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GENERALIZED TRANSVERSE MOMENTUM DISTRIBUTIONS*

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ABSTRACT

Simple models are used to argue that distributions in generalized transverse momentum variables can be used to study various aspects of the transverse spatial structure in production processes and distinguish between competing inelastic mechanisms.

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It is commonly assumed that an experimental study of transverse momentum distributions cannot yield much detailed information about production processes. This would seem to follow from the observation that the transverse momentum distribution of a particle exiting from a production process is strongly limited, with an almost universal shape which does not depend strongly on the energy or the type of particle involved.[1] This is in sharp contrast with the longitudinal distributions which show considerable structure, variation, and correlation. The latter aspects of inelastic processes have thus been the primary focus of theoretical concern. It is the purpose of this note to point out that there can be significant structure in certain transverse momentum distributions. This structure reflects, among other things, rescattering or absorptive effects in the production process. The short-range nature of these effects appears in the experimental distributions at large values of appropriately chosen combinations of transverse momenta. We discuss several simplified and quite different models for production processes in order to illustrate this physical point and to suggest new experimental analyses that can test for rescattering and short-range spatial structure. The models have been chosen to represent the alternative mechanisms for particle production incorporated in most contemporary theoretical models[2] (multiperipheral, pionization and diffractive fragmentation) in a way which indicates how experimental tests may be made of the fundamental assumptions of these models (lack of long-chain correlations, statistical independence and factorization).

First, consider the absorptive multiperipheral model of Figure 1.a in the eikonal approximation.[3] The amplitude for the production of N secondaries is of the form

$$M \propto \int d^2B\, A(\underset{\sim}{B}) \prod_{j=1}^{N+1} \left[\int d^2b_j T_j(\underset{\sim}{b}_j) e^{i\underset{\sim}{b}_j \cdot \underset{\sim}{k}_j} \right] \delta^{(2)}(\underset{\sim}{B} - \sum_i \underset{\sim}{b}_i) \quad . \tag{1}$$

$A(B)$ is an absorptive factor which takes into account rescattering of the initial particles and of the leading fragments of target and projectile, and k_j is the transverse momentum transfer along b_j. If there is no rescattering $A(B) = 1$. In the general case, any pair of nonadjacent final particles suffer rescattering and this situation will be mentioned later. In order to study the effects of absorption on various transverse momentum distributions, we take $T(b_i) = \exp(-b_i^2/2R_i^2)$ and $A(B) = 1 - \epsilon \exp(-B^2/2R_{12}^2)$. For convenience the radii are taken to be energy independent, and the parameter ϵ is not necessarily small. The calculations can easily be extended to more complicated subamplitudes which may more accurately reproduce specific two-particle interactions.[4]

The primary effects can most easily be seen by using the above forms and taking all radii equal. In this case, the momentum space vector conjugate to B in the Fourier sense is

$$\underset{\sim}{v} = \sum_{j=1}^{N+1} \underset{\sim}{k}_j$$

which may also be written as

$$\underset{\sim}{v} = \left(\frac{N+1}{2}\right) (\underset{\sim}{k} - \underset{\sim}{n}) + \sum_{j=1}^{N} \underset{\sim}{p}_j \left(\frac{N+1}{2} - j\right) \quad .$$

The cross section for the production of N secondaries as a function of v^2 is

$$\frac{d\sigma}{dv^2} \propto \frac{\exp\left(-R^2v^2/(N+1)\right)}{(N+1)} \left\{1 - \frac{\epsilon}{N+2} \exp\left(+R^2v^2/2(N+2)\right)\right\}^2 . \tag{2}$$

In deriving this expression, constraints on the transverse momenta due to energy conservation have been ignored. We have checked that this approximation yields substantially the same results as the computer-generated distributions presented below, in which all kinematic constraints are properly taken into account.

Absorption enhances the large v^2 region of the cross section, producing more events there than if ϵ were zero. Also, the slope of the small v^2 region in the above distribution decreases with increasing multiplicity while similar calculations show that the transverse momentum distributions of the secondaries do not change with N.

The rescattering of a particle shows up in its transverse momentum distribution as a break in the slope of that distribution. For example, in the $N = 1$ reaction the p_T^2 distributions of the leading particles should display breaks and flatten out. On the other hand, the secondaries are (slightly) affected by absorption between the leading particles only if the radii involved in the T-matrices vary substantially down the chain. In general, therefore, the p_T distribution of a secondary should not show as pronounced a change in slope as that exhibited by a leading particle unless it is itself strongly rescattered.

Monte Carlo generated distributions for the cases $N = 1$ ($\pi^- p \rightarrow \pi^- \pi^+ n$) and $N = 2$ ($\pi^+ p \rightarrow \pi^+ \pi^+ \pi^- \pi^+ p$) are shown in Figure 2.a, b, for incident pion energy of 16 GeV with $R^2 = 8$ and $\epsilon = 0$ (dotted) and $\epsilon = 1.5$ (dashed). The significant feature is the long tail of events[5] extending to several GeV^2. The dips are also due to absorption. We do not expect these dips to appear strongly in the data except in special circumstances since there are several mechanisms which may fill them. For example, the presence of incoherent contributions to the cross section, such as helicity flip processes, will in general tend to fill in the dips. This mechanism can be examined by exploring the energy dependence of the distribution. Another possibility is that some of the neglected real parts of the amplitude may prevent the appearance of the dip. For example, ϵ may be complex. Taking ϵ to be equal to $(3 + i)/2$ leads to the distribution shown by the solid lines in Figures 2.a, b.

We turn now to the contrasting eikonal model, pictured in Figure 1.b, in which the production of secondaries is an iterative process in the direct channel. This diagram is to be understood as an eikonal graph in that it represents a sum of Feynman graphs with all possible crossings of the vertical exchanges. The amplitude may be written as

$$M \propto \int d^2B\, A(\underset{\sim}{B}) \prod_{j=1}^{N} \int d^2b_j\, T_j(\underset{\sim}{b}_j)\tilde{T}_j(\underset{\sim}{B}-\underset{\sim}{b}_j) e^{i\underset{\sim}{k}_j\cdot\underset{\sim}{b}_j + i(\underset{\sim}{k}_j+\underset{\sim}{p}_j)\cdot(\underset{\sim}{B}-\underset{\sim}{b}_j)} \quad . \tag{3}$$

In addition to the transverse dependence of the amplitude expressed, there is a factor of $(i/2s)^{N-1}$ for the propagators of the outside particles between emissions, and other energy factors associated with the exchanged spin.

For simplicity, we take T_j and T_j as being the same form as in the multiperipheral model, with all R_j equal to the same constant. A(B) again represents the effects of absorption between the outside particles and we use the same expression here as in the previous model. The transverse momentum conjugate to B is now $w \equiv k - n$. For a given multiplicity, N, the distribution in w^2 is given by

$$\frac{d\sigma}{dw^2} \propto \frac{\exp(-R^2w^2/2N)}{N^2} \left\{1 - \frac{\epsilon \exp(R^2w^2/2N(N+2))}{1 + (2/N)}\right\}^2 \tag{4}$$

with the same approximations that led to Eq. (2). A Monte Carlo-generated distribution in the case N = 2 is shown in Figure 3.a and compared with the case $\epsilon = 0$.

Our second model has features which are quite different from the multiperipheral model. In the absence of absorption, the leading (outside) particles undergo a random walk in transverse momentum space; whereas, in the same limit, the multiperipheral model corresponds to a random walk in coordinate space. In the model of Figure 1.b then, the mean transverse momentum of a leading particle will grow with increasing multiplicity, as long as the radii are energy independent or only weakly dependent upon energy. There seems to be some experimental support for both the increase in transverse momentum of leading particles with increasing multiplicity and the increase in transverse momentum with longitudinal momentum for some reactions.[1]

We have discussed these two models to emphasize that the effects of absorption between leading particles have well-defined experimental signatures in very different models. While the optimum transverse variable depends upon the assumed production mechanism, allowing one to experimentally distinguish such mechanisms, the effects are clearly visible in certain other variables. For example, the distribution in the experimentally more accessible variable w^2 for the multiperipheral model is shown in Figure 3.b. The shoulder at large w^2 would be absent if there were no absorption. For low multiplicity events, v^2 itself can be effectively determined by ordering the particles by their longitudinal momenta or rapidity.

An analysis similar to the above has been performed for the diffractive fragmentation model of Figure 1.c. This model differs from the two previous eikonal models in that the secondary particles share substantial fractions of the longitudinal momenta of the incident particles. Several transverse momentum distributions are of particular interest. First, the distributions of the relative p_T^2 between particles from the same jet probes the spatial structure of the fragmentation process. For example, one might distinguish thermodynamic cluster decay from more correlated mechanisms. Second, the distribution in the relative p_T^2 between two particles from opposite jets explores primarily the rescattering shown in Figure 1.c. In general, for small p_T^2 the second distribution should be steeper than the first and should display breaks at larger p_T^2 in a way similar to the w^2 distributions and for the same reasons as discussed in the previous models. Third, the relative transverse cluster momentum explores the nature of the diffractive exchange responsible for the fragmentation.

Distributions in other combinations of transverse momenta can reveal not only the above effects but also other important details about the production mechanism. For example, the distribution in the relative transverse momentum, $(p_i - p_j)_T^2$, between two produced particles can be used to study rescattering between secondaries in all models.[6] Furthermore, within the context of a multiperipheral interpretation, studies of transverse momentum differences between nearest neighbors along the chain can provide tests of the subamplitudes used in the model and can allow examination of possible off shell effects. The short-range effects discussed above can be used to illuminate quite different facets of the production process. It would be interesting to question the relationship between final state quantum numbers and their spatial origin in the production region. For example, are mesons of opposite charge or strangeness produced with a smaller average spatial separation than mesons with the same charge or strangeness?

Physical production mechanisms are much more complex than the simple models discussed here or anywhere else. However, the fact is that structure in generalized transverse momentum distributions is a direct reflection of short-range effects in quite different models. This observation and the simple physical connection between p_T and impact parameter in the eikonal picture, makes the distributions in w^2, $(p_i - p_j)_T^2$, etc. useful both in exploring the gross transverse spatial structure[7] of production processes, but also in differentiating between alternative production mechanisms and the models used to describe them.

FIGURE CAPTIONS

Figure 1 (a) Absorptive multiperipheral model in the eikonal approximation.

(b) Absorptive iterative production model in the eikonal approximation.

(c) Absorptive fragmentation model.

Figure 2 Distributions in the variable v^2 for the multiperipheral model of Figure 1.a, where dotted lines indicate predictions for $\epsilon = 0$, dashed lines are for ϵ real (3/2), and solid lines are for absorption with ϵ complex $(3 + i)/2$.

(a) Production of one secondary (N = 1).

(b) Production of two secondaries (N = 2).

Figure 3 Distributions in w^2, with the same line designations as Figure 2, for

(a) iterative production model of Figure 1.b with production of two secondaries (N = 2), and

(b) multiperipheral model for N = 2.

FOOTNOTES

1. There are, however, regularities in transverse momenta which have been noted experimentally — particularly correlations between mean values of transverse momenta and longitudinal variables. See F. Turkot in Proceedings Topical Conference on High-Energy Collisions of Hadrons, CERN 68-7, Vol. 1 (January 1968); L. VanHove, Physics Reports 1, No. 7 (1971) 347-380; and J. Ballam et al., SLAC-PUB-900.

2. The correlated behavior described in this paper is not that predicted by dual models in which transverse momentum correlations between a pair of particles decay exponentially away with increasing relative rapidity as a result of the factorization properties of the models. The occurrence of the small distance effects we describe is relatively independent of rapidities and its study is intended to call into question the factorization properties of these, and other, models. Proper unitarization of dual models would presumably lead to effects similar to those we discuss. For a discussion of correlations in the DRM see C.-L. Jen, Kyungsik Kang, P. Shen, and C.-I. Tan, Phys. Rev. Letters 27, 458 (1971).

3. J. R. Fulco, R. L. Sugar, UCSB Preprint (June 1971).

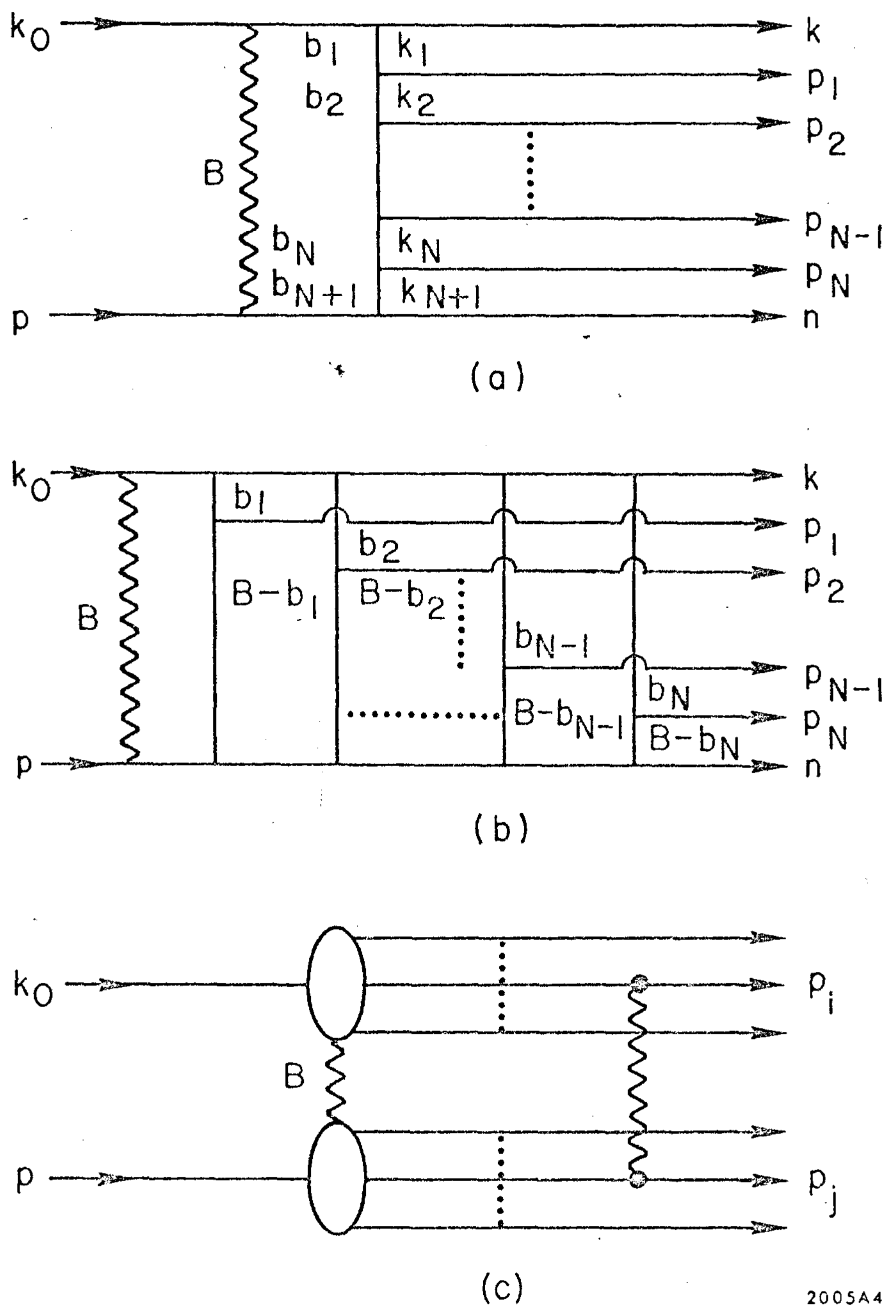
4. We have performed a detailed multi-regge analysis of the reaction $\pi^- p \rightarrow \pi^- \pi^+ n$ at 16 GeV, which gives a better fit to the longitudinal and mass spectra than the simple models described here. However, this more sophisticated model has the same qualitative features in all its transverse momentum distributions. The effects of absorption are equally distinctive.

5. One might expect that two-particle subamplitudes which are more precisely fit to the respective two-particle elastic or quasielastic data can provide a

tail. However, a sum of two Gaussians, one giving the strong forward peak, the other a long tail, was tried in the N = 1 case. This slightly modified the distributions, but absorption was still necessary to produce the shoulder at large v^2.

6. We have evaluated the N = 2 multiperipheral model with rescattering between leading particles and also between a secondary and a leading particle. The v^2 distribution was only slightly affected by the extra absorption but the distribution in the relative momentum associated with the other rescattering was significantly altered.

7. Although it is perfectly correct, it does not seem to clarify the physics to call this the Regge cut structure.



k0
k
b1
k1
p1
b2
k2
p2
B
pN-1
bN
kN
pN
bN+1
kN+1
p
n
(a)
k0
k
b1
p1
b2
p2
B-b1
B-b2
B
bN-1
pN-1
bN
B-bN-1
pN
B-bN
p
n
(b)
k0
pi
B
p
pj
(c)
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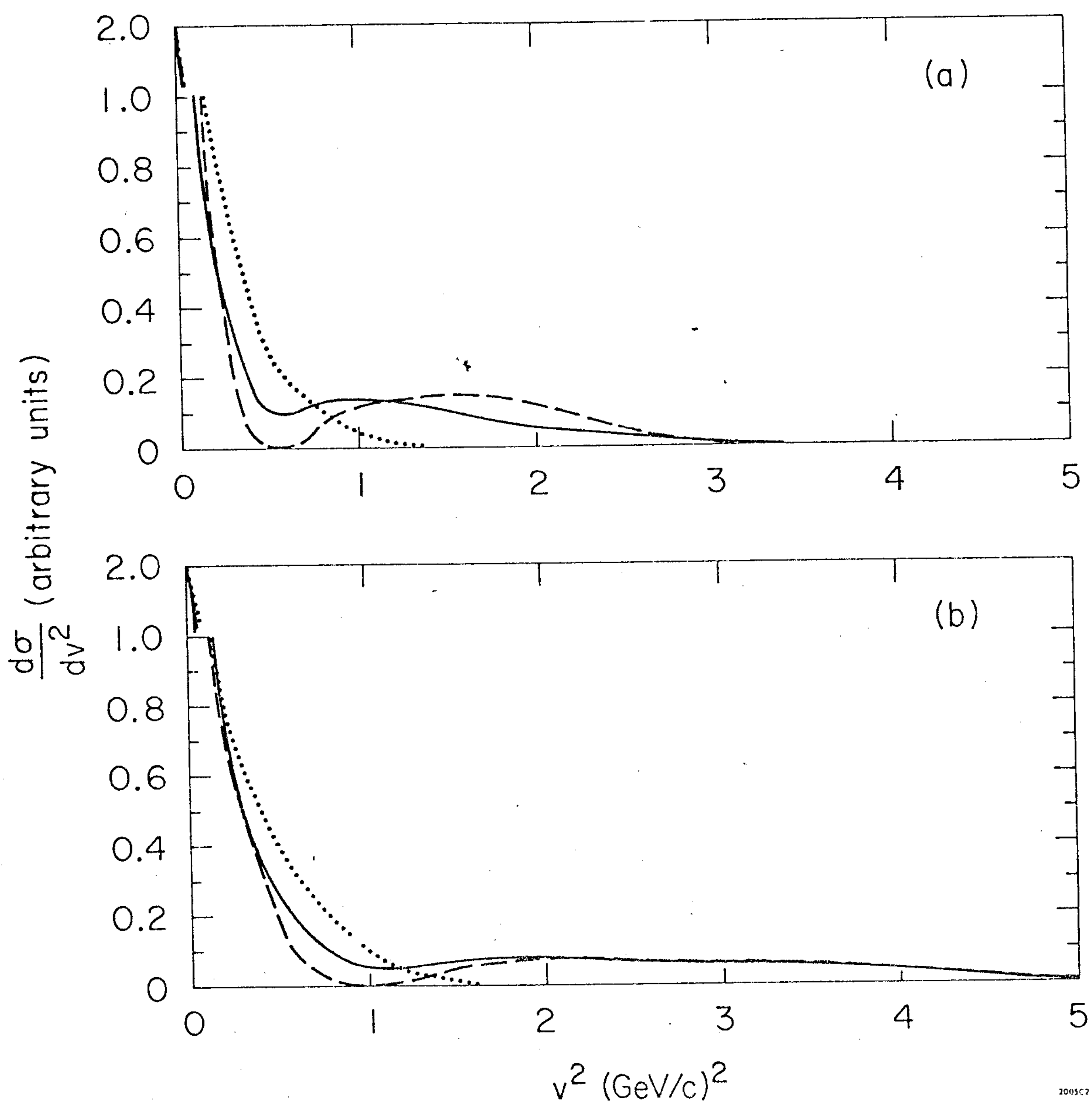
Fig. 1



(a)
(b)
2.0
1.0
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
$\frac{d\sigma}{dv^2}$ (arbitrary units)
v^2 (GeV/c)^2

Fig. 2

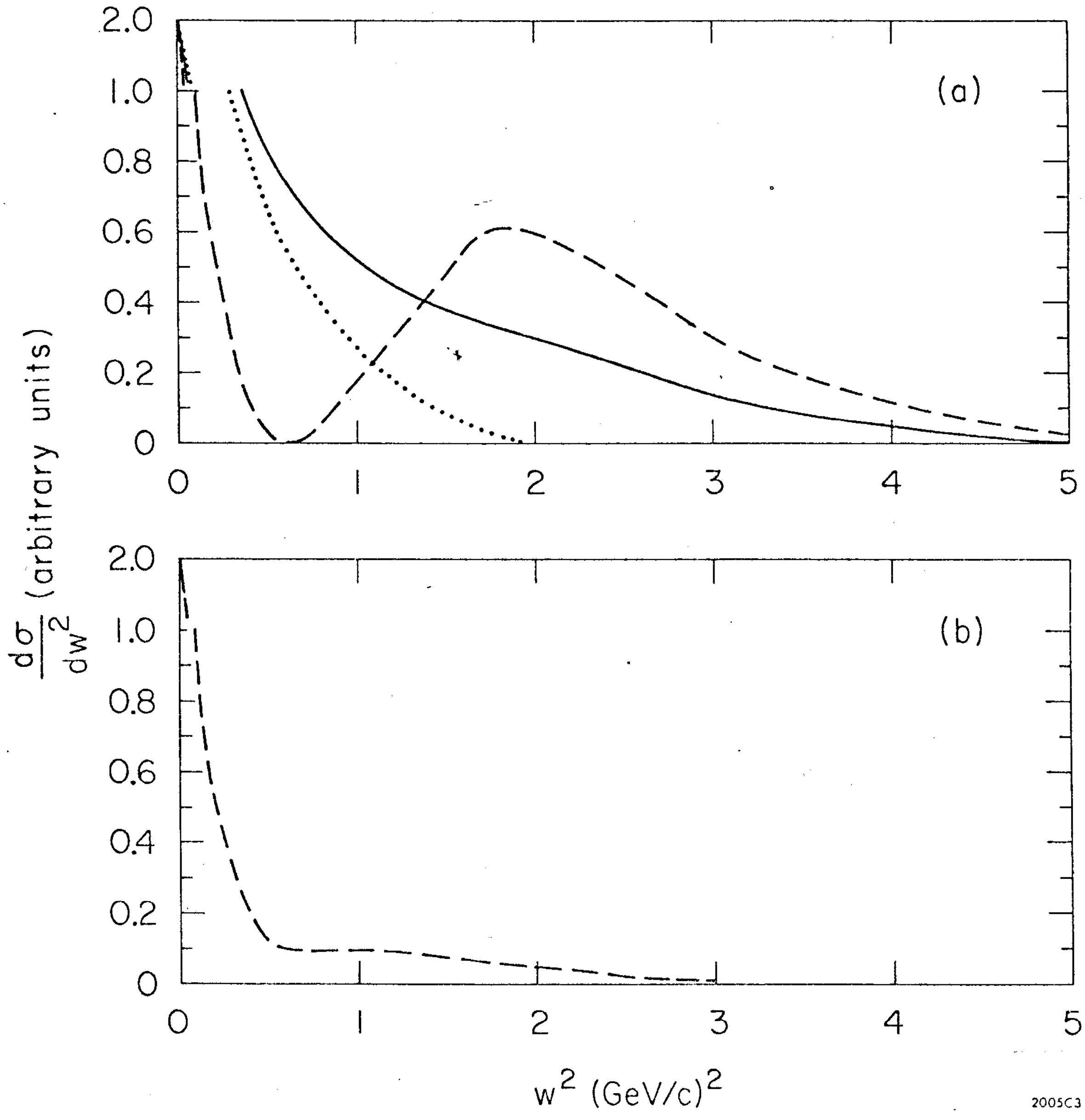

2.0
1.0
0.8
0.6
0.4
0.2
0
(a)
0
1
2
3
4
5
(b)
$\frac{d\sigma}{dw^2}$ (arbitrary units)
w^2 (GeV/c)^2
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Fig. 3