another model for the γ+γ amplitude which has been used by Berman, Bjorken, and Kogut[89] is based on direct parton pair production combined with rescattering corrections. This leads to an estimate of the deep inelastic electron photon scattering cross section similar to the above. This mechanism contains an extra "seagull" contribution to the structure functions of the photon, distinguishing it from a collection of hadrons. Further work, which combines the two approaches in a natural way is required. We also note that Berman et al.[89] predict the production of high transverse momentum hadrons in $\gamma\gamma \rightarrow$ hadrons even in the case of real photons. Unfortunately, the expected rate for such processes are very small in the high transverse momentum region where the parton effect can be seen. Similarly small rates are found in the super scaling region (large s, $|k_1^2|, |k_2^2|$) discussed in the parton model by Kingsley[90] and the Regge asymptotic limit by Efremov and Ginzburg.[91]

In general, there are great number of interesting measurements of $\gamma\gamma \rightarrow \pi\pi$ which will be possible using real and virtual photon γ-γ beams, such as photon mass dependences in $\gamma\gamma \rightarrow \pi^o$, $\pi^+\pi^-$.

<u>Other methods for measuring $\gamma\gamma \rightarrow$ hadrons</u>

There are two other important techniques which permit measurements and study of the processes $\gamma\gamma \rightarrow$ hadrons. The reaction

$$e^+e^- \rightarrow \pi^+\pi^-\gamma$$

where γ is a hard photon, has been discussed by Creutz and Einhorn[93] as a method to study the coupling of even charge conjugate states to one virtual plus one real photon. For $s=m_\phi^2$ this leads to the determination of the $\phi\epsilon\gamma$ coupling constant. The theoretical analysis is, however, somewhat complicated by the amplitudes in which the hard γ is emitted from a lepton line. Estimated cross sections are of order 1 to 10 nb.

The other important technique is the generalized Primakoff effect as discussed in the paper of Halprin, Primakoff and Anderson,[94] and extended recently by Stodolsky.[95] In this case the interaction of a photon beam with the Coulomb field of a target leads to extreme-forward production of a C=+ hadronic system. A typical form of the laboratory cross section is[95]

$$\frac{d\sigma}{dtdm^*} = \frac{\alpha Z^2}{4\pi} \frac{m^*}{k} \frac{1}{q_L^3} \sigma_{\gamma\gamma}(m^*) \sin^2 2\theta$$

with

$$t = |q^2| = q_T^2 + q_L^2$$

$$q_L = \frac{m^{*2} - m_k^2}{2\nu} = \frac{m^{*2}}{2\nu}$$

The observation of the target recoil at very small momentum transfer thus can yield measurements of $\sigma_{\gamma\gamma \to X}$. Background terms from normal photoproduction processes can usually be subtracted cleanly. The main difficulty is the observation of very small target recoils $(\Delta q_T < q_L)$ in a target with sufficient density to get sufficient counting rates.

It should also be noted that Coulomb production plays an important, even dominant, role in forward production processes with hadronic beams. This has been recently been emphasized by Stodolsky[95] and Berland, Dar, Eilan, and Franklin.[96] In the case of exchange reactions the latter authors note that the high energy dominance of Coulomb production (which increases logarithmically with laboratory energy) at any t may be used to measure general form factors and structure functions of mesons.

In any case, all of these $\gamma\gamma \to X$ techniques have the potential to greatly extend our knowledge of the electromagnetic interactions of hadrons. In particular, we can hope that in the near future much will be learned about the general Compton amplitude of the pion in all ranges of kinematics.

CONCLUSIONS AND SUMMARY

As we have seen there are a great number of new and important features of hadronic physics which can be studied via measurements of the process

$$\gamma\gamma \to \text{hadrons} \quad .$$

Fortunately it now appears that the investigation of this area is experimentally feasible, particularly via measurements of $ee \to eeX$ by colliding electron or positron colliding beams. Measurements of these processes will permit extraordinary checks on electromagnetic interactions involving two (real or virtual) photons, and will allow the mapping out of the electromagnetic production of the $C = +$ hadronic spectrum. In a sense the capabilities of the colliding beam facilities will be effectively doubled when these processes, which are complimentary to normal $C = -$ hadronic production, are studied.

In the two photon process we are involved with understanding and testing theory for the matrix element of two currents. In some cases this provides a unique test of theory; e.g., consequences of the Adler anomaly and low energy theorems in current algebra. Also, the study of two photon amplitudes allows tests of the extension of the parton model to the complete Compton amplitude, rather than just the imaginary part of the forward virtual amplitude in the scaling region. Production of two pions via two photons, also provides a new range of information on the fundamental pi pi interaction. In general the extension to two virtual photons, in which both target and projectile have variable mass, allows a great many new tests of hadron physics, in which we can probe short distance behavior with two localized probes.

The two photon process now makes electron-electron storage rings (such as that at DESY) particularly valuable because of freedom of background from the one-photon annihilation process. An interesting and perhaps practical way to reach very high CM energies (up to 50 GeV) in ee collisions would be to arrange collisions of the SLAC beam with itself, utilizing the proposed recirculation ring. Such a facility would be especially useful for new particle searches: heavy leptons, intermediate vector bosons, quarks, in fact all particles which can be pair produced in photon-photon annihilation.

A summary of the experiments discussed in this review is given in Table III.

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REFERENCES

1. J. C. Wesley and A. Rich (to be published) and Phys. Rev. Letters 24, 1320 (1970).
2. B. N. Taylor, W. H. Parker, D. N. Langenberg, Rev. Mod. Phys. 41, 375 (1969).
3. M. J. Levine and J. Wright, Phys. Rev. Letters 26, 1351 (1971).
4. S. Brodsky and T. Kinoshita, Phys. Rev. D3, 356 (1971).
5. J. Calmet and M. Perrottet (to be published).
6. J. A. Mignaco and E. Remiddi, Nuovo Cimento 60A, 519 (1969).
7. J. Aldins, T. Kinoshita, S. J. Brodsky, and A. Dufner, Phys. Rev. D1, 2378 (1970) and Phys. Rev. Letters 23, 41 (1969).
8. M. J. Levine, J. Comp. Phys. 1, 454 (1967).
9. J. Bjorken, J. Kogut, and D. Soper, Phys. Rev. D3, 1382 (1971).
10. S-J. Chang and S-K. Ma, Phys. Rev. 180, 1506 (1969).
11. D. Foerster, "The anomalous moment in the infinite momentum frame," Abstract 242.
12. H. Suura and E. H. Wickmann, Phys. Rev. 105, 1930 (1957);
    A. Petermann, Phys. Rev. 105, 1931 (1957);
    H. Elend, Phys. Letters 20, 682; 21, 720 (1966);
    A. W. Erickson and H. Liu (unpublished).
13. B. E. Lautrup and E. de Rafael, Phys. Rev. 174, 1835 (1968); Nuovo Cimento 64A, 322 (1969).
14. B. E. Lautrup, A. Peterman, and E. de Rafael, Nuovo Cimento 1A, 238 (1971).
15. See, e.g., M. Gourdin and E. de Rafael, Nucl. Phys. 10B, 667 (1969).
16. See S. J. Brodsky and J. D. Sullivan, Phys. Rev. 156, 1644 (1967) and references therein.

17. J. Bailey et al., Phys. Letters 28B, 28B, 287 (1968).

18. C. Jarlskog, preprint 1363-CERN (1971).

19. Further review and references may be found in S. J. Brodsky and S. D. Drell, Ann. Rev. Nucl. Sci. 20, 147 (1970), and E. de Rafael, B. Lautrup, and A. Petermann, Phys. Reports (1971) (to be published).

20. G. W. Erickson and D. R. Yennie, Ann. Phys. (N.Y.) 35, 27, 447 (1965).

21. G. W. Erickson, preprint "Improved Lamb Shift Calculation for all Values of Z" (1971).

22. A. M. Disiderio and W. R. Johnson, Phys. Rev. A3, 1267 (1971).

23. T. Appelquist and S. J. Brodsky, Phys. Rev. Letters 24, 562 (1970) and Phys. Rev. A2, 734 (1970).

24. M. F. Soto, Jr., Phys. Rev. Letters 17, 1153 (1966), and Phys. Rev. A2, 734 (1970). See also J. Weneser, R. Bersohn, and N. Kroll, Phys. Rev. 91, 1257 (1953).

25. A. Peterman, preprint CERN-Th. 1354 (1971) "A New Value for the Lamb Shift."

26. R. Barbieri, J. Mignaco, and E. Remiddi, preprint SNS 71/3 (1971), and Nuovo Cimento Letters 3, 588 (1970).

27. A. Peterman, Phys. Letters 35B, 325 (1971).

28. J. Fox, private communication.

29. B. Lautrup, A. Peterman, and E. de Rafael, Phys. Letters 31B, 577 (1970).

30. S. J. Brodsky, 1969 International Conference on Electron and Photon Interactions at High Energies, Daresbury.

31. C. W. Fabjan and F. M. Pipkin (to be published).

32. T. Crane et al., Phys. Rev. Letters 27, 474 (1971).

33. R. De Voe et al., Phys. Rev. Letters 25, 1779 (1970); D. Farant et al., University of Chicago preprint EFI-71-47 (1971).

34. J. F. Hague et al., Phys. Rev. Letters 25, 628 (1970).

35. E. D. Theriot et al., Phys. Rev. A2, 707 (1970), and private communication, V. W. Hughes.

36. E. de Rafael, private communication.

37. S. D. Drell and J. D. Sullivan, Phys. Letters 19, 516 (1966).

38. S. B. Crampton et al., Phys. Rev. Letters 11, 338 (1963).

39. T. Fulton, D. Owen, and W. Repko, Phys. Rev. Letters 26, 61 (1971).

40. D. R. Yennie, private communication, and see, e.g., S. Brodsky, Brandeis Lectures, 1969.

41. M. Levy and J. Sucher, Phys. Rev. 186, 1656 (1969); E. Brezin, C. Itzykson, and J. Zinn-Justin, Phys. Rev. D1, 2349 (1970); A. O. Barut and Z. Z. Aydin, Abstract 229, and references therein.

42. A. Nandy and R. Sawyer, preprint (1971).

43. I. T. Todorov, Phys. Rev. D3, 2351 (1971). References are also given here for earlier work on the quasipotential model.

44. H. Grotch and D. R. Yennie, Rev. Mod. Phys. 41, 350 (1969).

45. C. Fronsdal and R. W. Hoff, Phys. Rev. D3, 933 (1971).

46. M. Nauenberg, private communication.

47. Y. S. Tsai, SLAC preprint (1971).

48. E. J. Williams, Kgl. Danske Videnskab. Selskab. Mat.-Fys. Medd. 13, No. 4 (1934).

49. L. Landau and E. Lifshitz, Phys. Z. Sovietunion 6, 244 (1934).

50. S. J. Brodsky, T. Kinoshita, and H. Terazawa preprint CLNS-152 (April 1971), Phys. Rev. (to be published September 1971).

51. V. N. Baier and V. S. Fadin, Nuovo Cimento Letters 1, 481 (1971); preprints (Novosibirsk, 1971); Phys. Letters 35B, 156 (1971); JETP 13, 293 (1971).

52. M. Greco, preprint (LNF-71/1, January 1971). (Abstract 219)

53. H. Cheng and T. T. Wu, Phys. Rev. Letters 23, 1311 (1969); Phys. Rev. D1, 2775 (1970), and many references therein.

H. Cheng and T. T. Wu, preprint DESY 71/23 (May 1971).

54. The complete expression for $N(\omega)$, as given by Dalitz and Yennie (Ref. 55) (see also Refs. 56, 50) is

$$N(\omega) = \frac{\alpha}{\pi}\left\{\frac{E^2+E'^2}{E^2}\left(\ell n\left(\frac{E}{m_e}\right)-\frac{1}{2}\right)+\frac{(E-E')^2}{2E^2}\left(\ell n\,\frac{2E'}{E-E'}+1\right)\right.$$
$$\left.+\frac{(E+E')^2}{2E^2}\,\ell n\,\frac{2E'}{E+E'}\right\} \quad .$$

55. R. H. Dalitz and D. R. Yennie, Phys. Rev. 105, 1598 (1957).

56. R. B. Curtis, Phys. Rev. 104, 211 (1956).

57. N. A-Romero, A. Jacçarini, and P. Kessler, C. R. Acad. Sci. Ser. B269, 153, 1129 (1969); Abstracts 104, 259 and 260; the same authors and J. Parisi, Nuovo Cimento Letters 4, 933 (1970); Phys. Rev. D3, 1569 (1971); J. Parisi, These detroisieme cycle (Paris, 1970).

58. F. E. Low, Phys. Rev. 120, 582 (1960).

59. F. Calogero and C. Zemach, Phys. Rev. 120, 1860 (1960).

60. V. E. Balakin, V. M. Budnev, and I. F. Ginzburg, Pis'ma Zh. Eksp. Teor. Fiz. 11, 559 (1970) [JETP Letters 11, 388 (1970)];

V. M. Budnev and I. F. Ginzburg, preprint TP-55, Novosibirsk (1970).

61. S. J. Brodsky, T. Kinoshita, and H. Terazawa, Phys. Rev. Letters 25, 972 (1970).

62. D. H. Lyth, Phys. Letters B30, 195 (1971).

63. C. Carlson and W. K. Tung, preprint (Chicago, 1971).

64. F. J. Yndurain (private communication).

65. R. L. Goble and J. L. Rosner, preprint (Minnesota, June 1971).

66. B. Schrempp-Otto, F. Schrempp and T. F. Walsh, Abstract 177.

67. A. Bramon and M. Greco, Abstract 216.

68. H. Kleinert, L. P. Staunton and P. H. Weisz, Abstract 128.

69. S. L. Glashow, R. Jackiw and S. Shei, Phys. Rev. 187, 1916 (1969); G. Gounaris, Phys. Rev. D1, 1426 (1970); A. Baracca and A. Bramon, Nuovo Cimento 51A, 873 (1967); M. Gourdin (to be published); L. H. Chan, L. Clavelli and R. Torgerson, Phys. Rev. 185, 1754 (1969).

70. E. Bertolucci et al., submitted to the International Conference on Elementary Particles, Amsterdam, 1971.

71. A. Litke, Thesis, Harvard University (1970).

72. V. G. Serbo, Pis'ma Zh. Eksp. Teor. Fiz. 12, 50 (1970) [JETP Letters 12, 39 (1970)].

73. R. W. Brown and I. J. Muzinich, preprint BNL 15713 (February 1971).

74. J. Sandweiss, private communication.

75. E. de Rafael, private communication.

76. For a review of this puzzle see N. Christ and T. D. Lee, Phys. Rev. D4, 209 (1971), and references therein.

77. S. L. Adler, Phys. Rev. 177, 2426 (1969).

78. H. Terazawa, Phys. Rev. Letters 26, 1207 (1971).

79. R. Aviv, N. D. Hari Dass, and R. F. Sawyer, Phys. Rev. Letters 26, 591 (1971), and UCSB preprint to be published.

80. E. S. Abers and S. Fels, Phys. Rev. Letters 26, 1512 (1971).

81 T. Yao, Phys. Letters 35B, 225 (1971).

82. K. Wilson, private communication.

83. I wish to thank P. Roy for discussions on this point.

84. T. M. Yan, preprint SLAC-PUB-928 (July 1971).

85. T. Das et al., Phys. Rev. Letters 18, 759 (1967).

86. T. F. Walsh, preprint DESY 71/15 (1971).

87. S. J. Brodsky, T. Kinoshita, and H. Terazawa, Phys. Rev. Letters 27, 280 (1971).

88. J. D. Bjorken and E. Paschos, Phys. Rev. 185, 1975 (1969).

89. S. Berman, J. D. Bjorken, and J. Kogut, SLAC preprint (to be published).

90. R. Kingsley, preprint DAMTP 71/29 (July 1971).

91. A. U. Efremov and I. F. Ginzburg, Dubna preprint (1971).

92. A simple estimate of the total $ee \to ee\pi^+\pi^-$ cross section in the range of interest can be made. For $4m_\pi^2 < s < 8m_\pi^2$,

$$Q^2_{min} < |q_1^2| < Q^2_{max}, \quad |q_2^2 - q_1^2| < 0.2\,|q_1^2|$$

one loses a factor

$$\sim \frac{1}{2} \times 0.2 \times \int_{Q^2_{min}}^{Q^2_{max}} \frac{F(Q^2)}{Q^2} \frac{1}{\log^2 E^2/m_e^2} \sim 10^{-3}$$

relative to the equivalent photon result for $\sigma_{ee \to ee\pi^+\pi^-}$.

93. M. J. Creutz and M. B. Einhorn, Phys. Rev. Letters 26, 341 (1970).

94. A. Halprin, H. Primakoff and C. Anderson, Phys. Rev. 152, 1295 (1966) and references therein.

95. L. Stodolsky, Phys. Rev. Letters 26, 404 (1971); N. Jurisic and L. Stodolsky, SLAC-PUB-794 (1970), and references therein.

96. G. Berland, A. Dar, G. Eilan, J. Franklin, Abstract 152, and references therein.

TABLE I

Compilation of Contributions to the Lamb Shift: $2S_{1/2}$-$2P_{1/2}$ in Hydrogen
(compiled by de Rafael, Lautrup, Peterman[19])

Order	Description and References	Correction, Units $\frac{\alpha}{\pi}(Z\alpha)^4\frac{m}{6}$	Numerical Value in MHz
$\alpha(Z\alpha)^4 m$	2nd order self-energy	$\left(-2\log(Z\alpha)+\frac{m}{M}+\frac{11}{24}-\log\frac{K_0(2,0)}{K_0(2,1)}\right)\left(1-3\frac{m}{M}\right)$	1009.920
$\alpha(Z\alpha)^4 m$	2nd order magnetic moment	$\frac{1}{2}\left(1-2.75\frac{m}{M}\right)$	67.720
$\alpha(Z\alpha)^4 m$	2nd order vac. polarization	$-\frac{1}{5}\left(1-3\frac{m}{M}\right)$	- 27.084
$\alpha(Z\alpha)^5 m$	2nd order binding	$(Z\alpha)(3\pi)\left(1+\frac{11}{128}-\frac{1}{2}\log 2+\frac{5}{192}\right)\left(1-3\frac{m}{M}\right)$	7.140
$\alpha(Z\alpha)^6 m$	4th order binding + higher orders	$(Z\alpha^2)(a+b\log(Z\alpha)^2+c\log^2(Z\alpha)^2)+(Z\alpha)^3\pi(9.56)$	- 0.372
$\alpha^2(Z\alpha)^4 m$	4th order self-energy	$3\frac{\alpha}{\pi}0.470$	0.444
$\alpha^2(Z\alpha)^4 m$	4th order magnetic moment	$\frac{\alpha}{\pi}(-0.328)$	- 0.102
$\alpha^2(Z\alpha)^4 m$	4th order vac. polarization	$\frac{\alpha}{\pi}\left(\frac{41}{54}\right)$	- 0.239
$\alpha(Z\alpha)^4\frac{Zm}{M}m$	Recoil corrections	$\frac{Zm}{M}\left(a_1+b_1\log(Z\alpha)^2\right)$	0.359
$\alpha(Z\alpha)^4\left(\frac{mR_p}{e}\right)^2 m$	Proton size	$\frac{\pi^2}{2}\left(\frac{mR_p}{e}\right)^2$	0.125
			TOTAL = 1057.911 ± 0.011

The value of the Bethe logarithm $\log(K_0(2,1)/K_0(2,0))$ is that evaluated by Schwartz and Tiemann. The constants a, b, c, a_1 and b_1 are the following:

$$a=-\frac{4\pi^2}{3}-4-4\log^2 2-0.28\pm0.5;\quad b=\frac{55}{48}4\log 2;\quad c=-3/4;\quad b_1=1/4;\quad a_1=2\log\frac{K_0(2,1)}{K_0(2,0)}+\frac{97}{12}.$$

TABLE II

Precision Tests of Lamb Shift Calculations (in MHz)

(compiled by G. W. Erickson[21])

	interval	theory (±1σ)	experiment (±1σ)		$\frac{\text{theory-exp}}{\sigma}$
H	$2S_{1/2} - 2P_{1/2}$	1057.912 ± 0.011	1057.90 ± 0.06	a	+0.2
			1057.77 ± 0.06	b	+2.3
	$2P_{3/2} - 2S_{1/2}$	9911.123 ± 0.031	9911.17 ± 0.04	c	-0.9
			9911.25 ± 0.06	d	-1.9
			9911.38 ± 0.03	e	-6.0
	$3S_{1/2} - 3P_{1/2}$	314.894 ± 0.009	314.810 ± 0.052	i	+1.5
D	$2S_{1/2} - 2P_{1/2}$	1059.272 ± 0.025	1059.28 ± 0.06	f	-0.1
			1059.00 ± 0.06	b	+4.2
He^+	$2S_{1/2} - 2P_{1/2}$	14,044.78 ± 0.61	14,045.4 ± 1.2	g	-0.5
			14,040.2 ± 1.8	b	+2.4
	$3S_{1/2} - 3P_{1/2}$	4184.42 ± 0.18	4183.17 ± 0.54	h	+2.2
	$3P_{3/2} - 3S_{1/2}$	47,843.39 ± 0.23	47,844.05 ± 0.48	h	-1.2

a R. T. Robiscoe and T. W. Shyn, Phys. Rev. Letters 24, 559 (1970), with uncertainty as assigned by TPL, Ref. (2).

b Experiment done before 1958 and discussed in Ref. (2), which gives the one standard deviation uncertainty used here.

c B. L. Cosens and T. V. Vorburger, Phys. Rev. A2, 16 (1970).

d T. W. Shyn, T. Rebane, R. T. Robiscoe, and W. L. Williams, Phys. Rev. A3, 116 (1971).

e S. L. Kaufman, W. E. Lamb, Jr., K. R. Lea, and M. Leventhal, Phys. Rev. Letters 22, 507, 806 (1969).

f B. L. Cosens, given in Ref. (2).

g M. A. Narasimham, given in Ref. (2).

h D. L. Mader, M. Leventhal, and W. E. Lamb, Jr., Phys. Rev. A3, 1832 (1971).

i C. Fabjan and F. M. Pipkin, to be published.

TABLE III

Summary of Two Photon Experiments Using ee → eeX

$\gamma\gamma$ COLLIDING BEAMS

[Real γ + Real γ] (including polarized γ's for parity determinations)

$\gamma+\gamma \to e^+e^-, \mu^+\mu^-, e^+e^-e^+e^-$	QED checks	"SLAC + SLAC"
$\to \bar{\ell}\ell^*, \overline{W}W, q\bar{q}$	Particle Searches	
$\to \pi^o, \eta^o, \eta', \delta, A_2$, etc.	Narrow resonances Tests of F.E.S.R., symmetry	
$\to \pi^+ + \pi^-$	$\pi\pi$ phases σ, f_0, current algebra	
$\to K^+, K^-$	Superconvergence relations Born term	
→ odd # pions $\pi^o\pi^o\pi^o$, $\pi^o\pi^+\pi^-$	Current algebra, PCAC at threshold	
→ pion pairs $\pi^+\pi^-\pi^+\pi^-$	Current algebra, PCAC at threshold and ρ-ρ	
→ all	$\sigma_{\gamma\gamma}$ total, Pom fact., Vec. Dom.	
→ all + one high transverse	Parton model	

[Virtual γ + Real γ]

$\gamma+\gamma \to$ all	Inelastic e-scattering on a γ-target (scaling, parton models)
$\gamma+\gamma \to$ all + π (threshold)	Current algebra, axial vector commutator
$\gamma+\gamma \to \pi^+ + \pi^-$	General structure
$\gamma+\gamma \to \pi^o$	Slope at space-like q^2

[Virtual γ + Virtual γ]

$\gamma+\gamma \to \pi^+\pi^-$	Integrand in δm_π^2 + general structure
$\gamma+\gamma \to$ all	$\gamma+\gamma$ amplitude (measure 6 W's) Super scaling

FIGURE CAPTIONS

1. Distinct contributions to the sixth order anomalous moment of the electron. (a) Fermion loop contributions (these also contribute to the difference of electron and muon momenta in sixth order). (b) Non-fermion loop contributions.

2. (a) Hadronic contribution to the muon moment.
   (b) Relation to e^+e^- annihilation.

3. (a) W boson contribution to the muon moment.
   (b) Fermi neutrino pair contribution to the muon moment.

4. Contributions to the fourth order vertex of the electron.

5. Proton polarization contributions to the ground state hyperfine splitting of hydrogen.

6. The two photons process for pair production by $e^- - e^\pm$ colliding beams. Diagram (a) dominates for $m_e/E \rightarrow 0$ and/or in the case when both scattered leptons are detected in the forward direction.

7. The total cross sections for $ee \rightarrow eeX$, with $X = \pi^o$, η, $\pi^+\pi^-$ and $\mu^+\mu^-$. The cross section for π^o and η are exact and calculated without form factors (see Fig. 8). The two photon cross sections for $\pi^+\pi^-$ and $\mu^+\mu^-$ are calculated in the equivalent photon approximation.

8. Comparison of the equivalent photon approximation and exact Feynman diagram calculations of $\sigma_{ee \rightarrow ee\pi^o}$, $\sigma_{ee \rightarrow ee\eta^o}$. The solid line is calculated using a 3-point $F_{\mu\nu} \overline{F^{\mu\nu}} \phi_\pi$ vertex. The dashed line indicates the effect of ρ-dominance form factors in the k_1^2 and k_2^2 variables. The dashed-dot line is the equivalent photon result using the Dalitz-Yennie form. Note that the three curves will converge if the scattered leptons are constrained to scatter in the forward direction. The calculation and curves are from Brodsky, Kinoshita and Terazawa, Ref. 50.

9. Kinematics of the process $ee \rightarrow ee\pi^+\pi^-$.

10. The differential cross section for $ee \rightarrow ee\pi^+\pi^-$ illustrating the dependence on the coplanarity angle ψ. (See Fig. 9.) The calculation uses the entire Feynman amplitude (Born approximation). The gradual fall-off in ψ indicates that coplanarity restrictions are not sufficient for separating $ee \rightarrow ee\pi^+\pi^-$ from $e^+e^- \rightarrow \gamma \rightarrow \pi^+\pi^-$ + neutrals. From Brodsky, Kinoshita and Terazawa, Ref. 50.

11. Effects of hadronic interactions on the total two photon cross sections. Curves are given for $\sigma_{ee \rightarrow eeX}$ in (a) the Born approximation, (b) a complete isotropic (J=0) contribution from the sigma pole plus the isotropic (L.E.T.) part of the Born amplitude with m_σ = 700 MeV, Γ_σ = 400 MeV and 600 MeV, and (c) estimated cross sections for multihadron production with assumed threshold $s_{th} = (3m_\pi)^2$ and $(4m_\pi^2)$. From Brodsky, Kinoshita and Terazawa, Ref. 50.

12. Unitarization of the s-wave in $\gamma\gamma \rightarrow \pi\pi$ due to a σ resonance. The s-matrix pole is at 550 MeV. The solid curve is the ratio of s-wave Born and constrained s-wave Born contributions to $\sigma_{\gamma\gamma \rightarrow \pi^+\pi^0}$. The dashed curve is the prediction for I=2. From Goble and Rosner, Ref. 65.

13. Goble-Rosner predictions for $\gamma\gamma \rightarrow \pi^+\pi^-$. σ_s is the unitarized s-wave contribution, σ_{tot} includes the remaining $\ell \geq 2$ pair contributions. The lower curves are calculated from Born approximation. From Goble and Rosner, Ref. 65.

14. Goble-Rosner prediction for $d\sigma/d\beta$ ($ee \rightarrow ee\pi^+\pi^-$) as a function of pion velocity β. The dashed curve is the unitarized s-wave contribution. The solid line includes other partial waves from Born approximation. From Goble and Rosner, Ref. 65. Γ = 420 MeV.

15. Goble-Rosner prediction for $d\sigma/d\beta(ee \to ee\pi^+\pi^-)$ (see Fig. 14 for labelling). In this calculation the current algebra constraint on $\pi\pi \to \pi\pi$ at s=0 is not enforced. From Goble and Rosner, Ref. 65.

16. Predictions for $\sigma_{ee \to eeX}$ assuming finite energy sum rules and various pole-dominance assumptions, given by Braman and Greco, Ref. 67.

17. Relationship between $ee \to ee\pi^+\pi^-$ and the order α self-energy contribution to the pion mass, discussed by T. M. Yan (Ref. 84). The integrand $F(Q^2)$ in the Cottingham formula for $m^2_{\pi\pm} - m^2_{\pi_0}$ can be obtained from the matrix element of $ee \to ee\pi^+\pi^-$ for $k_1^2 = k_2^2 = -Q^2$, assuming a soft-pion extrapolation $k^{\mu}_{\pi^-} \to 0$.

18. Inelastic electron scattering on a photon target, discussed in Refs. 86 and 87. The photon "target" is obtained from the equivalent photon spectrum $d\omega/\omega\ N(\omega, \theta_{max})$ from the one lepton in $ee \to eeX$ which is detected in the forward direction $0 < \theta < \theta_{max}$.

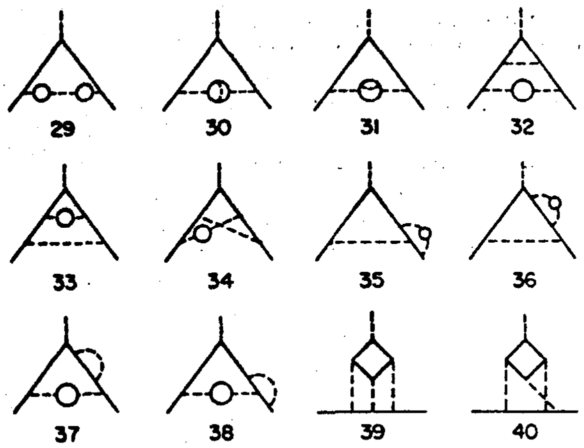
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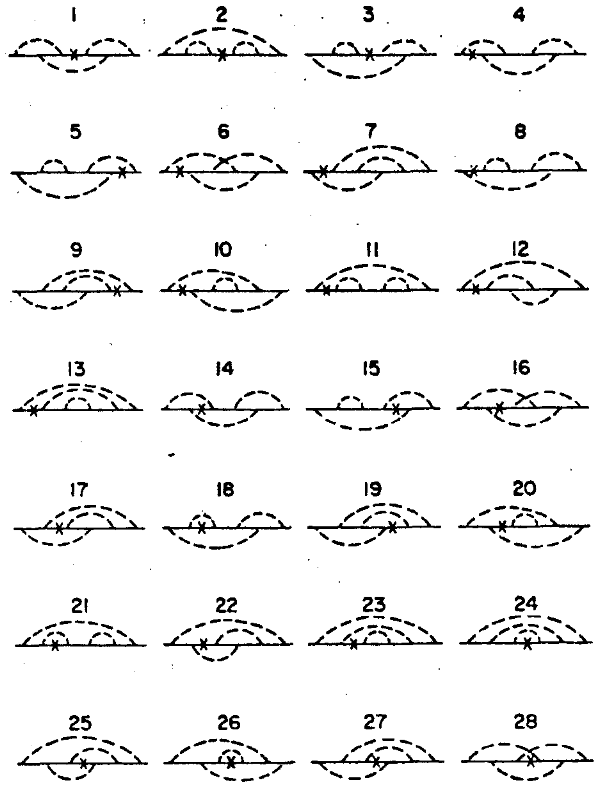
Fig. 1A


1
2
3
4
5
6
7
8
9
10
11
12
13
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15
16
17
18
19
20
21
22
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24
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26
27
28

Fig. 1B

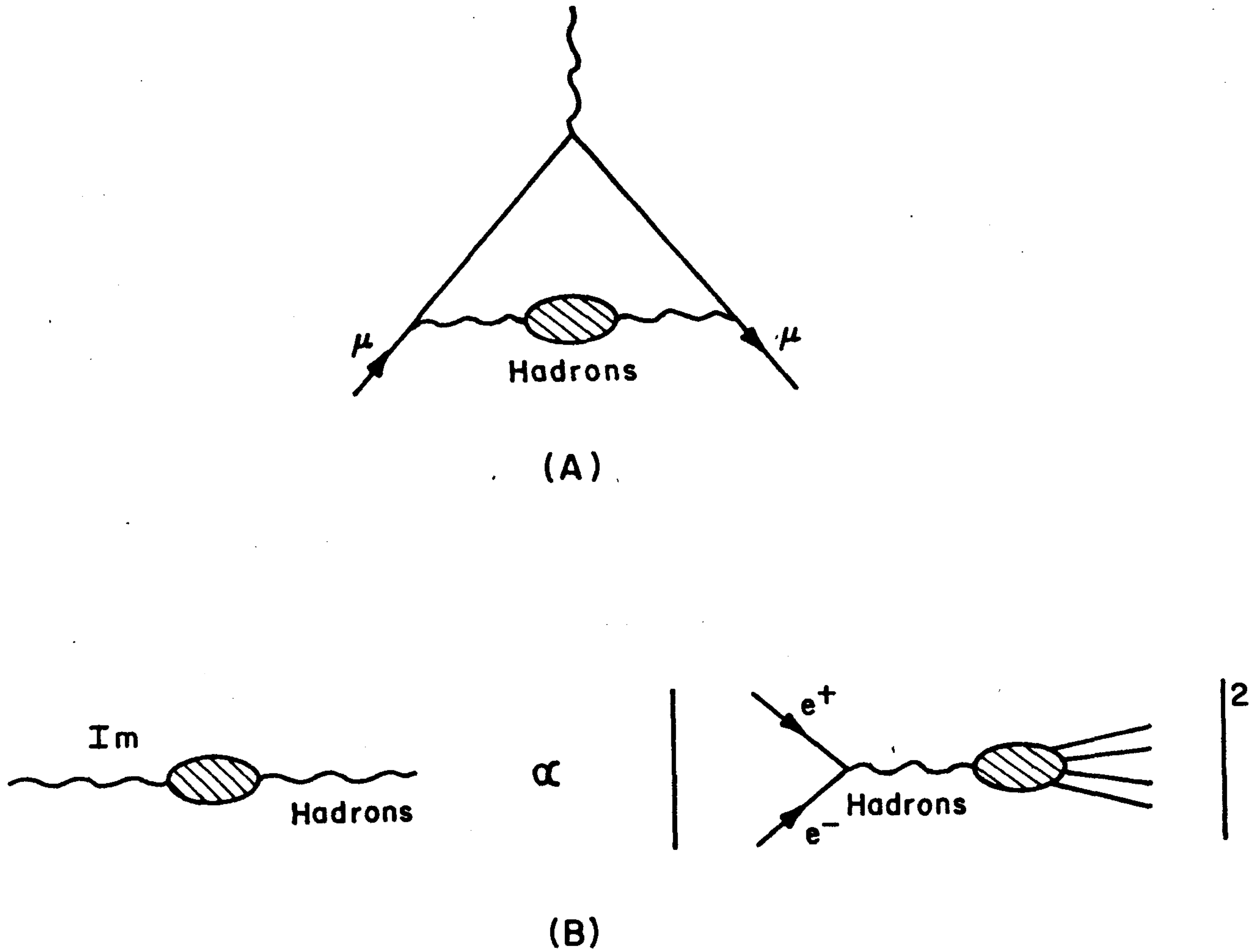
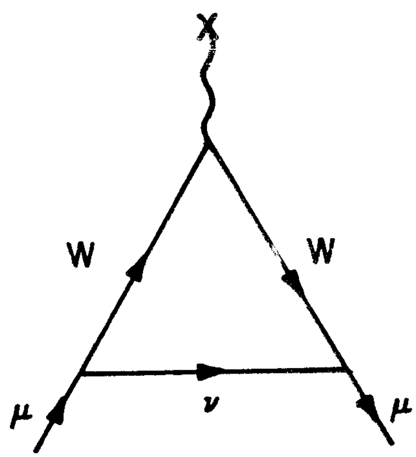
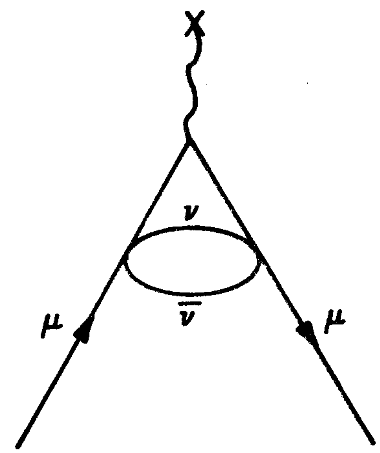


Fig. 2



A



B

Fig. 3

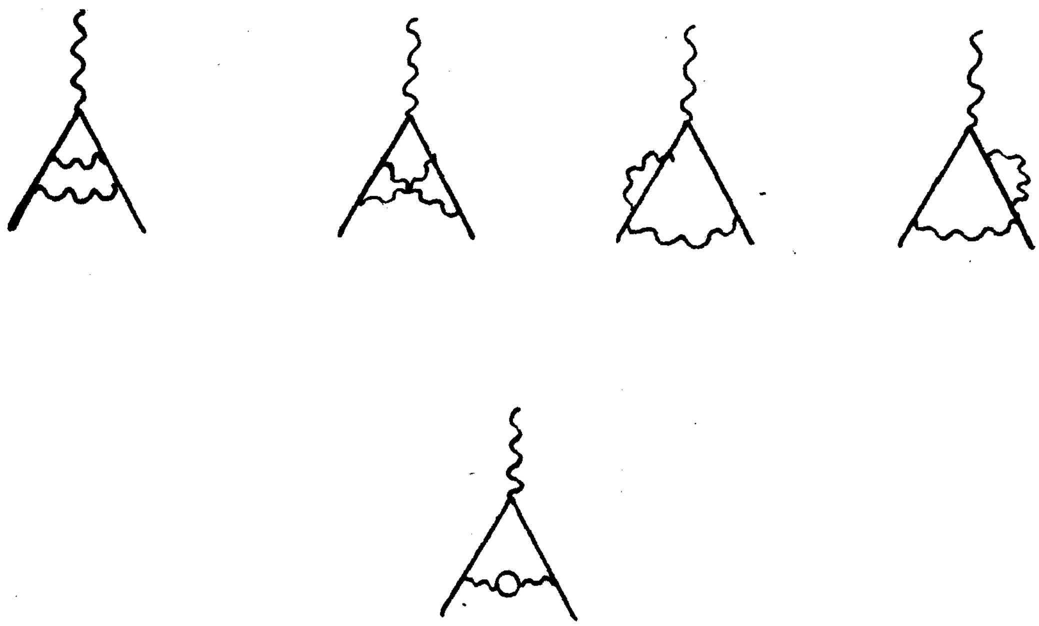


Fig. 4

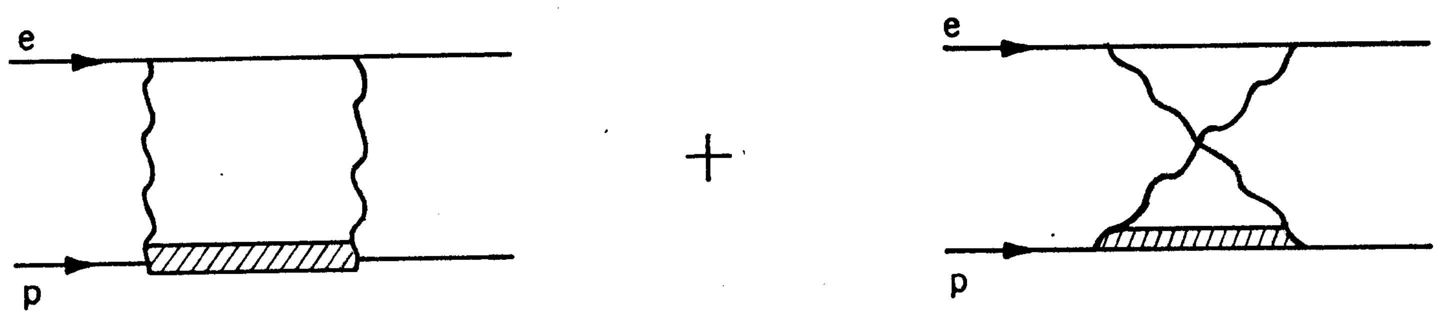


Fig. 5

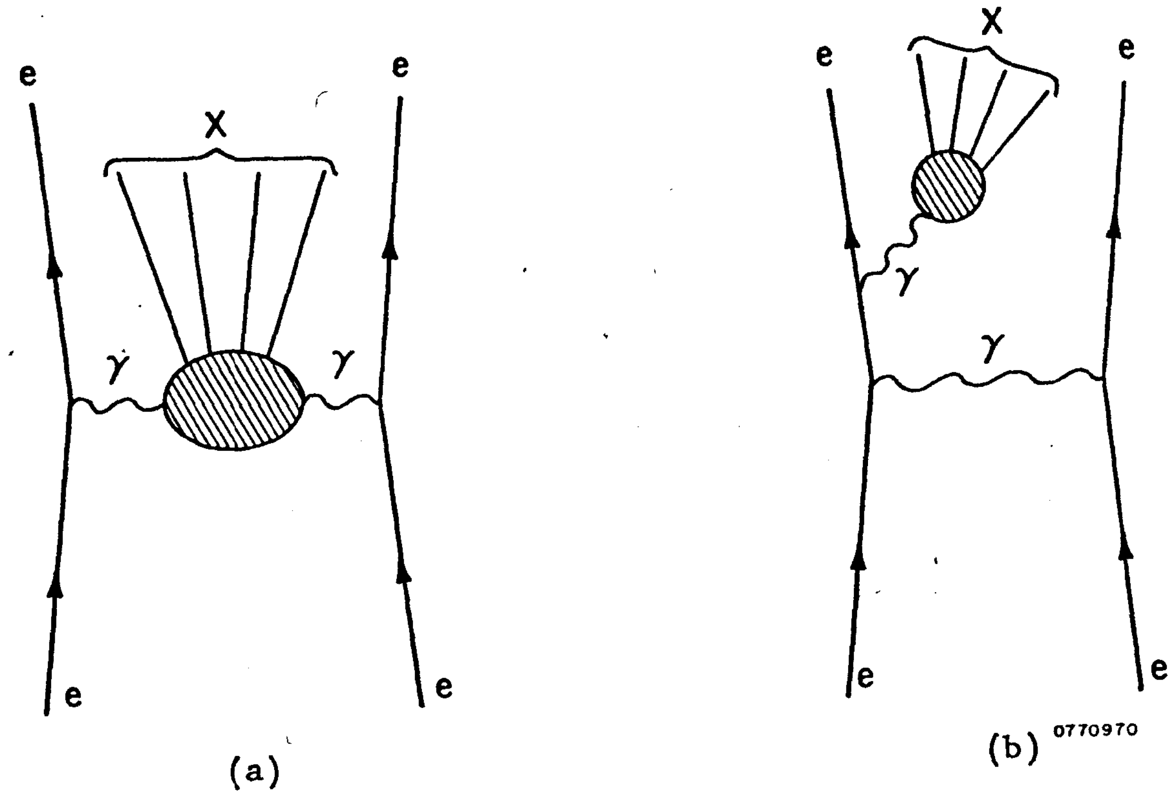
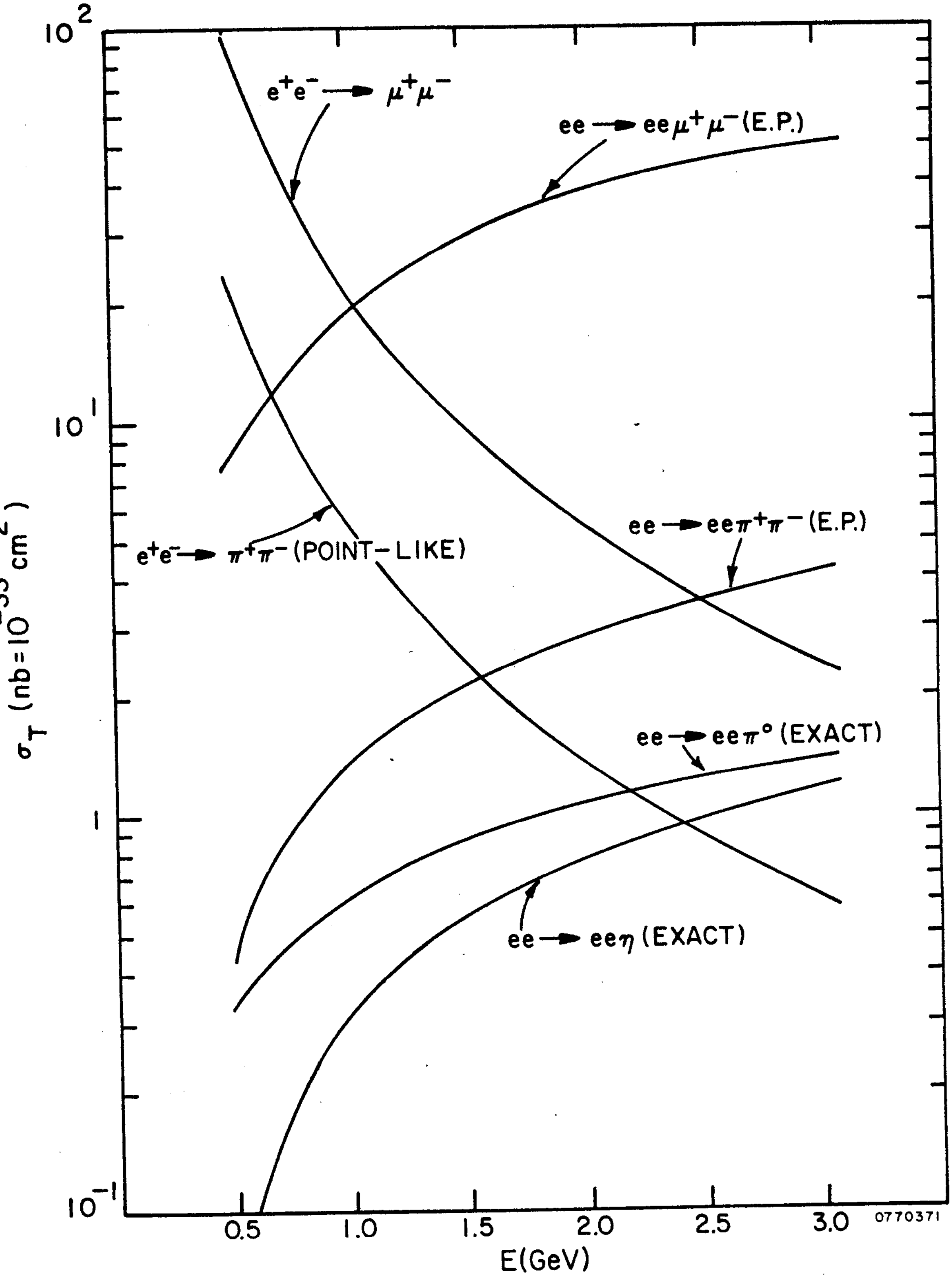


Fig. 6



$e^+e^- \rightarrow \mu^+\mu^-$
$ee \rightarrow ee\mu^+\mu^-$ (E.P.)
$e^+e^- \rightarrow \pi^+\pi^-$ (POINT-LIKE)
$ee \rightarrow ee\pi^+\pi^-$ (E.P.)
$ee \rightarrow ee\pi^0$ (EXACT)
$ee \rightarrow ee\eta$ (EXACT)
σ_T (nb=10^{-33} cm^2)
10^2
10^1
1
10^{-1}
0.5
1.0
1.5
2.0
2.5
3.0
E(GeV)
0770371

Fig. 7

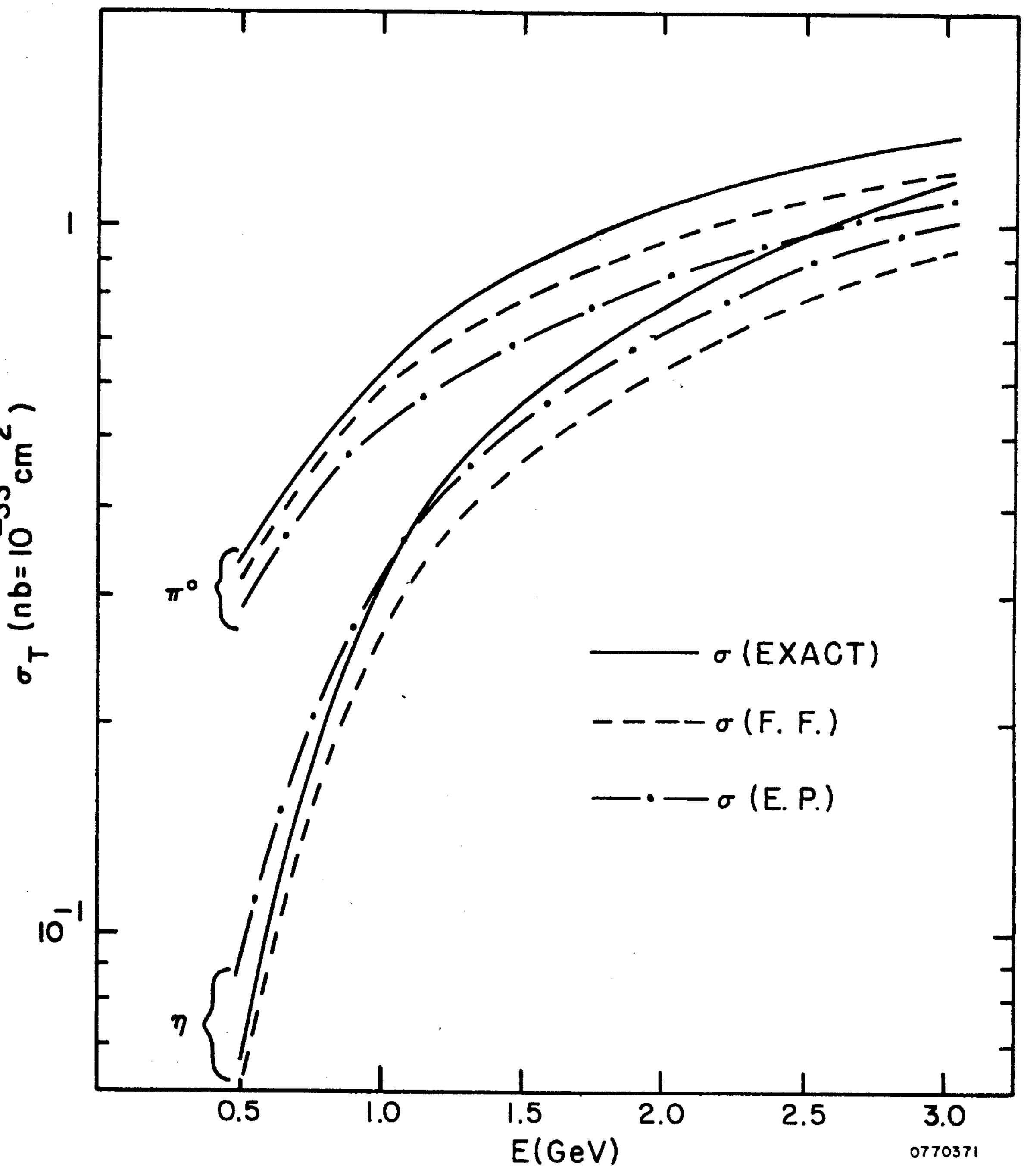

σ (EXACT)
σ (F. F.)
σ (E. P.)
π°
η
1
10⁻¹
σ_T (nb=10⁻³³cm²)
0.5
1.0
1.5
2.0
2.5
3.0
E(GeV)
0770371

Fig. 8

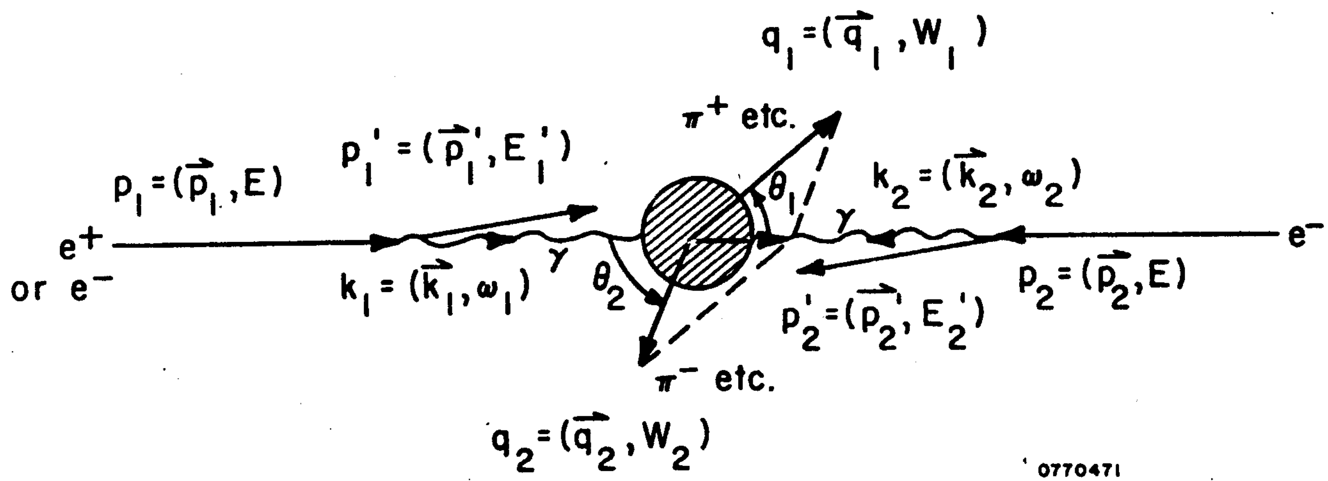
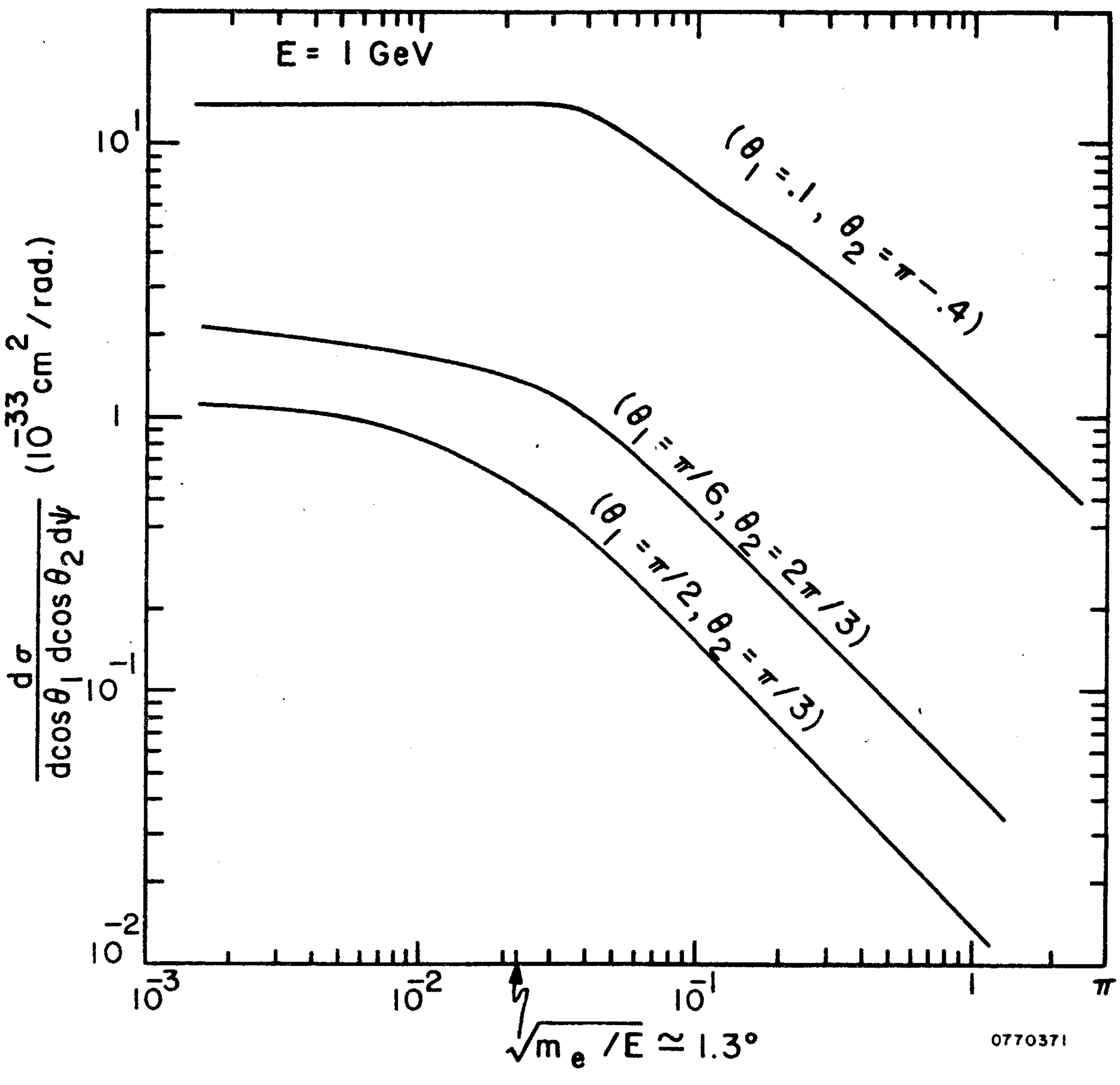
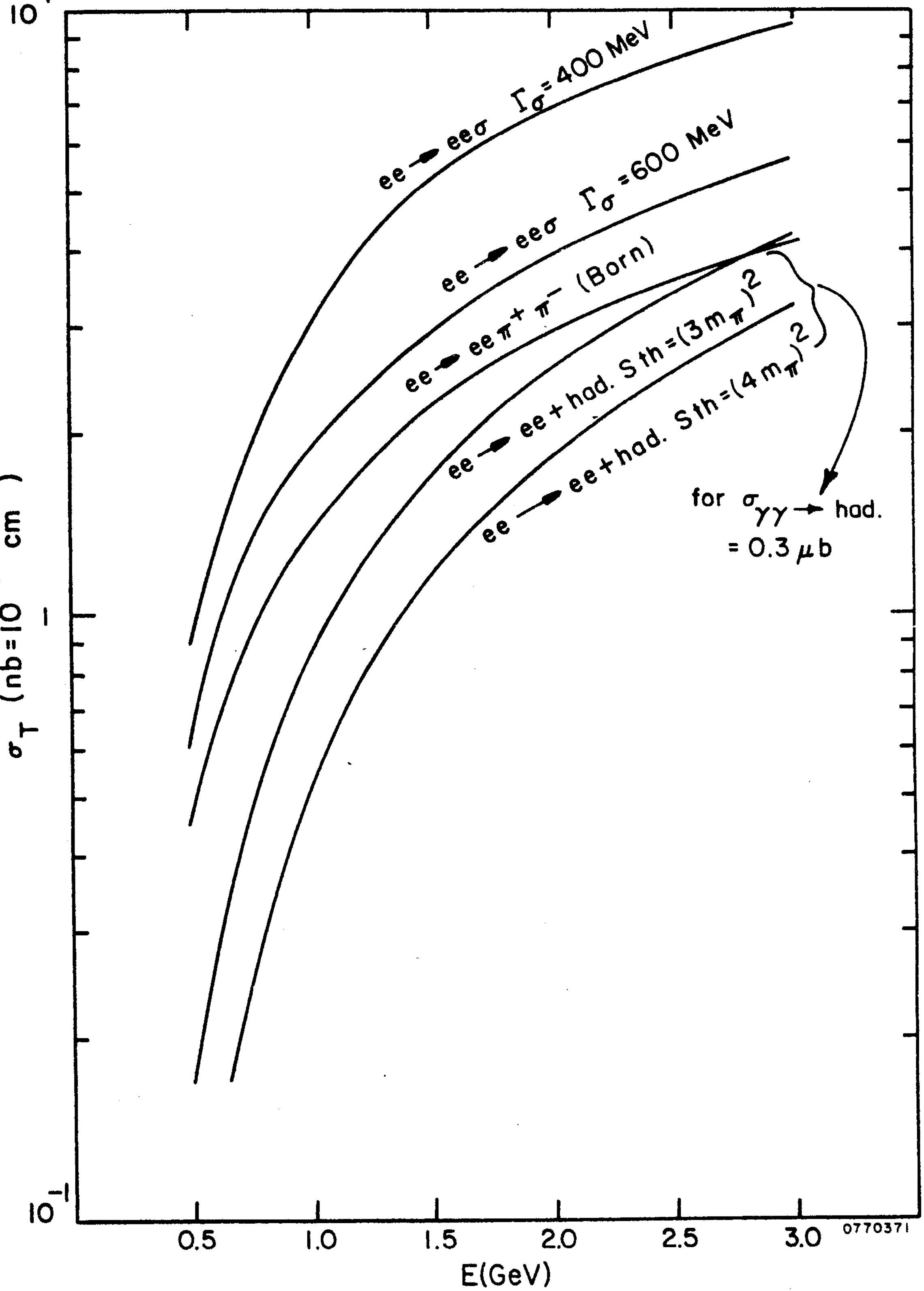

q1 =(q1 ,W1)
π+ etc.
p1' = (p1', E1')
p1 =(p1 ,E)
θ1
k2 = (k2, ω2)
γ
e+
or e−
e−
k1 = (k1, ω1)
γ
θ2
p2 = (p2,E)
p2' = (p2', E2')
π− etc.
q2 = (q2, W2)
0770471

Fig. 9



E = 1 GeV
$(\theta_1 = .1,\ \theta_2 = \pi - .4)$
$(\theta_1 = \pi/6,\ \theta_2 = 2\pi/3)$
$(\theta_1 = \pi/2,\ \theta_2 = \pi/3)$
$\frac{d\sigma}{d\cos\theta_1\, d\cos\theta_2\, d\psi}$ $(10^{-33} cm^2/rad.)$
10^1
1
10^{-1}
10^{-2}
10^{-3}
10^{-2}
10^{-1}
1
π
$\sqrt{m_e/E} \simeq 1.3°$
0770371

Fig. 10



$ee \to ee\sigma \quad \Gamma_\sigma = 400$ MeV
$ee \to ee\sigma \quad \Gamma_\sigma = 600$ MeV
$ee \to ee\pi^+\pi^-$ (Born)
$ee \to ee +$ had. $S_{th} = (3m_\pi)^2$
$ee \to ee +$ had. $S_{th} = (4m_\pi)^2$
for $\sigma_{\gamma\gamma \to \text{had.}} = 0.3\ \mu b$
σ_T (nb = 10^{-33} cm^2)
10^1
1
10^{-1}
0.5
1.0
1.5
2.0
2.5
3.0
E(GeV)
0770371

Fig. 11

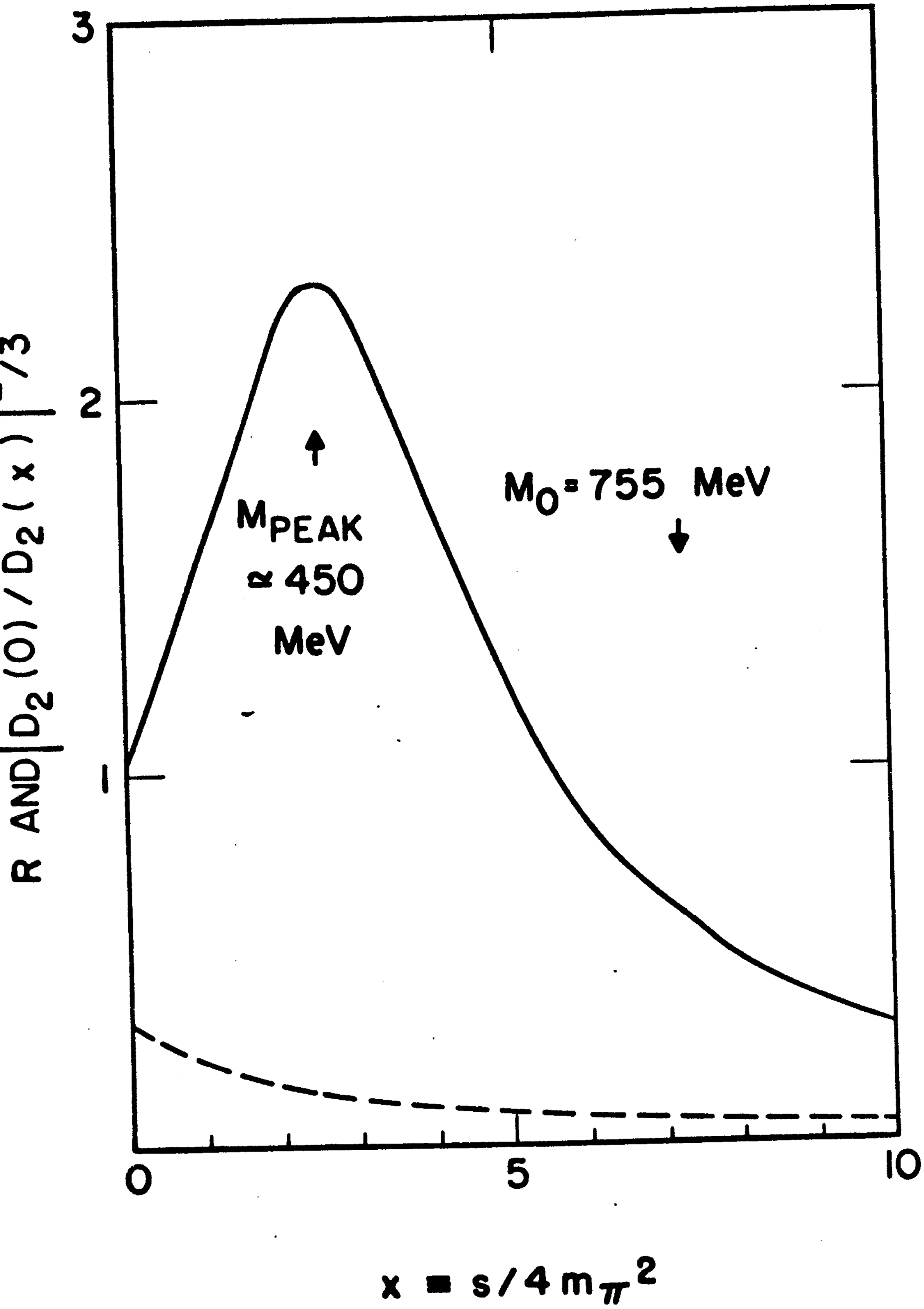
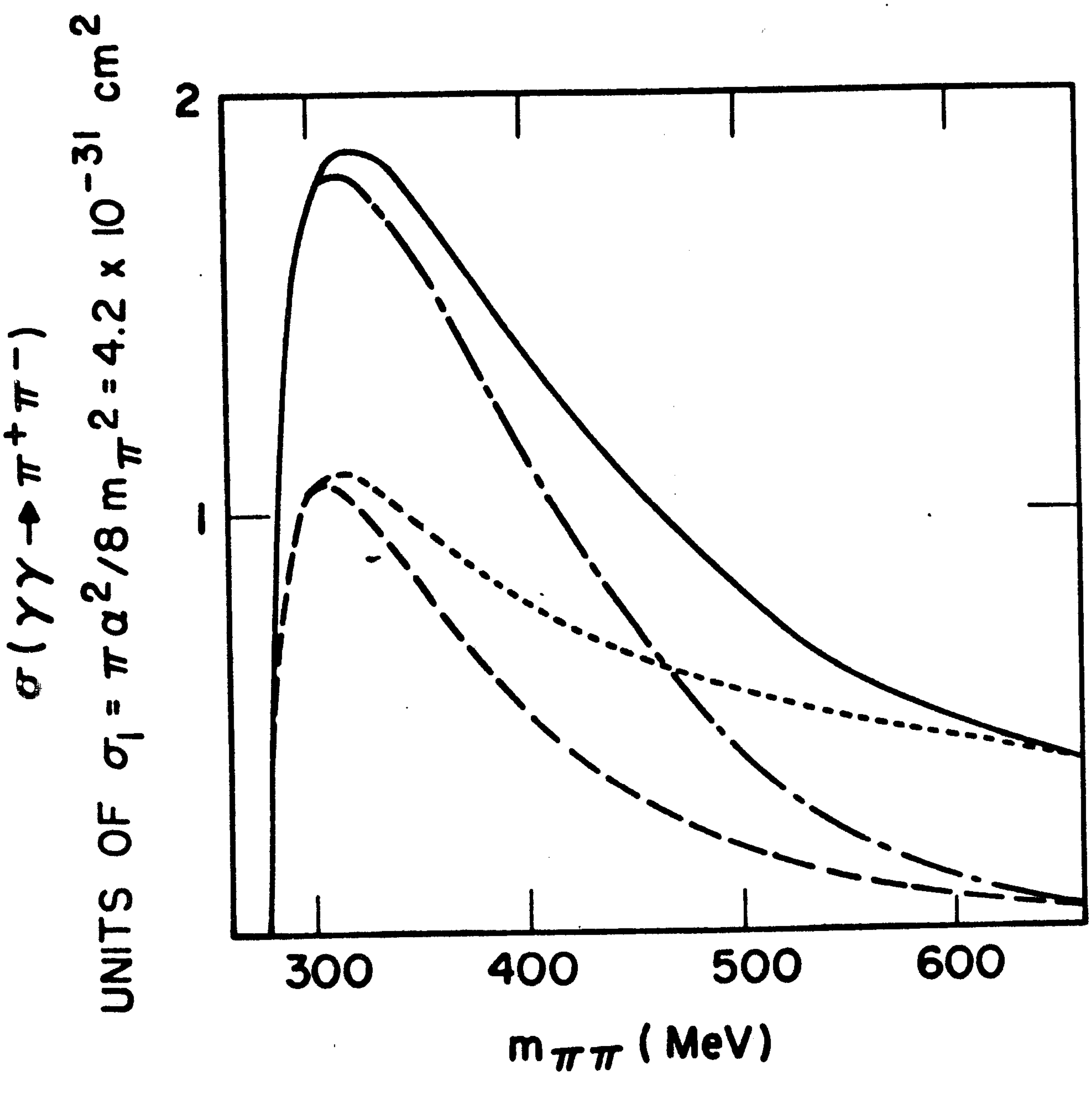
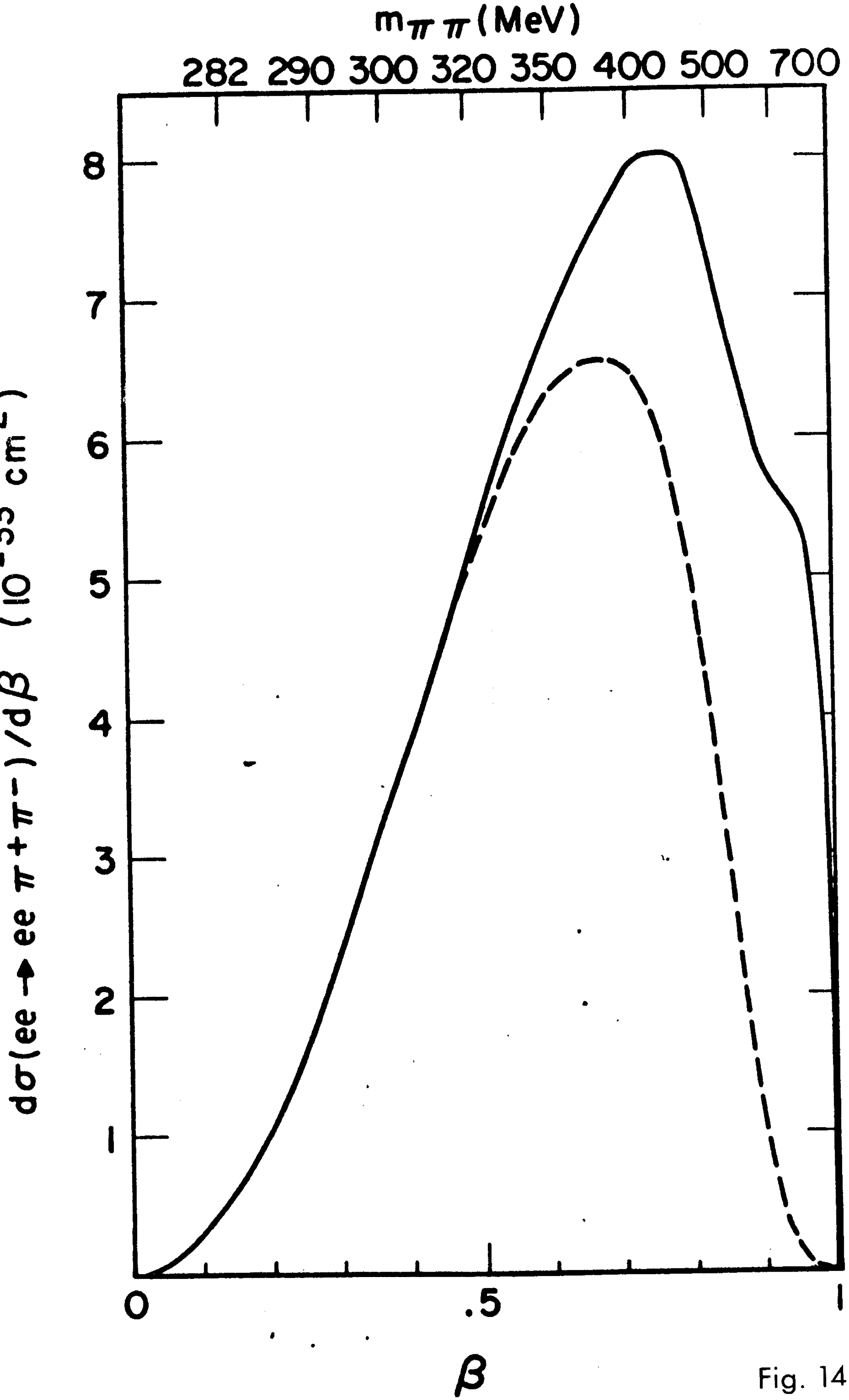

3
2
1
0
5
10
R AND $|D_2(0)/D_2(x)|^{2/3}$
M_{PEAK}
≃ 450
MeV
M_0 = 755 MeV
$x \equiv s/4m_\pi^2$

Fig. 12



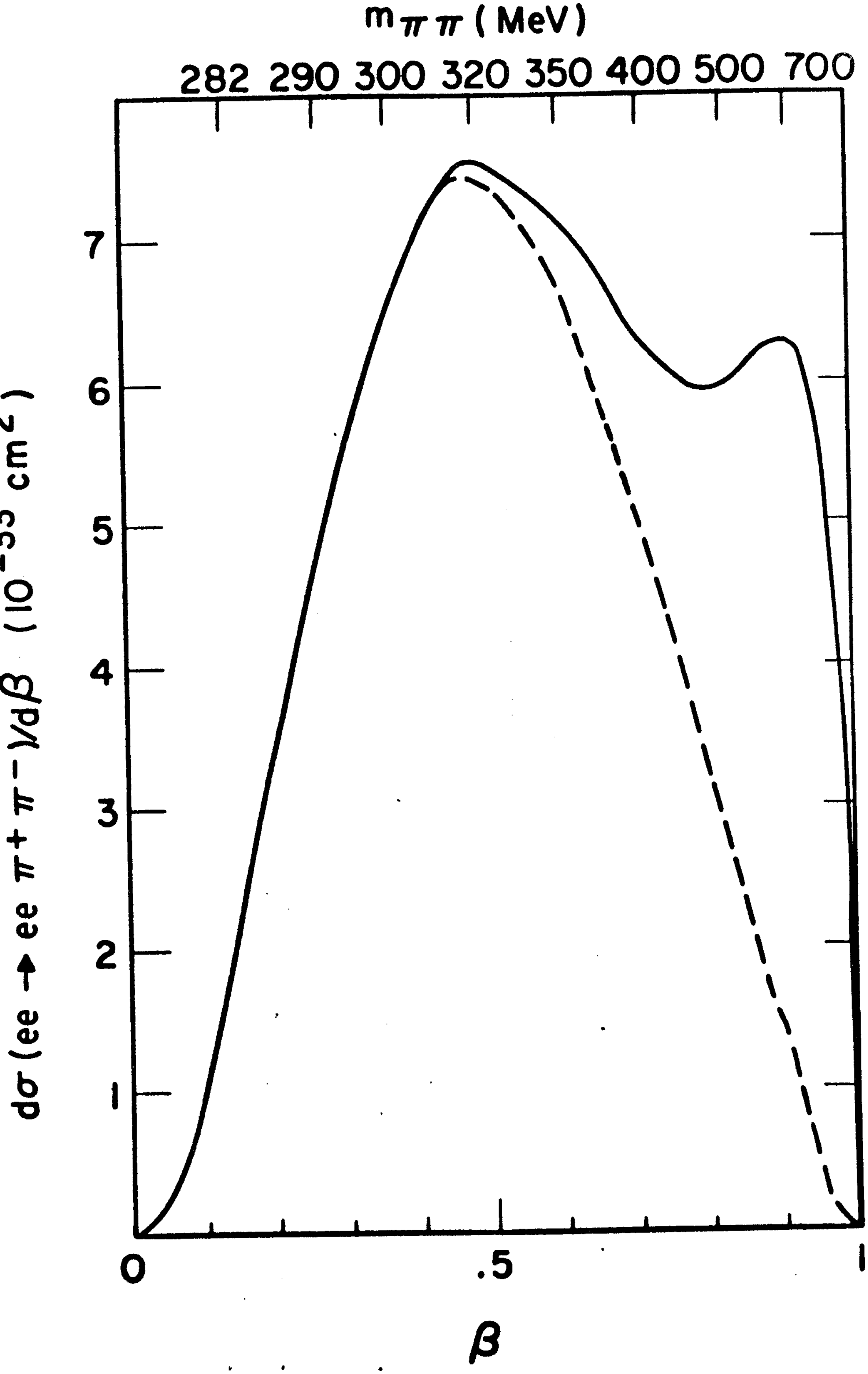
2
1
$\sigma(\gamma\gamma \rightarrow \pi^+\pi^-)$
UNITS OF $\sigma_1 = \pi\alpha^2/8m_\pi^2 = 4.2 \times 10^{-31}$ cm^2
300
400
500
600
$m_{\pi\pi}$ (MeV)

Fig. 13



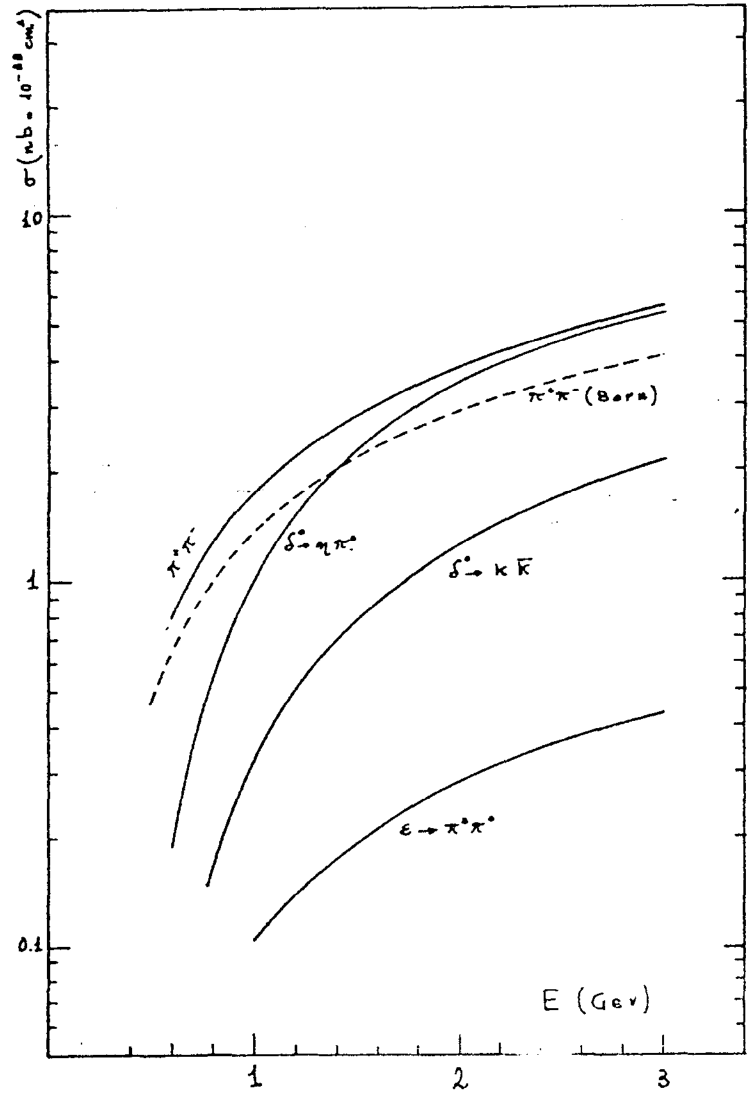
m_ππ (MeV)
282 290 300 320 350 400 500 700
8
7
6
5
4
3
2
1
dσ(ee → ee π+π−)/dβ (10^−33 cm^2)
0
.5
1
β

Fig. 14

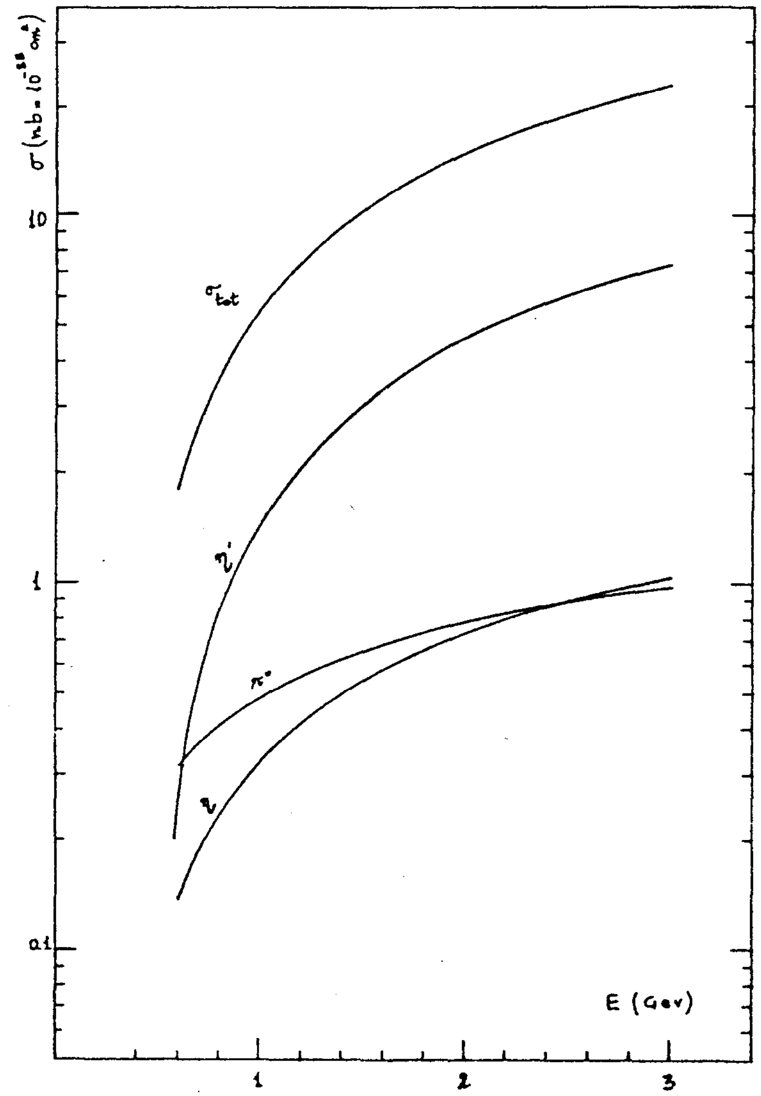


$m_{\pi\pi}$ (MeV)
282 290 300 320 350 400 500 700
$d\sigma(ee \to ee\,\pi^+\pi^-)/d\beta$ (10^{-33} cm^2)
1
2
3
4
5
6
7
0
.5
1
β

Fig. 15

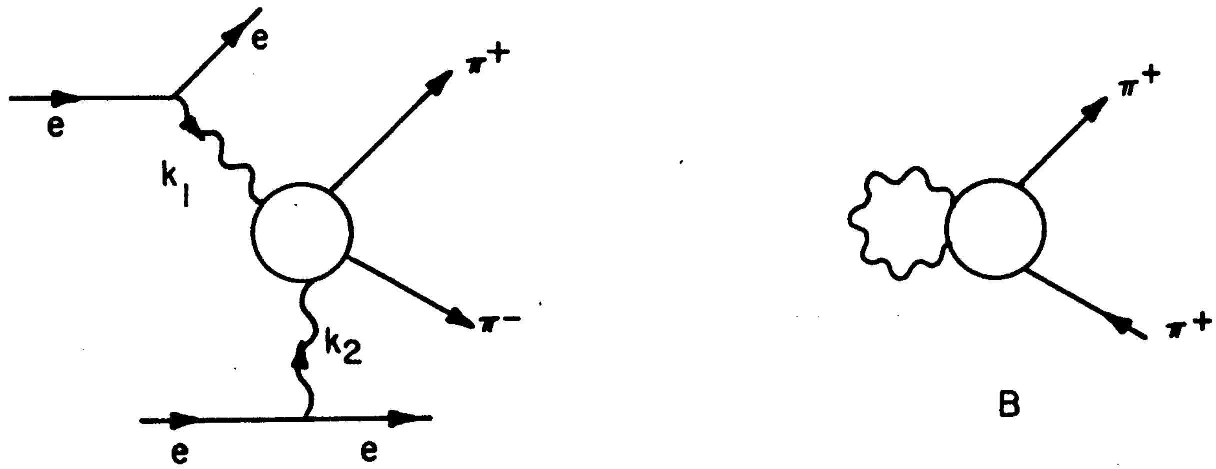


Total cross sections for δ and ε production. The values m_ε = 0.8 GeV and m_δ = 1 GeV have been used.



Total cross section for production of π^0, η and η'. σ_{tot} refers to the total hadronic two photon cross section.

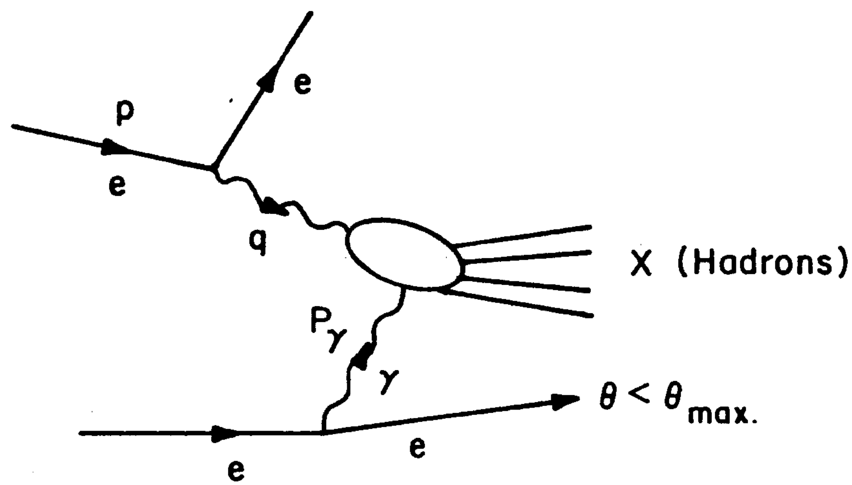
Fig. 16



A

B

Fig. 17



$$\nu = q \cdot P_\gamma \;, q^2 = -Q^2 \;, y = \nu / p \cdot P$$

Fig. 18