

COMPARISON OF THE INCLUSIVE  $\pi^-$  DISTRIBUTIONS  
FROM  $\gamma p$ ,  $K^+ p$ , AND  $pp$  COLLISIONS\*

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ABSTRACT

The inclusive reactions  $\gamma p \rightarrow \pi^- + \text{anything}$ ,  $K^+ p \rightarrow \pi^- + \text{anything}$  and  $pp \rightarrow \pi^- + \text{anything}$  are compared for small  $\pi^-$  laboratory momenta. We discuss in detail their relative shapes and magnitudes and present a theoretical interpretation of these characteristics.

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The concepts of limiting fragmentation<sup>1</sup> and factorization<sup>2</sup> have been proposed as characteristic properties of inclusive reactions. According to the first, the double differential cross section  $\frac{\partial^2 \sigma}{\partial P_L \partial P_T}$ , evaluated in the laboratory system, should become independent of energy for small  $P_L$ . According to the second, if the Pomeron trajectory dominates, then when this limit is attained, the differential cross section for a given target should be independent of the projectile particle, when divided by  $\sigma_T$  (the asymptotic total cross section).

In this letter, we present laboratory distributions  $\frac{1}{\sigma_T} \frac{\partial^2 \sigma}{\partial P_L \partial P_T}$  for low-momentum  $\pi^-$  from the reactions

$$\gamma p \rightarrow \pi^- + \text{Anything} \quad (\bar{E}_\gamma = 10.4 \text{ GeV}) \quad (1)$$

$$K^+ p \rightarrow \pi^- + \text{Anything} \quad (P_{K^+} = 11.8 \text{ GeV/c}) \quad (2)$$

$$pp \rightarrow \pi^- + \text{Anything} \quad (P_p = 28.5 \text{ GeV/c}) \quad (3)$$

We compare the relative shapes and normalizations of these distributions.

Previously, it has been shown<sup>3</sup> that reaction (1) decreases by about 30% over the  $E_\gamma$  range from 5.5 to 15 GeV, and so has not reached a limiting distribution in the target-fragmentation region at these energies.<sup>4</sup> An energy-dependent study of reaction (2) has not been made. The distribution for reaction (3) is approximately independent of energy over the range 13.5-28.5 GeV/c.<sup>5</sup> Thus the data at hand consist of two reactions, (1) and (2), at about the same incident energy, and a third, reaction (3), that is almost independent of energy.

The data of reaction (1) consist of 39,000 events (58,000  $\pi^-$  tracks) with  $E_\gamma$  in the range 6-18 GeV studied in the SLAC 2-meter streamer chamber using an 18-GeV bremsstrahlung beam. To obtain sufficient statistics for a detailed comparison, we have combined these data to obtain a single distribution at an effective  $\bar{E}_\gamma = 10.4 \text{ GeV}$ .<sup>3</sup> The data of reaction (2) consist of 280,000 events

(360,000  $\pi^-$  tracks) produced by an 11.8 GeV/c  $K^+$  beam in the LRL-SLAC 82-inch liquid hydrogen bubble chamber.<sup>6</sup> The data of reaction (3) consist of 12,200 events (25,000  $\pi^-$  tracks) studied in the BNL hydrogen bubble chamber at incident momentum 28.5 GeV/c.<sup>5</sup> For all three reactions, the measured momenta of the negative tracks were used without kinematic fitting and were assumed to be  $\pi^-$ . The overall normalization errors are estimated to be 13%, 5%, and 5% for reactions (1), (2) and (3), respectively.

On Fig. 1, we plot the laboratory distributions  $\frac{1}{\sigma_T} \frac{\partial \sigma}{\partial P_L}$  for all three reactions as functions of  $P_L$  (uppermost points). Beneath these data, we show  $\frac{1}{\sigma_T} \frac{\partial^2 \sigma}{\partial P_L \partial P_T}$  for 0.1-GeV/c intervals of  $P_T$ . The values  $\sigma_T = 0.1 \text{ mb}$ <sup>7</sup>,  $17.5 \text{ mb}$ <sup>8</sup> and  $39.8 \text{ mb}$ <sup>9</sup> were assumed for the  $\gamma p$ ,  $K^+ p$ , and  $pp$  asymptotic total cross sections, respectively. The errors shown in Fig. 1 are statistical and do not reflect uncertainties in overall normalization of the three experiments nor in the asymptotic cross sections used.<sup>10</sup>

From Fig. 1 we see that:

- (a) The three reactions have similar shapes in  $\frac{1}{\sigma_T} \frac{\partial \sigma}{\partial P_L}$  and in  $\frac{1}{\sigma_T} \frac{\partial^2 \sigma}{\partial P_L \partial P_T}$  for small laboratory momenta ( $|P| < 0.5 \text{ GeV/c}$ ),<sup>11</sup>
- (b) Statistically significant differences in shape are present at larger  $|P|$ ; in particular, the  $\gamma p$  distribution is relatively larger at large  $P_T$  and small  $P_L$ .<sup>12</sup>
- (c) Where differences are statistically significant, the  $\gamma p$  distribution is largest throughout the region shown, and the  $K^+ p$  distribution is smallest.

We can arrive at a simple quantitative statement about these observed differences in magnitude in the fragmentation region by integrating the distributions over all  $P_T$  and negative  $P_L$ . This region of backward  $\pi^-$  is likely to be richer in target fragmentation than the entire range of Fig. 1. We find  $0.031 \pm 0.004$ ,  $0.0180 \pm 0.0008$ , and  $0.0132 \pm 0.0006$  for  $\gamma p$ ,  $pp$ , and  $K^+ p$ , respectively. Thus

these integrals are in the ratios  $\gamma p : pp : K^+ p = (2.4 \pm 0.3) : (1.36 \pm 0.10) : 1$ . The two errors quoted reflect the overall normalization in the  $K^+ p$  experiment, combined with the error in the  $\gamma p$  and  $pp$  experiments, respectively. From their relative magnitudes we see that if factorization is to hold at high energies, at least two of the reactions have not reached their limiting values.<sup>13</sup>

We have no explanation for the relatively small deviations between the  $K^+ p$  and  $pp$  distributions (20-40%), but we advance a tentative explanation for the rather large differences in magnitude and shape between the  $\gamma p$  reaction and the other two. This difference can be discussed in the framework of Mueller.<sup>2</sup> The quantum numbers of the  $\pi^+ pK^+$  and  $\pi^+ pp$  systems are exotic,<sup>14</sup> and hence are characterized by Pomeron factorization. The  $\pi^+ p\gamma$  system, on the contrary, is non-exotic and hence the  $\pi^-$  distribution will also receive contributions from non-Pomeron exchanges that do not limit at these energies. The distribution from such a nonexotic reaction has been conjectured<sup>2</sup> to be of the form  $f(P_L, P_T) + g(P_L, P_T)/\sqrt{s}$ . We can test this conjectured energy dependence<sup>15</sup> using the  $\gamma p$  cross sections at the energies of Ref. 3. In Ref. 3, the integrated distributions for reaction (1) for  $P_L < 0.0$  were evaluated at photon energies of 5.5, 7.5, 10.5 and 15 GeV. A decreasing trend in the values was observed:  $0.038 \pm 0.004$ ,  $0.038 \pm 0.003$ ,  $0.031 \pm 0.003$ ,  $0.025 \pm 0.003$ . In addition to these four values, we take the value found above for the  $K^+ p$  integrated distribution as a conjectured asymptotic value for the  $\gamma p$  reaction. All five values are fit to a function of the form  $A + B/\sqrt{s}$ . We obtain  $A = 0.013 \pm 0.001$ , and  $B = 0.082 \pm 0.008$  GeV with a  $\chi^2$  of 2.6 for 3 degrees of freedom. Hence the data are consistent with an interpretation in which the nonexotic  $\gamma p$  reaction will fall with increasing energy to eventually limit at the distribution for the exotic  $K^+ p$  reaction (The  $\gamma p$  data are also consistent with the  $pp$  cross section as a limit.)

These differences in magnitude and also the relative differences in shape between the  $\gamma p$  reaction on the one hand and the  $K^+ p$  and  $pp$  reactions on the other hand can be formulated in terms of a specific dynamical model, the Regge-exchange model.<sup>2,16,17</sup> In this model (see Fig. 2), the proton emits the  $\pi^-$  and propagates as a Reggeized  $\Delta^{++}$  in the t-channel; this  $\Delta^{++}$  then scatters with the projectile particle. The cross section for this process is given by<sup>16,17</sup>

$$d^3\sigma = \frac{d^3P}{\pi E} \left( \frac{s}{M^2} \right)^{2\alpha(t)-1} \beta^2(t) \sigma_B(M^2) \quad (4)$$

where  $\alpha(t)$ ,  $\beta(t)$  are the trajectory and residue of the exchanged Reggeon, and  $\sigma_B$  is the total cross section for the  $\Delta^{++}$  — beam interaction. If the Pomeron dominates,  $\sigma_B$  is a constant  $b$ ; otherwise,  $\sigma_B$  will also include a decreasing resonance term of the form  $a/M$ . In terms of the  $\pi^-$  momentum  $P_L$  and energy  $E$ , and the target mass  $m$ , we have  $M^2 \approx s \left[ 1 + \frac{P_L - E}{m} \right]$ .

In Ref. 17, detailed fits to  $\pi^-$  spectra from  $pp$  counter data at energies ranging from 12.4 to 30 GeV/c were obtained; expressions for  $\alpha(t)$ ,  $\beta^2(t)$  and  $\sigma_B$  are given there. The prediction of Eq. (4), integrated over  $P_T$  with these parameters, is shown in Fig. 1. We see that Eq. (4) describes the  $pp$  data for small  $P_L$  ( $P_L \leq 0.1$  GeV/c) reasonably well.<sup>18</sup>

For the  $\gamma p$  reaction, we have  $\sigma_B = b + a/M$ . We would then predict the ratio of the  $\gamma p$  and  $pp$  inclusive normalized distributions to be  $1 + C \left[ 1 + \frac{P_L - E}{m} \right]^{-1/2}$ . The parameter  $C$  is an average of  $(ab^{-1}s^{-1/2})$  over the bremsstrahlung spectrum. To test this conjecture, we compare the ratios of the  $\gamma p$  and  $pp$  distributions of Fig. 1 to this prediction. An adequate fit to the data is obtained with  $C = 1$ . The experimental ratios and the predicted curves are shown in Fig. 3.<sup>10</sup> We see that at small  $P_T$  and large  $P_L$  (corresponding to large  $M$ ) both experiment and theory give a ratio of 2. At large  $P_T$  and small

$P_L$  (small  $M$ ), the model predicts a ratio larger than 2 because of the low- $M$  resonances in the  $\Delta^{++}\gamma$  process. This feature qualitatively agrees with the experimental data, though the data suggest perhaps a sharper increase at small  $M$  than that provided by the term  $a/M$ .<sup>19</sup>

In summary, we conclude that in the target fragmentation region the  $K^+p$  and  $pp$  distributions are similar in magnitude and shape to within 20-40%, but that the  $\gamma p$  distribution: (1) has a magnitude over two times larger than the  $K^+p$  and  $pp$  distributions and (2) falls with increasing energy, perhaps to eventually limit at the  $K^+p$  distribution, and (3) at the energy studied has a different relative shape attributable to the nonexotic contributions.

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10. We point out that data of Reaction (1) may contain some  $e^-$  contamination of tracks arising from  $e^+e^-$  production. Such contamination would be present only in the  $0.0 < P_T < 0.1 \text{ GeV}/c$  interval and only for  $P_L > 0.0$ . The other  $P_T$  intervals would be unaffected.
11. We also remark that the double distributions do not factorize into separate functions of  $P_L$  and  $P_T$ .
12. Moreover, significant differences can be seen at large  $P_L$ ; this corresponds to  $x > 0$  in the c. m. and is beyond the proton fragmentation region. The c. m. variable  $x$  is defined in Ref. 1(b) as  $x \equiv 2 \cdot P_L(\text{c. m.})/\sqrt{s}$ .
13. We also have evaluated the distributions, integrated over all  $P_T$  and over  $P_L < 0.5 \text{ GeV}/c$ . We find these quantities to be  $0.43 \pm 0.01$ ,  $0.174 \pm 0.009$  and  $0.204 \pm 0.010$ , respectively. The latter two values may be compared with the values found in Ref. 9:  $0.20 \pm 0.02$ , and  $0.22 \pm 0.02$ , respectively.



The corresponding ratios based on the present experiment are

$\gamma p : K^+ p : pp = (2.6 \pm 0.3) : (1.20 \pm 0.08) : 1$  for this more extended range in  $P_L$ . However, note that this region of  $P_L$  includes contributions from  $x > 0$  (see Fig. 1).

14. By exotic we mean that no resonances are known to decay into these particles (see Ref. 2).
15. A similar analysis has been performed for the reaction  $\pi^- p \rightarrow \pi^- + \text{anything}$ , by W.D. Shephard, J.T. Powers, N.N. Biswas, N.M. Cason, V.P. Kenney, R.R. Riley, D.W. Thomas, J.W. Elbert and A.R. Erwin, Notre Dame-Wisconsin preprint (unpublished).
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The intercept  $\alpha(0)$  and its relation to the  $\pi^- p$  backward elastic cross section in the intermediate energy range ( $s = 3 - 8 \text{ GeV}^2$ ) is discussed in this work.
18. The curve of Fig. 1 represents the contribution to the total  $\pi^-$  distribution that comes from the single  $\pi^-$  produced by the baryon exchange mechanism of Fig. 2. This process is related through duality to the backward decay of the various  $N^*$  resonances of the proton. In this picture, the remaining portion of the spectrum in Fig. 1 then arises from the  $\pi^-$  produced by other mechanisms.
19. Thus the model of Fig. 3 could lead one to use the ratio of the inclusive distributions to study the nature of the  $\gamma \Delta^{++}$  total cross section. Here, we have taken this cross section to be of the form  $b + a/M$ .

## FIGURE CAPTIONS

1. Differential cross sections  $\frac{1}{\sigma_T} \frac{\partial^2 \sigma}{\partial P_L \partial P_T}$  for  $\gamma p \rightarrow \pi^- + \text{anything}$  (open circles),  $K^+ p \rightarrow \pi^- + \text{anything}$  (closed circles), and  $pp \rightarrow \pi^- + \text{anything}$  (squares) as a function of  $P_L$  for a range of intervals in  $P_T$ . Also shown is  $(d\sigma/dP_L) / \sigma_T$  (uppermost points). The curve is the theoretical prediction of Eq. (4) for the  $pp$  reaction (see text). The diagonal lines at the left are intended to distinguish the  $P_T$  selections at small  $P_L$ . The approximately-vertical broken lines show the value of  $P_L$  corresponding to  $x = -0.5$  and  $x = 0.0$ , respectively, where  $x \equiv 2 \cdot P_L(\text{c. m.}) / \sqrt{s}$ .
2. Regge model for proton fragmentation in proton- and photon-induced inclusive reactions.
3. The ratio  $\Omega_{\gamma p}$  of the normalized distributions. The curve indicates the prediction of Eq. (4) over the entire  $P_L, P_T$  region with  $C = 1$  (see text); solid portion indicates the  $P_L, P_T$  region for which Eq. (4) adequately accounts for the  $\pi^-$  distribution in the  $pp$  reaction; dashed portion indicates the prediction of Eq. (4) over the rest of data.

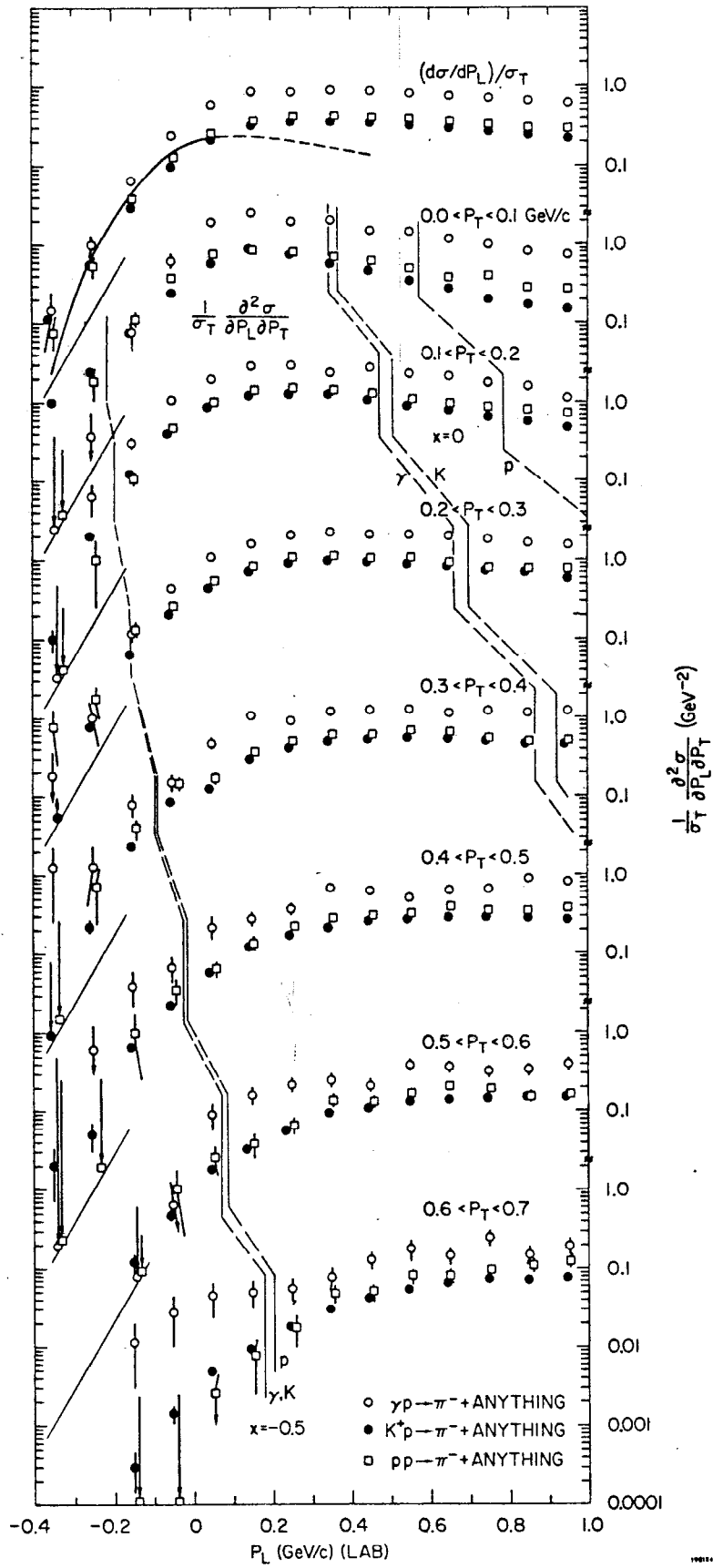


Fig. 1

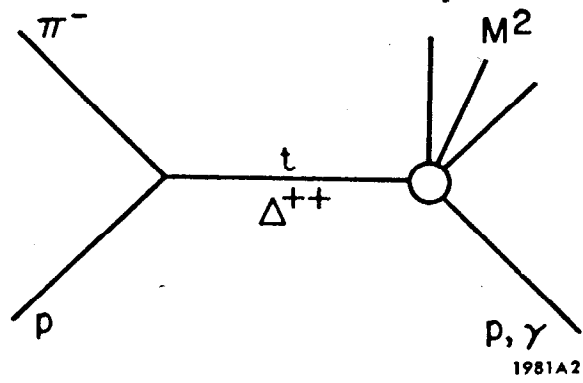


Fig. 2

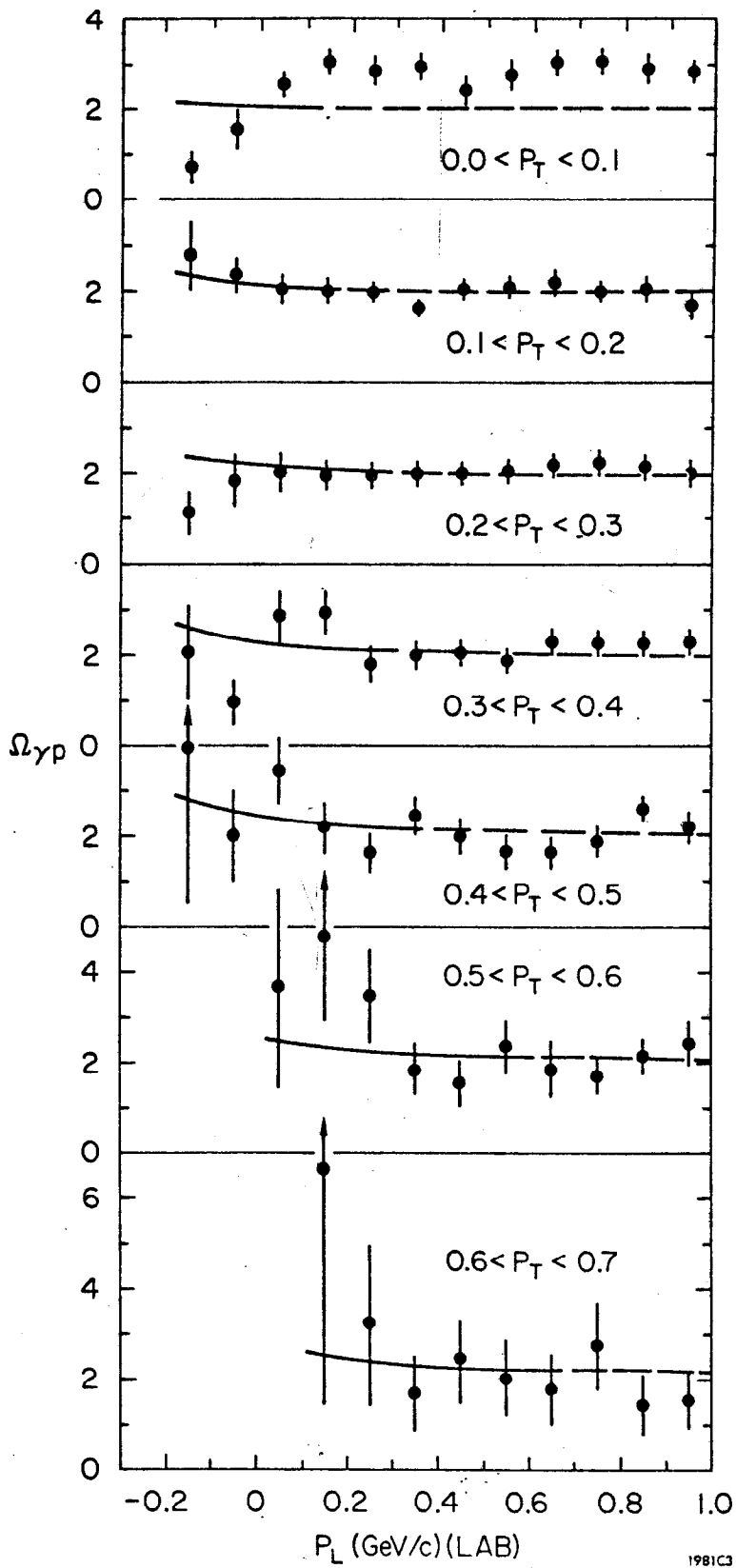


Fig. 3